

A NEW UNIFIED EQUATION FOR WIND-DRIVEN SURFICIAL OXYGEN TRANSFER INTO STATIONARY WATER BODIES

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ABSTRACT. Wind-driven surficial oxygen transfer into stationary water bodies plays an important role in analyzing the fate and biochemical processes of environmentally important gases. This article (1) reviews research on oxygen and other gas transfers into non-moving, open water bodies, and (2) presents the synthesis of a new, unified equation for oxygen mass transfer coefficients based on gas transfer data published during the last 50 years. Both theoretical and empirically derived oxygen coefficients were reviewed using data derived from investigations in controlled wind tunnels, floating reaeration devices in open waters, and natural open waters. To facilitate the comparative analyses, gas transfer coefficient correlations for other gases were normalized to oxygen, and wind speeds were normalized to 10 m height. Wind was the major turbulence agent facilitating the gas transfer processes. Generally, low wind speed did not significantly influence the transfer coefficients. However, the transfer coefficients increased, even exponentially, with higher wind speeds. There were large variations among existing transfer coefficient correlations. Nonetheless, sufficient gas transfer data have been published in the last five decades to provide a solid basis for synthesizing a new unified oxygen transfer coefficient formula to estimate surficial oxygen transfer into treatment lagoons. The new unified equation for wind-driven surficial oxygen transfer is a function of Schmidt number, wind speed, and temperature.

Keywords. Aeration, Lagoons, Oxygen transfer, Wastewater, Wind.

Wind produces turbulence in stationary water bodies, facilitating gas exchange processes, and the gas exchange velocity increases with wind speed (Banks, 1975; Crusius and Wanninkhof, 2003; Liss and Merlivat, 1986; Yu et al., 1977). Wind-driven oxygen transfer into animal waste treatment lagoons through the water-air interface plays an important role in analyzing the complex biochemical processes responsible for ammonia removal in the lagoons. It is one of the key factors that critically influences the design and operation of these lagoons. The oxygen transfer characteristic of treatment lagoons can also be used to estimate other important gas exchanges, such as ammonia emissions. Because oxygen mass transfer through the air-water interface is mostly liquid-phase controlled, the oxygen mass transfer coefficient can be used to better estimate the individual liquid-phase mass transfer coefficients of fugitive chemicals for the surface impoundment emission model of the U.S. EPA's WATER9 wastewater treatment computer program (U.S. EPA, 1994). In addition, the surficial oxygen transfer information is an integral part of the emission model for estimating VOC volatil-

ization rates from surface aeration tanks of municipal wastewater treatment plants (Hsieh et al., 1993). This emission model estimates VOC volatilization rates based on a proportional relationship of mass transfer coefficients between VOCs and oxygen. This VOC emission model could be further improved by accounting for the additional oxygen transfer by the wind blowing over the aeration tank surface.

This article (1) briefly reviews the theory and existing correlations for surficial oxygen transfer, and (2) presents the synthesis of a new, unified equation for an oxygen mass transfer coefficient based on the compiled gas transfer data from the literature.

METHODOLOGY

In developing the new unified correlation, we first compiled the K vs. wind speed observations of the water bodies where wind was the major turbulence-causing agent. We attempted to obtain raw data directly from published articles, and we were successful in most cases. However, a few authors reported their findings only in graphical forms. In those cases, we used the TatuGIS Viewer (version 1.1.1.166, TatuGIS, Gdynia, Poland) in order to estimate the values of data from the scanned images. These compiled data points include the transfer coefficients of the liquid-side controlled gases (O_2 , SF_6 , N_2O , CO_2 , and ethyl ether) obtained from various laboratory wind tunnels, floating reaeration devices in open water, and natural open waters. The transfer coefficients of these gases were normalized to O_2 and the wind speed to 10 m reference height (U_{10}) using the Schmidt number ratios and the seventh-root or logarithmic wind profiles.

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THEORY

The gas absorption rate into a unit surface area of a water body is traditionally expressed as a product of a thermodynamic driving force and a mass transfer coefficient. The driving force is the concentration gradient in the air and/or water phases. The total impedance of this interfacial mass transfer process can be viewed as a sum of two mass transfer resistances (liquid phase and gas phase) in series. For sparingly soluble gases such as oxygen, the liquid-phase mass transfer resistance controls the entire absorption process. As a result, oxygen absorption flux into open water bodies is expressed only with the liquid-phase concentration gradient (or oxygen deficit):

$$j_{O_2} = K_L \cdot (C_S - C) \quad (1)$$

where

j_{O_2} = mass flux of oxygen absorbed into water surface (kg m⁻² s⁻¹)

K_L = overall liquid-phase oxygen mass transfer coefficient (m s⁻¹)

C_S = dissolve oxygen concentration in equilibrium with air (kg m⁻³)

C = bulk water concentration of dissolved oxygen (kg m⁻³).

The bulk dissolved oxygen concentration and the surface area are easily measured. Thus, the problem of predicting oxygen absorption from the atmosphere to lagoons using equation 1 reduces to the determination of the overall liquid-phase mass transfer coefficients. Albeit simple in mathematical form, the prediction of the oxygen absorption rate using equation 1 is far from simple because the overall liquid-phase mass transfer coefficient (hereafter referred to as the transfer coefficient) depends on a complex array of hydrodynamic and aerodynamic conditions, along with molecular transport properties of gases. Efforts to quantify the transfer coefficient have been significant. Since the beginning of the 20th century, researchers around the world have attempted to relate the molecular transport properties of gases, hydrodynamic conditions, and aerodynamic conditions to the transfer coefficient through various hypotheses.

The earliest hypothesis, known as the classic two-film theory, was based on the observation that heat transfer could be formulated with a thin film of fluid surrounding the heat-transferring objects (Lewis and Whitman, 1924). The film theory assumes that a thin, stagnant layer of fluid exists near every interface, and the entire mass transfer resistance occurs in this film. Gas molecules cross the stagnant layer via molecular diffusion, a very slow transport process. In the bulk fluid phase, the film theory assumes a complete mixing without any concentration gradient, thus no mass transfer resistance. The film theory provides a mathematically very simple relationship (eq. 2) between diffusivity of gas molecules and the film thickness, which depends on hydrodynamic or aerodynamic conditions:

$$K_L = \frac{D}{\delta} \quad (2)$$

where

D = molecular diffusivity of a gas (m² s⁻¹)

δ = thickness of the stagnant layer (m).

Albeit simple in concept and mathematics, the existence of the stagnant layer of fluid at the interface is not very convincing, especially under turbulent conditions. Higbie (1935) modified the film theory with periodic exposure of small fluid portions (typically referred to as the penetration of fluid packets in the literature) to the interfacial layer from the bulk fluid. The gas molecules diffuse into these fluid portions while staying at the interface; then, the diffused gas molecules are carried into the bulk liquid with the fluid portions. A few years later, Danckwerts (1951) extended Higbie's penetration hypothesis with a statistical residence time distribution of the fluid portions at the interface. This theory is commonly referred as the surface renewal theory because the interfacial surface is viewed as being continuously renewed with the fluid portions. The transfer coefficient of the surface renewal theory is expressed as:

$$K_L = \sqrt{D \cdot r} \quad (3)$$

where r is the surface renewal rate (s⁻¹).

These film-based theories elucidate simple physical pictures of gas absorption processes. However, direct application of these theories for predicting transfer coefficients is severely limited because the values of δ and r are unknown. These parameters can only be estimated after values of transfer coefficients become known, not the other way around.

Unlike the simple film-based mass transfer theories, Deacon (1977) utilized the turbulent boundary layer theory in an attempt to accurately describe the picture of the interfacial mass transfer process for a smooth water surface. He used the velocity profile of the turbulent air flowing over a smooth, rigid wall, which was previously determined by Reichardt (Hinze, 1959). He then assumed that the same shear stress existed both in air and water interfaces. Adopting Reynolds' analogy (Cussler, 1984), i.e., that all turbulent Reynolds is due to eddies and is independent of any diffusion coefficient, thermal conductivity, or viscosity, Deacon (1977) developed the following transfer coefficient equation for non-reactive gases:

$$K_L = 0.082 \cdot Sc^{-2/3} \left(\frac{\rho_a}{\rho_w} \right)^{1/2} u_{*,a} \quad (4)$$

where

ρ_a = air density (kg m⁻³)

ρ_w = water density (kg m⁻³)

$u_{*,a}$ = friction velocity (m s⁻¹)

Sc = Schmidt number (ν/D)

ν = kinematic viscosity of water (m² s⁻¹).

Equation 4 predicted values of oxygen transfer coefficients observed at relatively low wind conditions reasonably well (Liss, 1973). However, for higher wind conditions, the water surface becomes rough with the onset of ripple waves and the validity of using this equation quickly diminishes.

Notwithstanding the limited successes of the above theories, no single theory has yet completely described the dependence of the transfer coefficient on the entire array of complex environmental factors. As a result, most researchers have relied heavily on empirical relationships of the transfer coefficient to pertinent environmental factors.

EMPIRICAL EQUATIONS FOR TRANSFER COEFFICIENTS

The applicable empirical equations for predicting oxygen transfer coefficients for open water bodies have been predominately derived from reaeration and other gas exchange studies on ponds, lakes, and estuaries. In these mostly stationary water bodies, wind is the primary turbulence agent for facilitating gas transfer. The U.S. EPA (Bowie et al., 1985) compiled a number of oxygen transfer coefficient correlations for lakes from the literature. These formulas were plotted against wind speed, as shown in figure 1. All these formulas were related with wind speed except that from Di Toro and Connolly (1980). Figure 1 shows that there is a high degree of variation among these formulas. We also recognized that some authors, such as Downing and Truesdale (1955), Liss (1973), and Kanwisher (1963), only reported data, not formulas. It was not clearly stated how Bowie et al. (1985) obtained these formulas, but most likely they produced the formulas from fitting the data.

This investigation was also expanded to include studies involving other gases. The experimental methods for these studies consisted of laboratory wind tunnels of varying sizes, floating aeration tents, floating aeration pools, and deliberate tracer methods. Table 1 shows the empirical transfer coefficient formulas derived from these various experimental methods.

The wind tunnel method uses a wind tunnel to measure the effects of wind on the oxygen or other gas transfer in a water tank. The size of wind tunnels used in the past ranged from few meters to 100 m long (Wanninkhof and Bliven, 1991). The floating reaeration tents were made of polythene sheet

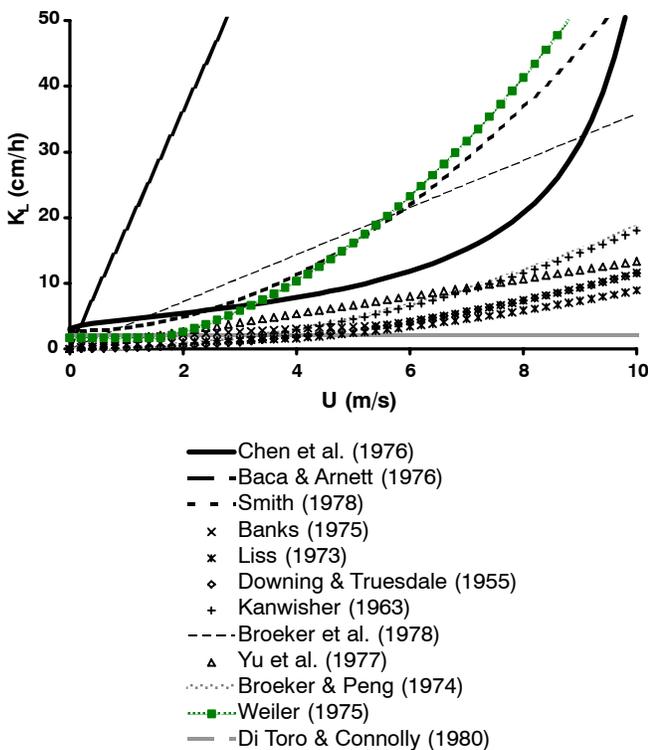


Figure 1. O₂ transfer coefficients for lakes according to table 3-11 of Bowie et al. (1985). Heights of wind speed were not reported.

(about 1 ft in diameter), and field aeration coefficients were measured in the Thames estuary near London Bridge during 1954 (Thames Survey Committee, 1964). Oxygen transfer coefficients were estimated by comparing the decline of oxygen in the floating-tent headspace with the dissolved oxygen deficit in the water column for the duration of the reaeration test (about 2 h). Floating reaeration pools were used by Yu et al. (1977) in order to estimate reaeration coefficients of the Spruce Run reservoir in Clinton, New Jersey.

The deliberate tracer method uses a conservative tracer such as sulfur hexafluoride (SF₆) for non-invasively measuring gas transfer coefficients in lakes and reservoirs (Wanninkhof et al., 1985, 1987). Sulfur hexafluoride is an ideal tracer gas for gas transfer studies because of the following characteristics: (1) biochemically inert, (2) present in background concentrations of less than 10⁻³ M in natural waters, (3) similar solubility and diffusivity values to that of O₂ and CO₂, (4) no adsorption to sediment, and (5) detected at less than 10⁻¹⁶ M. Sulfur hexafluoride has become an established tracer for atmospheric transport and oceanic diffusion studies (Brown et al., 1975; Watson and Liddicoat, 1985; Ledwell et al., 1986).

COMPARISON AMONG EMPIRICAL EQUATIONS

The empirical transfer coefficient formulas previously discussed were developed based on wind speeds measured at various heights above the water surface and with gases other than oxygen. In order to compare these formulas, it is necessary to normalize the wind speed at a reference height and to convert the transfer coefficient formulas for oxygen at a reference temperature. The following sections describe the normalization process.

Normalization of Wind Speed at a Reference Height

It is customary for researchers to relate the transfer coefficient with the wind speed at a reference height of 10 m. If the wind speed was not measured at the 10 m reference height, then the wind speed can be normalized to 10 m assuming the logarithmic or the seventh-root wind profiles (Crusius and Wanninkhof, 2003; Downing and Truesdale, 1955; Liss and Merlivat, 1986; Lunney, 1983; Upstill-Goddard et al., 1990; Wanninkhof and Bliven, 1991; Yu et al., 1977):

$$\frac{U_{10}}{U_z} = \frac{\ln\left(\frac{10}{z_0}\right)}{\ln\left(\frac{z}{z_0}\right)} \quad (23)$$

where z_0 is the surface roughness length (m).

Empirical values of z_0 for different surface characteristics are listed in Sutton (1953) and Arya (2001). For instance, the surface roughness parameter is 0.001 cm for smooth water surfaces, mud flats, and ice. If the surface roughness length cannot be estimated with a reasonable accuracy, the well-known "seventh-root law" may be used to estimate the 10 m wind speed. The seventh-root law describes the variation of wind speed with height in a turbulent boundary layer on a flat plate, as modified from the power-law expression originally suggested by L. Prandtl (Arya, 2001):

Table 1. Empirical equations for gas transfer coefficients.

Expression		Eq.	Gases	Experimental Method	References
K_L (m/s) = $4.19 \times 10^{-6} U^{1/2}$	for small U ^[a]	5	O ₂	Wind tunnel	Banks (1975)
K_L (m/s) = $1.80 \times 10^{-6} U$	for medium U ^[a]	6			
K_L (m/s) = $0.32 \times 10^{-6} U^2$	for large U ^[a]	7			
where U = wind speed at a reference height (m/s) K_L = transfer coefficient (m/s).					
$\frac{k_2}{(k_2)_0} - 1 = 0.2395 \cdot U_{20\text{cm}}^{1.643}$		8	O ₂	Wind tunnel	Mattingly (1977), Bennett and Rathbun (1972)
where k_2 = wind-produced reaeration coefficient (h ⁻¹) $(k_2)_0$ = transfer coefficient predicted by Bennett and Rathbun (1972):					
$(k_2)_0 = 0.101 \frac{U_w^{0.607}}{H^{1.689}}$					
U_w = average water velocity (m/s) H = average water depth (m) $U_{20\text{cm}}$ = wind speed at 20 cm above the water surface (m/s).					
K_L (m/s) = $3.7 + 0.79 \cdot U_1$		9	O ₂	Floating reaeration tents	Thames Survey Committee (1964)
where U_1 = wind speed at 1 m (mph)					
K_L ^[b] (cm/h) = $2.59 \cdot U_{10} - 7.6$	for 4.2 m/s < U_{10} < 9 m/s	10	O ₂	Floating pool reaeration	Yu et al. (1977, 1983)
$K_{L,600}$ (cm/h) = $0.17 \cdot U_{10}$	for $U_{10} \leq 3.6$ m/s	11	SF ₆ , CO ₂	Wind tunnel, deliberate tracer method	Liss and Merlivat (1986)
$K_{L,600}$ (cm/h) = $2.85 \cdot U_{10} - 9.65$	for 3.6 m/s < $U_{10} \leq 13$ m/s	12			
$K_{L,600}$ (cm/h) = $5.9 \cdot U_{10} - 49.3$	for $U_{10} > 13$ m/s	13			
where $K_{L,600}$ = transfer coefficient of CO ₂ (Sc for CO ₂ at 20°C = 600)					
$K_{L,600}$ (cm/h) = $1.11U_{10} + 0.35$	for $U_{10} < 9.5 \pm 3$ m/s	14	SF ₆	Deliberate tracer method	Upstill-Goddard et al. (1990)
$K_{L,600}$ (cm/h) = $2.53U_{10} - 13.09$	for $U_{10} > 9.5 \pm 3$ m/s	15			
$K_{L,600}$ (cm/h) = 1.0	for $U_{10} < 3.7$ m/s	16	SF ₆	Deliberate tracer method	Crusius and Wanninkhof (2003)
$K_{L,600}$ (cm/h) = $5.14 \cdot U_{10} - 17.9$	for $U_{10} \geq 3.7$ m/s	17			
$K_{L,600}$ (cm/h) = $0.32 \cdot U_{10}^{1.73}$		18	SF ₆ , N ₂ O, CH ₄	Deliberate tracer method	Wanninkhof and Bliven (1991)
$K_{L,600}$ (cm/h) = $0.45 \cdot U_{10}^{1.6} \left(\frac{Sc}{600} \right)^{-0.5}$		19	SF ₆	Deliberate tracer method	Macintyre et al. (1995)
K_L (cm/h) = $0.31 \cdot U_{10}^2 \left(\frac{Sc}{660} \right)^{-0.5}$		20	¹⁴ C ₂	¹⁴ C inventory	Wanninkhof (1992)
K_L (m/s) = $2.788 \cdot 10^{-6} \left(\frac{D_{O2}}{D_{EE}} \right)^{2/3}$	for $0 \leq U_{10} < 5$ m/s	21	Ethyl ether	Wind tunnel	Lunney et al. (1985)
K_L (m/s) = $2.611 \cdot 10^{-7} U_{10}^2 \left(\frac{D_{O2}}{D_{EE}} \right)^{2/3}$	for $U_{10} > 5$ m/s, $F/D > 51.2$	22			
where F/D = fetch to depth ratio					

[a] Assumed $U_{10} < 3$ m/s for small U , 3 m/s < $U_{10} < 8$ m/s for medium U , and $U_{10} > 8$ m/s for large U .

[b] We refitted their data to develop this formula.

$$\frac{U_z}{U_{10}} = \left(\frac{z}{10}\right)^{1/7} \quad (24)$$

Although the seventh-root law does not have a sound theoretical basis, as does the logarithmic wind profile, it provides simple and reasonable fits to the observed wind profiles in the atmospheric boundary layer. In this article, equation 23 was used if the z_0 values were reported in published reports; otherwise, equation 24 was used to normalize wind speeds at various heights to U_{10} .

Normalization of Different Gases to O_2 at $20^\circ C$

Results of dimensional analysis and many empirically and/or theoretically determined transfer formulas suggest that the transfer coefficient can be expressed as a product of Sc^{-n} and the turbulence of water (Cussler, 1984; Deacon, 1977; Jahne et al., 1987; Macintyre et al., 1995). Assuming that the turbulence does not change significantly over small temperature differences, the transfer coefficients of different gases can be related with the ratio of the Schmidt numbers of the gases:

$$\frac{K_{L1}}{K_{L2}} = \left(\frac{Sc_1}{Sc_2}\right)^{-n} \quad (25)$$

where

- n = power index
- $K_{L,1 \text{ or } 2}$ = transfer coefficient of gas 1 or 2 (L/T)
- $Sc_{1 \text{ or } 2}$ = Schmidt number of gas 1 or 2.

The values of n vary from 1 to 1/2 depending on hydrodynamic and boundary conditions as well as the theories assumed (Hsieh et al., 1993; Rainwater and Holley, 1983; Watson et al., 1991; Whitmore and Corsi, 1994). The two-film theory suggests $n = 1$, while the penetration/surface renewal theories suggest $n = 0.5$. Applying the boundary-layer theory for laminar flow over a flat plate, Cussler (1984) showed the dependence of transfer coefficients on Schmidt number as $Sc^{-2/3}$. Turbulent boundary-layer flow over a smooth wall (eq. 4) also produced a dependence of $Sc^{-2/3}$ as for laminar boundary-layer flow. However, the solid wall analogy of water surface in deriving equation 4 was argued against by Csanady (1990) because horizontal motions ("slip") were possible for the free water surface. He showed that for a solid boundary, the value of n should be 2/3, whereas for a fluid boundary, n should be 1/2 based on the surface divergence model. In another interpretation, Jahne et al. (1987) concluded that n changed from 2/3 to 1/2 with the increase in friction velocity. They suggested that the change in n reflected the transition from a rigid to a free surface.

This change in n values was utilized by Crusius and Wanninkhof (2003). They used $n = 2/3$ for a wind speed below 3 m/s and $n = 1/2$ for higher wind speeds, reflecting the turbulent boundary layer theory for the smooth water surface regime and the surface renewal theory for rough surface regimes. In order to normalize and compare the empirical formulas developed for different gases, we used the same approach as Crusius and Wanninkhof (2003), i.e., the transfer coefficients for other gases were normalized to O_2 at $20^\circ C$ using equation 26 with n values of 2/3 and 1/2 for the corresponding two wind regimes. The Schmidt numbers of the gases were estimated using third-order polynomial equations with temperature, as reported by Wanninkhof (1992).

Comparison of Various Transfer Coefficient Correlations Normalized to O_2 at $20^\circ C$

The transfer coefficient equations for K_L in table 1 normalized to O_2 at $20^\circ C$ were plotted together against U_{10} (fig. 2). For the range of wind speeds typically encountered in treatment lagoons, i.e., 0 to 10 m s⁻¹, these correlations predict the oxygen transfer coefficient from 0 to about 34 cm h⁻¹. This range of oxygen transfer coefficient is even wider than that for treatment wetlands, i.e., 0.2 to 20.8 cm h⁻¹ (Kadlec and Knight, 1996). The range of transfer coefficients predicted by these correlations also widens with wind speed. For instance, these correlations predict a range of transfer coefficients from 0.6 to 7.5 cm h⁻¹ at $U_{10} = 3$ m s⁻¹. Whereas at $U_{10} = 10$ m s⁻¹, the range increases from 5.7 to 34.6 cm h⁻¹. This variation in the oxygen transfer coefficient can be illustrated with the range of the oxygen mass absorbed into the treatment lagoons per unit water surface area (i.e., surficial oxygen flux). The surficial oxygen fluxes estimated from these correlations vary from 123 to 757 kg O_2 ha⁻¹ d⁻¹ at a wind speed of 10 m s⁻¹, assuming near-zero dissolved oxygen concentrations in the bulk of the treatment lagoons. This lack of agreement among these formulations calls for a new unified transfer coefficient relationship. Fortunately, the published data from the last five decades can be effectively used to create this new relationship.

NEW UNIFIED OXYGEN TRANSFER EQUATION

A total of 297 transfer coefficient data points were compiled from gas exchange studies conducted in laboratory wind tunnels of varying sizes and various field sites. The compiled data points were obtained from publications as early as 1955 and as recent as 2003. When these gas transfer coefficient data were plotted against U_{10} (fig. 3), it showed a general trend of transfer coefficients monotonically increasing with wind speed, although somewhat scattered at higher wind speeds. Transfer coefficients were not significantly influenced by lower wind speeds.

If these data were fitted empirically with U_{10} only, as frequently done by previous researchers with their individual data, then the effects of other important variables on the

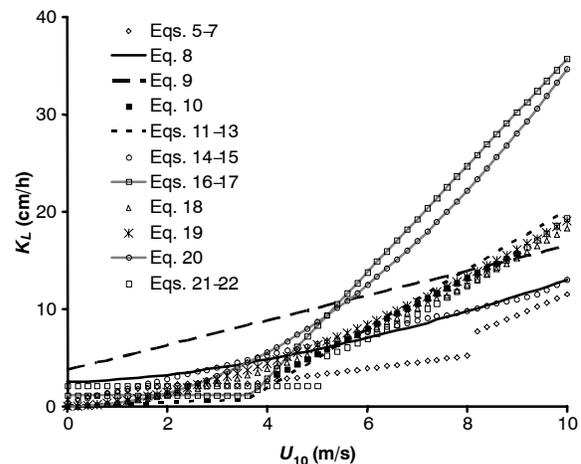


Figure 2. Comparison of existing gas transfer correlations normalized to O_2 at $20^\circ C$.

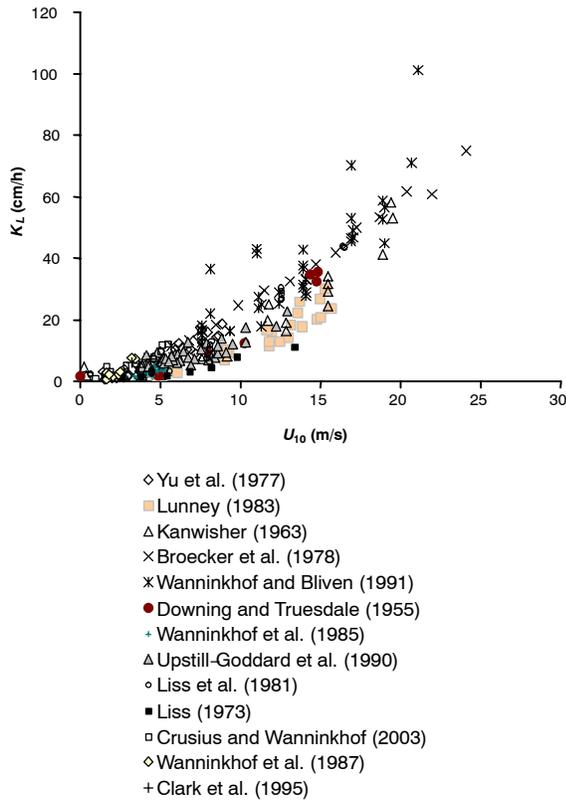


Figure 3. Compiled gas transfer coefficient data plotted against to U_{10} .

transfer coefficients would not be revealed. Both the molecular diffusivity of oxygen and wind-induced shear stress on water surface play important roles in the surficial oxygen transfer process, and they should be included in the new equation. In our initial search for an appropriate mathematical model to meaningfully fit the compiled data with these important variables, we selected the transfer coefficient equation (eq. 4) developed by Deacon (1977). It included: (1) the effects of molecular diffusivity in the form of Schmidt number, and (2) the wind-induced shear stress in the form of friction velocity multiplied by the square root of the air and water density ratio.

We then examined if equation 4 could adequately predict the compiled transfer coefficients. Whereas use of equation 4 requires knowledge of the friction velocity, and many authors did not measure the friction velocity, we estimated its values under the experimental conditions using the correlation developed by Csanady (1997). As shown in figure 4, predicted transfer coefficients were significantly lower than the observed data, especially for higher wind speeds. However, the poor prediction at high wind speed was expected because the model originally assumed the water surface as a smooth rigid surface. At lower wind speeds, the calm water surface may resemble the smooth rigid surface. This idealized assumption is not valid at higher wind speeds due to formation of ripples in the water surface.

Although Deacon's model was not directly applicable for higher wind speeds, it suggested to us that the Schmidt number (Sc) and the friction velocity would be important correlating variables for the new mass transfer coefficient equation. Furthermore, we assumed that transfer coefficients would be a function of $Sc^{-1/2}$ as accorded with the surface

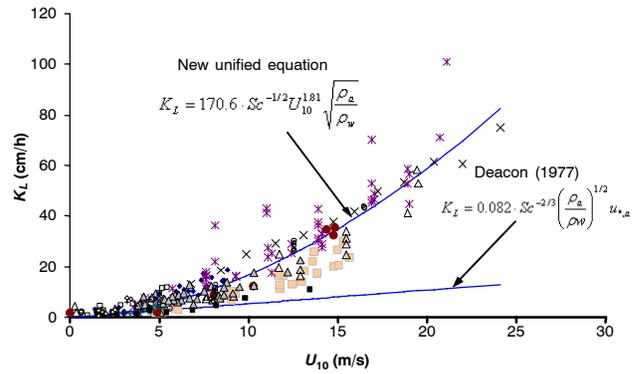


Figure 4. Predicting compiled transfer coefficient with the new equation and Deacon's model.

renewal theory. However, the friction velocity is not a convenient parameter to measure; it needs to be estimated from vertical wind profiles. Therefore, we assumed that the friction velocity could be expressed as a simple power function of U_{10} . Based on these considerations, the following mathematical formulation was proposed to fit the compiled data:

$$K_L = aSc^{-1/2}U_{10}^b\left(\frac{\rho_a}{\rho_w}\right)^{1/2} \quad (26)$$

The two arbitrary constants of the non-linear equation (eq. 26) were estimated using the MS Excel Solver function. The resulting equation is:

$$K_L \text{ (cm/h)} = 170.6 \cdot Sc^{-1/2}U_{10}^{1.81}\left(\frac{\rho_a}{\rho_w}\right)^{1/2} \quad \text{for } U_{10} > 0 \quad (27)$$

The transfer coefficient (K_L) is in cm/h and the 10 m wind speed (U_{10}) is in m/s. The compiled transfer coefficient data fit the new equation quite well, as evidenced by the coefficient of determination of 0.92.

The new equation will be very useful in the design and operation of treatment lagoons. It predicts the transfer coefficient of oxygen as well as other sparingly soluble gases (i.e., liquid-side controlled gases) under specific weather conditions. The mass flux rate of atmospheric oxygen absorbing into the treatment lagoon through its water surface can be estimated based on the lagoon size and the operating dissolved oxygen concentration of the lagoon. In addition, this surficial oxygen mass flux, expressed as a continuous function of environmentally and operationally relevant variables, will be useful for developing mathematical models of treatment lagoons (Ro et al., 2006).

LIMITED VERIFICATION OF EQUATION 27

Equation 27 was evaluated for agreement with recent wind tunnel aeration data by Chu and Jirka (2003). They used a 20 m long wind tunnel and utilized nitrogen bubbling instead of a chemical method to deoxygenate water so that any uncertainty concerning residual deoxygenating reaction could be avoided. Oxygen transfer coefficients due to pure wind-driven flow were measured and corrected to 20°C. The free-stream wind speed associated with their oxygen transfer coefficients were measured at 0.4 m above water surface. After normalizing their wind speeds to U_{10} with the

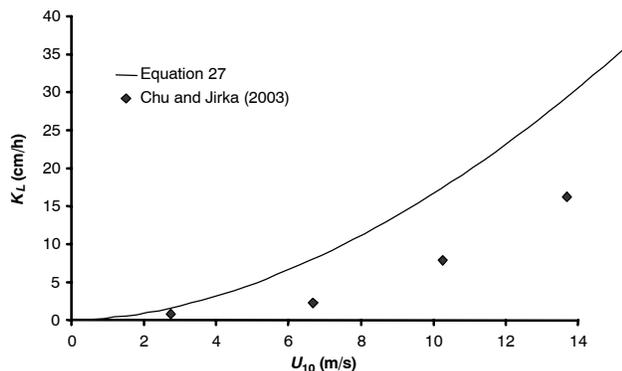


Figure 5. Comparison of predicted K_L with the measured K_L for oxygen transfer reported by Chu and Jirka (2003).

seventh-root law (eq. 24), we compared their measured oxygen transfer coefficient data with equation 27. As shown in figure 5, their data were lower than the values predicted by equation 27. We speculate that this difference arose from the fact that the 20 m wind tunnel provided limited fetch for waves to fully develop as compared to field conditions (Wanninkhof et al., 1985). Nevertheless, equation 27 reasonably agreed with their wind-driven oxygen transfer coefficients.

CONCLUSION

The existing formulations for the transfer coefficient did not compare well to each other. After normalizing wind speed at the reference height of 10 m and various gases to oxygen at 20°C, these formulas predicted wide ranges of the oxygen transfer coefficient. Fortunately, substantial gas transfer data have been published in the last five decades. We synthesized a new unified equation for oxygen mass transfer into treatment lagoons based on 297 mass transfer data points published in the last 50 years. The new empirical equation is a function of Schmidt number, wind speed, and water and air temperatures. The transfer coefficient increases with the 1.81th power of wind speed, similar to many previous observations of quadratic wind speed dependence. The accuracy of the new equation was partly validated with independent wind tunnel oxygen transfer data. This new equation can be used for reasonable estimation of atmospheric oxygen transfer into open water bodies, such as treatment lagoons and surface impoundments.

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