



## Superstatistics of hydro-climatic fluctuations and interannual ecosystem productivity

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[1] Ecosystems driven by hydro-climatic fluctuations at different time scales can be interpreted as non-equilibrium dynamical systems. Here we explore the propagation of daily and interannual rainfall fluctuations through the soil-plant system using the theory of superstatistics. With the help of simplified stochastic models of rainfall, we show how interactions of daily and interannual rainfall fluctuations may qualitatively change the probability distributions of rainfall toward higher frequencies of extreme droughts and intense storms. This in turn is likely to induce marked changes in productivity of mesic ecosystems, while more xeric ecosystems might be insensitive or even benefit from them. This study provides a theoretical basis for predictions of ecosystem responses to the increased precipitation variability expected in future North American climate regimes. **Citation:** Porporato, A., G. Vico, and P. A. Fay (2006), Superstatistics of hydro-climatic fluctuations and interannual ecosystem productivity, *Geophys. Res. Lett.*, 33, L15402, doi:10.1029/2006GL026412.

[2] Hydro-climatic processes fluctuate on a wide range of time scales. Such fluctuations are often unpredictable from the standpoint of living systems, and adapting to them presents one of the grand challenges for the survival of individuals and for the stability and function of communities and ecosystems. A critical question is how fluctuations in hydro-climatic variables at different time scales change the frequency and intensity of extreme events, and how those changes affect ecosystems. There is a growing consensus that an increase in frequency and intensity of extreme hydro-climatic events will represent one of the most potent features of global climate change, with major possible social and biotic impacts [Allen and Ingram, 2002; Easterling et al., 2000; Weltzin et al., 2003]. Recent field experiments [Gutschick and BassiriRad, 2003; Knapp et al., 2002; Knapp and Smith, 2001] and modeling analysis [Porporato et al., 2004] have shown that extreme hydro-climatic events are capable of generating intense impacts on organisms in water-limited ecosystems.

[3] Understanding the adaptations and responses of organisms and ecosystems to hydro-climatic variability and extremes requires understanding the probabilistic structure of the critical hydro-climatic drivers. ‘Superstatistics’ can be applied to explain the qualitative variation in the distribution and extremes of external forcing varying at

multiple hierarchical scales [Beck, 2001; Beck and Cohen, 2003], describing the superposition of ‘local’, or short time-scale fluctuations, that combine to produce a higher-order overall distribution. The latter may be qualitatively different from the ‘local’ one, with dramatic repercussions on the statistics of extreme events. In mathematical terms, imagine that the intermittent, non-linear dynamics of a representative state variable,  $z$  (rainfall depth or interarrival time between storms in our case), at short time scales is described by the probability density function (pdf),  $p(z; \rho)$ , where  $\rho$  is a parameter. If  $\rho$  itself is varying so that the external forcing fluctuates at longer time scales,  $z$  is driven to a different/new ‘local’ equilibrium. When the timescale of variation in  $\rho$  is much larger than the one of  $z$ , the probability distribution function of  $z$  can be obtained as a superposition of temporary equilibria, that is,

$$p(z) = \int p(z|\rho)\varphi(\rho)d\rho, \quad (1)$$

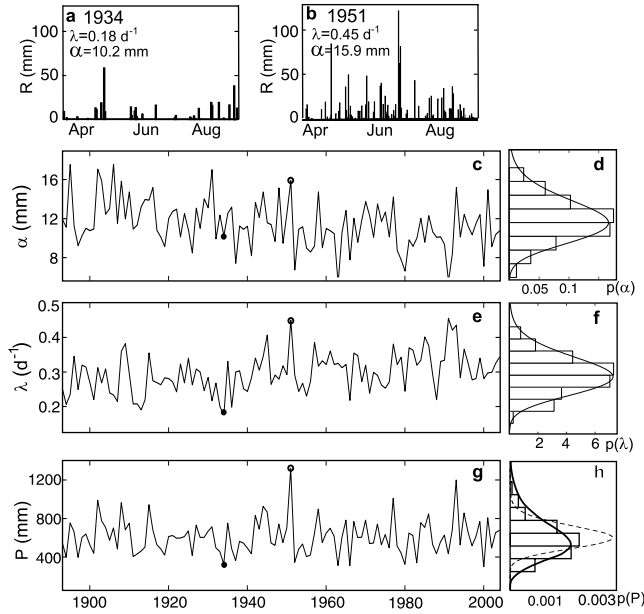
where  $p(z|\rho)$  is the conditional probability of  $z$  given  $\rho$ , while  $\varphi(\rho)$  is the distribution of  $\rho$  [Beck, 2001; Benjamin and Cornell, 1970].

[4] Water-limited ecosystems are excellent examples of externally-forced, non-equilibrium systems, and the conceptual framework of superstatistics can be readily applied to them. In fact, changes in the hydro-climatic regime from growing season to growing season are generally controlled by large-scale circulation patterns that are well known for their interannual and interdecadal variability [Goodin et al., 2003]. This is especially true for rainfall, where daily events are a pulsing process with random interarrival times and intensities (e.g., Figures 1a and 1b), with distributional statistics exhibiting marked interannual variability (Figures 1c, 1e, and 1g). Ecosystems tend to reach equilibrium with the daily forcing within each growing season, while year by year the interannual variability in daily forcing drives ecosystems toward new ‘local’ equilibria. If the adaptation times of the systems are relatively short, as it is often the case for water-limited ecosystems, their ‘global’ statistics describing the overall pdf of rainfall result from a superposition of the ‘local’ equilibria determined by fluctuations in the pdfs of daily rainfall (equation (1)).

[5] To proceed quantitatively we use a simple stochastic model of daily rainfall [Laio et al., 2001], that describes the occurrence of rainfall as a compound Poisson process with frequency of events  $\lambda$ . The distribution of times  $\tau$  between precipitation events is thus an exponential with mean  $1/\lambda$ ,  $p_\tau(\tau|\lambda) = \lambda \exp(-\lambda\tau)$ , and exponentially distributed rainfall amounts  $h$  with mean  $\alpha$ ,  $p_h(h|\alpha) = 1/\alpha \exp(-h/\alpha)$ . This model fits the observed daily rainfall data for individual growing seasons quite well. However, when applied to all

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**Figure 1.** Growing season (April to September) rainfall regime at Manhattan, KS ( $39^{\circ}12'N$ ,  $96^{\circ}35'W$ ; coop. id. 144972; data available online at [www.ncdc.noaa.gov](http://www.ncdc.noaa.gov)). Daily precipitation during (a) a very dry and (b) a very wet growing season. (c) Time series of mean depth of rainfall events,  $\alpha$  (mean 11.4 mm, standard deviation, s.d., 2.4 mm); (e) mean rate of storm arrival,  $\lambda$  (mean 0.30 1/d, s.d. 0.06 1/d); (g) total rainfall during the growing season,  $P$  (mean 612.4 mm, s.d. 180.3 mm). Frequency distribution and fitted two-parameter gamma distributions of (d)  $\alpha$  and (f)  $\lambda$  (parameters:  $a_h = 21.8$ ,  $b_h = 1.90$  1/mm;  $a_\tau = 27.4$ ,  $b_\tau = 91.9$  d). (h) Frequency distribution of total precipitation,  $P$ , and corresponding theoretical model (solid line); the pdf obtained assuming no interannual variability is also plotted for comparison (dashed line).

years combined, the model underestimates rainfall variability, based on the comparison of measured total precipitation with the one modeled including only daily fluctuations (Figure 1h). The strong interannual variability in the mean rainfall frequency and event depth ( $\lambda$  and  $\alpha$ , respectively) is even more evident in Figures 1c and 1e. Formally, interannual variability could be defined as the changes in the year-by-year statistics that cannot simply be explained as the result of different realizations of the same stochastic process describing intra-annual fluctuations. Interannual fluctuations in rainfall depth and frequency can be assumed to be independent; a good model for such interannual fluctuations (Figures 1d and 1f) [D'Odorico *et al.*, 2000] is a two-parameter gamma distribution

$$g_x(x) = \frac{b_x^{a_x}}{\Gamma(a_x)} x^{a_x-1} e^{-b_x x}, \quad (2)$$

where  $x$  stands for either  $\alpha$  or  $\lambda$ ,  $b_x$  is the scale parameter, and  $a_x$  is the shape parameter of the distribution.

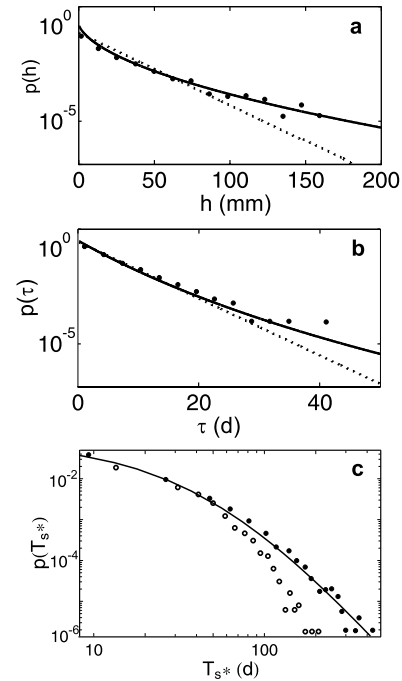
[6] The superstatistics of rainfall depth and interval between storms can be obtained analytically by means of equation (1), using the exponential distributions for daily fluctuations conditional on the parameter values, the dis-

tributions of which are described by equation (2). Accordingly, the pdf of rainfall depth  $h$  can be shown to be

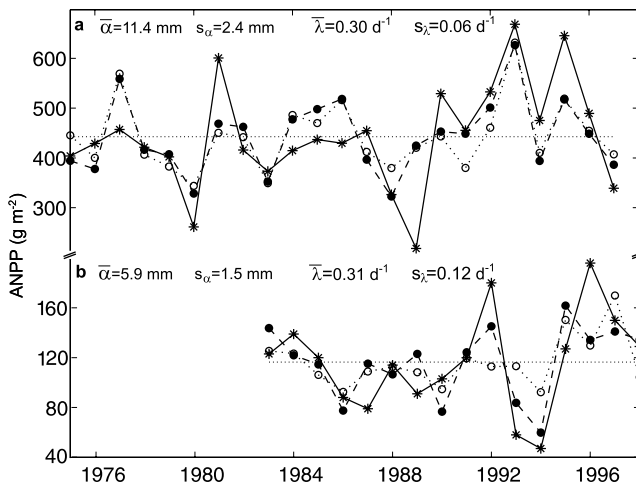
$$p(h) = \frac{2b_h^{(a_h+1)/2}}{\Gamma(a_h)} h^{(a_h-1)/2} K_{1-a_h} \left[ 2\sqrt{b_h h} \right], \quad (3)$$

where  $K_n$  is the Bessel function of order  $n$ th [Abramowitz and Stegun, 1965]. The tail of this pdf is similar to that of a stretched exponential distribution [Sornette, 2003] and therefore intermediate between a power-law (e.g., scaling behavior) and an exponential one. Interannual variability thus increases the frequency of extreme rainfall event depths with respect to the model including only daily variability. The obtained pdf (equation (3)) well describes the observed rainfall patterns, while the exponential distribution proves to be inadequate to statistically characterize extreme storm depths of all the years combined (Figure 2a). Similarly, the pdf of the intervals between storms,  $\tau$ , including interannual variability, can be obtained as

$$p(\tau) = \frac{a_\tau b_\tau^{a_\tau}}{(\tau + b_\tau)^{a_\tau+1}}, \quad (4)$$



**Figure 2.** (a) Pdf of rainfall depths and (b) interarrival times obtained as superstatistics of daily and interannual variability (continuous lines); comparison with the exponential pdf corresponding to no interannual variability (dotted lines), and observed frequency distributions at Manhattan, KS (closed symbols). (c) Pdf of the duration of periods of plant water stress at Manhattan, KS: Monte Carlo simulation (closed symbols); fitted Pareto distribution (continuous line); pdf obtained with no interannual variability (open symbols). The parameters of the stochastic soil moisture model [Laio *et al.*, 2001] are: soil type is silty loam ( $n = 0.47$ ,  $s_h = 0.14$ ,  $s_w = 0.16$ ,  $s^* = 0.35$ ,  $s_{fc} = 0.59$ ,  $K_s = 33$  cm/d,  $b = 5$ ,  $E_w = 0.01$  cm/d) and maximum vegetation transpiration is  $E_{\max} = 0.45$  cm/d.



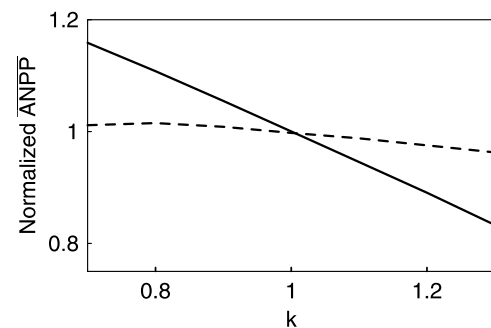
**Figure 3.** Time series of ANPP (stars) and estimated ANPP at (a) KNZ LTER site, KS, and (b) SGS LTER site, CO. Estimated ANPP is obtained with a multiple regression model using only the total rainfall over the growing season,  $P$  (open symbols), and including also the mean event depth,  $\alpha$ , and the mean interval between rainfall events,  $1/\lambda$  (close symbols). At KNZ the linear regression using the total precipitation resulted in  $r^2 = 0.40$  ( $237 + 0.30P + \xi$ ;  $\sigma_\xi = 85 \text{ g/m}^2$ ), while the multiple linear regression improved  $r^2$  up to 0.47 ( $437 - 6.2\alpha - 41\lambda^{-1} + 0.32P + \xi$ ;  $\sigma_\xi = 84 \text{ g/m}^2$ ). At SGS the linear regression with  $P$  yielded  $r^2 = 0.27$  ( $40 + 0.27P + \xi$ ;  $\sigma_\xi = 35 \text{ g/m}^2$ ), while the multiple linear regression gave  $r^2 = 0.53$  ( $173 + 28\alpha - 40\lambda^{-1} - 0.29P + \xi$ ;  $\sigma_\xi = 31 \text{ g/m}^2$ ). The random deviation from the linear regression,  $\xi$ , is practically Gaussian distributed, with zero mean and standard deviation  $\sigma_\xi$ . At both sites, the growing season is assumed to last from the beginning of April to the end of September. Annual ANPP data collected at KNZ and SGS LTER sites are available online at <http://intranet.lternet.edu/cgi-bin/anpp.pl> (PI: A.K. Knapp and M.D. Smith) [Knapp and Smith, 2001]. Note that in order to obtain meaningful statistics for KNZ we used rainfall records from Manhattan, KS, located approximately 12 km north of the LTER site. Daily rainfall records for SGS LTER site are available on line at <http://sgs.cnr.colostate.edu>.

which, interestingly, is a Pareto distribution with scale parameter  $b_\tau/a_\tau$  and shape parameter  $-1/a_\tau$  [Laherrere and Sornette, 1998]. Data from different sites in the USA show a wide range of values for  $a_\tau$ , always well above 2 (i.e., the distribution is not Lévy stable [Sornette, 2003]). The power-law tail implies more frequent extreme events with significant implications for the return time of drought periods, and thus repercussions for vegetation water stress, river streamflows, and water resource availability.

[7] Correct modeling of the hierarchical variation in rainfall frequency and depth is crucial for improving predictions of important attributes of water limited ecosystems. It has already been shown how interannual variability in rainfall can cause bimodal probability distributions of mean soil moisture [D'Odorico and Porporato, 2004; Ridolfi et al., 2000]. The distribution of soil moisture affects the distribution of plant water status which in turn directly impacts ecosystem function. In particular, rainfall daily and interannual fluctuations can be related to the duration

$T_{s^*}$  of periods in which soil moisture is below a threshold  $s^*$ , marking the onset of water stress. Figure 2c shows the probability distribution of  $T_{s^*}$  resulting from Monte Carlo simulations, using the stochastic soil moisture model [Laio et al., 2001] with and without the inclusion of interannual variability. The distribution of  $T_{s^*}$  with interannual variability is well described by a Pareto distribution (solid line in Figure 2c), the power-law behavior of which implies a higher frequency of long periods of plant water stress, resulting in reduced plant carbon assimilation and net primary productivity [Porporato et al., 2001].

[8] We illustrate these implications by examining two grassland ecosystems with different degree of water limitation, for which long series of aboveground net primary productivity (ANPP) are available: the shortgrass steppe, represented by the Shortgrass Steppe Long-Term Ecological Research (LTER) site (SGS) in northern Colorado, and the tallgrass prairie, represented by the Konza Prairie LTER site (KNZ) in eastern Kansas. ANPP at these sites is significantly correlated with interannual variability in total growing season rainfall ( $r^2 = 0.40$  and  $r^2 = 0.27$  for KNZ and SGS, respectively; Figure 3). The inclusion of mean frequency and intensity of rainfall along with total annual rainfall in a multiple linear regression improves the ANPP prediction, especially for the semi-arid shortgrass steppe site ( $r^2 = 0.47$  and  $r^2 = 0.53$  for KNZ and SGS, respectively). Climate models predict less frequent precipitation occurrence, but larger rainfall input per event due to more intense convective storms [Easterling et al., 2000]. We explored the consequences of such changes via Monte Carlo simulations by predicting ANPP using the multiple regression of Figure 3 and changing the parameters in the probability distributions of interannual variability (i.e., the two-parameter gamma distribution of equations (2)) while keeping the mean total precipitation constant. As shown in Figure 4, the results



**Figure 4.** Normalized mean ANPP under different rainfall scenarios corresponding to changes in interannual variability for constant mean rainfall totals, at KNZ (continuous line) and at SGS (dashed line). The hypothetical rainfall scenarios are generated such that the means of  $\alpha$  and  $\lambda$ ,  $\bar{\alpha}$  and  $\bar{\lambda}$ , are modified in opposite ways in order to keep the mean total precipitation constant,  $\bar{P} = \bar{\alpha} \bar{\lambda}$ . Thus values of the parameter  $k = \bar{\alpha}_{\text{projected}}/\bar{\alpha}_{\text{current}}$  higher than one correspond to increased mean intensity and reduced mean frequency of rainfall events, and vice versa for  $k < 1$ . The ANPP is obtained by means of the multiple regression models of Figure 3 that employ the annual mean depth of rainfall events  $\alpha$ , the annual mean rainfall frequency  $\lambda$ , and the cumulative precipitation during the growing season  $P$ .

predict a low impact of changes in rainfall patterns on ANPP at SGS, as opposed to a marked decrease in mean ANPP at KNZ. The latter finding agrees with results from a 4-year rainfall manipulation experiment [Knapp *et al.*, 2002]: however, the results of the regressive model cannot be directly compared to experimental data, due to the short duration of the experiment, which only provides a limited characterization of the link between interannual rainfall variability and ANPP. The diverging results between the two sites can be explained by the difference in total precipitation, which reflects a difference in mean storm depth ( $\bar{\alpha}$  for KNZ is twice that of SGS), rather than a difference in mean rate of storm arrival. As a consequence, at KNZ an increase in mean storm depth would likely result in increasing losses due to runoff and deep drainage combined with longer return intervals with greater soil moisture depletion, resulting in decreased net primary productivity, while at SGS more intense rainfall is likely to be more effectively used by the vegetation. The fact that the current vegetation at SGS is more adapted to water stress, as opposed to the more mesic KNZ prairie, could further enhance the differences in ecosystem response.

[9] In summary, we have shown that interannual variability of plant productivity is substantially explained by interannual patterns of rainfall total amount, frequency and storm depth. These patterns arise from superstatistics of rainfall at different time scales, which can lead to more likely extreme events in terms of intensity of storms and duration of droughts. Under different rainfall scenarios, our multiple regression model for ANPP as a function of rainfall patterns predicts a decrease of plant productivity, when event depth and interval among storms increase. This is particularly evident in mesic grasslands, where rainfall partitioning and plant water stress tend to be very sensitive to changes in timing and amount of rainfall, and clearly shows that the impact of interannual variability and climate change will be different depending on the type of water limitation of each ecosystem.

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