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Water transport in soils as in fractal media

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Abstract

Fractal scaling laws of water transport were found for soils. A water transport model is needed to describe this type of transport in soils. We have developed a water transport equation using the physical model of percolation clusters, employing the mass conservation law, and assuming that hydraulic conductivity is a product of a local component dependent on water content and a scaling component depending on the distance traveled. The model predicts scaling of water contents with a variable $x/t^{1/(2+\beta)}$ where β deviates from the zero value characteristic for the Richards equation. A change in the apparent water diffusivity with the distance is predicted if the apparent diffusivity is calculated using the Richards equation. An equation for the time and space invariant soil water diffusivity is obtained. Published data sets of five authors were used to test the scaling properties predicted by the model. The value of β was significantly greater than zero in almost all data sets and typically was in the range from 0.05 to 0.5. This exponent was found from regression equations that had correlation coefficients from 0.97 to 0.995. In some cases a dependence of β on water content was found indicating changes in scaling as the water transport progressed. © 1998 Elsevier Science B.V.

Keywords: Water movement in soil; Soil physical properties; Fractals; Unsaturated zone; Richards' equation; Soil characterization

1. Introduction

Laws of scaling of many soil properties have been found to be scale-invariant, i.e. the dependence of a property on the scale is the same over a range of scales. Fractal scaling has been found appropriate to express such scale independence for collections of soil particles and aggregates (Bartoli et al., 1991; Niemeyer and Ahl, 1991; Perfect and Kay, 1995; Tyler and Wheatcraft, 1992; Young and Crawford, 1991), microporosity and macroporosity (Bartoli et al., 1991; Brakensiek et al., 1992), soil pore surfaces (Sokolowska, 1989; Toledo et al., 1990), etc. In many aspects, soil is a fractal medium.

When the network of pores is fractal, transport in this network differs from the transport in the media with properties independent of scale. In particular, diffusion of solutes in a fractal pore network does not obey Fick's law, and the anomalous diffusion takes place instead (Gefen et al., 1983). When Fick's law is valid, the dependence of concentration on time t and distance x from the source of a solute can be expressed as a function of a single Boltzmann variable λ

$$\lambda = \frac{x}{t^{0.5}} \tag{1}$$

With anomalous diffusion, the dependence of concentration on time and distance can also be expressed as a function of a single variable ζ

$$\zeta = \frac{x}{t^{n}} \tag{2}$$

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where the exponent n can be both greater than and less than 0.5, depending on the fractal properties of the pore network where the transport occurs (Orbach, 1986). When $n > 0.5$, the diffusion spreading increases as the solute propagates from the source. If one tries to calculate Fick's diffusion coefficient from data on different stages of anomalous diffusion with $n > 0.5$, an increase in the diffusion coefficient will be found as the anomalous diffusion progresses. On the contrary, diffusion spreading will gradually slow down when the anomalous diffusion obeys Eq. (2) with $n < 0.5$. In this case, Fick's diffusion coefficient will decrease with time or distance of spreading. Anomalous dispersion of solutes has been observed in fractured rocks and is intensively studied in subsurface hydrology (Acuna and Yortsos, 1995; Neuman, 1990, 1995).

The transport of soil water is more complex than solute diffusion in water saturated media, because the soil water diffusivity depends on soil water content. The soil water transport similar to anomalous diffusion has been observed in horizontal soil columns. Ferguson and Gardner (1963) suggested that the soil water diffusivity may be a function of distance or some other variable as well as water content. Rawlins and Gardner (1963) found that soil water diffusivity should be considered a function either of time or of distance. Guerrini and Swartzendruber (1992) have demonstrated that the diffusivity decreases as time increases and the decrease obeys the power law. They found a solution of the equation of water transport in unsaturated soil with time-dependent diffusivity. Later these authors suggested that a fractal Brownian motion can be a model for the soil water movement (Guerrini and Swartzendruber, 1994). Pachepsky et al. (1995) conjectured that the fractal structure of soil pore space might cause an anomalous diffusion in soils.

The objectives of this work were to develop a model of soil water transport accounting for fractal pore networks, and to explore the extent the anomalous diffusion manifests itself in experiments on horizontal movement of water in soils.

2. Theory

We derive the equation of water transport in unsaturated fractal media following a derivation of the saturated water transport equation in fractal media given by Yortsos (1990). One-dimensional transport will be considered below. We start with the mass conservation equation in the form

$$\frac{\partial \theta}{\partial t} + \frac{\partial q}{\partial x} = 0 \tag{3}$$

where θ is the soil water content and q is the volume flow rate per unit area. We do not consider soil density changes during the water transport. We assume that the water flux in soil is related to the pressure gradient according the Buckingham–Darcy law:

$$q = -k\frac{\partial h}{\partial x} \tag{4}$$

where k is the unsaturated hydraulic conductivity and h is the hydraulic head.

The physical model of transport in fractal media that is used in this paper is called a site percolation network (Orbach, 1986). This model has been used first to describe transport of water and oil in sandstones and later was applied to transport phenomena in fractured media (Feder, 1988, p. 104; Yortsos, 1990). The percolation network is created on a rectangular grid. Each intersection is occupied by water at random with some probability p. Sites are said to be connected if they are adjacent along the vertical or horizontal direction. Each particle of the moving material can move with a constant rate with steps of an elementary length that are random in direction to adjacent sites of the infinite network. The remarkable property of this model is that the transport of an ensemble of the particles of the moving material follows the diffusion equation with the diffusion coefficient D_p dependent on the distance x from the starting point following the power law

$$D_p \propto x^{-\beta} \tag{5}$$

Here exponent β shows how the path of a particle of the moving material will deviate from the free diffusion path because of the presence of fractal solid phase. The more irregular the path is, the larger is the value of β. The exponent β can be negative in which case diffusion speeds up as the transport progresses (Mandelbrot, 1983).

Chang and Yortsos (cited in Yortsos, 1990) suggested that the power dependence on distance will hold for permeability of a fractured media in

the form:

$$k = mx^{-\beta} \tag{6}$$

where m is a local property of the medium and exponent β is the same as in Eq. (5) characteristic of the fractal percolation network where the transport occurs.

We will assume that Eq. (6) is valid for a soil provided that the local component of the hydraulic conductivity m is a function of the water content:

$$m = m(\theta) \tag{7}$$

Combining Eq. (3), Eq. (4), Eq. (6) and Eq. (7) in one equation, we obtain the equation of the one-dimensional water transport in unsaturated fractal porous medium:

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial x}\left(x^{-\beta} m(\theta)\frac{\partial h}{\partial x}\right) \tag{8}$$

We will focus on horizontal one-dimensional transport. In this case, the hydraulic head, h, is equal to soil matric potential, ψ. Soil water diffusivity will be defined as usual

$$D = k\frac{\mathrm{d}\psi}{\mathrm{d}\theta} = m(\theta)x^{-\beta}\frac{\mathrm{d}\psi}{\mathrm{d}\theta} = G(\theta)x^{-\beta} \tag{9}$$

where $G(\theta) = m(\theta)(d\psi/d\theta)$ is a local component of the soil water diffusivity. Then the equation of the horizontal water transport in unsaturated soil as in a fractal medium will be:

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial x}\left(x^{-\beta} G(\theta)\frac{\partial\theta}{\partial x}\right) \tag{10}$$

The presence of the independent variable in the hydraulic diffusivity term distinguishes this equation from the customary Richards' equation in which the diffusivity depends only on water content. Eq. [10] reduces to Richards' equation when $\beta = 0$.

An equation similar to Eq. (10) with G = const. was introduced and studied by O'Shaughnessy and Procaccia (1985) to describe diffusion on fractal objects. These authors found an analytical solution in which, for the one-dimensional case, the concentration was a function of the variable $xt^{-1/(2+\beta)}$. We will show now that the solution of the non-linear Eq. (10) for the water movement in a semi-infinite horizontal column is a function of the same variable.

We consider Eq. (10) with the following boundary and initial conditions:

$$\theta = \theta_b,\ x = 0,\ t \geq 0$$

$$\theta = \theta_i,\ x > 0,\ t = 0 \tag{11}$$

Now we will use the general Boltzmann-type variable ζ defined by Eq. (2). Assuming that θ depends only on ζ, we will have

$$\frac{\partial\theta}{\partial t} = \frac{\mathrm{d}\theta}{\mathrm{d}\zeta}\left(-n\frac{\zeta}{t}\right);\quad \frac{\partial\theta}{\partial x} = \frac{1}{t^n}\frac{\mathrm{d}\theta}{\mathrm{d}\zeta};$$
$$\frac{\partial}{\partial x}\left(x^{-\beta}G(\theta)\frac{\partial\theta}{\partial x}\right) = \frac{1}{t^n}\frac{\mathrm{d}}{\mathrm{d}\zeta}\left(x^{-\beta}G(\theta)\frac{1}{t^n}\frac{\mathrm{d}\theta}{\mathrm{d}\zeta}\right) \tag{12}$$

Replacing partial derivatives in Eq. (10) with expressions from Eq. (12), we have:

$$-n\frac{\mathrm{d}\theta}{\mathrm{d}\zeta}\zeta = \frac{\mathrm{d}}{\mathrm{d}\zeta}\left(\frac{t^{1-2n}}{x^{\beta}}G(\theta)\frac{\mathrm{d}\theta}{\mathrm{d}\zeta}\right)$$
$$= \frac{\mathrm{d}}{\mathrm{d}\zeta}\left(\frac{t^{1-(2+\beta)n}}{\zeta^{\beta}}G(\theta)\frac{\mathrm{d}\theta}{\mathrm{d}\zeta}\right) \tag{13}$$

If the exponent n is related to β as

$$n = \frac{1}{2+\beta} \tag{14}$$

then Eq. (13) reduces to:

$$-\frac{1}{2+\beta}\frac{\mathrm{d}\theta}{\mathrm{d}\zeta}\zeta = \frac{\mathrm{d}}{\mathrm{d}\zeta}\left(\zeta^{-\beta}G(\theta)\frac{\mathrm{d}\theta}{\mathrm{d}\zeta}\right) \tag{15}$$

This equation has to be solved with the boundary conditions

$$\theta(0) = \theta_b,$$

$$\theta(\infty) = \theta_i \tag{16}$$

which follow from Eq. (11). When the pore space does not exhibit fractal properties, $n = 0.5$, $\zeta = \lambda$ and Eq. (15) coincides with the well-known consequence of the Richards' equation (Philip, 1955).

Thus, if Eq. (10) is valid, water content will depend solely on the variable $\zeta = xt^{-1/(2+\beta)}$ during flow in a semi-infinite horizontal column:

$$\theta = \theta\left(\frac{x}{t^{\frac{1}{2+\beta}}}\right) = \theta\left(\frac{x}{t^n}\right) \tag{17}$$

Note that a result equivalent to Eq. (17) was obtained earlier by Guerrini and Swartzendruber (1992) who

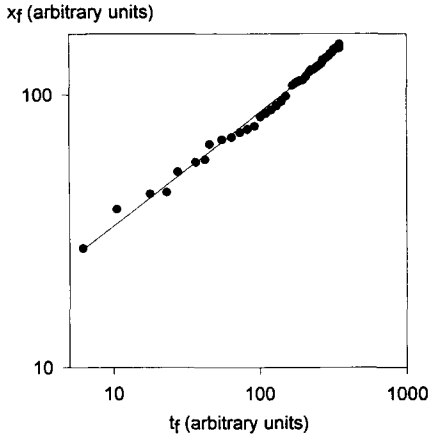

Fig. 1. Distance to the wetting front as a function on time in experiments of Gardner and Widtsoe (1921).

assumed the soil water diffusivity to be a product of a function of water content and an inverse power function of time.

Transport properties of the fractal media characterized by Eq. (10) are defined by the parameter β and by the function $G(\theta)$. The most convenient relationship to find values of β is a corollary of Eq. (17) in the form

$$x = \zeta(\theta)t^n \tag{18}$$

Here $\zeta(\theta)$ is the inverse function to $\theta(\zeta)$ and n depends on β according to Eq. (14). The "Materials and methods" section below presents techniques of using Eq. (18) to find a value for β from various types of data on water transport in horizontal soil columns. If the value of β is known, the local component of the diffusivity, $G(\theta)$ can be found using an integral of Eq. (15):

$$G(\theta) = -\frac{1}{2+\beta}\zeta^{-\beta}\frac{\mathrm{d}\zeta}{\mathrm{d}}\theta\int\theta_i\theta\zeta\mathrm{d}\theta \tag{19}$$

where θ_i is the initial water content. With $\beta = 0$, when $D = G(\theta)$, this equation reduces to the known consequence of the Richards' equation (Philip, 1955):

$$D(\theta) = -\frac{1}{2}\frac{\mathrm{d}\lambda}{\mathrm{d}}\theta\int\theta_i\theta\lambda\mathrm{d}\theta \tag{20}$$

To find the function $G(\theta)$ when the dependence of θ of ζ is known, one can solve Eq. (19) numerically using the technique proposed by Philip (1955) to solve Eq. (20).

3. Materials and methods

We used published data on water transport in a horizontal soil column to test the applicability of Eq. (17) and Eq. (18) and to find values of the parameter β.

Data from two publications related the distance to wetting front to the time of observations. Gardner and Widtsoe (1921) studied the transport of water, and Nielsen et al. (1962) studied transport of water and oil in soil. Air-dry soil was packed in columns. A negative pressure was held constant at one of the columns' end. The largest distance where the wetting front was observed was 50 cm. Positions of the wetting front were measured visually.

To test the applicability of Eq. (18) with data on wetting front advance, we plotted distance-on-time dependencies in log-log scale. The exponent n in Eq. (18) was then a slope of the regression line log x on log t. Figs 1 and 2 show plots of data from papers of Gardner and Widtsoe (1921) and Nielsen et al. (1962), respectively.

Rawlins and Gardner (1963) studied movement of water in horizontal soil columns using gamma ray

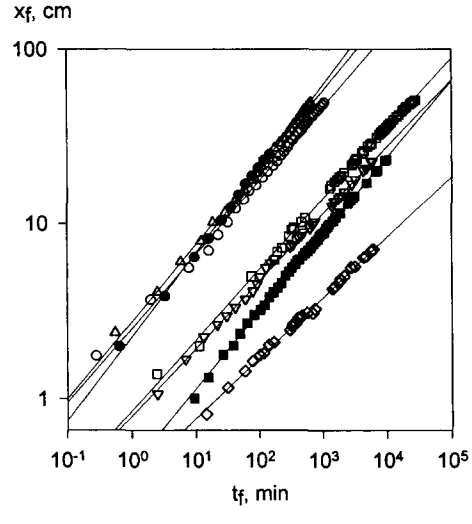

Fig. 2. Distance to the wetting front as a function on time in experiments of Nielsen et al. (1962); ○, Columbia silt loam wet at −50 mb; □, Columbia silt loam wet at −100 mb; ●, Columbia silt loam wet with oil at −2 mb; ■, Columbia silt loam wet with oil at −38 mb; △, Hesperia sandy loam wet at −2 mb; ▽, Hesperia sandy loam wet at −50 mb; ◇, Hesperia sandy loam wet at −100 mb.

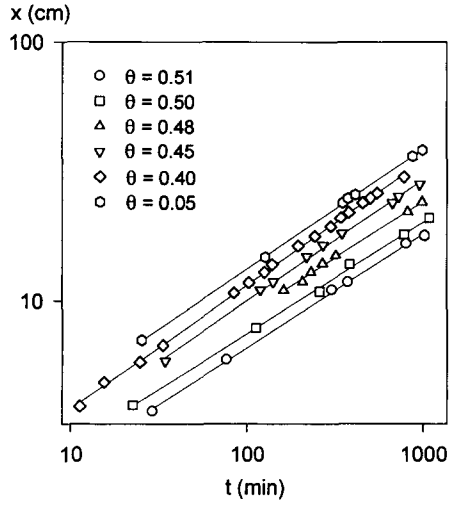

Fig. 3. Distance-on-time dependencies for several volumetric water contents θ in experiments on water movement in horizontal soil column, from work of Rawlins and Gardner (1963).

attenuation. The largest distance where the water content was measured was 40 cm. They presented data on the dependence of the distance at which a particular water content had been reached on time when this water content is reached. To test applicability of Eq. (18) with these data, we plotted distance-on-time dependencies in log–log scale for several water contents. Fig. 3 shows these plots. The exponent n in Eq. (18) was then a slope of the regression line $\log x$ on $\log t$ calculated for each water content separately.

Ferguson and Gardner (1963) studied movement of water in air-dry soil using gamma ray attenuation. The largest distance where the water content was measured was 27.5 cm. They presented dependencies of water content on time for several distances from the inlet. To use this data in testing Eq. (18), we fitted regression equation

$$t = a + \frac{b}{(c-\theta)^d}$$

to time–moisture curves for each of the observation distances separately. Fig. 4(a) shows the measured data and the regression curves. After that we took a set of water content values 0.05, 0.10, ..., 0.40, and calculated the time when these water contents were reached for each observation distance. Distance–time data points were plotted in log–log coordinates as shown in Fig. 4(b). The exponent n has been found separately for each water content as a slope of regression of $\log x$ on $\log t$. The largest value of water content (0.40) was selected to provide at least three distances at which the water content was reached.

Smiles et al. (1978) studied movement of water and solutes in sand–kaolinite mixtures. They carried out their set A of experiments with distilled water, set B with 10^{-3} kg L^{-1} solution of KCl, and set C with 10^{-2} kg L^{-1} solution of KCl. The largest observation distance was 22 cm. The authors presented dependencies of water content on the Boltzmann variable λ found at three observation times. To use these data, we fitted their dependencies of λ on water content

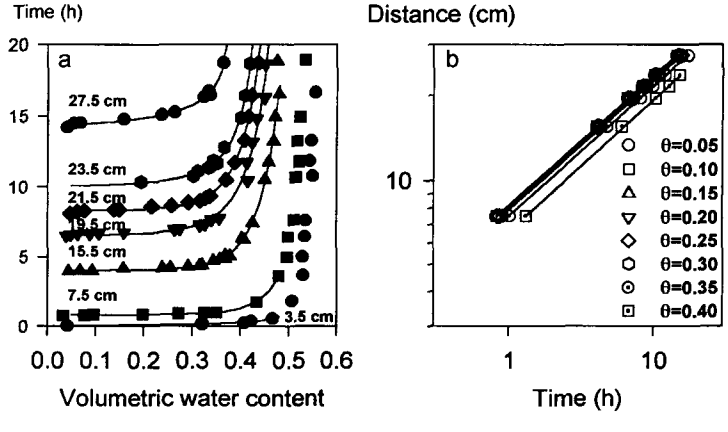

Fig. 4. Water movement in a horizontal soil column in experiments of Ferguson and Gardner (1963); (a) time of reaching various water contents at several distances from the source; (b) distance-on-time dependencies for several volumetric water contents θ.

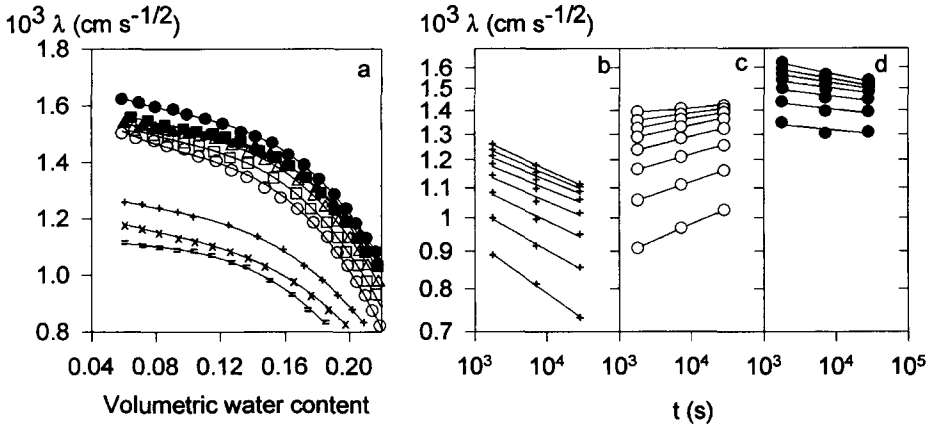

Fig. 5. Water movement in horizontal soil columns in experiments of Smiles et al. (1978); (a) Boltzmann variable λ as related to water contents at 1800 s (+, ○, ●), at 7200 s (X, □, ■) and at 28 800 s (= , △, ▲) in data sets A (+, x, =), B (○, □, △) and C (●, ■, ▲), respectively; (b) (c) and (d) Boltzmann variable as a function of time at volumetric water contents 0.06, 0.08, ..., 0.20 from data sets A, B and C, respectively.

with cubic regression separately for each observation time. Fig. 5(a) shows the data and the regression curves. Then we selected a set of water contents 0.06, 0.08, ..., 0.20, found values of λ from the regression equations and plotted dependencies of λ on observation times for these water contents. These plots are shown in Fig. 5(b–d) for the three sets of experiments. To find the exponent n in Eq. (18), we calculated linear regressions $\log(\lambda) = A - B \log(t)$ for each water content. Since by definition $\log(\lambda) = \log(x)$

Table 1
Values of parameters n and β found from data on horizontal movement of water to soil columns

Data source	Soil	n (mean ± standard error)	β	Graph
Gardner and Widtsoe (1921)	Name and texture not reported	0.417 ± 0.006	0.398	Fig. 1
Nielsen et al. (1962)	Columbia silt loam wet at −50 mb	0.402 ± 0.003	0.488	Fig. 2, ○
	Columbia silt loam wet at −100 mb	0.425 ± 0.006	0.353	Fig. 2, □
	Columbia silt loam wet with oil at −2 mb	0.480 ± 0.008	0.083	Fig. 2, ●
	Columbia silt loam wet with oil at −38 mb	0.440 ± 0.003	0.273	Fig. 2, ■
	Hesperia sandy loam at −2 mb	0.440 ± 0.004	0.273	Fig. 2, △
	Hesperia sandy loam at −50 mb	0.384 ± 0.002	0.604	Fig. 2, ▽
	Hesperia sandy loam wet at −100 mb	0.344 ± 0.003	0.907	Fig. 2, ◇
Rawlins and Gardner (1963)	Salkum silty clay loam, $\theta = 0.51$	0.439 ± 0.007	0.278	Fig. 3
	Salkum silty clay loam, $\theta = 0.50$	0.430 ± 0.008	0.326	Fig. 3
	Salkum silty clay loam, $\theta = 0.48$	0.437 ± 0.011	0.288	Fig. 3
	Salkum silty clay loam, $\theta = 0.45$	0.467 ± 0.009	0.141	Fig. 3
	Salkum silty clay loam, $\theta = 0.40$	0.479 ± 0.003	0.088	Fig. 3
	Salkum silty clay loam, $\theta = 0.05$	0.461 ± 0.002	0.169	Fig. 3
Ferguson and Gardner (1963)	Salkum silty clay loam, $\theta = 0.05$	0.454 ± 0.002	0.203	Fig. 4(b)
	Salkum silty clay loam, $\theta = 0.10$	0.453 ± 0.002	0.208	Fig. 4(b)
	Salkum silty clay loam, $\theta = 0.15$	0.452 ± 0.003	0.212	Fig. 4(b)
	Salkum silty clay loam, $\theta = 0.20$	0.452 ± 0.003	0.212	Fig. 4(b)
	Salkum silty clay loam, $\theta = 0.25$	0.452 ± 0.003	0.212	Fig. 4(b)
	Salkum silty clay loam, $\theta = 0.30$	0.454 ± 0.003	0.203	Fig. 4(b)
	Salkum silty clay loam, $\theta = 0.35$	0.458 ± 0.004	0.183	Fig. 4(b)
	Salkum silty clay loam, $\theta = 0.40$	0.465 ± 0.006	0.151	Fig. 4(b)

Table 2
Values of parameters n and β found from data on horizontal movement of water to columns with kaolinite–sand mixtures in experiments of Smiles et al. (1978)

Volumetric water content	Set A		Set B		Set C	
	n[a]	β	n	β	n	β
0.06	0.455 ± 0.001	0.198	0.507 ± 0.000	− 0.027	0.480 ± 0.003	0.083
0.08	0.458 ± 0.005	0.183	0.512 ± 0.001	− 0.047	0.483 ± 0.004	0.070
0.10	0.459 ± 0.007	0.179	0.516 ± 0.001	− 0.062	0.485 ± 0.004	0.062
0.12	0.459 ± 0.009	0.179	0.519 ± 0.001	− 0.073	0.484 ± 0.004	0.066
0.14	0.457 ± 0.009	0.188	0.522 ± 0.001	− 0.083	0.488 ± 0.005	0.049
0.16	0.453 ± 0.007	0.208	0.525 ± 0.000	− 0.096	0.489 ± 0.006	0.045
0.18	0.444 ± 0.004	0.252	0.531 ± 0.001	− 0.115	0.490 ± 0.008	0.041
0.20	0.429 ± 0.003	0.331	0.540 ± 0.001	− 0.147	0.490 ± 0.008	0.041

[a] Mean ± standard error.

− 0.5 log (t), we could express log (x) as a function of log (t) in the form

$$\log(x) = (0.5 - B)\log(t) + A$$

Comparison of this equation with Eq. (18) shows that the exponent n can be found as $n = 0.5 - B$.

All data points were obtained by digitizing graphs found in the publications. The digitizing was made in triplicate. Coefficient of variation within the replications did not exceed 0.1%.

4. Results and discussion

Values of parameters n with standard errors and values of β estimated from mean n values are presented in Table 1 for soil columns, and in Table 2 for kaolinite–sand mixtures. Linear regression equations used to find the value of n had correlation coefficients equal to 0.99 in Fig. 1, not less than 0.995 in Figs 2, 3 and 4 and not less than 0.97 in Fig. 5.

Values of n were significantly different from 0.5 in almost all data sets that we considered except experiments of Smiles et al. (1978) with kaolinite–sand mixtures (set C at water contents of 0.18 and 0.20). There was no clear trend in change of values of n and β with water content. For example, whereas Columbia silt loam wetted with water showed a decrease in n as the water content increased, Hesperia sandy loam demonstrated a drastic increase in n along with an increase in water content. There was a decrease in values of n followed by an increase as water content of Salkum silty loam decreased from 0.51 to 0.05 in experiments of Rawlins and Gardner (1963). The values of n were remarkably stable in the same soil as the water content varied in the wide range studied by Ferguson and Gardner (1963). The kaolinite–sand mixtures of Smiles et al. (1978) showed the opposite apparent trends in changes of n with the water content. There was a decrease in the value of n in data set A, and an increase in the value of n in data sets B and especially C, as water content changed from 0.06 to 0.20. Kaolinite–sand mixture of the data set B was the only data set with values of n greater than 0.5 and negative values of β. Here diffusivity increased as water moved along the column.

Small deviations of the value of n from 0.5 result in relatively large differences in values of β. Values of β that we found mostly ranged between 0 and 0.4. In two cases for the Hesperia sandy loam we had values greater than 0.5. Our values of β were less than 0.8 and 1.5 for two-dimensional and three-dimensional percolation clusters, respectively. It is not surprising since the water movement in cylindric samples is confined and the wandering opportunities of particles are restricted. Nevertheless, since the geometrical constraints of the medium hinder the motion of particles, the propagation of the diffusion front is slowed (Giona, 1992).

Although the percolation cluster model represents a good physical model to simulate and interpret anomalous diffusion, it is not the only model useful for this purpose (Roman et al., 1989). The critical feature required for the occurrence of non-Fickian behavior

is a memory of the particle (Kinzelbach, 1987). Uncorrelated consecutive steps of a particle reflect the situation where the correlation length is smaller than the step length. This assumption has been used to derive the Richards' equation from basic principles of statistical mechanics (Bhattacharya et al., 1976). Non-Fickian behavior occurs when the length of the fluid path is on the same order as or smaller than the range of spatial correlation of inhomogenities. In this case a correlation between successive dispersive steps occurs. The anomalous diffusion may also have non-fractal interpretation. Guerrini and Swartzendruber (1992) found that the scaling given by Eq. (17), applicable in soils where the transport coefficient in the water transport equation depends on water content and time, decreases as a power of time. These authors have related the time-dependence to the accumulation of bulk density invariant changes in microstructure that can alter the geometry of the pore space.

Although Eq. (17) and Eq. (18) are consequences of the properties of Eq. (10), the validity of these equations does not prove the validity of Eq. (10). Testing Eq. (10) with other boundary conditions is needed to prove its applicability.

For the same soil water content, values of the soil water diffusivity decreased as the distance from the source increased (set B of Smiles et al., 1978, was the only exception). A study of solute dispersion in saturated geologic media by Neuman (1990) demonstrated an increase in solute dispersivities as the scale increased. This should not be viewed as a contradiction. There is no straightforward physical analogy between solute transport in saturated pore space and water transport in unsaturated pore space For any scale selected, the diffusivity as used in Eq. (10) remains a local property changing along the direction of the water movement, whereas the dispersivity is a single value associated with a particular scale.

The spatial arrangement of particles in soils has been successfully described using mass, volume, and surface fractal dimensions (Bartoli et al., 1991; Perfect and Kay, 1995). These dimensions, however, may not be relevant in estimating values of β. The value of β is related to the scaling of pore connectivity which does not have a direct relationship with mass or volume fractal dimensions. Lemaitre and Adler (1990) studied saturated viscous flows through two Menger sponges having similar fractal dimensions and found quite different water transport properties in these sponges. To estimate soil hydraulic conductivity, Crawford (1994) has suggested using the values of both the mass fractal dimension and the spectral dimension which is directly related to β.

Significantly different values of n and β were found for the same kaolinite–sand mixture in experiments A, B and C of Smiles et al. (1978). We hypothesized that the differences could be caused by different initial concentrations of KCl used to saturate samples which might result in a different rearrangement of particles within samples caused by the dispersion of clay material. The pore connectivity in data set B seemed to be better than in data sets A and C.

Since the connectivity of water-filled pores may be different at different water contents, there is no reason why the value of β should be constant over the range of water contents. The value of β was approximately constant over a range of water contents in some of the cases. However, β was increasing drastically as the water content in Hesperia sandy loam decreased.

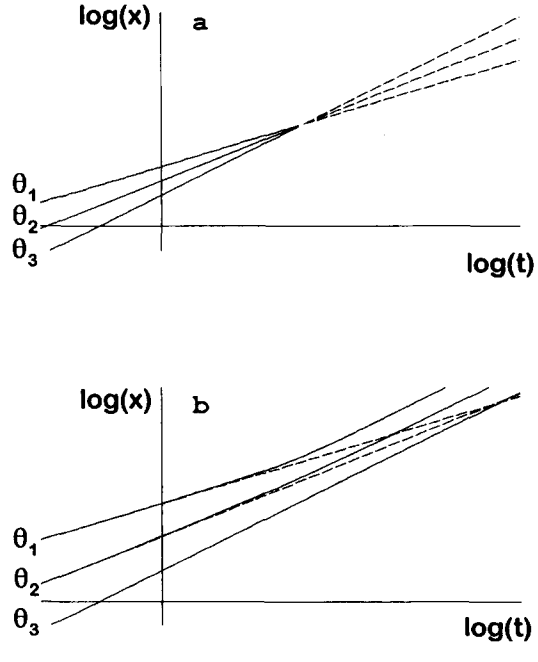

Fig. 6. Isomoisture lines in the log (x) – log (t) plane; (a) increase in slopes with the increase in moisture content; (b) hypothetic convergence of the isomoisture lines to the line of Fick scaling.

The light texture of this soil may be a reason of changes in a connectivity as the soil water pressure has dropped from −2 to −100 mb. Eq. (10) may still be valid but the solution in the form of Eq. (17) will not be applicable anymore.

The increase of β with θ can not continue indefinitely as water movement progresses. An example for three water contents $\theta_1 < \theta_2 < \theta_3$ is shown in Fig. 6(a). Each straight line in this figure has the equation $\log(x) = n(\theta)\log(t) + b(\theta)$ where both the slope n and the intercept b depend on θ. Since the slope n decreases as β increases, the straight lines drawn for different values of θ have to intersect eventually which means a non-physical step-wise jump in water content at the intersection point. The increase in $\beta(\theta)$ that can be traced, for example, for Hesperia sandy loam is probably a phenomenon specific for the observed region of the $\log(x) - \log(t)$ plane. We hypothesized that in a broader region we would see a convergence of all the isomoisture lines to the line with $n = 0.5$ corresponding to Fick's diffusion as shown in Fig. 6(b). This hypothesis is based on the idea of soil being fractal over a finite scale (correlation length) and the applicability of a classical description at larger scales (Yortsos, 1990). Some evidence supporting this hypothesis can be found in data of Fig. 5(b and d). The slopes of lines that would connect the second and the third data point for each water content seem to be smaller than slopes of straight lines that would connect the first and the second points. The eventual change of isomoisture lines to lines having the slope 0.5 may be also true for the case of n independent of water content, and that may limit the applicability of Eq. (17). The change of type of diffusion may not necessarily be related to the scale. Grinrod and Impey (1993) simulated solute transport in saturated two-dimensional fractal porous medium and found that at early stages an anomalous transport occurs, whereas the later breakthrough is predominantly Fickian. Changes of the diffusivity scaling as the water transport in soil progresses seem to be an interesting issue to explore. Another interesting topic is the scaling law for the vertical movement when the gravity as an external force is applied. General considerations of Roman et al. (1989) imply that the scaling variable ζ from Eq. (2) or Eq. (18) should be in this case replaced by the scaling variable $x/\log(t)$.

References

Acuna, J.A., Yortsos, Y.C., 1995. Application of fractal geometry to the study of networks of fractures and their pressure transient, Water Resour. Res., 31, 527.

Bartoli, F., Phylippy, R., Doirisse, M., Niquet, S., Dubuit, M., 1991. Structure and self-similarity in silty and sandy soils: the fractal approach, J. Soil Sci., 42, 167–185.

Bhattacharya, R.N., Gupta, V.K., Sposito, G., 1976. A Markovian stochastic basis for the transport of water through unsaturated soil, Soil Sci. Soc. Amer. J., 40, 465–467.

Brakensiek, D.L., Rawls, W.J., Logsdon, S.D., Edwards, W.M., 1992. Fractal description of macroporosity, Soil Sci. Soc. Amer. J., 56, 1721–1723.

Crawford, J.W., 1994. The relationship between structure and the hydraulic conductivity of soil, European J. Soil Sci., 45, 493–501.

Feder, J., 1988. Fractals. Plenum Press, New York.

Ferguson, H., Gardner, W.R., 1963. Diffusion theory applied to water flow data obrained using gamma ray absorption, Soil Sci. Soc. Amer. Proc., 27, 243–246.

Gardner, W., Widtsoe, J.A., 1921. The movement of soil moisture, Soil Sci., 11, 215–233.

Gefen, Y., Aharony, A., Alexander, S., 1983. Anomalous diffusion on percolating clusters, Phys. Rev. Lett., 50, 77–80.

Giona, M., 1992. IFS-simulation of transport phenomena on complex fractal media, Molecular Simulation, 8, 265–271.

Grinrod, P., Impey, M.D., 1993. Channeling and Fickian dispersion in fractal simulated porous media, Water Resour. Res., 29, 4077–4089.

Guerrini, I.A., Swartzendruber, D., 1992. Soil water diffusivity as explicitly dependent on both time and water content, Soil Sci. Soc. Amer. J., 56, 335–340.

Guerrini, I.A., Swartzendruber, D., 1994. Fractal characteristics of the horizontal movement of water in soils, Fractals, 2, 465–468.

Kinzelbach, W., 1987. Numerische Methoden zur Modelierung des Schadstofftransports im Grundwasser. Oldenburg Verlaf, M ünchen.

Lemaitre, R., Adler, P.M., 1990. Fractal porous media IV: three-dimensional Stokes flow through random media and regular fractals, Transport in Porous Media, 5, 325–340.

Mandelbrot, B.B., 1983. Fractal Geometry of Nature. W.H. Freeman, New York.

Neuman, S.P., 1990. Universal scaling of hydraulic conductivities and dispersivities in geologic media, Water Resour. Res., 26, 1749–1758.

Neuman, S.P., 1995. On advective transport in fractal permeability and velocity fields, Water Resour. Res., 31, 1455.

Nielsen, D.R., Biggar, J.W., Davidson, J.M., 1962. Experimental consideration of diffusion analysis in unsaturated flow problem, Soil Sci. Soc. Am. Proc., 26, 107–111.

Niemeyer, J., Ahl, C., 1991. Die Fractale Verteilung der Minerale und deren Bedeutung fur die Sorption, Mitt. Dtsch. Bodenk. Ges., 66, 217–274.

Orbach, R., 1986. Dynamics of fractal networks, Science, 231, 814–819.

O'Shaughnessy, B., Procaccia, I., 1985. Analytical solutions for diffusion on fractal objects, Phys. Rev. Lett., 54, 455–458.

Pachepsky, Ya.A., Polubesova, T.A., Hajnos, M., Sokolowska, Z., Jouzefaciuk, G., 1995. Fractal parameters of pore surface area as influenced by simulated soil degradation, Soil Sci. Soc. Amer. J., 59, 58–65.

Perfect, E., Kay, B.D., 1995. Application of fractals in soil and tillage research: a review, Soil Tillage Res., 36, 1–20.

Philip, J.R., 1955. Numerical solution of equations of the diffusion type with diffusivity concentration-dependent, Trans. Faraday Soc., 51, 885–892.

Rawlins, S.L., Gardner, W.H., 1963. A test of the validity of the diffusion equation for unsaturated flow of soil water, Soil Sci. Soc. Amer. Proc., 27, 507–511.

Roman, H.E., Bunde, A., Havlin, S., 1989. Anomalous transport on random fractal structures: stretched Gaussians, power-laws, and logarithmic time dependencies, Berichte der Bunsengesellschaft, Phys. Chem.; 93, 1205–1208.

Smiles, D.E., Philip, J.R., Knight, J.H., Elrick, D.E., 1978. Hydrodynamic dispersion during adsorption of water by soil, Soil Sci. Soc. Am. J., 42, 229–234.

Sokolowska, Z., 1989. On the role of energetic and geometric heterogeneity in sorption of water vapor by soils: application of a fractal approach, Geoderma, 45, 251–265.

Toledo, P.G., Novy, R.A., Davis, H.T., Scriven, L.E., 1990. Hydraulic conductivity of porous media at low water content, Soil Sci. Soc. Am. J., 54, 673–679.

Tyler, S.W., Wheatcraft, S.W., 1992. Fractal scaling of soil particle-size distributions: analysis and limitations, Soil Sci. Soc. Am. J., 56, 362–369.

Yortsos, Y.C., 1990. Heterogeneity description using fractal concepts. In: Modeling and applications of transport phenomena in porous media. Rhode Saits Genèse, Von Karman Institute for Fluid Dynamics, pp. 1–35.

Young, I.M., Crawford, J.W., 1991. The fractal structure of soil aggregates: its measurement and interpretation, Soil Sci., 42, 187–198.