Summary of the Modifications of Winter Hydrology Subroutines in WEPP

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The modifications of the winter hydrology subroutines of WEPP were to v2006.5. Winter hydrology subroutines were examined thoroughly and changes were made where problems were identified.

Solar Radiation on sloped surface

WEPP employed the algorithm of Lloyd and Swift (1976) to estimate solar radiation on sloped surface. In this algorithm, measured solar radiation on a horizontal surface is adjusted with a factor to estimate actual radiation on a sloped surface using Eq. 1–2 (Lloyd and Swift, 1976)

$$R_s = R_2 f \tag{1}$$

$$f = \frac{R_4}{R_3} \tag{2}$$

where R_s [w/m^2] is estimated solar radiation on a sloped surface, R_2 [w/m^2] is measured solar radiation on a horizontal surface, f [-] is the adjustment factor for solar radiation on sloped surface, R_3 [w/m^2] and R_4 [w/m^2] are potential solar radiation on a horizontal surface and a sloping surface, respectively. The potential solar radiation is a theoretical value defined as the radiant flux density incident on the surface of the earth without the presence of its atmosphere (Lloyd and Swift, 1976; Campbell and Norman, 1998).

In reality, incoming solar radiation may be absorbed, scattered, or reflected as it travels through the atmosphere. Naturally, solar radiation received by an object on the surface of the earth includes "direct radiation (directly from the sun), diffuse radiation (scattered by sky and clouds), and reflected radiation from terrestrial objects" (Campbell and Norman, 1998). Direct solar radiation is highly directional and irradiance at a surface is calculated using Lambert's cosine law (Campbell and Norman, 1998). However, the diffuse sky radiation and reflected ground radiation are in all directions (Campbell and Norman, 1998). The diffuse irradiance at a surface cannot be determined from directly applying Lambert's cosine law. Instead, an empirical equation is typically used as discussed below. On steep north-facing slopes, diffuse sky radiation can be a major fraction of the solar irradiance at the surface. Therefore, the method of estimating the adjustment factor for sloped surface in the WEPP model needs to be improved.

In v2008.4, total solar radiation on a hillslope is modeled with consideration of the impact of atmosphere. Specifically, solar radiation is separated into direct (beam) and diffusive radiation while the reflected radiation from other watershed elements is neglected. Atmospheric transmittance is applied in estimating the solar irradiances on a horizontal and a sloped surface, respectively, and these solar irradiances are in turn used to determined the adjustment factor. The direct and diffuse irradiance equations (Eq. 3–6) from Campbell and Norman (1998) are adapted to estimate solar irradiance received by a surface. Atmospheric transmittance is estimated using Equation 7–11. The adjustment factor for sloped surface is estimated following Eq. 12–13. These equations are implemented in the subroutine *sunmap.for*

$$S_t = S_b + S_d \tag{3}$$

$$S_b = S_{po} \tau^m \cos(\psi) \tag{4}$$

$$S_d = 0.3 S_{po} (1 - \tau^m) \cos(\psi) \tag{5}$$

$$m = \frac{P_a}{101.3\cos(\psi)} \tag{6}$$

$$S_t = R_2 = S_b + S_d = S_{po}\tau^m\cos(\psi) + 0.3\,S_{po}(1-\tau^m)\cos(\psi) \tag{7}$$

$$R_2 - 0.3\,S_{po}\cos(\psi) = 0.7\,S_{po}\tau^m\cos(\psi) \tag{8}$$

$$R_3 = S_{po}\cos(\psi) \tag{9}$$

$$R_2 - 0.3\,R_3 = 0.7\,R_3\tau^m \tag{10}$$

$$\tau^m = \frac{R_2 - 0.3\,R_3}{0.7\,R_3} \tag{11}$$

$$S_d = 0.3\,(1-\tau^m)R_3 \tag{12}$$

$$f = \frac{R_4\tau^m + S_d}{R_3\tau^m + S_d} \tag{13}$$

where S_t [w/m^2] is total solar radiation, S_b [w/m^2] is beam solar radiation, S_d [w/m^2] is diffuse solar radiation, S_{po} [w/m^2] is the extraterrestrial solar radiation, τ [-] is the atmospheric transmittance, m [-] is the optical air mass number, $P_a/101.3$ [-] is the ratio of the atmospheric pressure at the observation site to that at the sea level ($P_a/101.3$ is assumed equal to 1.0 in v2008.4), f, R_2, R_3 and R_4 [w/m^2] are as previously defined with R_3 and R_4 calculated using solar constant, latitude, zenith angle, slope angle, and Julian day following Lloyd and Swift (1976).

Cloud cover

Estimation of areal cloud cover in WEPP was based on Sutton's (1953) proposition as cited in Flanagan and Nearing (1995) that clouds reflect approximately 70% of solar radiation and transmit only 30% to the earth's surface. In v2006.5, cloud cover was negatively proportional to the ratio of measured to potential solar radiation as given by Eq. 14 (Flanagan and Nearing, 1995)

$$C_{cl} = \frac{1 - R_2/R_3}{0.7} \tag{14}$$

where C_{cl} [-] is cloud cover; R_2 [w/m^2] and R_3 [w/m^2] are measured and potential solar radiation on a horizontal surface, respectively.

In fact, R_3 in Eq. 14. should be the solar radiation received by the earth surface under clear sky, which, following Eq. 8, attains the maximum value of 80% (when the zenith angle is 0° and $m = 1$), instead of 100%, of potential solar radiation. Therefore, v2006.5 overestimates cloud cover by assuming that 100% of potential solar radiation reaches the earth surface.

In v2008.4, the influence of atmosphere was considered in estimating cloud cover. The maximum and minimum atmospheric transmittances are 0.75 for clear sky and 0.4 under cloudy conditions (Campbell and Norman, 1998). Eq. 15–17 are implemented in the subroutine *sunmap.for* of v2008.4 for estimation of cloud cover

$$R_2 = 0.3R_3 + 0.7R_3\tau^m \tag{15}$$

$$R_2/R_3 = (1 - C_{cl})\,(0.3 + 0.7\tau_{\max}{}^m) + C_{cl}\,(0.3 + 0.7\tau_{\min}{}^m) \tag{16}$$

$$C_{cl} = \frac{(0.3 + 0.7\tau_{\max}{}^m) - R_2/R_3}{0.7\,(\tau_{\max}{}^m - \tau_{\min}{}^m)} \tag{17}$$

where C_{cl} [-], R_2, R_3, m and τ are as previously defined, τ_{max} [-] and τ_{min} [-] are maximum (under clear sky) and minimum (under cloudy sky) atmospheric transmittance, respectively. Eq. 15 is directly from Eq. 10. Eq. 16 includes the fractions of solar radiation under clear sky and cloud cover, respectively. Eq. 17 follows immediately from Eq. 16.

Start time of storm events

In the WEPP model, rain or snow is determined based on hourly temperature. Pecipitation is snow when hourly air temperature is less than 0 °C. Therefore, precipitation start time is very important to the winter routines in determining the precipitation is rain or snow.

In the WEPP model, there are two formats available to input daily climate data. One allows to input daily precipitation and the characteristics of the precipitation (such as duration, time to peak and peak intensity). The other one is called breakpoint data and allows to input several pairs of time and accumulative precipitation at the time for the day. In WEPP v2006.5, a random number generator is used to generate the precipitation start time for both climate inputs. Actually, precipitation start time is given when breakpoint data is used. In WEPP v 2008.4, precipitation start time will be read from the climate inputs when breakpoint data is used.

Snow accumulation

WEPP adjusts snow depth and density in considering new falling and drifting snow, snow settling and snowmelt. Water will not leave the snow pack until snow density exceeds 350 kg/m^3.

There are two major problems identified in WEPP v2006.5 snow depth and density adjustment subroutine. (1) snow densification only occurs at no-precipitation hours, therefore, snow depth is over-estimated when snow continues for several days. (2) warm rain was not considered in the routine for snow depth and density adjustment, rain will by pass snow pack and directly infiltrate into the soil or form runoff. Both problems is corrected in WEPP 2008.4.

Snowmelt

Snowmelt is estimated using a modified equation based on Hendrick et al. (1971) for generalized basin snowmelt on an hourly bases (eq. 18–23). The model was developed form snowmelt equations by U.S. Army Corp of Engineers. The snowmelt is a sum of snowmelt caused by solar radiation, long-wave radiation, convection-condensation and warm rain.

$$M_{hr} = 0.0254\,(M_a + M_b + M_c + M_d) \tag{18}$$

$$M_a = 0.0607\,R_{hr}(1.0 - C_{can}) \tag{19}$$

$$M_b = (0.025/24)T_{hr} - (0.84/24)(1 - C_{cl})(1.0 - C_{can}) \tag{20}$$

$$M_c = v(0.0084/24)(1 - 0.8C_{can})(0.22T_{hr} + 0.78T_{dewhr})v_{adj} \quad (21)$$

$$v_{adj} = 1.0 - \frac{1.0}{\log\left[(z_v - z_d + z_r)/z_r\right]} \quad (22)$$

$$M_d = (0.007/24)P_{hr}T_{hr} \quad (23)$$

where: M_{hr} [m] is hourly melt water (m), M_a [in] is melt from hourly solar radiation, M_b [in] is melt from long-wave radiation exchange, M_c [in] is melt from convection and condensation, M_d [in] is melt from warm rain, R_{hr} [langley, cal cm^{-2}] is hourly solar radiation, T_{hr} [°C] is hourly air temperature, $T_{dew\,hr}$ [°C] is hourly dew point temperature P_{hr} [m] is hourly rainfall, C_{can} [-] is canopy cover, C_{cl} [-] is cloud cover, v [miles hr^{-1}] is wind velocity, v_{adj} [-] is wind velocity adjustment factor, z_v [m] is the height wind measured, z_d [m] is the height of the zero displacement of the wind profile, z_r [m] is the roughness of the surface.

In WEPP v2006.5, snow melts when hourly air temperature is greater than -4 °C. Instead of daily mean air temperature used in the Hendrick et al. (1971) model; WEPP v2006.5 uses hourly air temperature without properly considering the negative melt during cold period of the day. In WEPP v2008.4, snowmelt routine only be called when daily mean temperature is greater than 0°C and net melt is calculated at the end of a day in considering the negative melt of the cold hours.

In WEPP v2006.5, snowmelt from convection-condensation would be zero if wind velocity is approximately zero (eq. 21). Unfortunately, wind adjustment factor is coded wrong in WEPP v2006.5 which causes wind velocity always close to zero. Therefore, WEPP v 2006.5 underestimates the convection-condensation snowmelt. However, in heavily forested area where wind velocity is close to zero, convection-condensation term plays a major role in snowmelt. In WEPP v2008.4, ACE convection-condensation equation for heavily forested areas and the wind velocity adjustment factor equation were incorporated into the Herndrick et al. model (eq. 24–26).

$$\begin{aligned} M_c = v(0.0084/24)(1.0 - 0.8C_{can})(0.22T_{hr} + 0.78T_{dewhr})v_{adj} \\ + 0.8C_{can}(0.045/24)T_{hr} \end{aligned} \qquad \textit{when } v > 0. \quad (24)$$

$$M_c = 0.8C_{can}(0.045/24)T_{hr} \qquad \textit{when } v = 0. \quad (25)$$

$$v_{adj} = 1.57\, z_v^{-1/6} \quad (26)$$

Hourly dew-point temperature was incorrectly modeled to fluctuate diurnally following a sine curve in WEPP v2006.5. In v2008.4, input daily dew-point temperature is now used. In warm rain melt component , the rainfall is hourly rainfall amount. Therefore, It is wrong to divided rainfall amount by 24 in WEPP v2006.5. In v2008.4, the unnecessary division is removed.

Frost simulation

(1) layers

Instead of two large layers (tilled and untilled layer) in WEPP v2006.5 for frost simulation, v2008.4 divided each soil layer into 10 finer layers. Though WEPP frost simulation equation (eq. 27–29) has the term to account for the water movement to the frozen front. However, it is impossible to track the soil water movement using only tilled and untilled layer in simulation. Therefore, in WEPP v2008.4, forst simulation part is re-coded using finer layers to reveal the full function of WEPP frost simulation method.

$$\frac{L\Delta d_{fz}\theta}{\Delta t} = Q_{srf} - Q_{uf} \tag{27}$$

$$Q_{srf} = \frac{K_{srf}\Delta T_{srf}}{Z_{srf}} \tag{28}$$

$$Q_{uf} = K_{uf}\frac{\Delta T_{uf}}{Z_{uf}} + LK_w\frac{\Delta P}{Z} + \frac{C_{uf}dT_{uf}Z_{uf}}{\Delta t} \tag{29}$$

where: L [J m^{-3}] is the latent heat of fusion, Δt [s] is the time of freezing or thawing, Δd_{fz} [m] is the soil depth of frozen or thawed during Δt time period, θ is soil water content, Q_{srf}, Q_{uf} [W m^{-2}] are the heat flux through the snow-residue-frozen soil system and from unfrozen soil beneath the frozen zone, Z_{srf}, Z_{uf} [m] are the depth or thickness of the combined snow-residue-frozen soil layer and the depth of unfrozen soil to the point of stable temperature, K_{srf}, K_{uf} [W m^{-1} °C^{-1}] are the average thermal conductivity through the combined snow residue-frozen soil and through unfrozen soil, ΔT_{srf}, ΔT_{uf} [°C] are the temperature difference from surface across the snow-residue-frozen soil to frozen front of 0 degree isotherm and from frozen front across the unfrozen soil to the depth of stable temperature, K_w [m s^{-1}] is the unsaturated hydraulic conductivity of unfrozen soil, Z [m] is the thickness of the soil between the frozen front and the center of its adjacent unfrozen soil layer, ΔP_{uf} [m] is the difference of total water potential between the frozen front and the center of its adjacent unfrozen soil layer, C_{uf} [J m^{-3} °C^{-1}] is the heat capacity of the unfrozen soil, dT_{uf} [°C] is the change in temperature of a unit volume of unfrozen soil, and WEPP neglects the third term of the right hand side of equation (29) and assumes Z_{uf} a constant value of 1.0 meter.

Subroutines removed:

(a) subroutines for estimating unsaturated hydraulic conductivity (*set_hc.for* and *calchc.for*)
(b) main driver of frost simulation (*forst.for*)
(c) subroutine for calculating energy from the surface (*caqout.for*)
(d) subroutine for freezing (*cldfrz.for*)
(f) subroutine for thawing (*wmelt.for*, *tfwmelt.for*, and *mltfdp.for*)

Subroutines added:

(a) subroutines for estimating unsaturated hydraulic conductivity and soil water potential (*saxpar.for* and *Saxfun.for*)
(b) main driver of frost simulation (*forstN.for*)
(c) subroutine for freezing (*frzng.for*)
(d) subroutine calculating soil thawing from top (*mlttop.for*)
(f) subroutine calculating soil thawing from bottom (*mltbtm.for*)
(g) subroutine for calculating soil temperature below the frost zone (*tmpfun.for* and *tmpcft.for*)
(h) subroutine for determining soil water redistribution during frost period (*watdst.for*)
(i) subroutine calculating soil water exchange between winter subroutines and water balance as well as other subroutines (*frwatc.for*)
(j) subroutine for estimating saturated hydraulic conductivity of frozen soil (*frsoil.for*)

(2) energy and energy flux and frost simulation

In WEPP 2006.5, there were several place energy flux was used where energy amount should apply. I guess the authors of frost simulation in WEPP v2006.5 were confused by energy and energy flux. In WEPP v2008.4, a time factor, the difference between energy and energy flux, is clearly applied as the core concept in coding frost simulation.

IN WEPP v 2008.4, it is assumed when soil is frozen, all water of the frozen soil is in ice form. Two variables are implemented to implicitly record whether sandwich frost layer exists and where the sandwich layers are. When frost sandwich does not exists, energy fluxes from top and soil below are compared at the frozen front to determine wether freezing or thawing processes is on going. Otherwise the heat flux from soil below thaws the bottom of the frost and the energy from top thaws or freezes from top.

(3) soil temperature below frozen zone

In WEPP v2006.5, it is assumed that soil one meter below the frozen front is 7 °C. The assumption could be interpolated as soil temperature gradient is 7 °C m $^{-1}$. However, soil temperature and soil temperature gradient change with time and soil depth.

Temperature of a uniform soil can be represented by eq. 30 (Campbell and Norman, 1998). Soil temperature gradient at a time may be estimated by eq. 31 the partial derivative of soil temperature to depth . Figure 1 shows soil temperature and its temperature gradient in 0–3 meter soil for Morris, MN at end of December. The assumption of 7 °C m $^{-1}$ in WEPP v2006.5 is valid only when frost depth is around 1.2m (Fig. 1). In v2008.4, eq. 31 is used to estimate soil temperature one meter below the bottom of the frost zone with an assumption of 2 meter damping depth. Energy form soil below is set to zero if the estimated soil temperature is below 0 °C . Therefore, the frost routines in WEPP v2008.4 is also applicable for the permafrost area.

$$T(z,t) = T_{avg} + A_0 \exp(-z/D) \sin\left[\omega(t - t_0) - z/D)\right] \tag{30}$$

$$\frac{\partial T(z,t)}{\partial z} = A_0\,(-1.0/D)\,\exp(-z/D)\left(\sin\left[\omega(t - t_0) - z/D)\right] + \cos\left[\omega(t - t_0) - z/D)\right]\right) \tag{31}$$

where A_0 [°C] is the amplitude of yearly temperature fluctuation at soil surface, T_{avg} [°C] is yearly average temperature at soil surface, ω is $2\pi/365$, D [m] is damping depth, z [m] is soil depth, t [d] is the time variable in Julian day, and t_0 [d] is a phase adjustment to the time variable.

In v2008.4, the monthly temperature data in the climate input file is used to fit a sine curve for the parameters in eq. 30. Newton method is applied for a minimum square error in the curve fitting.

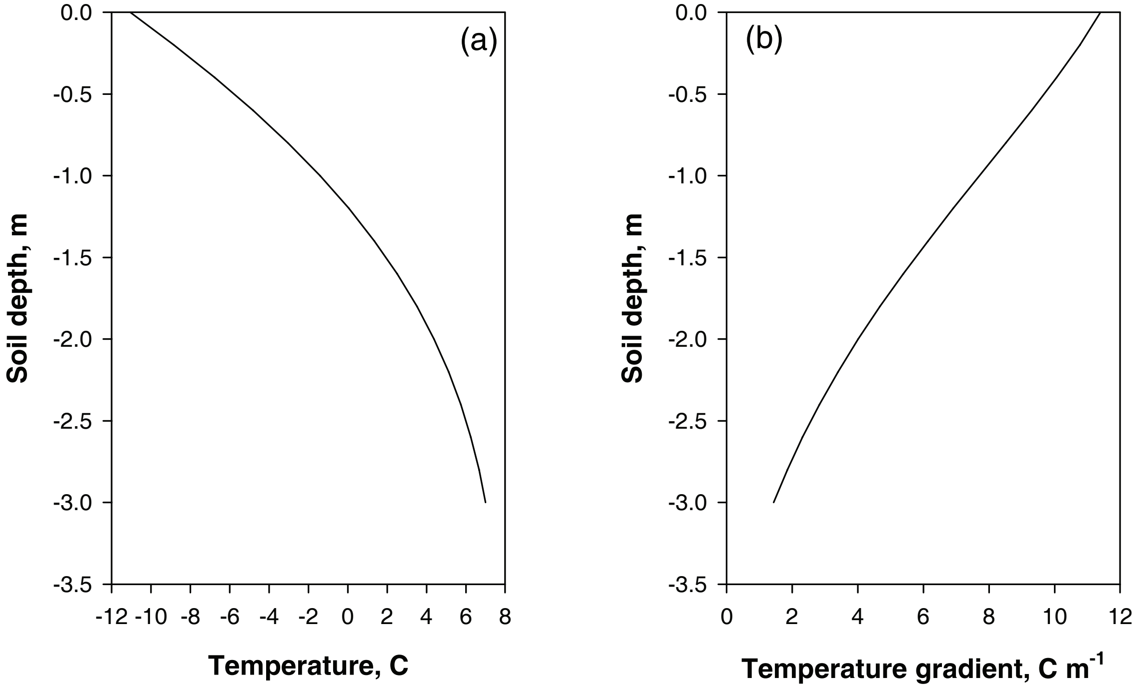

Figure 1, soil temperature and temperature gradient in top 0–3 meter when A_0 = 18 °C, T_{avg} = 6 °C, D = 2 m, t_0 = 110 day and t = 365 day

(4) unsaturated hydraulic conductivity and soil water potential

In WEPP v2006.5, the method of Prasanta Kalita's for determining the value of unsaturated hydraulic conductivity based soil tension in units of cm is used.

In WEPP v2008.4, the model of Saxton ans Rawls (2006) is used to estimate unsaturated hydraulic conductivity and total soil water potential based on soil water content and soil texture.

(5) saturated hydraulic conductivity of frozen soil

In WEPP v2006.5, saturated hydraulic conductivity of frozen soil is estimated by multiplying a frozen factor to the hydraulic conductivity of unfrozen soil. The frozen factor is an exponential function of the ratio between ice content and soil field capacity (equation 32), the factor is limited in between 0.1–2.0.

$$f_{fz} = 3.75\, e^{(-0.026\, \theta_{ice}/\theta_{fc})} \tag{32}$$

For the case study in the U.S. Pacific Northwest (Greer et al., 2006) showed that this factor has to be 100 times smaller for WEPP to generate the observed runoff vaulues.

In WEPP v2008.4, a different approach is used. It is assumed that ice occupies spaces the same as air of unsaturated soil. Therefore, the method of estimating unsaturated hydraulic conductivity of a soil is used to estimate the saturated hydraulic conductivity of frozen soil. The unsaturated hydraulic conductivity with soil water content of that porosity minus ice content is estimated following Saxton ans Rawls (2006). The estimated unsaturated hydraulic conductivity is then used as saturated hydraulic conductivity of the frozen soil.

(6) soil water potential of the freezing front

In WEPP frost simulation equation, the second term of the right hand side of equation (29) considers water upward movement from unfrozen soil to the frozen front. In WEPP v2006.5, this term was calculated wrong by directly using the tension of unfrozen soil instead of the total water potential difference between the frozen front and unfrozen soil.

In WEPP v2008.4, total water potential of unfrozen soil is estimated following Saxton ans Rawls (2006). The water potential of the frozen front is estimated using soil freezing depression point using generalized Clausius-Clapeyron equation following Kunio Watanabe's (associate professor, Mie University, Japan) suggestion. Soil freezing depression point usually is in a range of 0.01–0.25 °C (personal communication with Dr. Watanabe, 2008). The pressure potential at frozen front should be in the range of −1m to −25m. I prefer to set the soil water potential of the freezing front to −10m. Currently, the water movement to frozen front is turned off by setting frozen front water potential 0.

(7) soil water redistribution when frost exists

Soil water redistribution due to freezing and thawing is important to erosion simulation. In WEPP v2006.5, frost simulation is using a two layer system (tilled and untilled soil). It is not possible to track soil water content change due to freezing and thawing with only two thick layers. Therefore, no special water redistribution routine for frozen soil except the soil water content simulations in the daily water balance subroutine.

In WEPP v2008.4, a new subroutine for soil water redistribution when frost exists is added. In this subroutine, unsaturated soil water movement is estimated between the fine soil layers of frost simulation. The water movement simulation is hourly based using Darcy's law with estimated unsaturated hydraulic conductivity and soil water potential using Saxton ans Rawls (2006) model.

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