EFFECT OF VARIABLE LINEAR ELASTIC PARAMETERS FINITE ELEMENT PREDICTION OF SOIL COMPACTION

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ABSTRACT

An axisymmetic linear elastic finite element program was developed to investigate the effect that the two linear elastic parameters, Poisson's ratio and Young's modulus, had on soil compaction. This program was verified against Boussinesq's linear elastic theory. It was found that increased values of Young's modulus had no effect on the stress state in the soil mass but that strain levels were decreased. Increased values of Poisson's ratio increased the stress state and decreased the strain levels. The interaction of these two parameters point to the need to be able to vary both over the entire stress range.

INTRODUCTION

Methods of predicting soil compaction could enable farmers to select times to till or traffic fields when the soil was not in a highly compactable state. Many prediction methods are available but one that offers significant promise for modeling of soil compaction is finite element analysis. Finite element methods can accurately model complex loading geometries (tires, tracks, etc.), and the analysis can be performed on microcomputers. The computing power required to run finite element analysis previously restricted its use to large main-frame computers. Now, farmers own microcomputers that are powerful enough to solve difficult soil compaction problems.

Ideally, the detailed analysis of soil compaction would begin with an adequate constitutive relationship between applied stresses and resulting deformations. However, a constitutive relationship that takes into account all intricacies of agricultural soil has not been developed. Because of computational and theoretical limitations, the linear elastic assumption has been used, with the finite element method, to analyze soil compaction (with mixed success). Some difficulty with finite element analysis is expected because agricultural soil rarely behaves in a linear elastic manner. Compressive stresses usually cause agricultural soil to compact and the original volume of the soil is unrecoverable. However, until significant gains are made in describing soil constitutive relationships, some form of the linear elastic assumption will continue to be used with finite element analysis.

To make use of the linear elastic assumption, two input parameters are specified, Poisson's ratio and Young's modulus. These parameters vary depending upon soil texture, soil moisture content. soil history, the loading used, etc. The relationship between these parameters and soil compaction is not completely understood. Most studies assume that one or both of these parameters are constant (Girijavallabhan and Reese, 1968; Perumpral, 1969; Duncan and Chang, 1970; Naylor and Pande, 1981; Pollock et al., 1985). Young's modulus is usually determined by compressive laboratory tests on similar soils. Poisson's ratio is usually assumed to be a value between 0.3 and 0.45 based upon the clay content of the soil. Assumptions that must be made in determining these parameters warrant further investigation into the effect of these parameters upon finite element predictions of soil compaction. To do this, a finite element program must be developed that will allow the linear elastic parameters to be varied independently. The objectives of this research were to:

* Develop and verify a finite element program capable of predicting soil compaction and stress distribution,
* Compare predictions from a linear-elastic finite element model with results from a laboratory experiment, and
* Determine the effects of Poisson's ratio and Young's modulus on the finite element model's predictions.

DEVELOPMENT AND VERIFICATION OF FINITE ELEMENT PROGRAM

In finite element analysis of most soil mechanics problems, plane-stress, plane-strain, or axisymmetric descriptions have been used along with plane-strain triangular elements (Desai and Christian, 1977). An axisymrnetric model was chosen because this idealization is easily physically modeled in the soil bins at the National Soil Dynamics Laboratory (NSDL) by using a round plate to apply loads to the soil surface. Use of an axisymmetric model gives the ability to solve a three-dimensional problem with the added advantages of two-dimensional storage and computation time. This is important because a soil compaction prediction model that can be used with a microcomputer is desired.

A plane-strain, plane-stress finite element program (Desai and Abel, 1972) was modified for the axisymmetric application. Because axisymmehic geometry includes three principal directions (radial, tangential, and vertical), rather

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than the two associated with plane-strain or plane-stress analysis. basic changes had to be made in several of the finite element matrices (Raper, 1987; Bathe, 1982: Desai and Abel, 1972). The stress-strain or constitutive matrix had to be increased in size from a 3x3 to a 4x4. The resulting relationship is give” below:

$$\{\sigma\} = [C]\{\varepsilon\} \tag{1}$$

where
$\{\sigma\} = \{\sigma_{xx}\ \sigma_{yy}\ \sigma_{xy}\ \sigma_{zz}\}^T$,
$\{\varepsilon\} = \{\varepsilon_{xx}\ \varepsilon_{yy}\ \varepsilon_{xy}\ \varepsilon_{zz}\}^T$, and
$[C]$ = the stress-strain matrix.

For axisymmetric geometry, the [C] matrix is of the form (Bathe, 1982)

$$[C] = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1 & \frac{\nu}{1-\nu} & 0 & \frac{\nu}{1-\nu} \\ \frac{\nu}{1-\nu} & 1 & 0 & \frac{\nu}{1-\nu} \\ 0 & 0 & \frac{1-2\nu}{2(1-\nu)} & 0 \\ \frac{\nu}{1-\nu} & \frac{\nu}{1-\nu} & 0 & 1 \end{bmatrix} \tag{2}$$

where
E = Young's modulus, and
ν = Poisson's ratio.

The stress-strain matrix is multiplied by the strain matrix (Bathe, 1982), [B], the transposed strain matrix ,$[B]^T$, and then integrated over the volume of the element, V, to determine the stiffness matrix for the element, [K].

$$[K] = \int_V [B]^T[C][B]\,dV \tag{3}$$

The resulting system of equations to be solved is of the form

$$[K]\{\delta\} = [R] \tag{4}$$

where
$\{\delta\}$ = the nodal deformation vector, and
$\{R\}$ = the vector that contains the equivalent nodal forces for the element.

In these equations, the stiffness matrix, [K], and the force vector, {R}, are known from the problem definition. The only unknown is the nodal deformation vector, $\{\delta\}$. This vector is then solved for and used to calculate the element stresses and strains.

From examining the equations that comprise finite element analysis, it is hypothesized that one can determine the effects of modifying Young's modulus. Since Young's modulus is multiplied by the matrix in equation 2, and this matrix is multiplied by the strain matrix in equation 3, Young's modulus is directly linked to the nodal deformations which are solved for from equation 4. Doubling the value of Young's modulus decreases the nodal deformations by a factor of two. This in turn decreases the element strains by the same factor because they are calculated from the nodal deformations. The element strains (which have been reduced by a factor of two) are then multiplied by Young's modulus (which has bee” doubled) to calculate the element stresses (as in equation 1). Therefore, Young's modulus does not affect the stress levels. It only has the effect of decreasing the deformations and strains.

Unfortunately, it is not as easy to determine the effects of modifying Poisson's ratio. Because of the manner that Poisson's ratio is incorporated into the stress-strain relationship (see eq. 2), it is not intuitively obvious how its adjustment will affect the element stresses and strains. Simply doubling the value of Poisson's ratio will influence some directional deformations differently than others. All nodal deformations will be changed, but by different factors. The only method to determine the effect of Poisson's ratio on finite element analysis is to examine the predicted results.

During verification of this program, the solutions failed to converge acceptably “ear the central axis. This problem was caused by the triangular plane strain elements that were used in the model (Huebner and Thornton, 1982). A” inaccurate averaging method was used to determine this element's volume and surface area. The isoparametric linear quadrilateral element was then chosen because it used numerical integration to determine its volume and surface area. This element is easily numerically integrated by using Gauss-Legendre methods. Two subroutines were taken from Bathe (1982) that used the Gauss-Legendre methods of numerical integration and allowed the user to determine the order of integration desired. For the elements used in this program, Bathe advised using 2-point integration, eve” though the subroutines allowed 2-, 3-, or 4-point integration. The isoparametric linear quadrilateral element was also chosen because future research could include three-dimensional models and including this element would make the program mom general and easier to expand.

A grid, using these elements, was designed to allow a” axisymmetric pressure load to be applied near the center (fig. 1). This mesh was designed so that the total radial distance was 6 times greater than the radius of the pressure load. The factor of 6 was thought to be sufficient to introduce a semi-infinite soil medium that allowed large stresses and strains near the central axis without being affected by fixed boundary conditions. The mesh was composed of 169 elements and 196 nodes and extended radially from the centerline 1.44 m and vertically into the soil 1.44 m.

The finite element program with isoparametric quadrilateral elements was first tested in a gravity loading situation. This test was conducted to verify that all elements were properly linked and no discontinuities developed over the zone of interest. (The plane-strain triangular elements failed this test.) As can be seen in figure 2, the behavior of the isoparametric quadrilateral elements was linear. The vertical stresses were constant across the entire mesh. This behavior proved that all elements were responding as anticipated to a uniform gravitational load.

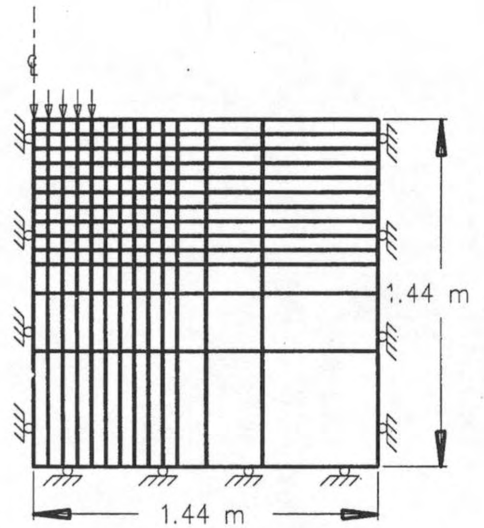

Figure 1–Axisymmetric finite element mesh that is used to model a flexible circular footing.

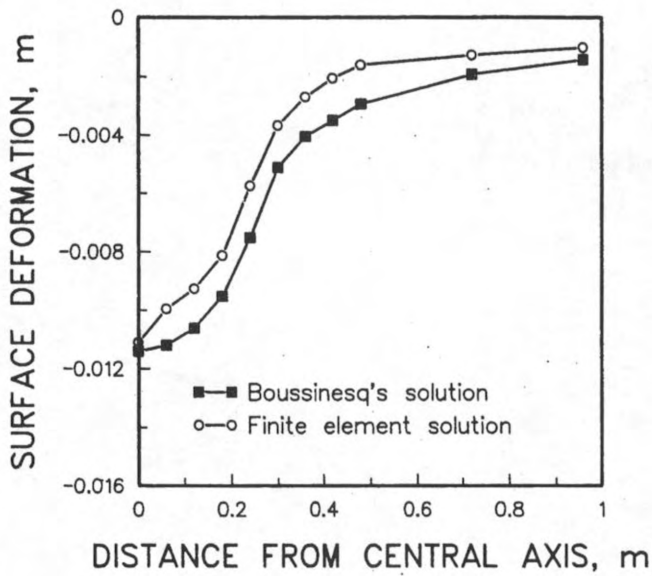

Figure 3–Comparison of finite element and Boussinesq surface settlement predictions for flexible circular footing loading.

The program was further verified by using a flexible circular footing problem. This footing problem is a common verification technique used in civil engineering applications (Girijavallabhan and Reese, 1968). The parameters necessary were the finite element mesh (fig. l), loading type and amount, and soil properties.

The loading was distributed evenly over the centermost four elements at the top of the mesh. The amount (125 kPa) and radius (0.24 m) of the pressure load corresponded to a similar problem solved by Pollock et al. (1985). The linear elastic parameters (E = 4788 kPa and v = 0.3) corresponded to the values used by Girijavallabhan and Reese (1968). They evaluated their program accuracy by plotting the surface settlement against the theoretical linear elastic equations developed by Boussinesq (Poulos and Davis, 1974). This same technique was used to evaluate performance of our program .

The flexible, circular loading problem was used to compare Boussinesq theory with our finite element program. Results from the two methods were in good agreement. The surface deformation predicted by the finite element method was close to, but slightly less than that predicted by Boussinesq theory (fig. 3). When the surface deformations were plotted against each other and fitted with a linear equation, there was statistical justification to accept the null hypothesis that the slope of the line was unity. A slope of other than unity would indicate that the finite element method would not predict the Boussinesq-predicted surface deformation. The vertical stress contours predicted by the finite element method (fig. 4) were slightly greater (12.4%) than those predicted by Boussinesq's theory but seemed reasonable and appeared to be of similar shape. Differences predicted by the two methods probably were because an infinite soil medium was assumed by Boussinesq's theory and a boundary existed on the finite element model. Our finite element model was therefore assumed to be working correctly.

METHODS AND MATERIALS

Estimations of linear-elastic parameters came from data obtained at the NSDL during development of the compaction model (Bailey et al., 1984). Analysis of data obtained from the.Norfolk sandy loam soil indicated that the stress-strain relationship behaved as shown in figure 5.

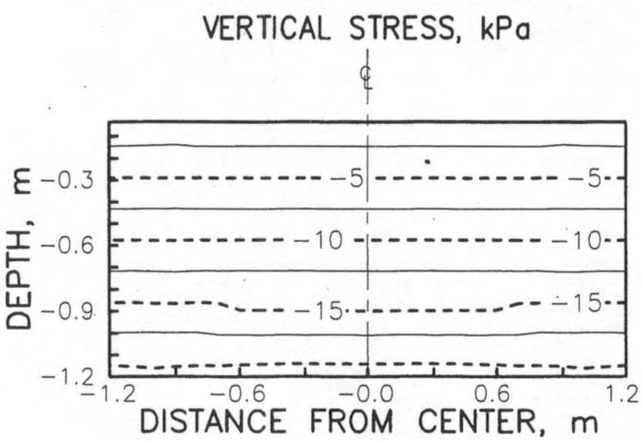

Figure 2–Contours of vertical stress obtained from finite element solution by using isoparametric linear quadrilateral elements for gravity loading.

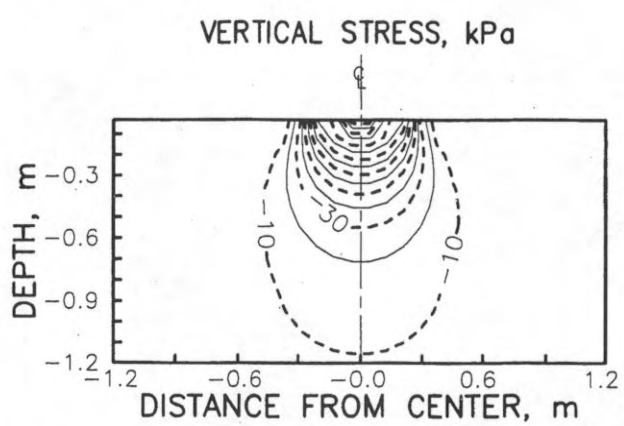

Figure 4–Contours of vertical stress obtained from finite element solution of flexible circular footing problem by using quadrilateral elements.

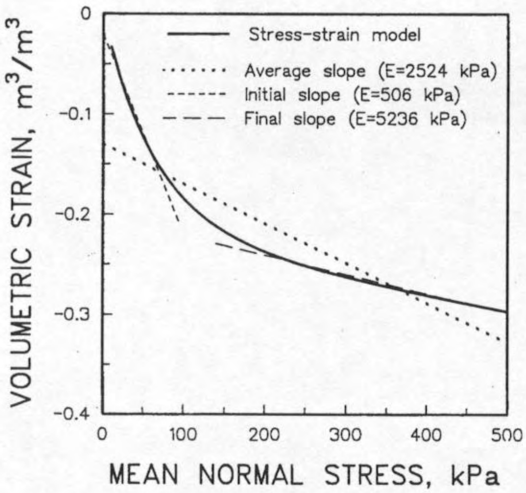

Figure 5–Stress-strain relationship for Norfolk sandy loam soil (from Bailey et al., 1984).

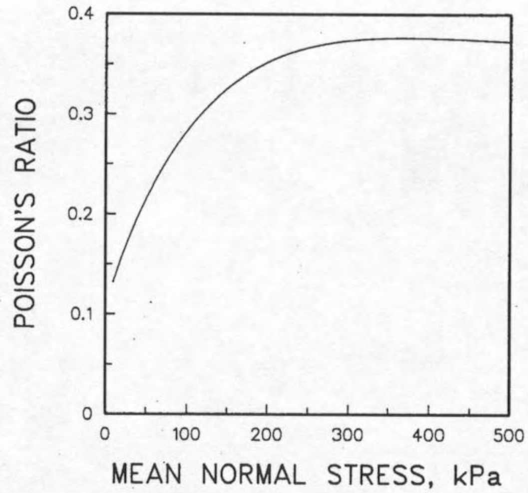

Figure 6–Effect of hydrostatic stress on Poisson's ratio for Norfolk sandy loam soil (from Bailey et al., 1984).

Three values of Young's modulus were determined from different regions of the curve. A minimum value of 506 kPa and a maximum value of 5236 kPa were calculated over the initial and final ranges of data, respectively. A median value of 2524 kPa was calculated by fitting a straight line over the entire range of data

Reasonable values of Poisson's ratio were more difficult to obtain. Data from research to develop the compaction model were collected under hydrostatic stress (equal stress in all three principal directions). This type of test would give equal axial strains and radial strains. However, a method was found (Duncan and Chang, 1970) that allowed values of Poisson's ratio to be calculated for small deviations from the hydrostatic stress state:

$$\nu = \frac{\Delta\varepsilon_1 - \Delta\varepsilon_v}{2\Delta\varepsilon_1} \tag{5}$$

where
ν = Poisson's ratio,
$\Delta\varepsilon_v$ = incremental volumetric strain, and
$\Delta\varepsilon_1$ = incremental axial strain.

This relationship was derived from the classic Poisson's ratio definition, which is the ratio of radial strain to axial strain. This equation, which uses axial strain and volumetric strain, allowed the data obtained by Bailey et al. (1984) from the Norfolk sandy loam soil to be used to calculate values of Poisson's ratio (fig. 6). This figure shows how Poisson's value varied from 0.13 to 0.38. Values that have been used for Poisson's ratio by other researchers were in the range of 0.30 to 0.45; however, our results suggested that smaller values should be considered. Therefore, three values were selected for Poisson's ratio; 0.13,0.25 and 0.38.

The finite element program was run with nine combinations of three levels of Young's modulus and three levels of Poisson's ratio. Variables predicted were mean normal stress, vertical stress, volumetric strain, and surface deformation.

A laboratory experiment was carried out in the soil bins at the NSDL to verify predicted results. A round steel plate with a 25 -kN load was used in the Norfok sandy loam soil bin. This load was thought to be the approximate maximum hat a tractor applied to the soil surface. Stress state transducers (Nichols et al., 1987) were used to measure stress levels in the soil beneath the steel plate at two initial depths: 15 cm and 25 cm. Although these transducers measured stresses in six directions, only the vertical stress measurements were used to compare against the finite element model.

RESULTS AND DISCUSSION

Values of stress obtained with the finite element method were significantly below those obtained in the soil bins. The shallow transducer (placed 15 cm below the surface) measured vertical stresses in the soil bins of 266.6 $\pm$ 172.9 kPa (for a 95% confidence interval) as compared to 59 - 61 kPa predicted by the finite element method for the different configurations of the two linear elastic parameters. The deep transducer (placed 25 cm below the surface) measured vertical stresses of 231.4 $\pm$ 17.1 kPa as compared to 58 - 63 kPa predicted by the finite element method. The surface of the soil in the soil bins was displaced 19.6 cm by the flat plate when the 25 kN load was exerted. The finite element predictions were much less than this, ranging from 0.4 to 5.8 cm.

Errors in predictions of vertical stress and surface deformation stem from inaccurate modeling due to the linear elastic assumption. Even when low numerical values of Young's modulus and Poisson's ratio were used, the high stresses and deformations found in the soil bin experiment could not be predicted. Even though values of stress, strain, and deformation were not predicted accurately by using the linear elastic assumption, certain important trends developed that could aid the understanding of the effect of each of these parameters on results predicted by finite element analysis.

When Poisson's ratio was increased from 0.13 up to 0.38 with a constant Young's modulus, the vertical stress level in the area beneath the flat plate increased substantially (figs. 7 and 8). and the strain levels decreased. When Young's modulus was increased from 506 kPa up to 5236 kPa with a constant Poisson's ratio, the stress levels

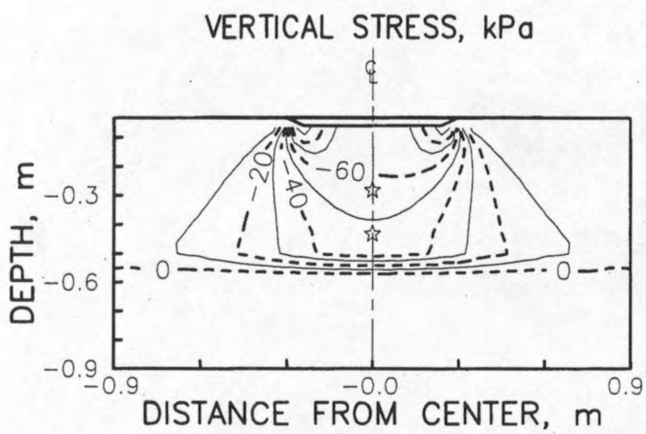

Figure 7–Vertical stress contours obtained with finite element method (Poisson's ratio = 0.13, Young's modulus = 506 kPa). Stars designate locations of detailed stress and strain analysis.

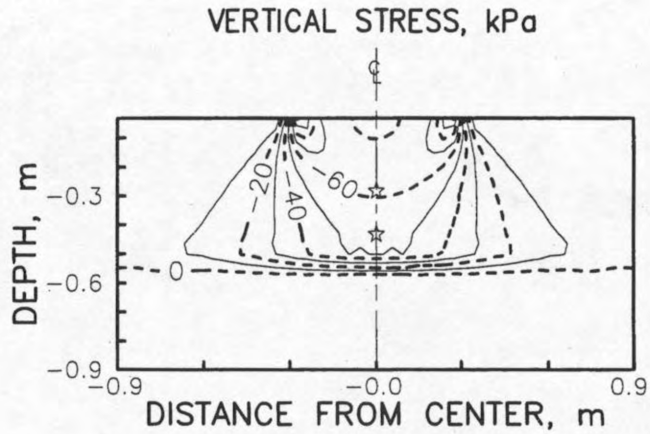

Figure 9–Vertical stress contours obtained with finite element method (Poisson's ratio = 0.38, Young's modulus = 5236 kPa). Stars designate locations of detailed stress and strain analysis.

did not change (figs. 8 and 9). but the strain levels decreased.

To help clarify these trends, vertical stress and volumetric strain were evaluated at depths of 28 cm and 43 cm beneath the center of the plate. The results arc shown in Tables 1 and 2.

At the 28 cm depth, when Poisson's ratio was increased from 0.13 to 0.38, the vertical stress increased 8.2%, 9.3%. and 9.4%, and the volumetric strain decreased 50.9%. 49.6%. and 49.6%, respectively, for Young's moduli of 506, 2524, and 5236 kPa At the 43 cm depth, the trend was even more significant. Vertical stress increased by 11.0%. 11.1%. and 11.2% and the volumetric strain decreased by 44.6%, 44.3%, and 44.3%. These differences were obtained by varying a linear elastic parameter Poisson's ratio) that oftentimes is taken to be constant.

Two other sets of predicted results were compared. It was found that when Poisson's ratio was held constant and Young's modulus was increased, vertical stress was not affected but volumetric strain was. At the 28 cm depth, as Young's modulus was increased from 506 kPa to 5236 kPa, the vertical stress changed -2.9%, 2.6%, and -1.6%. and volumetric strain decreased 90.8%. 90.7%. and 90.5%. respectively, for Poisson's ratios of 0.13, 0.25, and 0.38. At the 43-cm depth, the results were similar. Vertical stress also changed by -1.0%, 1.0%. and -0.8% and volumetric strain decreased by 90.4%. 90.3%. and 90.3%.

These trends were significant and surprising. The vertical stress in the soil is independent of changes in Young's modulus but increased as Poisson's ratio increased. Strain levels, however, depended on both Poisson's ratio and Young's modulus and decreased when either of these parameters increased. Volumetric strain was almost twice as sensitive to changes in Young's modulus as to changes in Poisson's ratio.

This research points out problems with nonlinear analysis in which Poisson's ratio is assumed to be constant and Young's modulus is varied. Arbitrarv choices of a constant Poisson's ratio determine the stress state in the soil, and even though Young's modulus is varied. the stress. state remains unchanged. These linear elastic parameters interact and determine the strain levels in the soil. To accurately depict soil compaction, both Young's modulus and Poisson's ratio should be variable over the entire soil volume being modeled.

CONCLUSIONS

1. An axisymmetric finite element program for microcomputers was developed that proved capable of analyzing soil compaction when a compressive load was applied. This finite element program was verified by comparisons with Boussinesq linear elastic theory.

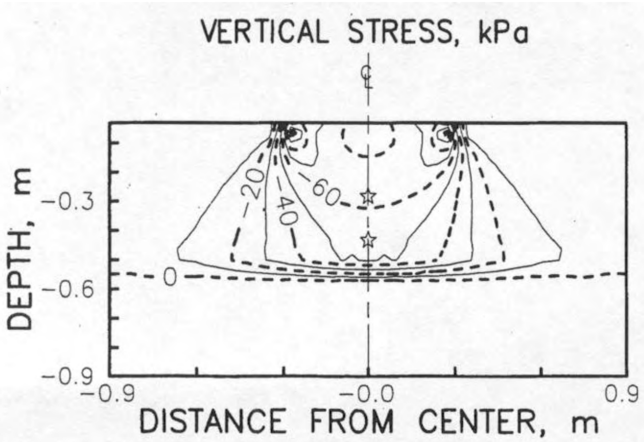

Figure 8–Vertical stress contours obtained with finite element method (Poisson's ratio = 0.38, Young's modulus = 506 kPa). Stars designate locations of detailed stress and strain analysis.

TABLE 1. Vertical stress values predicted for two depths beneath the center of a flat circular plate

Young's modulus	Poisson's ratio	Depth 28 cm	Depth 43 cm
–kPa–		–kPa–	–kPa–
506	0.13	57.1	47.4
	0.25	59.1	49.3
	0.38	62.2	53.2
2524	0.13	55.6	47.0
	0.25	57.7	48.8
	0.38	61.3	52.9
5236	0.13	55.5	46.9
	0.25	57.6	49.8
	0.38	61.3	52.8

TABLE 2. Volumetric strain values predicted for two depths beneath the center of a flat circular plate

Young's modulus	Poisson's ratio	Depth 28 cm	Depth 43 cm
kPa		—%—	—%—
506	0.13	7.86	6.46
	0.25	6.43	5.48
	0.38	3.86	3.58
2524	0.13	1.51	1.29
	0.25	1.24	1.10
	0.38	0.76	0.72
5236	0.13	0.72	0.62
	0.25	0.60	0.53
	0.38	0.36	0.35

2. Finite element prediction of soil compaction or stress state made by assuming constant values for the linear elastic parameters, Young's modulus, and Poisson's ratio correlated poorly with laboratory experiments.
3. Young's modulus and Poisson's ratio are both important in modeling soil compaction. Soil stress is dependent on Poisson's ratio, and soil strain is dependent on both Young's modulus and Poisson's ratio. Both variables should be allowed to change over the entire soil volume being modeled to obtain the best solution.

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