# High-Intensity Rainfall Rate Determination from Tipping-Bucket Rain Gauge Data

E. J. Sadler* and W. J. Busscher

## ABSTRACT

**High rainfall rates have a major impact on agriculture through runoff, erosion, and surface crusting and on other industries through sedimentation, failure of dams, and attenuation of radio waves. Traditional techniques used to measure and record rainfall have produced event totals or hourly rates. These are unsuitable for studying short-term impacts in brief storms (e.g., 27 mm in 30 min). The objective of this work was to develop an analytical procedure to provide millimeter per minute resolution in rainfall rate from short-term, tipping-bucket, rain gauge data. Counts of tips in 1-min intervals were assembled into an array of accumulated rainfall over time during an event. A cubic spline was fitted to the accumulated rainfall curve and then differentiated to yield the rate curve. Four synthetic rainfall patterns were used to test the technique by matching original and reconstructed curves. Regression of reconstructed rates on input rates resulted in $r^2$ ranging from 0.989 to 0.996. When applied to field data from 8 events (total: 209.5 mm) in July 1984 and 6 events (total: 139 mm) in July 1985, the technique described rainfall rate as a smooth, continuous function of time. This characteristic improves the suitability of the data for input to models of infiltration and runoff that adapt time steps to overcome numerical instability under rapidly changing conditions.**

RAINFALL rates during thunderstorms often exceed maximum infiltration rates of even well-drained soils. Surface water then exacerbates soil erosion, increasing sedimentation and loss of fertility. Runoff can cause flooding or, worse, hydrologic structure failure, resulting in significant personal and economic hardship. Intense rainfall also degrades radio communications (Ruthroff, 1970; Bodtmann and Ruthroff, 1974). Therefore, increased understanding of the rate of rainfall during storms could suggest potential solutions to these problems. Toward this end, extreme values of 1-min rainfall (Hershfield, 1972) and a method to determine 1-min values from U.S. Weather Service weighing gauge charts (Bodtmann and Ruthroff, 1976) have been reported.

Recent advances in solid-state circuitry have provided low-cost, robust data loggers, which have been incorporated into numerous weather stations. These loggers can automatically record either the number of tips of a tipping-bucket rain gauge per period or the time of each tip. Typically, recorded data have been daily or sometimes hourly totals. One known application included totals over 15-min periods (Sierra-Misco, Inc., 1986, personal communication). These devices were capable of providing more temporally resolved information (e.g., 1 mm min $^{-1}$), but a data analysis technique needed to be developed.

Williams and Erdman (1988) recorded the time of each tip, which provided the theoretical maximum information available from a tipping-bucket gauge. From this type of data, rainfall rate has usually been found by simply dividing the calibration of the gauge by the time between tips. More rigorous analyses have attempted to increase accuracy of tipping-bucket gauges, including corrections for the dynamic physical response of the bucket (C.L. Ruthroff, 1986, personal communication), for splashing at high rates by calibration at varied flow rates (Stange and Bender, 1984), and for splashing and jetting, also by calibration (Williams and Erdman, 1988). Other sources of error may not be so easily corrected; wind gusts, insect activity, and transient voltages caused by induction from nearby lightning strikes may also affect tipping buckets.

Regardless of accuracy, time-of-tip data may have serious limitations in some cases. Because the data logger must record the time of each tip, there must be sufficient on-line data storage to accomodate data from the maximum possible rainfall during the service interval. Uncertainty in data storage rate requires that remote stations be serviced more often than if the rate were known, that more storage be purchased, or that means be installed to interrogate the station automatically, all of which require additional expense. Additionally, the resulting variable-length data files may pose problems for computers lacking dynamic file allocation. These were not limitations for the dedicated system of Williams and Erdman (1988), but may prove so for weather stations currently in the field.

A compromise between temporal resolution and data storage requirements could be made. This compromise would allow data frequency to be increased about an order of magnitude, yet maintain storage requirements within limits. Here, the number of tips during a fixed duration, such as 1 min, would be reported. Output could be conditional upon rainfall, further reducing storage requirements. Although files would still be of variable length, there would be an upper limit for planning purposes. The objective for this work was to develop an analytical procedure to provide mm min$^{-1}$resolution in rainfall rate from this data.

## MATERIALS AND METHODS

The analytical procedure was written in FORTRAN-77 (American National Standard, X3.9-1978) on a minicomputer and has been transported to several industry-standard microcomputers. Requirements and performance of the procedure will be presented later in this paper. Researchers desiring to use the program can obtain the source code from the senior author.

Rainfall data were obtained with a tipping-bucket rain gauge (model 2500, Sierra-Misco, Inc., Berkley, CA)[1] (Calibration: 0.5 mm tip$^{-1}$) in a weather station. A microprocessor-based data collector (model CR7, Campbell Scientific, Inc., Logan, UT) recorded the number of tips that occurred

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[1]Mention of trademark, proprietary product, or vendor anywhere in this paper does not constitute a guarantee or warranty of the product by the USDA or the South Carolina Agric. Exp. Stn., and does not imply its approval to the exclusion of other products or vendors that may also be suitable.

each minute, if a tip occurred. These data were transferred to a minicomputer during regular daily processing.

Rain gauge data were summed into accumulated rainfall to give an ever-increasing function of time. This curve was expected to be more suited to curve fitting than the highly variable rate curve. Furthermore, periods with no tips should not appear as zero rates, but as physically realistic plateaus on the accumulated curve. This accumulated curve was then fitted with an interpolation technique, the choice of which depended upon the application, the degree of smoothing desired, and the availability of algorithms.

Several discussions of the relative worth of different interpolation methods are available in the literature; only a brief summary is included here. Linear, quadratic, cubic, and other interpolations pass through each point. Running averages, quadratic or cubic splines, sliding polynomials, and Fourier transforms provide some smoothing or low-pass filtering. The degree of the polynomial methods determines the smoothness of both the curve and its derivative. Linear interpolation produces a stairsstep rate curve. Quadratic techniques produce a sawtooth rate curve. Cubic and higher order techniques provide a smooth, continuous rate curve. A cubic spline (Kimball, 1976) smoothing method was chosen because it was necessary that the rainfall rate curve be smooth and continuous for use in models and because prior experience had shown splining to be satisfactory. This process is analogous to the use of sliding polynomials to derive the probability distribution function from the smoothed cumulative probability function (Thomas and Snyder, 1986), and of a cubic spline interpolation to derive uptake rates from nutrient concentration and biomass data for corn (Karlen et al., 1987; 1988).

During the summation of the raw data, individual rainfall events were isolated based on two user supplied parameters: (i) the minimum size event to analyze, set at 10 mm, and (ii) the maximum time between tips tolerated before initiating a new event, set at 10 min for the general case. The implications of these assumptions will be discussed later.

Within a rainfall event, a series of $X$-$Y$ pairs represented the cumulative rainfall amount and the time at the end of each interval. At any time of measurement, there will be some fraction of a tip volume in the bucket, which if neglected will constitute underestimation of the accumulated rainfall. To eliminate this bias, one could estimate either this fractional tip or the time at which the last tip occurred during the interval. The latter was done by assuming that tips occurred at a constant frequency within each 1-min interval and subtracting one-half of the average tipping time from the abscissa. The error associated with this assumption would likely be limited to 30 s in the worst case and would be expected to be insignificant during periods of moderate rainfall rates.

This leaves three unknown parameters of a rainfall event: (i) the start time, (ii) the finish time, and (iii) the final partial tip. The start time was unknown because the first data point represented the end of the minute during which the first tip occurred. This initial point was estimated by linear extrapolation from the first two data points back to the x-intercept, but with a slope one-half as large. This arbitrary choice of one-half the initially measured rate produced the best results in test cases. The final partial tip volume was assumed to be one-half tip, or 0.25 mm for the gauge used. One-half the final measured rate was extrapolated linearly to yield one-half tip, and that time was used for the endpoint. Zero rates at the beginning and end of the event were obtained by mathematically reflecting the points next to the end across the endpoints.

Finally, the cubic spline smoothing method (Kimball, 1976) was used to analytically describe cumulative rainfall as a function of time. In Kimball's (1976) procedure, the abscissa is divided into subranges by placing knots on the abscissa, and a least-square fit to a cubic polynomial is made to the data in each subrange. Individual polynomials were constrained to have equal first and second derivatives at these knots, which ensured smoothness of the final, aggregate curve. Kimball (1976) noted that knot placement was subjective, and experience was necessary to judge the number and placement.

Subjective placement was not efficient for a batch operation, which required an algorithm to place knots. Suitable accuracy could be obtained using two criteria and two constraints. The two criteria inserted knots based on abrupt changes and general trends. The two constraints allowed the user to reasonably control the stiffness of the spline. The first criterion inserted a knot if the absolute value of the second derivative, as estimated from the data, exceeded 0.9 mm $min^{-2}$. The second criterion placed knots at all points of inflection—if the second derivative, as calculated at two consecutive points, changed signs. A seven-point, central finite difference approximation was used to calculate the second derivative. Therefore, a point of inflection could not be just a one-point deviation.

The first constraint was that knots could not be placed at consecutive points; one was deleted and the other was moved to the midpoint. The second constraint was that knots could not be too widely spaced. A user specified maximum rainfall amount between knots, 5.0 mm, was converted to time by dividing by the event average rate. Knots were inserted if spacing exceeded that maximum time. These parameters resulted in a stiffness suitable for matching the test curves used. In some instances, short periods of rainfall rates much less than the event average resulted in too many knots being placed, with unrealistic fluctuations in the spline between the sparse data points. If the analysis of such a single event dictated, all knots could be selected from a user specified file. This allowed objective, automatic placement for the general case, yet retained the flexibility of subjective, interactive placement if desired.

Once knot placement was complete, data and knot arrays were entered into the cubic spline routine. The output of this routine was a set of coefficients for the cubic polynomial fit on each subrange:

$$Y = a_0 + a_1 X + a_2 X^2 + a_3 X^3$$

An example for a single rainfall event is shown in Table 1. These were used to generate tipping-bucket output and were reconstructed with the analytical method described above. All curves were integrated to give the accumulated rainfall as a function of time, evaluated at integer minutes, and truncated to an integer number of tips. Then, the amounts from the previous minutes were subtracted to provide the syn-

**Table 1. Coefficients of the series of cubic polynomials resulting from analysis of data of 26 July 1984, 1554 to 1826 h local standard time. Original number of points: 22. Number of internal knots: 11.**

| Subrange† | Low | High | $a_0$ | $a_1$ | $a_2$ | $a_3$ |
|---|---|---|---|---|---|---|
| 1 | 0.00 | 1.63 | 1.499 | −2.481 | −0.9601 | 0.0748 |
| 2 | 1.63 | 3.76 | 2.940 | −5.135 | 2.5895 | −0.2586 |
| 3 | 3.76 | 5.27 | −11.800 | 6.614 | −0.5321 | 0.0178 |
| 4 | 5.27 | 9.17 | −10.135 | 5.666 | −0.3523 | 0.0064 |
| 5 | 9.17 | 10.96 | −44.842 | 17.018 | −1.5900 | 0.0514 |
| 6 | 10.96 | 13.59 | 49.942 | −8.915 | 0.7751 | −0.0205 |
| 7 | 13.59 | 17.09 | 0.667 | 1.963 | −0.0253 | −0.0008 |
| 8 | 17.09 | 20.59 | −48.949 | 10.673 | −0.5350 | 0.0091 |
| 9 | 20.59 | 25.90 | 55.554 | −4.554 | 0.2045 | −0.0029 |
| 10 | 25.90 | 31.21 | −6.663 | 2.652 | −0.0737 | 0.0007 |
| 11 | 31.21 | 36.53 | 28.389 | −0.716 | 0.0342 | −0.0004 |
| 12 | 36.53 | 41.84 | 30.821 | −0.916 | 0.0397 | −0.0005 |

† The equation for each subrange was $Y = a_0 + a_1 X + a_2 X^2 + a_3 X^3$. The estimated start of the event was 17:52 h 45 s, and the spline axis origin was 17:51 h 40 s.

thetic tipping-bucket data. The first synthetic rainfall curve was a serpentine curve that represented an abrupt storm onset, a maximum of about 4 mm min$^{-1}$, and an exponential decay. Second, a ramp curve represented a rate that increased linearly to a maximum of twice the average, then decreased linearly. Third, a sine curve, representing a more variable rainfall rate, was built from a primary wave from 0 to 2 mm min$^{-1}$ added to a secondary wave ranging from 0 to 1 mm min$^{-1}$ three times during the event. These three curves resulted in 40 to 45 mm of rainfall in 30 min. Finally, another sine curve with 60 mm in 30 min was used to judge the effect of rainfall rate for the sine curve.

## RESULTS AND DISCUSSION

In this paper, results will be presented in the following order: (i) comparison of results against known inputs, including some observations of difficulties encountered; (ii) examples of analysis of real data, illustrating operational aspects; (iii) user specified parameters and manual knot placement; and (iv) performance characteristics of the procedure on several microcomputers.

An objective statistic to describe the goodness of fit between known and reconstructed rates is given in Table 2, which shows results of regression analysis of the procedure results on the inputs for the test cases. The test procedure is illustrated in Fig. 1, which is the third curve (the 45-mm sine case) in Table 2. For the test cases, the actual values of the rates and amounts were known, but this will not be the case for field data. The agreement for the test cases was verified by comparing the reconstructed rate to the known input, and was documented by the $r^2$ value of 0.990, intercept of −0.081 mm, and slope of 1.05 mm min$^{-1}$. Agreement for this case, which was the poorest overall for the test cases, is better than the tipping-bucket trace from which it was derived. This agreement occurs in spite of three difficulties illustrated in Fig 1. First, the output of a tipping-bucket gauge may be more variable than the actual rainfall rate. Therefore, the analysis must smooth through this variation to approximate the actual rate. Second, there is little basis for estimation of the actual start or finish times of the event. In the example, the initiation was estimated at 1.25 min after the actual start, but this error was minimized because the total rainfall at the first positive datum was matched. Finally, this synthetic event totaled exactly 45 mm, thus no partial tip remained in the bucket, so the analysis overstated the event total by one-half tip and the short time estimated for it to occur. These difficulties normally occur at times of low rates, minimizing errors relative to prevailing rates.

**Table 2. Results of regression analysis of reproduced rainfall rate against known input rates for four theoretical rainfall events.**

| Curve type | Duration | Total | $r^2$ | Intercept | Slope |
|---|---|---|---|---|---|
| | min | mm | | mm | mm min$^{-1}$ |
| Serpentine | 30 | 39 | 0.996 | 0.026 | 0.990 |
| Ramp | 30 | 45 | 0.989 | 0.050 | 0.973 |
| Sine | 30 | 45 | 0.990 | −0.081 | 1.050 |
| Sine | 30 | 60 | 0.995 | 0.006 | 0.993 |

Actual rainfall data from July of 1984 and 1985, each with several significant rainfall events, were analyzed and presented here to illustrate the results of this analysis. Summary statistics are presented in Table 3. In both data sets 72% of the total rainfall met the criteria for analysis; the remaining totals were < 10 mm. Coefficients for the polynomials resulting from analysis of the 26 July 1984 storm were shown in Table 1, and the curves are shown in Fig. 2. This brief, intense storm was characteristic of many summer storms in the southeastern Coastal Plain; its shape suggested the serpentine theoretical storm pattern used in the first test case. Knots for this analysis were placed by the automatic knot placement algorithm according to the standard criteria listed above. The main difference between the tipping-bucket output and the rate curve resulted from the smoothing of the data around 6 min. A knot could be added in this region if the smoothing were judged to be too coarse. However, such a decision

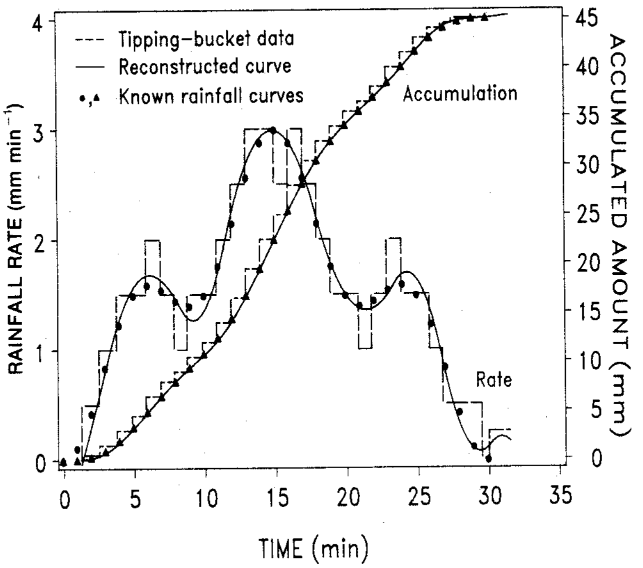


**Fig. 1. Reconstruction (solid lines) of known rainfall rate curve (symbols), from tipping-bucket data (stairstep). Circles are rates; triangles are accumulated amounts.**

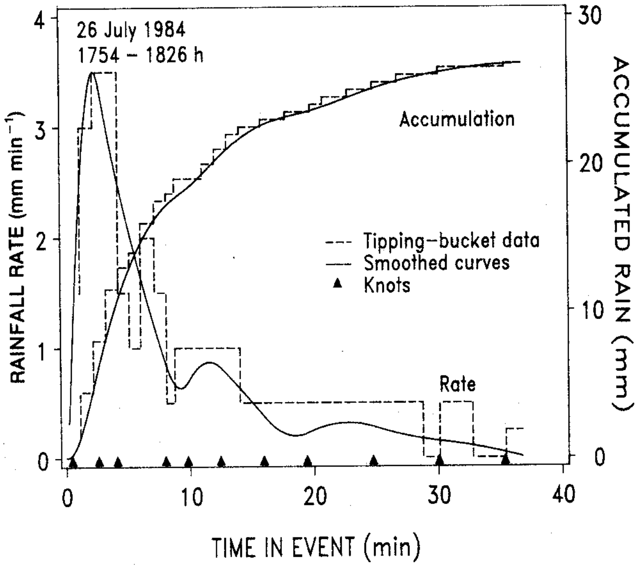


**Fig. 2. Analysis of rainfall of 26 July 1984. Smooth curves are results; discrete curves are tipping-bucket data. Triangles across the lower axis indicate positions of knots.**

would be subjective in the absence of knowledge of the true rainfall rate.

The 28 July 1985 data (Fig. 3) illustrated a longer, less intense rainfall event. It was above 1 mm $min^{-1}$ for about 20 min and below that for the remainding time. Automatic knot placement was judged too sparse in the 10 to 20 min region and too dense after 25 min. Subjective criteria for inserting knots included failure to match the rate curve to the tipping-bucket trace for a sustained period. Here, knots were inserted 10 to 20 min into the event, and some were removed later.

A single event from 21 July 1986, which partially relieved that summer's major drought, was an example of a highly variable, highly intense storm totaling 106 mm in < 1 h (Fig. 4). Because automatic knot placement allowed wide fluctuations during the calm period, knots were removed from 18 to 26 min to produce this curve.

Although the parameters used in these analyses resulted in acceptable operation for the four test cases and most of the real data analyzed, there were cases that were judged unacceptable. In general, these cases were the longer storms with calm intervals, such as in Fig. 3 and 4. The analytical procedure has a total of six parameters that control event separation and knot placement. The effects of these are discussed so that potential users can evaluate the procedure.

The minimum event size for analysis, 10 mm, was chosen for two reasons: (i) generally, little runoff or erosion occurs for smaller events; and (ii) such an event will have very few data points from which to infer information about high-frequency patterns in data using higher order interpolation. If these events were critical, one could use linear interpolation. The maximum time tolerated between tips, 10 min, resulted in event separation whenever the rainfall rate dropped below 0.05 mm $min^{-1}$ for this duration. Wider spacing between points allows oscillation between the points. This problem is worse when the sparse region is bounded by abrupt changes in the rate. Because this parameter was arbitrary, it was adjusted as desired. As an artifact, shorter values increase the number of events by making long, slow events such as drizzle appear as several smaller events.

The first knot placement criterion, the acceleration of the accumulated curve tolerated before a knot was inserted, was set at 0.9 mm $min^{-2}$. This caused insertion if the rate changed more than twice the resolution of the gauge. The spline was too flexible if this value was set below 1 tip $min^{-2}$, too stiff if set above 2 tips $min^{-1}$, and nearly stable between these values. The second knot placement criterion caused insertion at points of inflection, estimated where a 7-point central, finite-difference approximation changed signs. It is not

**Table 3. Event summaries for July 1984 and July 1985 rainfall analyses.†**

| Event number | Day of month | Day of year | Start time | Stop time | Event total | Number of points | Number of knots |
|---|---|---|---|---|---|---|---|
| | | | July 1984 | | | | |
| 4 | 13 | 195 | 1754 | 1912 | 47.5 | 50 | 17 |
| 7 | 13 | 195 | 2231 | 2255 | 10.0 | 18 | 7 |
| 16 | 15 | 197 | 1617 | 1629 | 14.0 | 13 | 5 |
| 33 | 21 | 203 | 1905 | 2058 | 49.0 | 73 | 26 |
| 36 | 23 | 205 | 1431 | 1500 | 12.0 | 18 | 6 |
| 39 | 25 | 207 | 1948 | 2020 | 37.0 | 28 | 12 |
| 40 | 25 | 207 | 2049 | 2125 | 13.5 | 21 | 5 |
| 46 | 26 | 208 | 1754 | 1826 | 26.5 | 22 | 13 |
| Total rainfall | | | | | 289.5 | | |
| Included in analyses | | | | | 209.5 | | |
| Excluded (< 10 mm total) | | | | | 80.0 | | |
| | | | July 1985 | | | | |
| 7 | 16 | 197 | 1341 | 1427 | 20.0 | 25 | 8 |
| 15 | 22 | 203 | 2210 | 2236 | 11.0 | 13 | 4 |
| 34 | 25 | 206 | 0318 | 0559 | 37.0 | 74 | 16 |
| 36 | 25 | 206 | 0625 | 0742 | 12.0 | 24 | 8 |
| 43 | 26 | 207 | 1950 | 2000 | 10.5 | 11 | 5 |
| 53 | 28 | 209 | 1356 | 1523 | 48.5 | 62 | 18 |
| Total rainfall | | | | | 193.5 | | |
| Included in analyses | | | | | 139.0 | | |
| Excluded (< 10 mm total) | | | | | 54.5 | | |

† Times and amounts are for original data and do not reflect estimations of initial or final times, nor of the final partial tip volume.

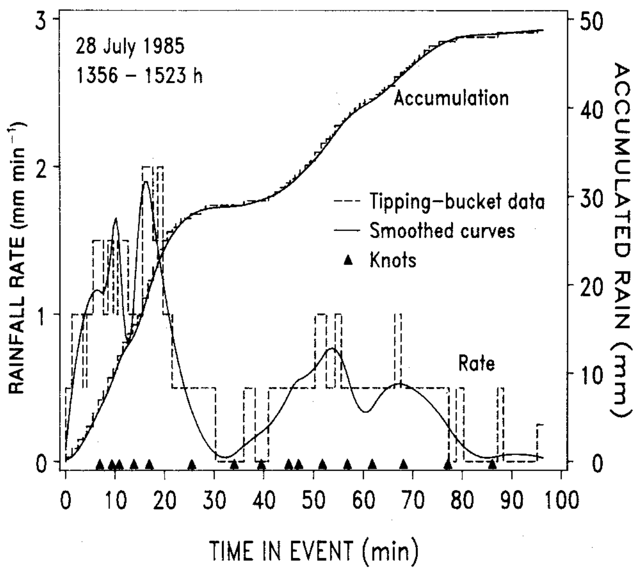


Fig. 3. Analysis of rainfall of 28 July 1985. Smooth curves are results; discrete curves are tipping-bucket data. Triangles across the lower axis indicate positions of knots.

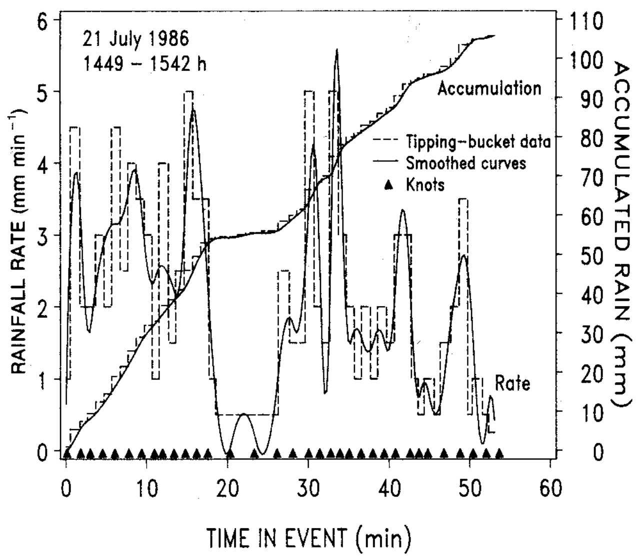


Fig. 4. Analysis of rainfall of 21 July 1986. Smooth curves are results; discrete curves are tipping-bucket data. Triangles across the lower axis indicate positions of knots.

advised that this be removed, although the spline could be made more flexible if the approximation were based on five points.

The first constraint on knot placement was an upper limit frequency of 1 $min^{-1}$, achieved by consolidation of adjacent knots to a single knot at the midpoint. Placing a knot at consecutive points would allow excessive fluctuations in the final curve, because it would essentially be forced through each point. Therefore, this constraint was not made user accessible. The second constraint placed a lower limit on frequency, or a maximum spacing between knots. This was done by converting a user specified maximum rainfall between knots, here 5.0 mm, to a mean value of the maximum time between knots by dividing by the average rate for the event. If, after the other steps were completed, a duration greater than this maximum existed, enough knots were added within the range to obey this constraint.

As mentioned above, the program as described performed acceptably for the four test cases and most of the real data. However, for cases such as shown in Fig. 3 and 4, manual adjustment appeared necessary to produce reasonable results. As those experienced with smoothing methods know, few general rules exist for fitting these data. However, an exception for rainfall analyses is that the rainfall rate must not drop below zero. This happens if large fluctuations occur at low rainfall rates, such as occurred near the middle of the event shown in Fig. 4. When this occurred, knots were removed and the remaining knots spread out. For the opposite problem, when the spline is too stiff, one should generally insert knots if the curve fails to match a sustained rate. This was the case for the 18 to 20 min region shown in Fig. 3, in which knots were added to increase the maximum rate near the peak. It is recommended that potential users experiment with both synthetic and actual data to develop experience with this technique.

The computing time required to analyze rainfall for the July 1984 data set ranged from a minimum of 38 s for the minicomputer to a maximum of over 1 h on a minimally configured microcomputer. This was reduced to a manageable requirement of about 5 min for a 1983 vintage, 8/16-bit microcomputer with the addition of a math coprocessor. A 1985 vintage 16-bit microcomputer required about 2.5 min with and 10 min without a math coprocessor. Finally, a 1987 vintage, 32-bit microcomputer required about 1 min, which compares favorably with that required by the minicomputer.

Examination of the time required for analysis of the eight individual events in this data set indicated that computing time increased parabolically with the number of subranges, but the second-order term coefficient was much $< 1$. Therefore, most desktop microcomputers would be capable of performing this analysis if they were equipped with a math coprocessor.

## SUMMARY AND CONCLUSIONS

A method to increase the temporal resolution of rainfall rates calculated from tipping-bucket rain gauge data was developed and tested using synthetic data. Reconstruction of rainfall rates from four theoretical storm curves resulted in explanation of 99% of the variation in input rates. Performance illustrated by actual data, for three storms presented in detail and data of July 1984 and 1985 presented in summary form, permits the reader to judge performance with field data. Hardware requirements were well within the capabilities of most desktop microcomputers. Both the cumulative rainfall and rate curves obtained were smooth and continuous as a result of the cubic spline interpolation. This characteristic makes the curves suitable for entry into models (e.g., infiltration and runoff) that may be sensitive to abrupt changes in slope of input curves.

Researchers desiring to analyze short-term rainfall data from the numerous weather stations in existence may request the source code for the program from the senior author.

## REFERENCES


Bodtmann, W.F., and C.L. Ruthroff. 1974. Rain attenuation on short radio paths: Theory, experiment, and design. Bell Teleph. Tech. J. 53:1329–1349.

Bodtmann, W.F., and C.L. Ruthroff. 1976. The measurement of 1 min rain rates from weighing raingage recordings. J. Applied Meteorol. 15:1160–1166.

Hershfield, D.M. 1972. Estimating the extreme-value 1-minute rainfall. J. Applied Meteorol. 11:936–940.

Karlen, D.L., R.L. Flannery, and E.J. Sadler. 1988. Aerial accumulation and partitioning of nutrients by corn. Agron. J. 80:232–242.

Karlen, D.L., E.J. Sadler, and C.R. Camp. 1987. Dry matter, nitrogen, phosphorus, and potassium accumulation by corn on Norfolk loamy sand. Agron. J. 79:649–656.

Kimball, B.A. 1976. Smoothing data with cubic splines. Agron. J. 68:126–129.

Ruthroff, C.L. 1970. Rain attenuation and radio path design. Bell Teleph. Tech. J. 49:121–135.

Stange, K., and A. Bender. 1984. A data acquisition system for measuring irrigation sprinkler intensities. Paper 84-2083, Am. Soc. Agric. Eng.

Thomas, A.W., and W.M. Snyder. 1986. Stochastic impacts on farming: I. Transformation of relevant variants. Trans. ASAE 29:128–134.

Williams, R.G., and M.D. Erdman. 1988. A low-cost computer interfaced rain gauge. Comput. Electron. Agric. 2:67–73.