PREDICTION OF SOIL STRESSES USING THE FINITE ELEMENT METHOD

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ABSTRACT

A finite element program has been developed that used a compaction model for agricultural soils developed at the National Soil Dynamics Laboratory (NSDL) end Auburn University to predict linear elastic parameters for each element in the model. Incremental leading was used by the finite element model to gradually load the soil so that these linear parameters could be varied many times ever the loading period. The finite element model was compared with data obtained from soil bin research. Results showed that a flat disc load was modeled well but a spherical disc load was not.

INTRODUCTION

The ability to predict soil compaction could enable farmers to till or traffic the soil when it is net in a highly compactable state or to estimate the damage being done to the soil structure by their excessive loading when tillage or traffic is necessary. One numerical technique that could be used to predict soil compaction is the finite element method (FEM). For almost 20 years this method has been touted as a way to solve soil mechanics problems (Girijavallabhan and Reese, 1968; Duncan and Chang, 1970; Perumpral, 1969; Pollock et al., 1985; Raper et al., 1987). But problems remain for agricultural engineers seeking to solve the soil compaction problem.

These problems stem from the complex nature of agricultural soil. Agricultural soil experiences much greater strain than soils that have typically been modeled by civil engineers using FEM. The nonlinear nature of agricultural soil is also a complicating. factor because it does not obey linear elastic theory and it exhibits plasticity.

Several attempts at using the FEM have been made to model agricultural soils with mixed success. Additional work is required to refine FEM before it can be used to accurately predict soil compaction. With recent advances at the NSDL and Auburn University in development of a constitutive relationship for agricultural soils (Bailey et al., 1984), portions of the necessary technology have now been developed to make the FEM a more successful technique for modeling of soil compaction.

Article has been reviewed and approved for publication by the Power and Machinery Div. of ASAE.

Contribution of USDA-Agricultural Research Service and Iowa State University. Journal Paper No. J-13088 of the Iowa Agric. and Home Econ. Exp. Stn., Ames; Project 2737.

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Therefore, the objectives of this research were to:

* Modify a linear elastic finite element program se that nonlinear behavior of agricultural soils under compressive loads can be modeled
* Verify the nonlinear FEM by comparing its results with a laboratory experiment on agricultural soil.

METHODS AND MATERIALS

MODEL DEVELOPMENT

A finite element computer program that uses isoparametric linear quadrilateral elements is usually bound by the assumption of linear elastic material properties. This assumption is unreasonable for agricultural soils which deform greatly under applied loads. One of the largest errors that has occurred from using the FEM to model soil compaction stems from the inaccurate definition of the relationship between stress end strain. A constitutive relationship must be used that accounts for nonlinear behavior of soil.

A new, basic, relationship has been proposed by the NSDL end Auburn University (Bailey et al., 1984) that meets two important physical constraints that others have failed to meet. The first physical constraint is that when a soil approaches its maximum bulk density, additional compressive stress causes a proportional change in volumetric strain. The second physical constraint is that when the initial state of applied stress is zero; the volumetric strain should also be zero. The model proposed is

$$\bar{\varepsilon}_v = (A + B\sigma_h)\left(1 - e^{-C\sigma_h}\right) \quad (1)$$

where

σ_h = hydrostatic stress,
A, B, C = compactability coefficients, and
$\bar{\varepsilon}_v$ = natural volumetric strain and is defined by

$$\bar{\varepsilon}_v = \ln\left(\frac{V}{V_o}\right) \quad (2)$$

where

V_o = initial volume,
V = current volume.

The compactability coefficients were determined by using a nonlinear curve-fitting technique on data obtained from hydrostatic loading of soil in a triaxial apparatus. This apparatus loaded the soil such that the applied stress in all three directions was the same value. The soil modeled in this experiment was a Norfolk sandy loam. The compaction model coefficients (A, B, and C from eq. 1 for this soil are, respectively, -0.242, -221×10^{-6} kPa^{-1}, and 0.0147 kPa^{-1} with a fitted initial bulk density of 1.2339 Mg/m^3.

Natural strain was used in this compaction model because of its advantages over engineering strain for agricultural soils. One advantage of natural strain is that large values of strain are more readily handled. Incremental natural strain is obtained by dividing the incremental change in length by the instantaneous length being considered. Each component of natural strain is related to the respective components of engineering strain by the equation (Gill and Vanden Berg, 1968):

$$\bar{\varepsilon} = \ln(1+\varepsilon) \qquad (3)$$

where ε is equal to the engineering strain.

To use equation 1 as a constitutive relationship in the finite element program, it must be written in terms of engineering strain. This relationship is necessary because of the linear elastic relationship between stress and strain that uses Young's modulus (E).

$$\sigma = E\varepsilon \qquad (4)$$

Taking the inverse of the natural log of each side of equation 3 results in

$$e^{\bar{\varepsilon}} = (1+\varepsilon) \qquad (5)$$

Rearranging gives

$$\varepsilon = e^{\bar{\varepsilon}} - 1 \qquad (6)$$

Substituting equation 6 into equation 1 gives another version of the hydrostatic compaction model.

$$\varepsilon_v = \exp\left[(A + B\sigma_h)\left(1 - e^{-C\sigma_h}\right)\right] - 1 \qquad (7)$$

This relationship gives the ability to use the stresses to calculate a new value of volumetric engineering strain. A plot of the engineering volumetric strain and natural volumetric strain vs. hydrostatic stress is given in figure 1 along with a plot of the bulk density vs. hydrostatic stress for a Norfolk sandy loam soil.

Equation 7 was incorporated into the finite element program by implementing the incremental loading technique discussed by Duncan and Chang (1970). The load was applied in several increments. This allowed linear elastic theory to be used because the calculated strains and stresses for each increment were small. These strain and stress values were accumulated over all load steps to obtain the total strain and stress for each element.

Because the soil becomes stiffer as load is applied, a method is needed to change the material properties between load steps. One method to accomplish this is to increase Young's modulus as the stress state of the element increases. This was done by using a tangential Young's modulus, defined as

$$E_t = \frac{d\sigma}{d\varepsilon} \qquad (8)$$

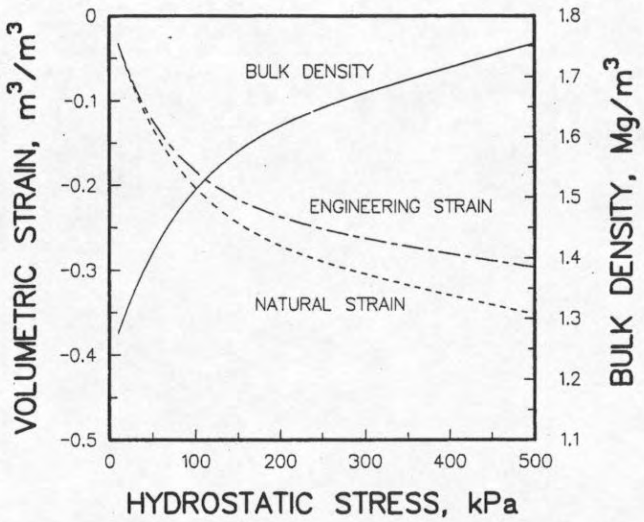

Figure 1–Variables obtained from compaction model developed by Bailey et al. (1984) plotted against hydrostatic stress.

This modulus continuously changes for equation 7 because of the nonlinearity of the stress-strain curve.

Because the elements in the soil mass will not be in a hydrostatic stress state, some other method must be used to calculate the value of stress that will determine the modulus of elasticity for the next load step. A solution is to use the mean normal stress value, σ_{mn}, where

$$\sigma_{mn} = \frac{(\sigma_x + \sigma_y + \sigma_z)}{3} \qquad (9)$$

σ_x, σ_y, and σ_z are the three components of stress in the radial, tangential, and vertical directions, respectively.

The compaction equation (eq. 7) gives strain as a function of stress. We can differentiate with respect to σ_{mn} and obtain an inverse relationship for the tangential Young's modulus.

$$E_t = \left[\frac{d\varepsilon_v}{d\sigma_{mn}}\right]^{-1} = \left\{\exp\left[(A + B\sigma_{mn})\left(1 - e^{-C\sigma_{mn}}\right)\right]\left[B + e^{-C\sigma_{mn}}(AC - B + BC\sigma_{mn})\right]\right\}^{-1} \qquad (10)$$

A plot of Young's modulus vs. mean normal stress is given in figure 2 for the Norfolk sandy loam soil.

Equation 10 was used after each load step to calculate a new value of Young's modulus for each element that depended upon the previous load step's stress. Theoretically, closer and closer approximations to the exact solution will be obtained as the load increment is decreased and the number of load steps is increased.

One very important point is that each element can be handled independently rather than a group of elements being assigned the same values of Poisson's ratio, ν, and Young's modulus, E. Each element's stress values will be calculated independently and these values will place each element at a particular position on the stress-strain curve. Therefore, each element has a unique value of ν and E

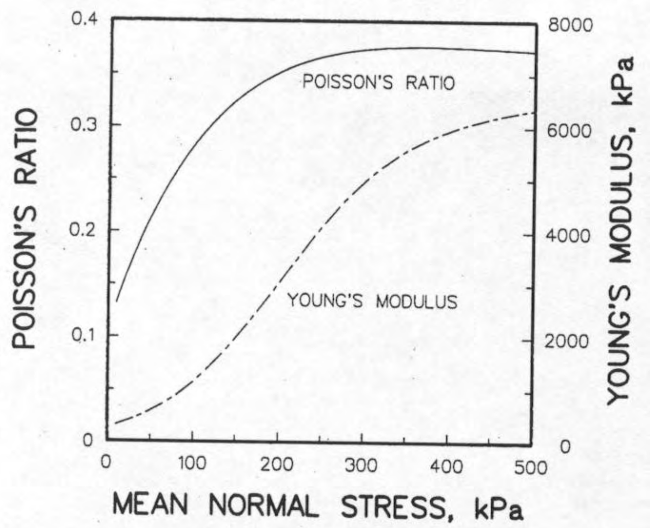

Figure 2–Poisson's ratio and Young's modulus obtained from compaction model (Bailey et al., 1984; Grisso et al., 1987) plotted against mean normal stress.

depending upon its current stress condition. Some prior research has used layers in order to simulate a stiffening soil profile with depth. The assumption of layers is not necessary with this program, unless the soil properties change as would occur when a subsoil with very different mechanical properties underlies the topsoil.

Duncan and Chang (1970) stated that two material properties are necessary to completely describe the mechanical behavior of any material under a general system of changing stresses. We now are able to calculate only one: i.e., Young's modulus. We can, however, use the following equation that was developed by Duncan and Chang (1970) to calculate values of Poisson's ratio, v, which depends upon the soil stress state.

$$\nu = \frac{\Delta\varepsilon_1 - \Delta\varepsilon_v}{2\Delta\varepsilon_1} \tag{11}$$

where

$\Delta\varepsilon_v$ = incremental volumetric strain
$\Delta\varepsilon_1$ = incremental axial strain

Applying a hydrostatic load will produce no difference between axial and volumetric strain because the stress in each direction is equal. Grisso et al. (1987) modified the Bailey et al. (1984) compaction model to include the effect of a deviatoric stress; i.e., the difference between the radial and axial stresses in a triaxial test. This model was also based on hydrostatic stress but an assumption is being made that the mean normal stress calculated from equation 9 is equivalent.

$$\bar{\varepsilon}_v = \beta(A + B\sigma_{mn})\left(1 - e^{-C\sigma_{mn}}\right) \tag{12}$$

where β is equal to a function of the deviatoric stress state

To remain compatible with the hydrostatic compaction model (eq. 7), only infinitesimal deviatoric stresses were used to calculate a value of Poisson's ratio. The mean normal stress calculated from equation 9 was used in Grisso's model (eq. 12). and the deviatoric stress was taken to be 0.99 and, 1.01 times this value. Appropriate values of strain were calculated from each of these situations and used to calculate Poisson's ratio for the finite element program. A plot of mean normal stress vs. Poisson's ratio is given in figure 2 for the Norfolk sandy loam soil.

A finite element program, entitled SOILPAK, was developed using all the techniques, models, equations, and assumptions previously discussed. This program was written in FORTRAN for use on a personal computer. The mesh that modeled the axisymmetric geometry of the soil is shown in figure 3. It has a radial boundary of 1.44 **m** and a lower boundary of -1.44 m. The load was applied to the top centermost four elements.

LABORATORY EXPERIMENT

An experiment was performed in the Norfolk sandy loam soil bin at the NSDL. The soil was prepared to mirror the same soil condition that was used for the development of the compaction model. The soil in the bin was prepared with a moisture content of 6.1% and in an initially loose condition.

Four different profiles of penetrometer readings were taken before the laboratory experiment was started in the soil bins. At the conclusion of the experiment, bulk density values were obtained. Measurements showed a layer of soil with increased penetration resistance and increased bulk density at depths ranging from 480 mm to 600 mm. An average hardpan depth was assumed at a depth of 540 mm for the FEM.

Two loading devices were used to apply loads to the soil surface by using the traction research vehicle at the NSDL. The first device was a flat steel plate and the second device was a spherical disc (fig. 4). The spherical disc was used to eliminate the stress concentrations that could occur near the edge of the flat plate.

A full load of 25* kN and a half load of 12.5 kN were applied to each of the discs. These loads are similar to that commonly exerted by tractor tires.

Values of soil stress were obtained by stress state transducers (SST) (Nichols et al., 1984). These transducers are capable of measuring the stress in six directions which allow the principal stress state in the soil to be computed

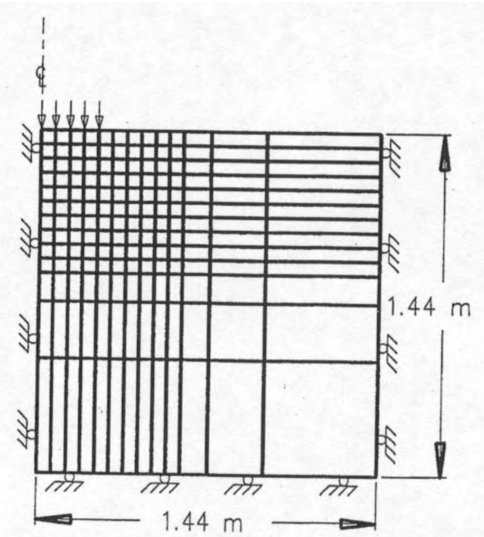

Figure 3–Axisymmetric finite element mesh that is used to model the soil medium.

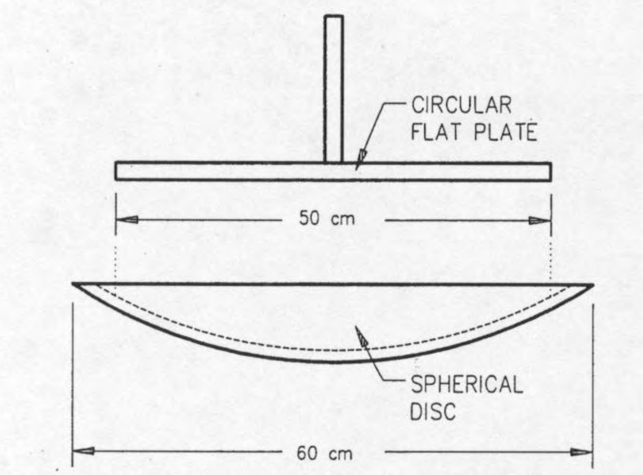

Figure 4–Flat plate and spherical disc that were used to apply loads to soil.

The SST gives the ability to examine horizontal stresses that are often considered to be negligible. One method of examining both the horizontal stress and the vertical component of stress was by using the mean normal stress (eq. 9).

The SSTs were placed at two locations in the soil. One, the center SST, was placed at the center of the axisymmetric load. Another, the radial SST, was placed at a distance of 20 cm from the center of the load. The SSTs at each location were also placed at two depths. In configuration 1, the center SST was placed at a depth of 15 cm and, the radial SST, at a depth of 25 cm (fig. 5). In configuration 2, the depths were reversed.

Three replications were performed for each of the runs. This amounted to a total of two loading devices by two loads by two stress cell configurations by three replications, which equaled 24 runs. These runs were done over a period of three days at the NSDL. Between the runs, the soil surface was covered by plastic to reduce moisture loss.

Approximately 100 samples of all stress data were obtained for the full load, and approximately 50 samples of

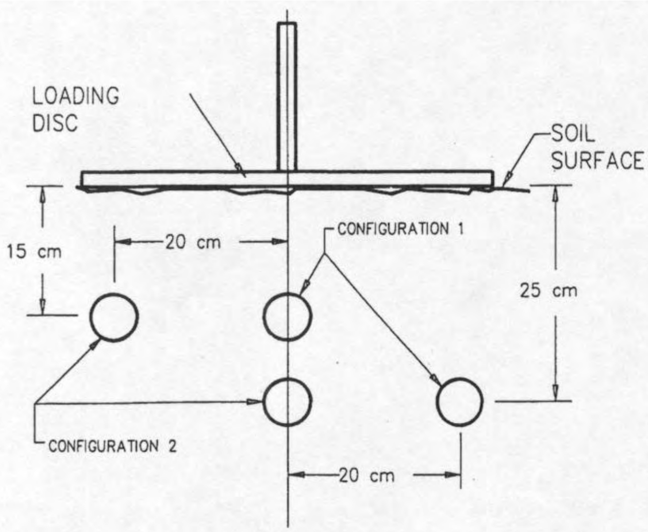

Figure 5–Positions at which stress state transducers were placed in the soil bin.

all stress data were obtained for the half load. The displacements of the soil surface and of the SSTs were obtained at the conclusion of the load application.

The values of the final surface displacement and the shape of the loading &vice were used to determine loading of the finite element mesh. The total surface displacement was split into multiple load steps. Values of stresses and strains were obtained at the centroid of each element for each respective load step. The stresses obtained at the element centroids were compared with the values obtained from the SSTs in the soil bin experiment.

RESULTS AND DISCUSSION

Table 1 shows the maximum displacements of the soil surface and average values obtained for the mean normal stresses for each of the transducers. The stress values obtained at the maximum loads were averaged from three different quantities and the stress values obtained from the half loads were averaged from six different quantities. The reason for more values at the half loads was that these values were also obtained when the model was loaded to the full amount. Also shown are 95% confidence intervals computed for each value. Large confidence intervals were

TABLE 1. Average maximum surface displacements and mean normal stresses* obtained from soil bin experiment

Treat-ment†			Center SST		Radial SST	
	Maxi-mum surface dis-place-ment	95% confi-dence interval	Mean normal stress	95% confi-dence interval	Mean normal stress	95% confi-dence interval
	---------cm-------		----------kPa----------			
111			106.1 ±	65.8	83.1 ±	50.6
112			88.5 ±	24.0	61.2 ±	46.8
AVER-AGE	19.6 ±	0.7				
121			41.7 ±	23.4	35.6 ±	11.5
122			40.0 ±	6.5	21.6 ±	11.2
AVER-AGE	12.9 ±	1.3				
211			66.6 ±	39.6	43.4 ±	13.9
212			53.9 ±	20.1	37.1 ±	5.6
AVER-AGE	19.5 ±	0.7				
221			31.9 ±	11.3	20.3 ±	6.9
222			29.8 ±	8.8	15.5 ±	3.8
AVER-AGE	14.0 ±	1.2				

* Six values were used to obtain average and C.I. for surface displacements; three values were used to obtain average stress and C.I. for 25-kN load; and six values were used to obtain average stress and C.I. for 12.5-kN load.

† First digit of treatment number
  1 = Flat disc
  2 = Spherical disc
Second digit of treatment number
  1 = 25 kN load
  2 = 12.5 kN load
Third digit of treatment number
  1 = Shallow transducers
  2 = Deep transducers

due to the large variance in the vertical stresses and few replications.

The FEM predicted similar values of volumetric strain for all load conditions. A maximum volumetric strain value of 36% was predicted beneath both the center of the flat plate and the spherical disc when the 25-kN load was applied This large value clearly showed that small strain theory could not be used without the incremental loading approach. When the load was reduced to 12.5 kN, the maximum volumetric strain decreased to 28% beneath the flat plate and 32% beneath the spherical disc.

FLAT DISC WITH 25-kN LOAD

All predicted mean normal stress contours were fairly close to the experimentally measured values (fig. 6). The only stress predicted with the FEM that did not lie within the confidence interval for the measured values was the center SST placed at the deepest depth (Table 1). The experimental values obtained at this position had a mean and confidence interval of 88.5 $\pm$ 24.0 kPa with the predicted value being approximately 62 kPa.

FLAT DISC WITH 12.5-kN LOAD

When the half load was applied to the soil surface with the flat plate, the stress contours appeared much as those from the full load (fig. 7). However, the stress magnitudes were roughly half the full load values. Also the transducers were not displaced as much as when subjected to the full load. The mean normal stress contours were. predicted very accurately, and each fell within the small confidence interval given at this lower loading level. For the half load, the central values were predicted more closely than the radial values, but all were within the confidence intervals.

SPHERICAL DISC WITH 25-kN LOAD

Stress concentrations at the device edge were much less due to the geometry of the spherical disc (fig. 8). Overall, the stress contours predicted by the FEM were greater than the measured values, although a portion of the predicted contours fit within the 95% confidence interval of the measured points. Predicted stress values failed to fall within acceptable limits of measured stress for either of the radial transducers. The upper radial SST had a mean and

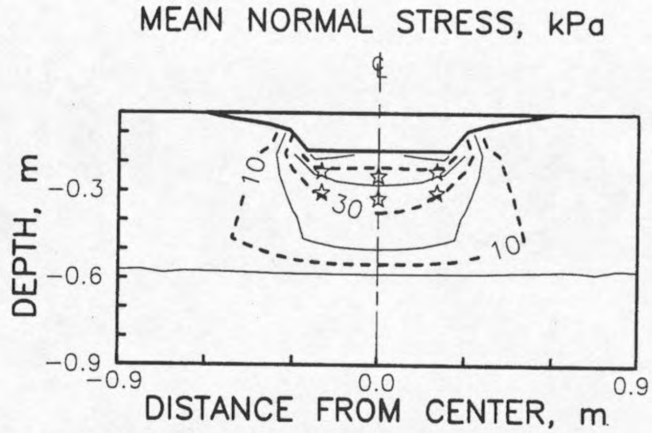

Figure 7–Mean normal stress contours predicted by finite element method when a flat disc applied a 12.5-kN load. A hard pan was assumed to be located at a depth of 540 mm, and stars indicate the positions of each SST.

confidence interval of 43.4 ± 13.9 kPa. The finite element predicted value was approximately 65 kPa. The deeper radial SST had a mean and confidence interval of 37.1 ± 5.7 kPa and a finite element predicted value of 50 kPa.

SPHERICAL DISC WITH 12.5-kN LOAD

The mean normal stress contour plot (fig. 9) did not accurately predict the experimental data. All predicted contours were relatively close to the experimentally measured values, but the only one that was within acceptable limits was the deep center SST. The shallow center SST had a mean and 95% confidence interval of 31.9 $\pm$ 11.3 kPa and a finite element predicted value of 47 kPa. The deep radial SST had a mean and 95% confidence interval of 15.5 $\pm$ 3.8 kPa and a finite element predicted value of 27 kPa while the shallow SST had a mean and confidence interval of 20.3 $\pm$ 6.9 kPa and a predicted value of 38 kPa.

SUMMARY AND CONCLUSIONS

The finite element program was modified to take into account the nonlinear constitutive relationship developed at the NSDL and Auburn University. A technique was used

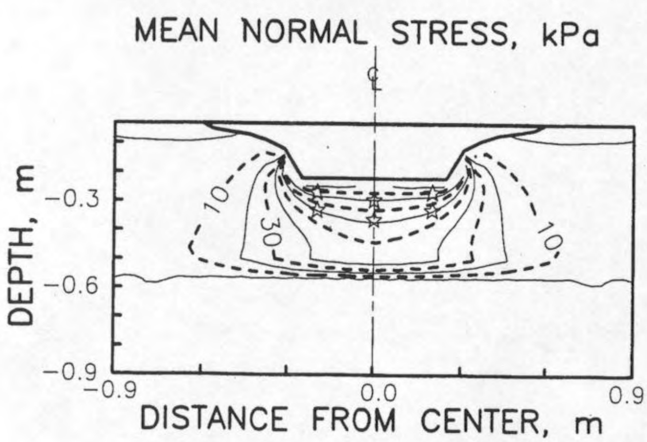

Figure 6–Mean normal stress contours predicted by finite element method when a flat disc applied a 25-kN load. A hard pan was assumed to be located at a depth of 540 mm, and the stars indicate the positions of each SST.

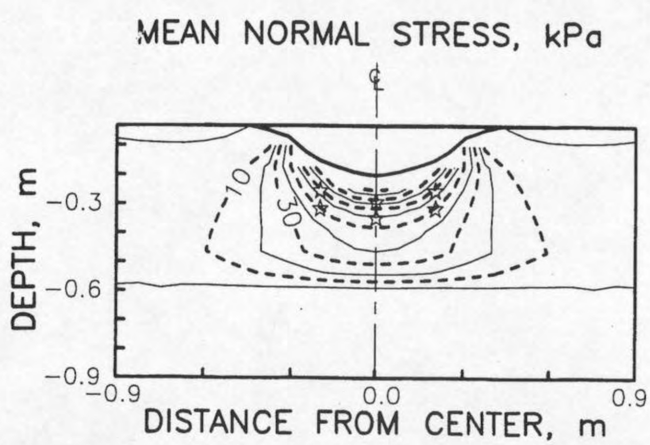

Figure 8–Mean normal stress contours predicted by finite element method when a spherical disc applied a 25-kN load. A hard pan was assumed to be located at a depth of 540 mm, and the stars indicate the positions of each SST.

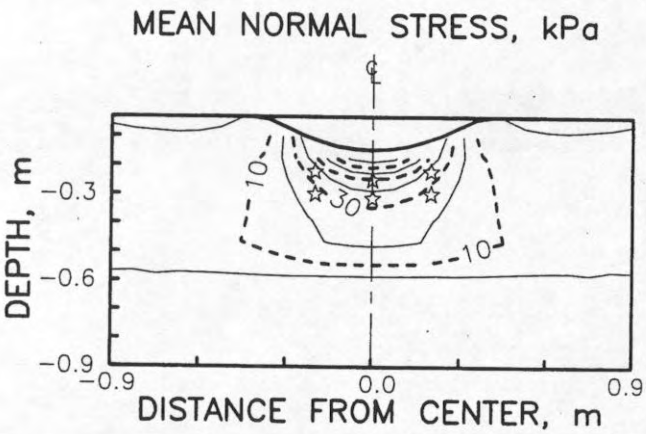

Figure 9–Mean normal stress contours predicted by finite element method when a spherical disc applied a 12.5-kN load. A hard pan was assumed to be located at a depth of 540 mm, and the stars indicate the positions of each SST.

whereby incremental values of Young's modulus and Poisson's ratio were calculated for each element based upon each element's stress state.

The nonlinear enhancements of the finite element program were evaluated by appropriate experimental verification at the NSDL. The FEM predicted mean normal stress within a 95% confidence interval of measured values for all except one of the transducers when the flat plate was used to load the soil. When the spherical disc was used, the FEM had more difficulty and was not able to predict any of the radial transducers accurately. It only missed predicting one of the center SSTs within acceptable limits, however, and this was at the lower loading level.

The mean normal stress levels generated by the low load on the spherical plate probably were not predicted as accurately because of the much lower experimental values that were obtained. The finite element program slightly overpredicted the stress levels in the elements and, for this situation, was outside the 95% confidence interval. The problems that were envisioned resulting from the stress concentrations near the edge of the flat plate never materialized. These stress concentrations probably even helped the finite element model predict accurate values.

Another likely reason for poor prediction of stress beneath the spherical plate is that the stress-strain model doesn't include all the mechanics of soil compaction, such as shear stress. More recent advances in this model should contribute to better predictions of soil compaction.

Improvements should be. made in the FEM to implement a more exact soil compaction relationship. Possibly the isotropic assumption could be modified to account for directional properties of soil: i.e., values of Poisson's ratio and Young's modulus could be predicted for each respective direction.

Continued enhancements to the FEM and the compaction model should enable researchers to accurately predict stress levels and the amount of compaction that the soil will undergo for a given applied load.

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