MEANDER SHAPE AND THE DESIGN OF STABLE MEANDERS

James R. Hagerman[1] and John D. Williams[2]

ABSTRACT: When natural stream channels are relocated or restored, meander designs are frequently calculated from empirical relationships between channel size or discharge, degree of sinuosity, and meander plan-form descriptors. Because these parameters are interdependent, certain parameters must be chosen arbitrarily as independent variables, and trial and error is necessary to produce a final design. Theoretical models of meander plan form provide insights into the nature of stable meanders, but the use of these models for design has been limited. A methodology that makes the interdependency of parameters explicit would simplify and improve the design process for rehabilitation projects. The sine-generated curve model proposed by Langbein and Leopold (1966) provides a complete description of stable meander plan form, in which meander shape can be considered a function of sinuosity and a scale factor. We applied this model to propose a design procedure that reduces the number of independent design variables and the need for iterative solutions.
KEY TERMS: rehabilitation, stream channel morphology, meander, channel design

INTRODUCTION

In recent decades, we have gained greater appreciation of the natural hydraulic, hydrologic, and biological functions of streams and floodplains (e.g. L.R. Johnston Associates, 1992). In particular, the value of meanders in streams for providing aquatic habitat and in maintaining stable stream pattern and function has been generally recognized. These factors, along with concern for the water quality impacts of soil material from eroding stream banks, has led to the development of a small industry specializing in streambank stabilization and to numerous projects involving restoration of meanders to straightened streams, addition of meandering low-flow channels to trapezoidal flood channels, mitigation of impacts to meanders from construction or development, and stabilization of banks damaged by adjacent or upstream land uses. Project goals vary from full ecosystem restoration to protection of the built environment against damage caused by bank erosion. Between these two extremes are projects that provide habitat improvement or replacement, or other local rehabilitation of stream structure and function. Restoration projects seek a stable but dynamic stream form with natural rates of erosion and deposition, while the goal of stabilization projects is to halt erosion while maintaining other stream functions (such as sediment transport and habitat). Meander plan-form restoration or reconstruction can be a part of projects with any of these goals.

Hasfurther (1985) identifies four general methods for designing the plan form of a meander: the carbon copy technique, in which a disturbed meander is replaced exactly as it was before disturbance (e. g. Brookes, 1987); the “natural” approach, in which a stream is allowed to form its own channel; the “systems” approach, which is an empirical method that derives channel characteristics from a detailed study (including Fourier analysis) of undisturbed streams in the surrounding area; and empirical equations, which relate watershed area or discharge to meander characteristics (e. g. Rinaldi and Johnson, 1997).

The above techniques have their applications, but they also have limitations. A carbon copy requires that the initial stream pattern was stable, watershed characteristics have not changed, and the necessary information is available to reconstruct the original pattern. The “natural” approach can require a long period of time to reach form stability, and can have significant impacts on the stream system during the period of adjustment. The “systems” approach can be very data-intensive and require a large expenditure of resources and time. Empirical equations for meander plan-form characteristics are considered to be applicable to all streams (Leopold and Wolman, 1960). Meander wavelength, belt width (or

[1] Technical Specialist, Tennessee Valley Authority, WT 10D, 400 West Summit Hill Dr., Knoxville, TN 37902
[2] Hydrologist, USDA-Agricultural Research Service, PO Box 370, Pendleton, OR 97801

amplitude), radius of curvature, and sinuosity are presented as being related to each other and to cross-section characteristics, such as bankfull width (Williams, 1986). However, choosing values of these parameters based on these general relationships can require trial-and-error adjustments to reach a consistent design (Federal Interagency Stream Restoration Working Group, 1998).

METHODS

Meander Models

Langbein and Leopold (1966) argue that the sine-generated curve, originally developed as a formulation of a most probable random walk path, minimizes the variance from mean downstream direction (compared to sinusoidal, joined segments of circles, and parabolic curves), and therefore this relationship "underlies(s) the stable form of meanders." Fit between this model and field measurements was found to be good. Ferguson (1973) provides further discussion of meander shape models.

The direction of the sine-generated curve changes as a sinusoidal function of distance along the curve path and takes the form:

$$\phi = \omega \sin (2\pi d/M) \qquad (1)$$

where ϕ is the angle of the curve from parallel with the X-axis, ω is the angle at which the curve crosses the X-axis, (Figure 1), d is the distance along the curve path, and M is the total path length for one wavelength (ϕ and ω in radians; d and M in consistent linear units).

By curve-fitting a numerical solution to (1), Langbein and Leopold developed an equation for ω (in degrees) as a function of K:

$$\omega = 125\{(K-1)/K\}^{0.5} \qquad (2)$$

where K is sinuosity, defined as path length divided by wave length:

$$K = M/\lambda \qquad (3)$$

and λ is wave length of the curve along the X-axis (in consistent linear units). Using this result, the same authors approximated radius of curvature (Rc, in consistent linear units) for the sine-generated curve:

$$Rc = (\lambda K^{1.5})/[13(K-1)^{0.5}] \qquad (4)$$

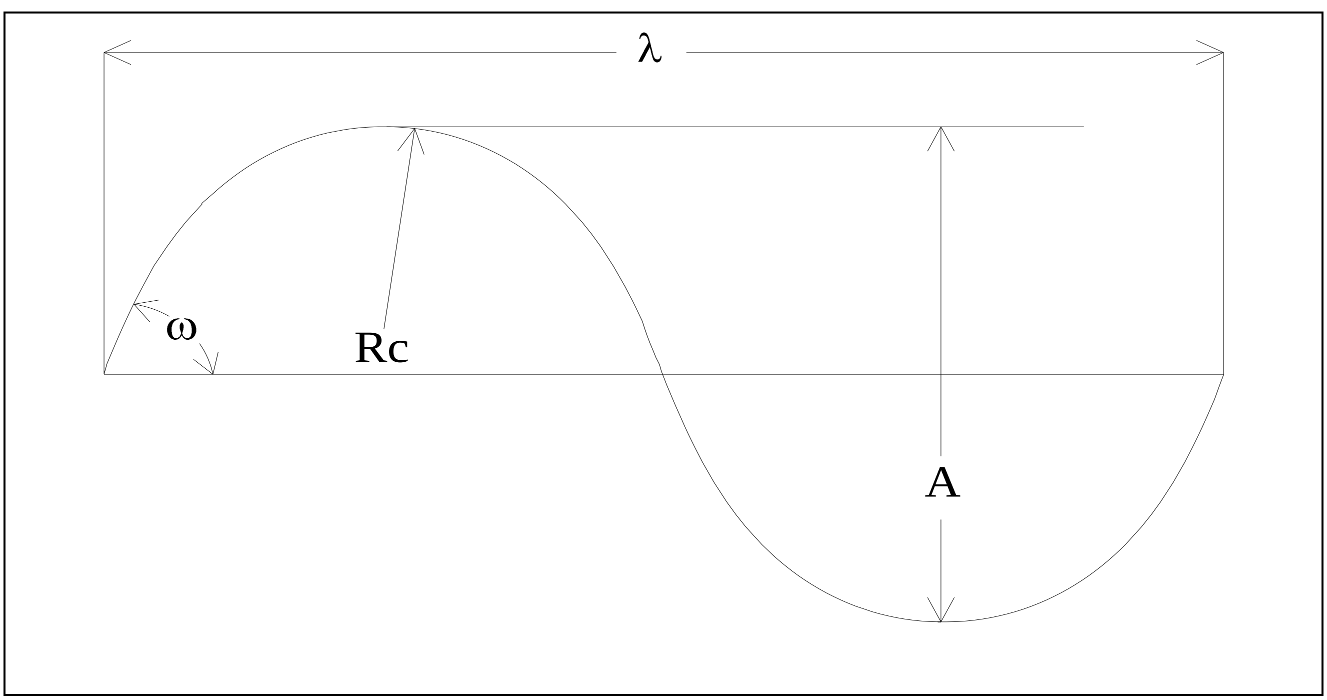

Figure 1. Variable definitions for describing meander shapes (Sinuosity [K] = 1.5).

Equation (1) describes curve direction rather than position, so it is difficult to apply this equation directly to meander plan-form design. Equation (4) is sometimes used in design, but radius of curvature alone does not fully describe the curve shape. Empirical relationships are frequently used to fill this gap (as in Rosgen, 1988), but wave amplitude is also needed to match meander shape to design sinuosity.

In order to complete the practical description of meander shape as defined by the sine-generated curve, we calculated the curve path by stepwise approximation. Equation (1) was calculated in a spreadsheet program using 200 steps for a full wave. An iterative process was used to find ω and meander amplitude (A) as a function of K. Curves were prepared from these results and displayed with their best-fit lines, along with Rc calculated from (4) and scaled by λ (Figures 2, 3, and 4). The previously published approximation for ω shows a good fit (Figure 2).

The best fit found for A/λ as a function of K is

$$A/\lambda = 6.0625Q^3 - 5.1279Q^2 + 2.509Q + 0.0005 \qquad (5)$$

where Q = (K-1)/K. For ease of calculation, the relationship

$$A/\lambda = 0.9743 \ln(K) + 0.0803 \qquad (6)$$

can be used for K ∗ 3 (Figure 3).

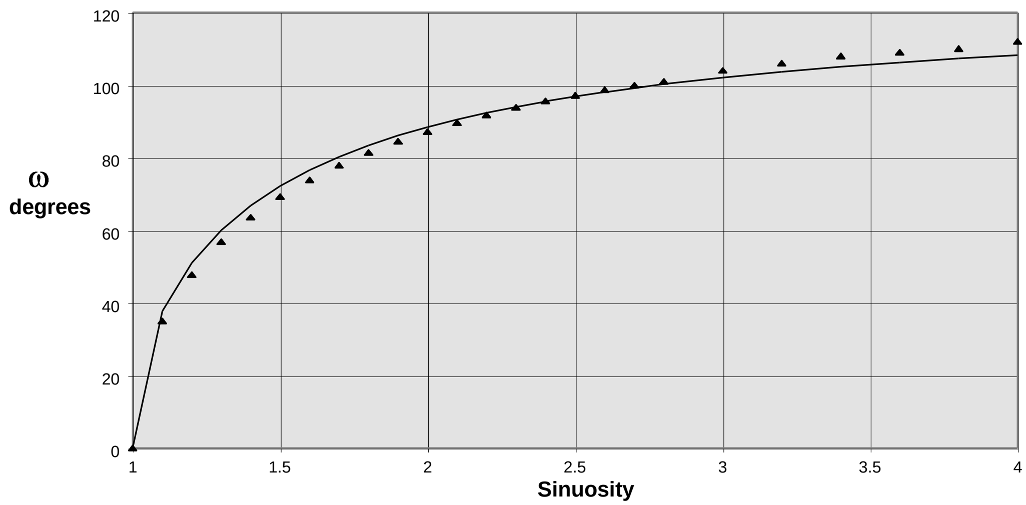

Figure 2. Angle at which the sine-generated wave crosses the X-axis (ω) as a function of sinuosity. Points from numerical solution of sine-generated curve, fitted line is from Langbein and Leopold (1966); equation (2) in this article.

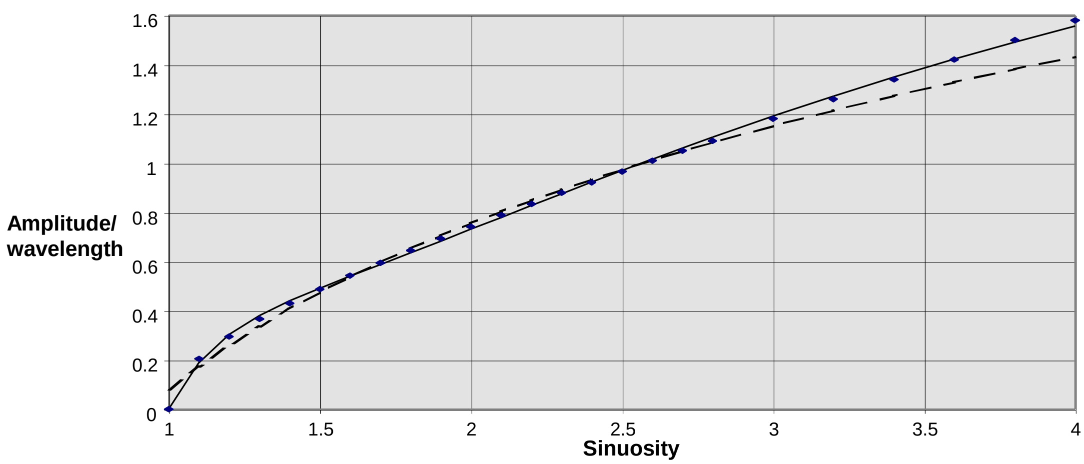

Figure 3. Ratio of sine-generated wave amplitude to wavelength (A/λ) as a function of sinuosity. Points from numerical solution, solid line is equation (5), dashed line is equation (6).

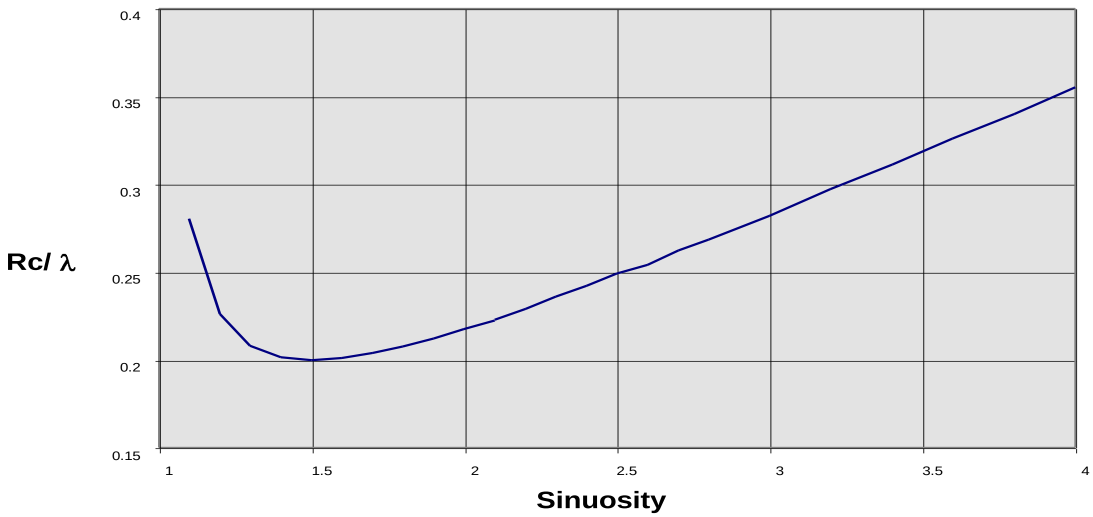

Figure 4. Ratio of radius of curvature to wavelength (Rc/λ), from Langbein and Leopold (1966), equation (4).

DISCUSSION

Considerable attention has been given to developing design procedures for channel cross sections. In addition to the empirical methods discussed above that use existing natural streams as a model, methods have been developed that attempt to mathematically account for bed load transport (e.g. Jackson and Van Haveren, 1985; Hey, 1997). Discussion of these techniques is beyond the scope of this paper, which is limited to meander plan form. However, it is important to begin the plan-form part of the design process with a channel slope (S), cross-section, and width (B) that transport channel-maintaining flows and bed load, can be supported by the bank materials, and provide transport continuity with upstream and downstream portions of the stream. The parameters S and B, along with valley slope at the restoration site, are used in the design of meander plan form.

The relationship between meander wave length and stream width is considered to be a fundamental property of meandering streams (Leopold and Wolman, 1960), although there may be some regional variation in behavior (Rinaldi and Johnson, 1997). Conventionally, the channel width calculated in the first step of the channel design process is related to λ by the equation from Leopold and Wolman (1960):

$$\lambda = 11.0B^{1.01} \quad (7)$$

or the equation from Williams (1986):

$$\lambda = 7.5B^{1.12} \quad (8)$$

Where stream width (B) and wave length (λ) are in meters. Sinuosity is calculated from valley slope (S_v) and channel slope (S) by a variation of equation (3):

$$K = S_v/S \quad (9)$$

Because the shape of the sine-generated curve is considered here as a function of K and λ, the meander plan can now be specified using ω, Rc, and A, which are calculated using equations (2), (4), and (5) or (6), respectively. A line segment is drawn (or flagged on the ground) at an angle of ω to the X-axis at X = 0 and at X = λ/2. The center of a circle of radius Rc is placed at the X,Y coordinates of (λ/4, A/2-Rc). A smooth curve using these construction curves creates the new stream centerline (Figure 5). This process is mirrored across the X axis beginning at X = λ/2 to continue the curve.

Because a tighter radius of curvature in bends creates higher local shear stress (Federal Highway Administration, 1988) care should be taken to keep the value of the ratio of radius of curvature to channel width (Rc/B) above those that would cause excessive bank erosion in the materials at the site. It should be noted that the Rc/λ curve (Figure 4) reaches a minimum of 0.2 at K = 1.5, and therefore if λ > 8B, Rc/B can be smaller than the generally recommended minimum value of 2.5 (Leopold and Wolman, 1960; Williams, 1986). Radius of curvature does not vary rapidly with K, so adjustment of Rc is feasible. In fact, the designer has some latitude to vary local curve parameters to take advantage of nonhomogeneities in materials. However, the resulting curve length should be checked to make sure that the sinuosity (and therefore slope) specified in the design is retained.

An alternate application for this procedure is a situation in which meander belt width is constrained, such as in the creation of a low-flow channel within a flood channel (e.g. Morris, 1996). In this case, λ is related to channel width as above, but equation (6) is solved for K. Rc and ω are then calculated from K to produce a stable meander shape.

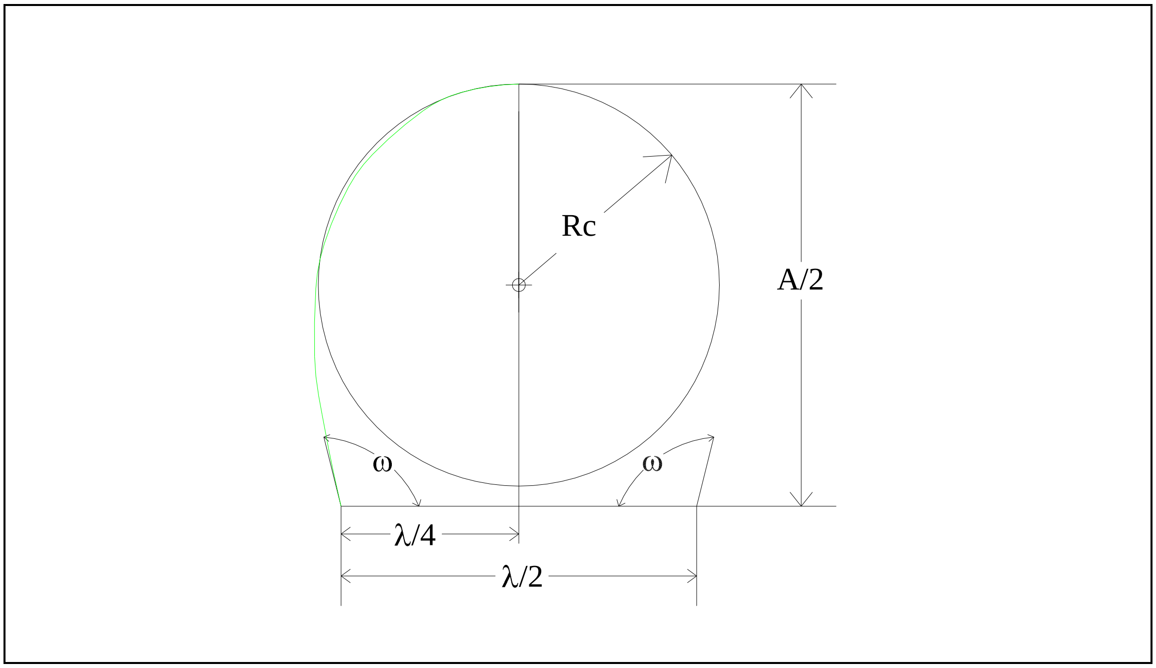

Figure 5. Meander construction from calculated parameters. For K=3: ω = 1048 (from equation [2]); Rc/λ = 0.282 (equation [4]); A/λ = 1.18 (equation [5] or [6]). To construct the curve, a line segment is drawn at an angle of ω to the X-axis at X = 0 and at X = λ/2. The center of a circle of radius Rc is placed at the X,Y coordinates of (λ/4, A/2-Rc). A smooth curve using these construction curves creates the new stream centerline.

CONCLUSIONS

The procedure described for calculating channel parameters uses the shape of the sine-generated curve to reduce the degrees of freedom and the need for iterative solutions in meander plan-form design. This procedure can be used as a substitute or as an adjunct to other plan-form design procedures.

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