Comment on 'The Concise Formulation of Diffusive Sorption of Water in a Dry Soil' by Wilfried Brutsaert

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Recently, Brutsaert [1976] showed very effectively how the various approximate solutions for the sorption problem are related and which solutions are the most accurate. This is a valuable contribution to the theory of unsaturated soil water flow and is the type of information that hydrologists need from mathematicians and other theoreticians dealing with flow in porous media. I wholeheartedly agree that we need 'solutions that satisfy the dual requirement of physical consistency and computational simplicity.' His conclusion that 'for most practical problems the need for expensive numerical procedures and tedious perturbation and iterative analytical methods is probably obviated' is, unfortunately, true only for the sorptivity problem. Hopefully, some day this can be said for the hydrologic unsaturated soil water flow problem as a whole.

Brutsaert obviously considers as his primary audience hydrologists concerned with basin-wide application of soil water flow theory. To be successful in this, one must move from complex to more simple models. Whereas the bulk of his paper certainly does simplify without sacrificing physical principles, the philosophy expressed in the Introduction, Discussion, and Concluding Remarks is, in my opinion, exactly the opposite. That is, Brutsaert appears to advise hydrologists to obtain sorptivity measurements as I have recently shown to be possible [Talsma, 1969]. Sorptivity is not an intrinsic soil property but is dependent on both the initial water content \( \theta_i \) (or pressure head \( H_i \)) and the water content \( \theta_w \) (or pressure head \( H_w \)) at the absorption interface. Obviously, Brutsaert had in mind absorption under ponding or from saturation when he developed his equations in terms of \( S \) (later in the paper, he identifies \( k_0 \) as the saturated hydraulic conductivity), but all his equations are also valid for \( \theta_i < \theta_w \). That is, \( S = 1 \) does not necessarily mean that \( \theta_i = \theta_w \). Therefore any one of the solutions in Table 1 can be used to derive \( D(\theta) \) from \( A_0(\theta_i, \theta_w = \text{const}) \). I used [Dirksen, 1975a] the linearized solution with a weighted mean diffusivity with \( \gamma = 0.67 \), corresponding to (12), 'Crank's [1956] result.'

Brutsaert found that (33') results in a far smaller error compared with an exact solution than any of the other approximate solutions and then states that Parmelee [1975] found the same equation to be superior to Crank's result. Brutsaert and Parmelee used (33') to obtain only one value, the saturated sorptivity, from a complete \( D(\theta) \) function. Both found a very small error for that value. In my reply to Parmelee [Dirksen, 1975b], I pointed out that going in the opposite direction from \( A_0(\theta_i, \theta_w = \text{const}) \) to \( D(\theta) \) the errors involved will generally vary with \( \theta \) or \( D(\theta) \). For three hypothetical soils with an exponential relationship between \( D \) and \( \theta \) the average value \( D(\theta)/D_{\text{exp}} \) for (33') was nearly perfect for each soil. But the variation of \( D(\theta)/D_{\text{exp}} \) with \( \theta \) (the 'slope' of a linear regression curve) was much larger for (33') than for (14) (\( \gamma = 0.67 \) ). This results in the same maximum error of about 3.5% for both cases. Figure 1 of Dirksen [1975b] shows that the absolute values of \( D(\theta) \), the 'slope,' and the magnitude of the fluctuations all decrease with increasing \( \theta \). Results were best when \( \theta \) was varied between 0.61 for a clay and 0.63 for a sand.

To evaluate the above results, one should remember that this reversed route involves a differentiation, which is inherently less accurate than the integration required in deriving \( A_0 \) from \( D(\theta) \). Furthermore, the accuracy of these results depends on the accuracy of the areas under the curves of Figure 1 of Gardner and Mayhugh [1958]. To evaluate the latter would require a check of the original work of Wagner [1952] and Philip [1955]. The amount of time involved in doing this is not warranted, especially since the weighted mean diffusivity solution can be generalized and tested against the exact solution of (26), which Brutsaert used to evaluate all of his approximate solutions. Whereas, strictly speaking, 'Crank's result' refers to the weighted mean diffusivity solution with \( \gamma = 1 \), (12) can be written in the general form

\[
A_0 = \left[ 4(1 + \gamma)/\pi \right] \int_0^1 S D dS \left[ S \right]^{1/2} 
\]

Substitution of the diffusivity for the exact solution (24) in this equation yields (27), with

\[
F_1 = \left[ 2(1 + \gamma)(m + 1) \right]^{1/2} \left[ \frac{1}{m + \gamma + 1} - \frac{1}{(m + 1)(2m + \gamma + 1)} \right]^{1/2} 
\]

Setting this \( F_1 \) equal to \( F_1 = (m + 1)^{-1/2} \) of the exact solution gives a quadratic equation \( ay^2 + by + c = 0 \), with

\[
a = (2/\pi)(m + 1) - 1 \\
b = (4/\pi)m^2 + [(10/\pi) - 3]m + [(6/\pi) - 2] 
\]
One root of this equation corresponds to the value of $\gamma = \gamma_{\text{exact}}$, for which the weighted mean diffusivity becomes exact. In Figure 1 of the present paper, $\gamma_{\text{exact}}$ is plotted versus $m$. This result could be added to Brutsaert's Table 1. For $m = 3$ to $m = 20$, $\gamma_{\text{exact}}$ varies from 0.594 to 0.576. These values are slightly lower than the best values for $\gamma$ cited above for exponential diffusivities, and they show an opposite trend from coarse (large $m$) to fine (small $m$) soils. This shows that (33') is not superior to the weighted mean diffusivity solution if $\gamma$ is allowed to vary rather than is held constant at $\gamma = \frac{1}{2}$.

In terms of Dirksen's [1975b] Figure 1 the 'slope' of (33') is the same as that for $\gamma = 0.50$, while all values for $D(\theta)$ are 0.954 times those for $\gamma = 0.50$. Thus (33') cannot satisfy the 'slope' for the values of $\gamma_{\text{exact}}$. However, it can be seen from this figure that a curve for $\gamma = 0.5825$, which is the value of $\gamma_{\text{exact}}$ for the typical value of $m = 8$, and the curve for (33') will be so close together that any remaining differences will have no practical significance. The relationship between $m$ and $\gamma_{\text{exact}}$ in Figure 1 is based on an exact analytical solution. Therefore if a weighted mean diffusivity, rather than (33'), is used to derive $D(\theta)$ from $A_{\theta}(\theta_0)$, $\gamma$ can best be chosen from Figure 1 and the textural classification of the soil involved.

In conclusion, Brutsaert's paper is very helpful in evaluating the relative merits of the various approximate solutions of the soil water flow problem, but his practical application is misleading. Instead, the paper enhances the merits of determining soil water diffusivity functions from a series of sorptivity measurements. Finally, the generalized form of the weighted mean diffusivity solution can be made exact by a proper choice of $\gamma$, making it even more accurate than (33').

REFERENCES


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