

Lacunarity and Fractal Analyses of Soil Macropores and Preferential Transport Using Micro-X-Ray Computed Tomography

Lifang Luo and Henry Lin*

Quantification of soil macropore networks and solute transport patterns is important to enhance our understanding of preferential flow in structured soils. We investigated soil macropore structure and solute transport dynamics in an intact soil column of 76-mm diameter and 265-mm length. Five positions (each with a thickness of 10.7 mm) in the soil column were scanned with a voxel resolution of 78.1 by 78.1 by 86.7 μm using micro-x-ray computed tomography. The scanning was done at three stages after the soil column was saturated: (i) before tracer (KI) introduction, (ii) 6 min after tracer introduction, and (iii) 78 min after tracer introduction. The macropore network and tracer distribution were reconstructed at the five scanned positions. Relative lacunarity functions (RLFs) and pore fractal dimensions, in both two and three dimensions, were calculated. Distinct macropore characteristics and flow patterns were observed at the five positions. The biopores were active in solute transport because of their high continuity and low tortuosity. Positive logarithmic trends were found between the fractal dimension and the volume percentage of the macropores and tracer distribution. Generally, the lacunarity function reflected the size distribution of macropores and the spatial pattern of flow and transport. The RLFs indicated that the tracer distributions exhibited more self-similarity than the macropore networks and that the representative elementary volume had not been reached for the three-dimensional macropore networks within the size range investigated (cube size ≤ 63). Lacunarity has diagnostic value in characterizing soil macropore structure and flow pattern and may be coupled with the fractal dimension to better describe and model soil structural properties.

ABBREVIATIONS: CT, computed tomography; REV, representative elementary volume; RLF, relative lacunarity function.

SOIL MACROPORES play a critical role in preferential water movement and solute transport (Beven and Germann, 1982). Effective techniques for characterizing preferential flow and transport, and their relationship to soil macropore structure remain elusive, however. New methods to improve the quantification of soil macropore structure and preferential flow patterns are important to enhance our understanding of the processes involved.

Fractal theory is one of the most widely used methods to quantify soil structure and flow patterns (e.g., Baveye et al., 1998; Young et al., 2001). The fractal dimension has been used to describe soil physical properties such as bulk density, pore size distribution, pore surface area, and soil aggregation, as well as soil physical processes such as water retention, percolation, diffusion, and solute transport (e.g., Perfect and Kay, 1995; Zeng et al., 1996; Perret et al., 2003; Anderson et al., 2000; Perrier et al., 1996, 1999; Giménez et al., 1997; Hatano et al., 1992).

Even though the fractal dimensions of two objects are the same, their structures can be quite different (Pendleton et al.,

2005). As Mandelbrot (1982) pointed out, the fractal dimension alone is not sufficient to describe the geometry and properties of “lacunar” fractals; another parameter, which he termed *lacunarity*, is necessary. Lacunarity measures the deviation of a geometric object from the translational invariance or homogeneity and can be considered as a scale-dependent index of heterogeneity (Plotnick et al., 1993). In terms of fractal geometry, the prefactor in the following power-law relationship is associated with the lacunarity, while the exponent corresponds to the fractal dimension (Mandelbrot, 1982):

$$\text{Property} = (\text{prefactor})(\text{scale})^{\text{exponent}} \quad [1]$$

Lacunarity reflects the fraction of space occupied by the feature of interest, the degree of dispersion and clustering, the presence of self-similarity or randomness, and the existence of hierarchical structure (Plotnick et al., 1993). One of the problems involved with the application of fractal theory is that power-law scaling is not necessarily indicative of a structure exhibiting self-similarity (Baveye and Boast, 1998; Young et al., 1997). The lacunarity curve of a fractal should be a straight line on a double logarithmic scale (Allain and Cloitre, 1991). This property could be a quite useful feature to tell whether self-similarity exists or not.

Despite the promising potential of lacunarity as an index to characterize different structures or spatial patterns, it has received relatively little attention in the soil science and hydrology literature. Lacunarity has been applied to differentiate the structures and spatial patterns of both fractals and nonfractals, such as landscape and land use (Plotnick et al., 1993), temporal changes in

Dep. of Crop and Soil Sciences, 116 ASI Building, The Pennsylvania State Univ., University Park, PA 16802. Received 16 Jan. 2008. *Corresponding author (henrylin@psu.edu).

Vadose Zone J. 8:233–241
doi:10.2136/vzj2008.0010

© Soil Science Society of America

677 S. Segoe Rd. Madison, WI 53711 USA.

All rights reserved. No part of this periodical may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopying, recording, or any information storage and retrieval system, without permission in writing from the publisher.

sediment transport (Plotnick et al., 1996), and pore space geometry in heterogeneous porous media (Kim et al., 2007). Zeng et al. (1996) calculated the two-dimensional fractal dimension and lacunarity of soil bulk density using medical x-ray computed tomography (CT). They found that lacunarity was more sensitive to soil heterogeneity than the fractal dimension and suggested that the joint distribution of fractal dimension and lacunarity would better discriminate soil structure. Until now, however, lacunarity analysis of soil macropore structure and preferential flow pattern has remained unexplored.

With advances in experimental techniques and numerical analyses, CT has become an attractive technology for soil scientists to nondestructively observe soil macropores, solute transport, and their interactions in real time (Anderson et al., 1990, 1992; Heijs et al., 1995; Perret et al., 1999, 2000; Luo et al., 2008). The intensive information gained from CT images provides a great opportunity to conduct fractal and lacunarity analyses of soil macropores and flow patterns. The CT techniques can provide three-dimensional information by synthesizing a series of two-dimensional CT images using certain algorithms (Perret et al., 1999) and thereby facilitate the comparisons between dimensions—a feature especially important for anisotropic media. Since macropore and tracer distributions are essentially three-dimensional and apparent anisotropy exists in well-structured soils (Perret et al., 1999; Luo et al., 2008), it seems more sensible to evaluate their properties in three dimensions. Perret et al. (2003) calculated two- and three-dimensional pore fractal dimensions of macropore networks using box-counting (two dimensions) and cube-counting (three dimensions) techniques. They found positive logarithmic trends between the fractal dimension and macroporosity. The relationship between the two- and three-dimensional fractal dimensions was not clear, however, mainly because of insufficient data. Gibson et al. (2006) compared the two- and three-dimensional fractal codimensions of soil aggregates and found that two-dimensional fractal dimensions could represent three-dimensional ones reasonably well.

The objectives of this study were to: (i) reconstruct and quantify three-dimensional macropore structure and preferential tracer transport in real time in an intact soil column; (ii) characterize a soil macropore network and solute distribution patterns along the soil depth using fractal dimensions and lacunarities; and (iii) examine if macropore networks and flow patterns are fractal and if lacunarity has diagnostic value in characterizing soil macropore structure and preferential flow pattern where fractal dimension alone could not.

Materials and Methods

Soil

An intact soil column, 76 mm in diameter and 265 mm in length, was taken from the R.E. Larson Agricultural Research Center of The Pennsylvania State University located at Rock Springs in Centre County, Pennsylvania. The soil series is mapped as a Hagerstown silt loam (a fine, mixed, semiactive, mesic Typic Hapludalf), which is one of the most important agricultural soils in Pennsylvania. The field has been planted with corn (*Zea mays* L.), soybean [*Glycine max* (L.) Merr.], and alfalfa (*Medicago sativa* L.) in rotation during the past few decades. All plant

residues have been left in the field after harvesting. The details of our soil sampling procedures can be found in Luo et al. (2008). Three soil horizons, the Ap1 (0–11 cm), Ap2 (11–21 cm), and Bt (21–26 cm), were identified in the soil column collected (Fig. 1). Basic soil properties are listed in Table 1.

Industrial Computed Tomography Unit

An industrial CT unit (OMNI-X, Bio-Imaging Research, Lincolnshire, IL) was used in this study. The system has a 225-kV micro-focus x-ray generator and a 225-mm-diameter image intensifier. The x-ray source and detector are fixed and the scanned object rotates (Fig. 2a). The sample is rotated 360° to receive the x-ray beam (polychromatic) while the detector provides intensity views to the data acquisition computer. One rotation takes about 10 to 15 min, depending on the number of views taken. In this research, up to 41 slices were acquired in a single rotation with 3600 views. High-resolution images (1024 by 1024 pixels) were acquired at the output to the computer system. We used different kinds of filters or wedges during the tuning-up procedure before

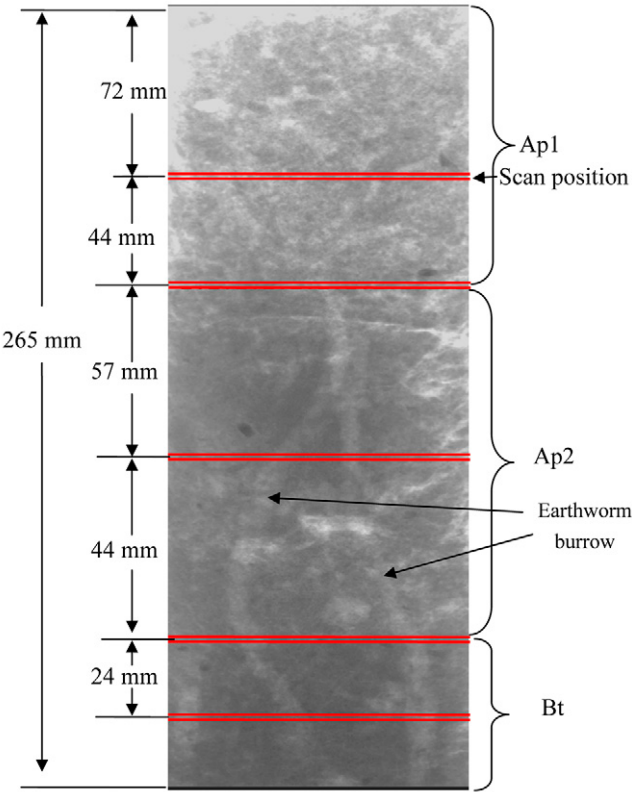


FIG. 1. A digital radiograph of the intact soil column investigated in this study. The higher the soil density, the darker the image. Two earthworm borrows are clearly noticeable that originated in the Ap1 horizon, passing the plow pan of the Ap2 and into the Bt horizon.

TABLE 1. Basic soil properties of the soil column studied. The numbers in parentheses are one standard deviation.

Horizon	Depth	Organic matter	Sand	Silt	Clay	$K_{sat}^{\dagger}$	Bulk density	Total porosity
	cm		%			cm/min	g/cm ³	m ³ /m ³
Ap1	0–11	4.5	19.3	64.8	15.9	0.051 (0.032)	1.43 (0.03)	0.46
Ap2	11–21	3.5	17.1	68.1	14.8	0.045 (0.114)	1.50 (0.06)	0.43
Bt	21–26+	1.9	22.9	51.7	25.4	0.035 (0.041)	1.61 (0.04)	0.39

[†] K_{sat} , saturated hydraulic conductivity.

our experiment to minimize noise and to obtain the best image quality. A digital radiography (DR) image was also collected using the same CT unit to monitor the whole column during the experiment. The DR image reproduces a two-dimensional projection of the whole object exposed to the x-ray beam, and the x-ray attenuation values represent the effect of the object's density and thickness (see example shown in Fig. 1).

Experimental Procedure

The experimental setup is shown in Fig. 2b. The soil column was housed in a polyvinyl chloride pipe and satiated from the bottom by raising the water (0.005 mol/L CaSO_4 solution) table gradually. This saturation process lasted for 4 d. The soil at saturation was scanned at a resolution of 78.1 by 78.1 by 86.7 μm at five positions, namely the Ap1, Ap2, and Bt horizons, the boundary between the Ap1 and Ap2 horizons (referred to as Ap1–Ap2), and the boundary between the Ap2 and Bt horizons (referred to as Ap2–Bt) (Fig. 2b). For each position, 123 images were obtained (about 10.7 mm thick in total). After scanning, 0.005 mol/L CaSO_4 solution was supplied to the top end of the soil column at a constant flow rate of 6.7 mL/min using an automatic pump. When the outflow rate became constant, the 0.005 mol/L CaSO_4 solution was replaced with a 60 g/L KI solution. An automatic fraction collector was used to collect the center and outer outflows separately (see Fig. 2b) at 1-min intervals. After introducing the KI solution for 6 min, the pump and the valve at the outlet tube were turned off to stop the flow. Scanning at the same five positions was repeated to observe the solute distribution. After this scanning, the KI solution leaching was restarted and scanning was repeated at the same five locations at 78 min. A total of 528 mL of the KI solution was applied during this experiment, which was about one pore volume of the soil column.

Data Analysis

The images (1024 by 1024 pixels) were cut to exclude the area outside the soil column using ImageJ, Version 1.39 (Rasband, 2002). The macropore threshold value, determined using the maximum entropy threshold algorithm in ImageJ 1.39, was used to segment the images. The images were also visually inspected to ensure that a reasonable threshold value was used. The tracer distribution was reconstructed by subtracting the images taken when the soil was satiated with the 0.005 mol/L CaSO_4 solution from the images taken after the introduction of the 60 g/L KI solution, then dividing it by the difference in x-ray attenuation of the 0.005 mol/L CaSO_4 solution and the 60 g/L KI solution (which is 459). After subtraction, the medium filter, a commonly used image-processing method to reduce the noise while preserving the edge, was used to minimize noise (Jassogne et al., 2007). The binary images were obtained with the following relationship:

$$\frac{\text{CT}(x, y, z, t) - \text{CT}(x, y, z, t = 0)}{\text{CT}_{\text{KI}} - \text{CT}_{\text{water}}} > 30\% \quad [2]$$

where $\text{CT}(x, y, z, t)$ is the x-ray attenuation value for a voxel during solute replacement at time t , $\text{CT}(x, y, z, t = 0)$ is the x-ray attenuation value when the soil was satiated with water (0.005 mol/L CaSO_4 solution) at the beginning of the experiment, and CT_{KI} and CT_{water} are the x-ray attenuation values of the 60 g/L KI solution and the 0.005 mol/L CaSO_4 solution, respectively. The

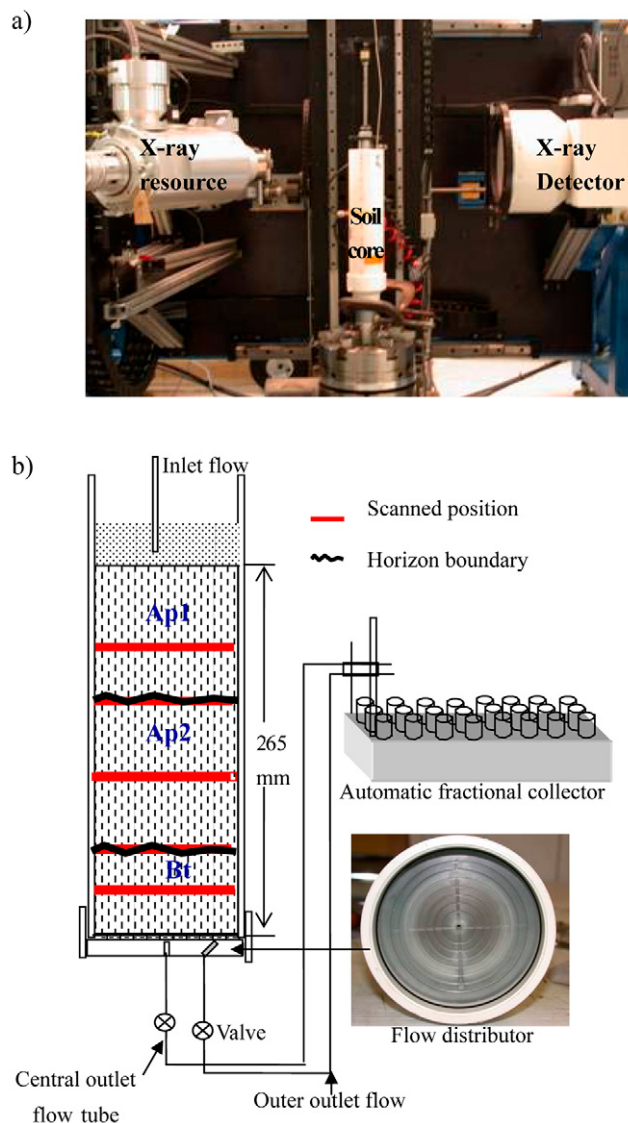


FIG. 2. (a) The industrial computed tomography unit used in this study and the position of the soil column used in the experiment; and (b) the experimental setup used to scan the five positions in the soil column while the solute transport experiment was being conducted simultaneously.

threshold (i.e., 30%) reasonably segmented regions with apparent changes in x-ray attenuation values and excluded noise in the background. We relied on visual inspection, which is very important to ensure that a reasonable threshold value is used despite the existence of many algorithms. After reconstruction, the macropore networks and tracer distributions were visualized using Amira, Version 3.1 (TGS Inc., San Diego).

After image segmentation, the macroporosity, tracer volume percentage, average hydraulic radius, pore fractal dimension, and lacunarity of macropores and tracer distribution in both two and three dimensions were calculated. The hydraulic radius was calculated as the ratio of the pore volume to the pore surface area. Fractal dimensions were calculated using box-counting (two dimensions) or cube-counting (three dimensions) methods (Perret et al., 2003). By covering the feature with boxes or cubes with side dimensions of r , the fractal dimension (D) can be estimated from the slope of $\ln[N(r)]$ against $\ln(r)$ with the following relationship:

$$N(r) = \left(\frac{1}{r}\right)^D \quad [3]$$

where $N(r)$ is the number of boxes intersecting the feature of interest. The three-dimensional third-iteration prefractal Menger sponge with a known fractal dimension of 2.727 was generated to test our algorithm (in this case, the solid phase was counted), which resulted in a fractal dimension of 2.718 (error = 0.3%). The fractal codimension (H) can be used to generalize and compare fractal dimensions in one, two, or three dimensions. The fractal codimension is defined as the difference between the Euclidian dimension (E) and the fractal dimension (Perret et al., 2003):

$$H = E - D \quad [4]$$

Lacunarity was calculated by a method introduced by Allain and Cloitre (1991). The gliding-box algorithm was used, in which a box or cube of size r moves or “glides” within the image, covering all of the pixels or voxels. The lacunarity (L) was calculated as a function of r :

$$L(r) = \frac{\sum_m m^2 P(m, r)}{\left[\sum_m m P(m, r)\right]^2} \quad [5]$$

where $P(m, r)$ is the probability that a box or cube of size r contains m pixels (in two dimensions) or voxels (in three dimensions) of interest. Since the first moment $\sum_m m P(m, r)$ is equal to the mean (u) of the probability distribution function $P(m, r)$, and the second moment $\sum_m m^2 P(m, r)$ is equal to the sum of the variance (σ^2) and the square of the mean of $P(m, r)$, Eq. [5] is easier to understand when rewritten as (Pendleton et al., 2005)

$$L = \left(\frac{\sigma}{u}\right)^2 + 1 \quad [6]$$

Lacunarity is a function of three factors: (i) the gliding box or cube size, (ii) the fraction (p) of the space occupied by the feature, and (iii) the spatial distribution or structure of the feature. First, as the box or cube size increases, the variance decreases. Thus, the same image will have lower lacunarities as the box size increases. When lacunarities decrease toward unity (i.e., $\ln[L(r)] = 0$) and remain constant afterward, this suggests that r has approached the representative elementary volume (REV) of the material under investigation. For example, in Fig. 3, the size of the REV is reached when $\ln(\text{box size})$ is >2.5 (i.e., box size = 10) for the “small” image, and at 4 (box size = 63) for the “middle” image. This is consistent with what Grossman and Reinsch (2002) suggested, that is, that the REV is 10 to 30 times that of the largest feature. Second, with similar spatial patterns, the lower the fraction, the higher the lacunarity. Specifically, when $r = 1$, $L = 1/p$. Finally, with the same fraction level, a higher lacunarity represents a higher degree of clustering or clumping. Figure 3 shows the lacunarity curves for images with the same size (216 by 216 pixels), the same fraction ($p = 0.5$), but different patterns. The lacunarities are highest for the image with large-size features and vice versa. The lacunarities decrease quickly toward zero when approaching the block size (36 by 36 pixels for the large, 6 by 6 pixels for the middle, and 1 by 1 pixel for the small images in Fig. 3). The hierarchy image in Fig. 3 is similar to a true fractal and therefore its lacunarity curve is nearly linear except near the point where $\ln(\text{box size})$ is equal to zero (Plotnick et al., 1993). As Allain and Cloitre (1991) pointed out, the lacunarity curve for a fractal should be a straight line.

To calculate the two-dimensional lacunarities, boxes with side dimensions of 1, 2, 3, 4, 6, 10, 16, 25, 40, 63, 100, 123, 158, 251, 398, and 631 pixels were used to “glide” over the two-dimensional images (820 by 820 pixels). Three images, the 20th, 40th, and 60th of the 123 images obtained at each scanned position, were selected to calculate the two-dimensional lacunarities, the mean of which was used to represent the overall two-dimensional lacunarities for each position. Because of the limited number of images (i.e., 123), a narrower size range of cubes (with side dimensions of 1, 2, 3, 4, 6, 10, 16, 25, 40, and 63 voxels) was used to “glide” through the three-dimensional images (820 by 820 by 123 voxels) and calculate the three-dimensional lacunarities. The lacunarities of the third-iteration prefractal of the Menger sponge and the binary image with small, randomly distributed pores were also calculated as references.

Since the overall shapes of the lacunarity curves depend on the degree of clustering or clumping and are independent of the fraction (Plotnick et al., 1993), the relative

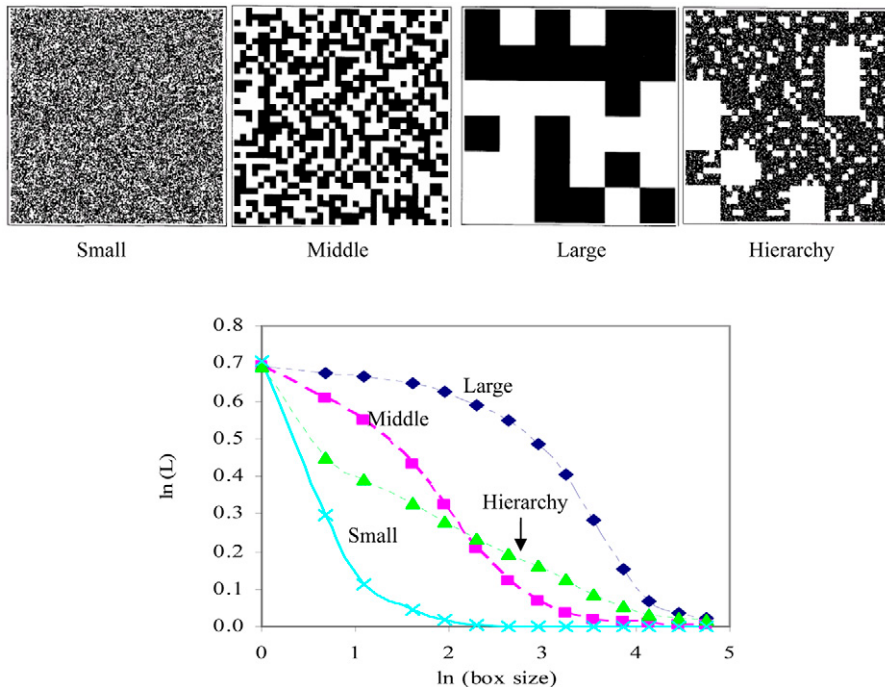


FIG. 3. Lacunarity (L) curves for images with the same size (216 by 216 pixels), the same fraction ($p = 0.5$), but different patterns (i.e., small-, middle-, and large-size features, and hierarchy with three size levels similar to fractals) (modified from Plotnick et al., 1993).

lacunarity function on a logarithmic scale, RLF, was defined and calculated as

$$RLF = -\frac{\ln[L(r)]}{\ln(p)} \quad [7]$$

Therefore, the influence of the fraction on lacunarity was excluded so that the shape of the lacunarity function and its corresponding spatial pattern could be better evaluated. In addition, integration was used to calculate the area under the curve of the RLF. This parameter represents the overall lacunarity level and provides a single value to facilitate comparisons among the different features.

To compare the fractal codimension and lacunarity in different dimensions (i.e., two and three dimensions), the TTEST procedure in MATLAB (MathWorks, Natick, MA) was used with a probability level of 0.01.

Results and Discussion

Reconstruction of Macropore Network and Tracer Distribution

Both the macropore network and solute distribution in the soil column varied considerably with depth (Fig. 1 and 4). This is in part due to the horization and different soil structures in this soil (Fig. 1). The macropores in the Ap1 horizon were rather evenly distributed compared with the two subsurface horizons. The macropores formed by earthworm burrows and roots constituted a significant proportion of the Ap2 and Bt horizons and their interface (Ap2–Bt). The macropores formed by earthworms were relatively large, round in shape, and highly continuous. While the macropores formed by roots were also highly continuous and round in shape, they were much smaller and more variable in size. Interaggregate macropores, such as those formed by freezing and thawing or wetting and drying, were generally smaller and more randomly and less continuously distributed in the soil column. As expected, the overall macroporosity in this soil column decreased dramatically from about 12% in the Ap1 horizon to <4% in the Ap2 horizon and increased slightly to >4% in the Bt horizon (Table 2). The lowest macroporosity in the Ap2 horizon can be related to the platy structure of this plow-pan layer.

Although the entire soil column was saturated from the bottom up gradually for four consecutive days, 3.9 to 17.8% of the

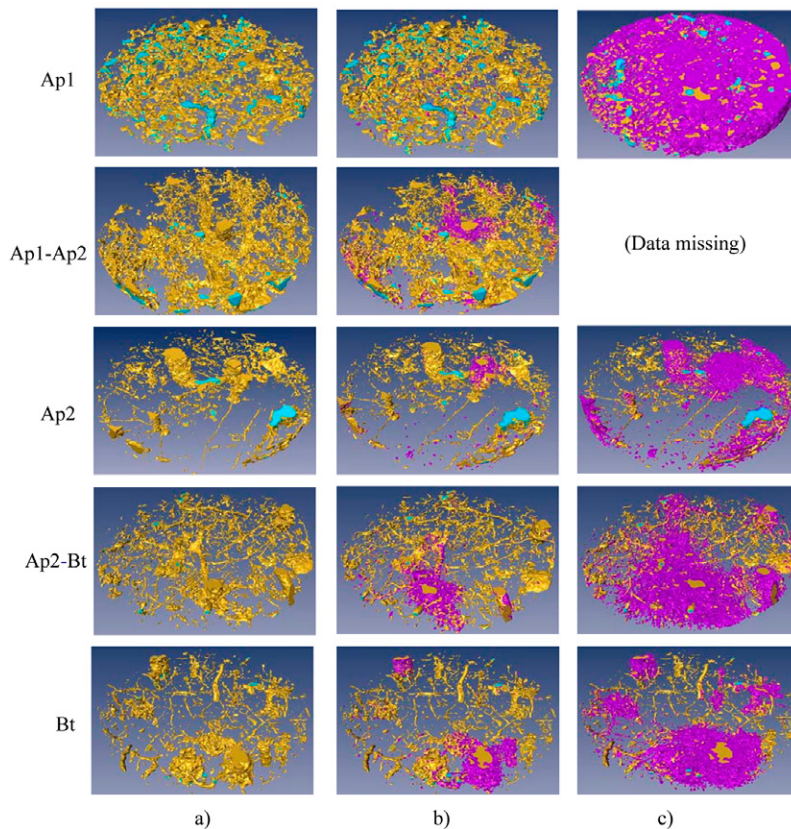


FIG. 4. A three-dimensional visualization of the five scanned positions in the soil column: (a) macropores under saturated condition before the tracer experiment (yellow color indicates water-filled pores and blue color indicates air-filled pores), (b) tracer distribution (in pink color) at 6 min, and (c) tracer distribution (in pink color) at 78 min. Note that the tracer distribution at 6 min in the Ap1 horizon was partly overlapped by the macropores and thus cannot be clearly seen.

TABLE 2. Properties of soil macropore networks and solute distributions in the soil column studied. The numbers in parentheses are one standard deviation, R^2 is the coefficient of determination (i.e., the goodness of fits associated with the estimation of the fractal dimensions), and RLF is relative lacunarity function.

Property	Ap1	Ap1–Ap2	Ap2	Ap2–Bt	Bt
Macroporosity, %	12.37	11.61	3.58	4.07	4.11
Entrapped air, %	2.20	1.45	0.34	0.16	0.55
Mean hydraulic radius, mm	0.026	0.026	0.039	0.033	0.037
Tracer volume at 6 min, %	7.79	4.73	1.53	3.37	2.59
Tracer volume at 78 min, %	41.43	No data	7.59	11.67	8.04
Mean fractal dimension in two dimensions					
Macropores	1.52 (0.04) $R^2 = 0.99$	1.41 (0.03) $R^2 = 0.99$	1.26 (0.05) $R^2 = 0.98$	1.23 (0.03) $R^2 = 0.99$	1.26 (0.01) $R^2 = 0.98$
Tracer at 6 min	1.32 (0.08) $R^2 = 0.99$	1.20 (0.05) $R^2 = 0.99$	1.17 (0.06) $R^2 = 0.96$	1.12 (0.07) $R^2 = 0.98$	1.15 (0.01) $R^2 = 0.97$
Tracer at 78 min	1.75 (0.04) $R^2 = 1$	No data	1.35 (0.05) $R^2 = 0.99$	1.40 (0.04) $R^2 = 0.99$	1.33 (0.01) $R^2 = 0.99$
Fractal dimension in three dimensions					
Macropores	2.61 $R^2 = 0.991$	2.47 $R^2 = 0.993$	2.23 $R^2 = 0.990$	2.28 $R^2 = 0.987$	2.26 $R^2 = 0.981$
Tracer at 6 min	2.41 $R^2 = 0.987$	2.26 $R^2 = 0.989$	2 $R^2 = 0.990$	2.14 $R^2 = 0.983$	2.03 $R^2 = 0.988$
Tracer at 78 min	2.86 $R^2 = 0.996$	No data	2.35 $R^2 = 0.997$	2.50 $R^2 = 0.996$	2.37 $R^2 = 0.997$
Integration of RLF in two dimensions					
Macropores	1.94	2.38	2.72	2.42	2.64
Tracer at 6 min	2.10	2.50	3.11	2.62	2.92
Tracer at 78 min	1.70	No data	2.77	2.52	2.76
Integration of RLF in three dimensions					
Macropores	1.69	2.13	2.61	2.19	2.54
Tracer at 6 min	1.88	2.33	3.01	2.50	2.82
Tracer at 78 min	1.45	No data	2.66	2.36	2.64

macropores contained air bubbles (Table 2). Hence, we termed the soil moisture condition “saturated” (<100% pore space filled with water) instead of “saturated” (100% pore space filled with water). After introducing the KI tracer for 6 min, the solute was distributed relatively evenly (but not completely) in the Ap1 horizon (Fig. 4b). It then moved preferentially through the underlying horizons via some continuous macropores. The earthworm burrows and root channels were highly active in the flow and transport because of their high continuity and low tortuosity. The solute distribution in the subsurface was clustered along the active macropores (Fig. 4b). After 78 min, the solute had moved from the main preferential flow paths to connected local macropores and the surrounding matrix (Fig. 4c). Again, the solute distribution in the Ap1 horizon was more homogeneous. The effluent breakthrough curve showed a quick increase in relative concentration after the tracer was first introduced. Relative concentrations were ~78% for the central outflow and ~65% for the outer flow at approximately one pore volume, indicating a high degree of preferential flow.

Fractal Dimensions

The fractal dimensions of the macropore network and solute distribution (in both two and three dimensions) varied with depth in a way similar to the changes in macroporosity and tracer volume percentage (Table 2). The fractal dimensions of the Ap1 horizon (1.52 in two dimensions and 2.61 in three dimensions for macropores, 1.32 in two dimensions and 2.41 in three dimensions for the tracer distribution at 6 min, and 1.75 in two dimensions and 2.86 in three dimensions for the tracer distribution at 78 min) were always the highest. The three-dimensional fractal dimensions of the Ap2 horizon were always the lowest, i.e., 2.23 for macropores, 2.00 for the tracer distribution at 6 min, and 2.35 for the tracer distribution at 78 min. These values are within the range of fractal dimensions reported in the literature for macropore networks (Peyton et al., 1994; Gantzer and Anderson, 2002; Perret et al., 2003) and for tracer distributions (Hatano et al., 1992). They suggest an underlying pore–solid fractal model (Perrier et

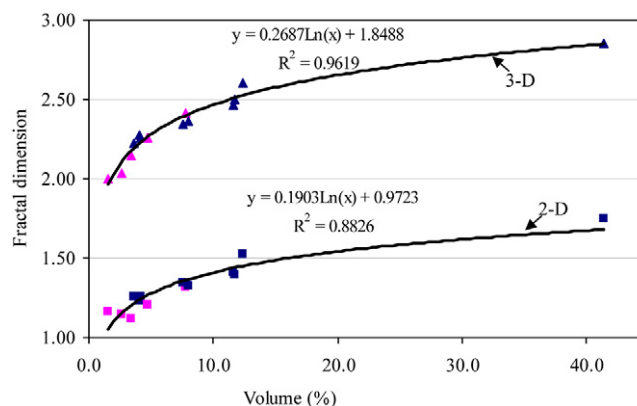


FIG. 5. The relationship between the fractal dimension of the macropores (pink) and tracer distributions (blue) in two (2-D) or three dimensions (3-D) and the volume percentage of the macropores and tracer distributions.

al., 1999) for the soil structure in this column. The fractal dimensions in two and three dimensions were closely correlated with macroporosity and tracer volume percentage with time. Positive logarithmic trends were observed in both two ($R^2 = 0.88$) and three dimensions ($R^2 = 0.96$) (Fig. 5), which is consistent with the results of Peyton et al. (1994), Perret et al. (2003), and Zeng et al. (1996). Table 3 shows significant linear correlations between macroporosity and the tracer volume percentages at 6 and 78 min, and between the two- and three-dimensional fractal dimensions of the macropore network and tracer distribution. These results suggest that macropores were the major control on the flow and transport processes observed in this study.

The difference between the fractal codimensions in two and three dimensions was not statistically significant in this study at the 0.01 probability level for both the macropores and the tracer volume percentages at 6 and 78 min. This result is consistent with research reported by Gibson et al. (2006), and indicates that the three-dimensional fractal codimension can be estimated by the two-dimensional fractal codimension.

TABLE 3. Pearson correlation coefficients between different soil properties investigated in this study. Since the difference between the fractal codimensions in two and three dimensions was not significant, only fractal dimensions in three dimensions are included here (RLF is relative lacunarity function).

Soil property	no.	1	2	3	4	5	6	7	8	9	10	11	12
Macroporosity	1												
Mean hydraulic radius	2	-0.94*											
Tracer volume at 6 min	3	0.89*	-0.89*										
Tracer volume at 78 min	4	1.00**	NS†	0.98*									
Fractal dimension in three dimensions													
Macropores	5	0.97**	-0.92*	0.97**	0.98**								
Tracer at 6 min	6	0.92*	-0.95*	0.98*	0.98*	0.98**							
Tracer at 78 min	7	0.97*	-0.98*	0.99**	0.99*	0.98*	1.00**						
Integration of RLF in two dimensions													
Macropores	8	NS	NS	-0.99**	-0.97*	-0.93*	-0.97**	-1.00**					
Tracer at 6 min	9	NS	0.92*	-0.98**	NS	-0.93*	-0.98**	-0.98*	0.98**				
Tracer at 78 min	10	-0.98*	0.97*	-0.99*	-0.99**	-0.99*	-0.99**	-1.00**	0.99*	0.96*			
Integration of RLF in three dimensions													
Macropores	11	NS	0.90*	-0.97**	NS	-0.92*	-0.98**	-0.99*	0.99**	0.99**	0.97*		
Tracer at 6 min	12	NS	0.93*	-0.98**	-0.95*	-0.94*	-0.99**	-0.98*	0.98**	1.00**	0.97*	0.99**	
Tracer at 78 min	13	-0.98*	0.97*	-0.99*	-0.99**	-0.99*	-1.00**	-1.00**	0.99**	0.97*	1.00**	0.98*	0.98*

* Significant at $P < 0.05$.

** Significant at $P < 0.01$.

† NS, not significant at $P < 0.05$.

Lacunarities

Relative lacunarity functions for the three-dimensional macropore network and tracer distribution with time are shown in Fig. 6. The RLFs for the third-iteration prefractal Menger sponge (labeled *fractal*) and the image with small, randomly distributed pores (labeled *random*) are also included in Fig. 6 for reference. As expected, the RLF of the third-iteration prefractal Menger sponge was nearly linear except where $\ln(\text{cube size})$ was close to zero. The RLF for the image with small, randomly distributed pores declined very quickly as cube size increased. In contrast, all of the RLFs for the macropore networks were rather convex in shape (Fig. 6a), suggesting that middle- to large-size pores existed and that the pore size distribution was not random. The RLFs for the macropore networks, however, varied with soil depth. The RLF of the macropore network in the Ap1 horizon was the lowest among the five positions investigated because of its relatively evenly distributed but smaller pores (Fig. 4a). On the other hand, the RLFs for the macropore networks in the Ap2 and Bt horizons were higher than those of other horizons, probably due to the large and continuous biopores in both horizons (Fig. 4). The RLFs for the macropore networks at the two horizon boundaries (i.e., Ap1–Ap2 and Ap2–Bt) fell between the curves for the Ap1 and Ap2 or Bt horizons. The shapes of the lacunarity curves imply that the macropore network may be close to fractal within a given size range, and Euclidean outside of that range.

The relative lacunarity curves for the tracer distribution at 6 min in the Ap1 horizon and the two boundaries were almost linear (Fig. 6b), again suggesting the existence of self-similarity across a given size range. In the Ap2 and Bt horizons, the RLFs for the tracer distribution at 6 min were markedly convex (Fig. 6b), reflecting a higher degree of clumping of the tracer in and around the large biopores and the occurrence of apparent preferential flow (Fig. 4b). The relative lacunarities for the tracer distribution at 78 min generally decreased with more advection and diffusion time (Fig. 6c). The RLF of the tracer distribution at 78 min in the Ap1 horizon became concave as more tracer moved into smaller pores (Fig. 6c and 4c). The relative lacunarity curve for the tracer distribution in the Ap2 and Bt horizons was close to linear, with more pores being filled with the tracer. This suggests more similarity to a true fractal in the spatial organization of the solute. Overall, the RLFs for the tracer distributions at 6 and 78 min were closer to an ideal fractal than those of the macropores within the range of cube sizes investigated.

The integrations of the RLFs in two and three dimensions are presented in Table 2. Interestingly, the integration of the RLF for the image with small, randomly distributed pores was equal to unity. The integration of the RLF quantifies the overall level of relative lacunarity and is sensitive to the size distribution of macropores and the degree of clustering or clumping of the tracer distribution. The integrations of the RLFs for the macropores were positively correlated with the mean hydraulic radius (Table 3). Accordingly, the integrations of the RLF for the macropores in the Ap2 horizon were the highest among the five positions scanned, 2.72 in two dimensions and 2.61 in three dimensions, which corresponded with the largest mean hydraulic radius (0.039 mm) among the five positions. The integrations of the RLFs for the tracer distribution at 6 min were higher than those for the tracer distribution at 78 min, corresponding to the clustering of the tracer in and around large biopores at early times.

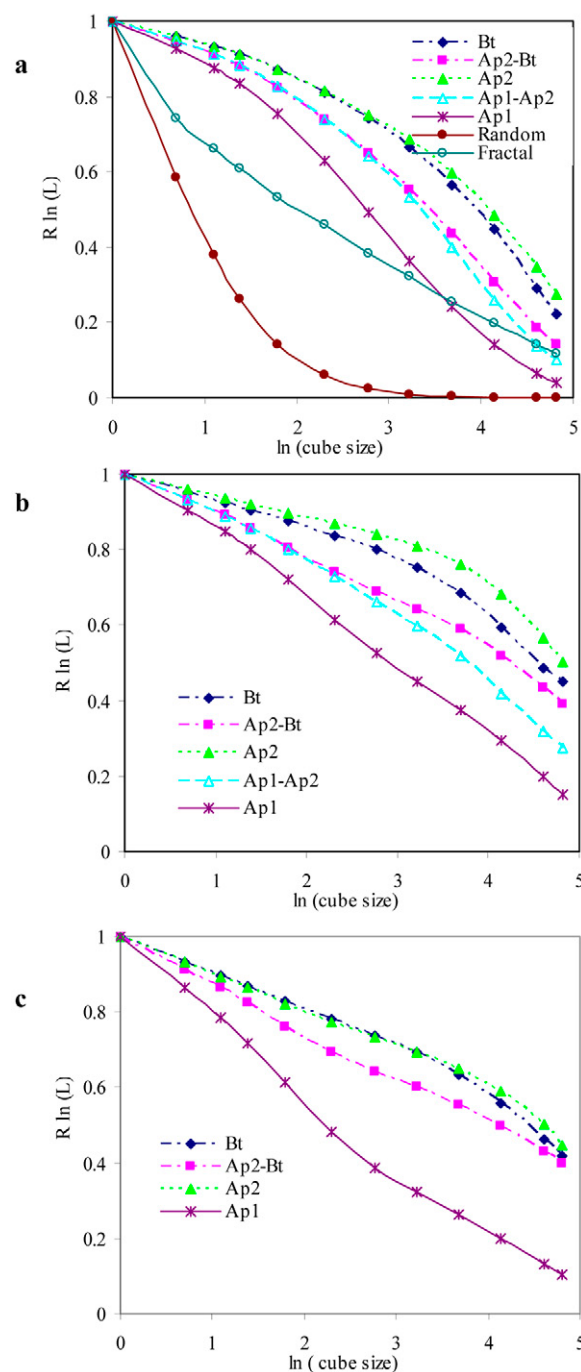


FIG. 6. Log-log plot of three-dimensional relative lacunarities [$R \ln(L)$] vs. gliding cube size for (a) the macropore network, (b) tracer distribution at 6 min, and (c) tracer distribution at 78 min. Also shown in (a) are the relative lacunarity functions (RLFs) for the third-iteration prefractal of a Menger sponge (labeled *fractal*) and for the image with small, randomly distributed pores (labeled *random*).

The integrated RLF parameter is a helpful index to evaluate the degree of preferential flow and solute transport in different soils with similar flow conditions.

Two-dimensional RLFs for the macropore network and tracer distribution were very close to the three-dimensional values within the same size range (≤ 63) (Fig. 6 and 7). The statistical difference of the integration of the RLF in two and three dimensions was significant at the 0.01 probability level, however, because the two-dimensional relative lacunarities were slightly but

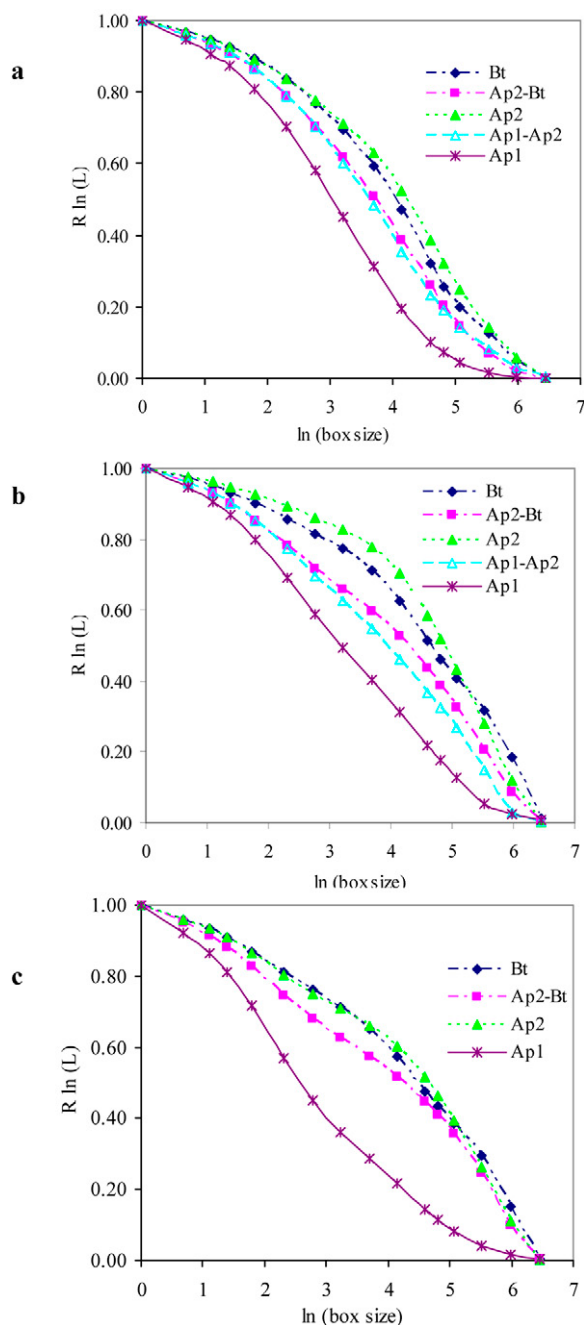


FIG. 7. Log-log plot of two-dimensional relative lacunarities [$R \ln(L)$] vs. gliding box size for (a) the macropore network, (b) tracer distribution at 6 min, and (c) tracer distribution at 78 min.

consistently greater than the three-dimensional values for both the macropores and the tracer distributions at 6 and 78 min (Table 2). According to the lacunarity curves, the REV had not been reached for the three-dimensional macropore networks within the size range investigated (cube size ≤ 63), probably because of their high heterogeneity (Fig. 6); however, the REV might have been met for the two-dimensional macropore network, especially for the Ap1 horizon since the relative lacunarity decreased toward zero at the point where $\ln(\text{box size}) = 5.8$ (i.e., box size = 330, or 26 mm) and remained very low afterward (Fig. 7). Table 3 also shows a strong negative linear correlation between the fractal dimension and the integration of the RLF in both two and three dimensions. Armatas et al. (2002) found a similar relationship

between the fractal dimension and the lacunarity. More research is needed, however, to investigate whether such a relationship exists for other soils.

Summary and Conclusions

Soil macropore structure and tracer transport in real time were reconstructed using an industrial CT scanner at five representative positions of an intact structured soil column (i.e., the Ap1, Ap2, and Bt horizons and their boundaries). Each position in the soil column displayed a different macropore characteristic and flow pattern. The earthworm burrows and root channels were highly effective in facilitating water flow and tracer transport because of their high continuity and low resistance. Macropore volume, mean pore hydraulic radius, fractal dimension, and relative lacunarity curves in two and three dimensions were computed from the CT images. The results demonstrated the dominant control of macroporosity on flow and transport under the “saturated” experimental conditions used in this study. Positive logarithmic trends were found between the fractal dimensions in either two or three dimensions and the volume percentage of macropores and tracer distributions. There was no statistically significant difference between the fractal codimensions in two and three dimensions. The RLFs of the macropore network and the tracer distribution with time varied with soil depth. The tracer distributions with time were closer to true fractals than the macropore networks based on their RLFs. The three-dimensional relative lacunarities were slightly lower than the two-dimensional values.

Overall, the lacunarity has several positive attributes: (i) the lacunarity function is sensitive to structural differences and reflects the size and spatial distribution of the features—it thus has the potential to differentiate one type of structure or pattern from another; (ii) the lacunarity function, as opposed to one single value (e.g., a fractal dimension), can be calculated to obtain size-dependent information; (iii) the lacunarity parameter can be applied to both fractal and nonfractal features; (iv) lacunarity is helpful in determining if self-similarity exists and if a REV for a porous media can be defined; and (v) the algorithm used to calculate the lacunarity is simple to implement. Since the lacunarity is scale dependent, caution is needed when comparing structures at different image resolutions. The relationship between the fractal dimension and lacunarity is worthy of further study. In particular, the lacunarity functions may be coupled with fractal parameters, as indicated in Eq. [1], to better model heterogeneous soil structural functions.

ACKNOWLEDGMENTS

We thank Dr. Avrami Grader and Dr. Phil Halleck of Penn State's Center for Quantitative Imaging for their help and advice on the CT scanning. This work was partially supported by the USDA National Research Initiative (Grant no. 2002-35102-12547) and the USDA Higher Education Challenge Competitive Grants Program (Grant no. 2006-38411-17202). We appreciate the valuable comments provided by Dr. Ed Perfect that helped enhance the quality of this paper.

References

- Allain, C., and M. Cloitre. 1991. Characterizing the lacunarity of random and deterministic fractal sets. *Phys. Rev. A* 44:3552–3558.
- Anderson, A.N., J.W. Crawford, and A.B. McBratney. 2000. On diffusion in fractal soil structures. *Soil Sci. Soc. Am. J.* 64:19–24.
- Anderson, S.H., R.L. Peyton, and C.J. Gantzer. 1990. Evaluation of constructed and natural soil macropores using x-ray computed tomography. *Geoderma* 46:13–29.

- Anderson, S.H., R.L. Peyton, J.W. Wigger, and C.J. Gantzer. 1992. Influence of aggregate size on solute transport as measured using computed tomography. *Geoderma* 53:387–398.
- Armatas, G.S., K.M. Kolonia, and P.J. Pomonis. 2002. Morphometry of porous solids: Lacunarity, fractal dimensions, connectivity, and some topological similarities with neurons. *Langmuir* 18:10421–10429.
- Baveye, P., and C.W. Boast. 1998. Concepts of “fractals” in soil science: Demixing apples and oranges. *Soil Sci. Soc. Am. J.* 62:1469–1470.
- Baveye, P., C. Boast, O. Susumu, J.I. Parlange, and T. Steenhuis. 1998. Influence of image resolution and thresholding on the apparent mass fractal characteristics of preferential flow patterns in field soils. *Water Resour. Res.* 34:2783–2796.
- Beven, K., and P. Germann. 1982. Macropores and water flow in soils. *Water Resour. Res.* 18:1311–1325.
- Gantzer, C.J., and S.H. Anderson. 2002. Computed tomographic measurement of macroporosity in chisel-disk and no-tillage seedbeds. *Soil Tillage Res.* 64:101–111.
- Gibson, J.R., H. Lin, and M.A. Bruns. 2006. A comparison of fractal analytical methods on 2-and 3-dimensional computed tomographic scans of soil aggregates. *Geoderma* 134:335–348.
- Giménez, D., E. Perfect, W.J. Rawls, and Ya. Pachepsky. 1997. Fractal models for predicting soil hydraulic properties: A review. *Eng. Geol.* 48:161–183.
- Grossman, R.B., and T.G. Reinsch. 2002. Bulk density and linear extensibility. p. 201–254. *In* J.H. Dane and G.C. Topp (ed.) *Methods of soil analysis*. Part 4. SSSA Book Ser. 5. SSSA, Madison, WI.
- Hatano, R., N. Kawamura, J. Ikeda, and T. Sakuma. 1992. Evaluation of the effect of morphological features of flow paths on solute transport by using fractal dimensions of methylene blue staining pattern. *Geoderma* 53:31–44.
- Heijs, A.W.J., J. Delange, J.F.T. Schoute, and J. Bouma. 1995. Computed tomography as a tool for nondestructive analysis of flow patterns in macroporous clay soils. *Geoderma* 64:183–196.
- Jassogne, L., A. McNeill, and D. Chittleborough. 2007. 3D-visualization and analysis of macro- and meso-porosity of the upper horizons of a sodic, texture-contrast soil. *Eur. J. Soil Sci.* 58:589–598.
- Kim, J.W., E. Perfect, and H. Choi. 2007. Anomalous diffusion in two-dimensional Euclidean and prefractal geometrical models of heterogeneous porous media. *Water Resour. Res.* 43:W01405, doi:10.1029/2006WR004951.
- Luo, L.F., H.S. Lin, and P. Hellack. 2008. Quantifying soil structure and preferential flow in intact soil using x-ray computed tomography. *Soil Sci. Soc. Am. J.* 72:1058–1069.
- Mandelbrot, B.B. 1982. *The fractal geometry of nature*. W.H. Freeman, New York.
- Pendleton, D.E., A. Dathe, and P. Baveye. 2005. Influence of image resolution and evaluation algorithm on estimates of the lacunarity of porous media. *Phys. Rev. E* 72(4):041306, doi:10.1103/PhysRevE.72.041306.
- Perfect, E., and B.D. Kay. 1995. Applications of fractals in soil and tillage research: A review. *Soil Tillage Res.* 36:1–20.
- Perret, J.S., S.O. Prasher, and A.R. Kacimov. 2003. Mass fractal dimension of soil macropores using computed tomography: From the box-counting to the cube-counting algorithm. *Geoderma* 54:569–579.
- Perret, J., S.O. Prasher, A. Kantzas, and C. Langford. 1999. Three-dimensional quantification of macropore networks in undisturbed soil cores. *Soil Sci. Soc. Am. J.* 63:1530–1543.
- Perret, J., S.O. Prasher, A. Kantzas, and C. Langford. 2000. A two-domain approach using CAT scanning to model solute transport in soil. *J. Environ. Qual.* 29:995–1010.
- Perrier, E., N. Bird, and M. Rieu. 1999. Generalizing the fractal model of soil structure: The pore–solid fractal approach. *Geoderma* 88:137–164.
- Perrier, E., M. Rieu, G. Sposito, and G. de Marsily. 1996. Models of the water retention curve for soils with a fractal pore size distribution. *Water Resour. Res.* 32:3025–3031.
- Peyton, R.L., C.J. Gantzer, S.H. Anderson, B.A. Haefner, and P. Pfeiffer. 1994. Fractal dimension to describe soil macropore structure using x-ray computed tomography. *Water Resour. Res.* 30:691–700.
- Plotnick, R.E., R.H. Gardner, W.W. Hargrove, K. Prestegard, and M. Perlmutter. 1996. Lacunarity analysis: A general technique for the analysis of spatial patterns. *Phys. Rev. E* 53:5461–5468.
- Plotnick, R.E., R.H. Gardner, and R.V. O'Neill. 1993. Lacunarity indices as measures of landscape texture. *Landscape Ecol.* 8:201–211.
- Rasband, W. 2002. *ImageJ: Image processing and analysis in Java*. Available at rsb.info.nih.gov/ij/index.html (verified 15 Oct. 2008). Res. Serv. Branch, Natl. Inst. of Health, Bethesda, MD.
- Young, I.M., J.W. Crawford, A. Anderson, and A. McBratney. 1997. Comment on number-size distributions, soil structure, and fractals. *Soil Sci. Soc. Am. J.* 61:1799–1800.
- Young, I.M., J.W. Crawford, and C. Rappoldt. 2001. New methods and models for characterising structural heterogeneity of soil. *Soil Tillage Res.* 61:33–45.
- Zeng, Y., C.J. Gantzer, R.L. Payton, and S.H. Anderson. 1996. Fractal dimension and lacunarity of bulk density determined with x-ray computed tomography. *Soil Sci. Soc. Am. J.* 60:1718–1724.