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ANALYTICAL REPRESENTATION OF CROSS-SECTION HYDRAULIC PROPERTIES

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ABSTRACT

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An alternative power function representation of hydraulic properties for one-dimensional flow routing in channels with compound cross-sections is put forward. The hydraulic properties of interest are cross-section flow area, wetted perimeter and conveyance factor. The independent variable is flow depth. The alternative power function representation is continuous with flow depth and it follows the trend of suddenly changing hydraulic property values at overbank elevation. It is applicable to a large range of complex section shapes which may include overbank channels, natural levees and minor channel irregularities such as sand bars and small terraces where traditional simple power functions have failed. The alternative power function representation is simple and highly efficient from the computational point of view. Its performance is illustrated for several actual compound cross-sections. Efficiency and extended capabilities make it an attractive procedure for one-dimensional channel flow routing in drainage networks where hydraulic properties of many cross-sections need repeated evaluation.

INTRODUCTION

Hydraulic properties of channel cross-sections (hereafter referred to as HP) are required for numerical channel flow routing. HP of interest are generally cross-sectional area, wetted perimeter (or top width) and conveyance factor. They are a function of stage, and therefore, require repeated evaluation during flow routing as stage varies with discharge. This calls for an efficient, yet accurate, scheme to quantify HP. A reliable approach is the tabular representation of HP with interpolation for flow depths between tabulated values (Cunge, 1975; Cunge et al., 1980). This procedure becomes storage intensive when a large number of cross-sections are considered such as in the hydrologic evaluation of complex drainage networks. A more expedient approach replaces the tabulated values by smooth curves and evaluates these for desired flow depths. This approach is well suited for regular concave cross-section shapes that display smooth changes in top width with flow depth. One practical and popular scheme is the power function approximation of HP with flow depth as the independent variable (Li et al., 1975; Simons et al., 1982). This representation is simple and highly efficient from the computational point of view. It allows the use of an analytical Newton–Raphson solution technique for backwater profile computations (Brown, 1982). Its disadvantage is a poor performance for irregular or compound cross-sections (Cunge, 1975; Cunge et al., 1980).

The simple power function approximation has subsequently been adapted to account for overbank flow (Brown, 1982). Two sets of coefficients are used, one for main channel flow conditions, and the other for overbank flow conditions. Even though the adaptation resulted in a better fit of the data, a discontinuity occurs when coefficients are switched as the free water surface reaches and exceeds overbank elevation. This may result in stability problems during numerical solution of the flow routing equations.

A further adaptation forced the power function to go through the HP value of the main channel at overbank elevation, thus eliminating the discontinuity (Brown, 1982). However, this can lead to serious errors in the estimation of HP values for overbank flow conditions because it forces the power function through two points, namely origin and main channel HP at overbank elevation and, therefore, leaves only one degree of freedom to approximate the actual data. Even though some of these limitations have been recognized (Posey, 1950; Cunge, 1975), the power function approximation remains an effective approach in numerical channel flow routing, particularly when a large number of cross-sections must be repeatedly evaluated as for channel flow routing in complex drainage networks.

In this paper practical numerical relations to determine representative HP for one-dimensional flow routing in channels with compound sections are presented and an alternative power function representation is put forward. The proposed alternative is more accurate than the traditional power function, and it is applicable to complex cross-sections with overbank channels where traditional power functions have generally failed. Applicability and limitations of this alternative are discussed, and the performance is demonstrated for field situations.

APPROACH AND LIMITATIONS

The HP which are considered in this paper are: cross-section flow area, wetted perimeter and conveyance factor. The power function approximation applies for fully turbulent and one-dimensional uniform open channel flow assumptions. A compound cross-section is assumed to consist of a single main channel subsection and an optional right and left overbank subsection. Natural levees may be present along the main channel. Overbank flow establishes as the free water surface in the main channel exceeds overbank or levee elevation. Minor irregularities in the shape of the compound section, such as sand bars or small terraces may be present. The proposed method is primarily targeted towards natural channel sections with smooth changes in top width with flow depth as opposed to man-made rectangular or trapezoidal canals. Examples of compound sections, natural levees, and shape irregularities for which this method applies are given throughout this paper. Finally, Manning's uniform flow formula is used to relate section geometry and channel flow parameters.

CROSS-SECTION DEFINITION AND DISCRETE REPRESENTATION

A channel section generally consists of two subsections: a main channel subsection and an optional right and/or left overbank channel subsection. For an effective numerical evaluation of its hydraulic properties, a section is discretized into simple elements. An element is defined as the portion of the section between two consecutive break points. Break points are points defined by a pair of x, y coordinates on the profile of a section as shown in Fig. 1. The x value is the horizontal distance from some arbitrary reference point, and the y value is the elevation or the vertical distance from some arbitrary reference elevation. Beginning of overbank channels are defined by the break points separating main channel and overbank subsections. When overbank elevations on both sides of the main channel are not equal, the lower of the two elevations determines the beginning of overbank flow.

A section discretization is shown in Fig. 1 for cross-section 24 on the Little Washita River, Oklahoma. The break points for the section in Fig. 1 are given in Table 1. The section is defined by a main channel flanked by two overbank channels. The main channel is about 80 m wide and the overbanks are about 3 times as wide as the main channel. Overbank channel flow begins at stage elevations above the overbank elevation of 359.6 m a.s.l.

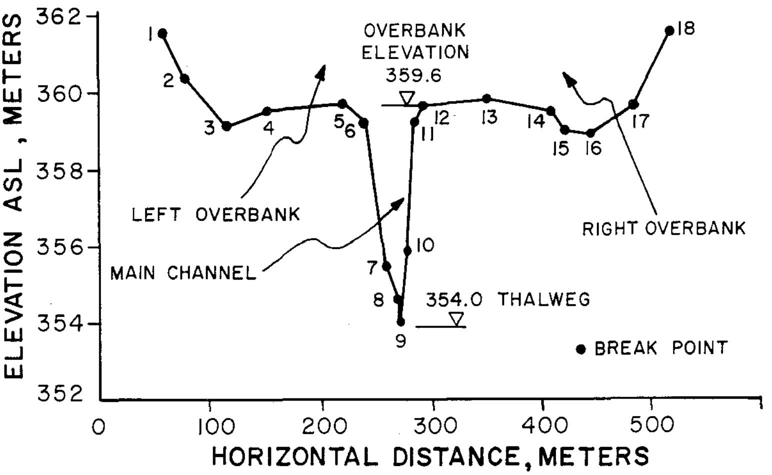

Fig. 1. Cross-section 24, Little Washita River, Oklahoma. Beginning of left and right overbank channels defined by break points 5 and 12, respectively. Horizontal to vertical distortion is 34 to 1.

TABLE 1

Break points for cross-section 24, Little Washita River, Oklahoma

Break point number	Horizontal reference x (m)	Elevation y, a.s.l. (m)	Break point number	Horizontal reference x (m)	Elevation y, a.s.l. (m)
1	61.3	361.5	10	277.7	355.9
2	77.7	360.4	11	285.0	359.2
3	114.3	359.1	12	292.6	359.6
4	152.4	359.5	13	349.0	359.8
5	211.8	359.7	14	406.9	359.5
6	240.8	359.2	15	419.1	359.0
7	258.5	355.5	16	443.5	358.9
8	270.4	354.6	17	483.1	359.6
9	271.9	354.0	18	517.6	361.5

DEFINITION OF HYDRAULIC PROPERTIES

Flow area is the cross-sectional area of the flow normal to the direction of flow; wetted perimeter is the length of the line of intersection of the channel wetted surface with a cross-sectional plane normal to the direction of flow; hydraulic radius is flow area divided by wetted perimeter (Chow, 1959). For a discretized section, such as shown in Fig. 1, the above parameters are computed as follows;

$$A = \sum_{i=1}^{NS} A_i \tag{1}$$

$$W = \sum_{i=1}^{NS} W_i \tag{2}$$

$$R = A/W \tag{3}$$

where: A is flow area; W is wetted perimeter; R is hydraulic radius; i is section element counter; NS is number of section elements at or below the free water surface. Manning's uniform flow relation is used to express discharge and flow velocity as a function of cross-section hydraulic parameters:

$$Q = \frac{1}{n} A R^{2/3} S^{1/2} \tag{4}$$

where: Q is discharge; n is Manning's channel roughness value; R is hydraulic radius; S is slope of energy line in longitudinal direction.

The conveyance is a measure of water carrying capacity of a cross-section. With Manning's uniform flow formula, it is defined as (Chow, 1959):

$$K = \frac{1}{n} A R^{2/3} \tag{5}$$

where K is conveyance, and n is Manning's roughness coefficient. The conveyance is evaluated for an entire subsection and not as a summation over section elements because the latter violates the uniform velocity distribution assumption and can lead to inaccurate results. Indeed the evaluation of flow velocity with Manning's equation applied to each section element results in a different flow velocity value for each element. This is inconsistent with the postulate of a single mean flow velocity implied by the application of the uniform flow formula. In the particular context of this paper, this means that the conveyance cannot be evaluated for each section element and then summed over the entire subsection to produce a conveyance value that is consistent with the initial assumptions.

For compound cross-sections total conveyance is the sum of the main and overbank channel conveyance values. This follows from the observation that for a section with both deep and shallow subsections, such as a river in flood stage with overbank flow, the flow in each portion is quite different and should be computed separately (Posey, 1950; Chow, 1959). Therefore, total conveyance correctly equals the sum of the conveyance of the main and overbank channels:

$$K_c = K_m + K_o \tag{6}$$

A one-dimensional representation of hydraulic radius for compound sections is given as a conveyance weighted average over the subsections:

$$R_c = \frac{K_m R_m + K_o R_o}{K_c} \tag{7}$$

where subscript c stands for compound section, m for main channel subsection, and o for overbank subsection. The corresponding value of the coefficient of equivalent roughness (Manning's n) is found by manipulating eqns. (5)–(7) to yield:

$$n_c = \frac{A\, R_c^{2/3}}{K_c} \tag{8}$$

Representative hydraulic radius and equivalent roughness, as given by eqns. (7) and (8), respectively, assures continuity and a smooth increase in total conveyance as the free water surface reaches and rises above overbank elevation. Furthermore, total conveyance value, when computed with eqns. (7) and (8), is identical to the sum of the main channel and overbank conveyance values. Therefore, the same total discharge is obtained from Manning's equation with parameters R_c and n_c as if the previously stated approach by Chow (1959) and Posey (1950) for compound cross-sections were followed. Therefore, eqns. (7) and (8) are equivalent one-dimensional representations of hydraulic radius and roughness for compound cross-sections.

As a final note, the equivalent hydraulic radius for compound sections is not necessarily a monotonous increasing function of flow depth. This follows from the one-dimensional and uniform flow assumption. A decrease may occur as flow transits from predominantly main channel to predominantly overbank flow. Main channel flow conditions are characterized by a comparatively deep and narrow main channel configuration with large hydraulic radius at bank full stage, whereas overbank flow conditions are characterized by comparatively shallow and wide flood plains with smaller hydraulic radius. Thus, one should expect a gradual transition of the equivalent hydraulic radius from a large to a smaller value as flow depth increases to establish significant overbank flow. This decrease is followed by a renewed increase as flow depth continues to increase. The change in the value of the equivalent hydraulic radius with flow depth is shown in Fig. 2 for the cross-section depicted in Fig. 1. Because of its functional behavior with flow depth the equivalent hydraulic radius cannot be approximated by a power function and must be evaluated using eqn. (7).

The equivalent roughness for compound sections is constant for main channel flow conditions, because resistance to flow is given by a single value for the main channel. As overbank flow establishes and dominates, it rapidly converges to a value determined by eqn. (8) (Fig. 2). Because the equivalent roughness is essentially two constant values with a short transition range, the power function is not an appropriate approximation and the equivalent roughness value should be evaluated using eqn. (8).

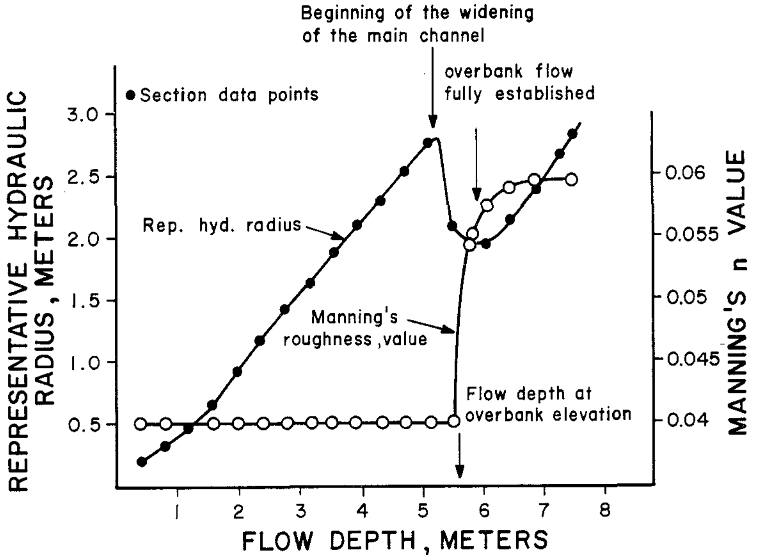

Fig. 2. Change in value of the representative hydraulic radius and Manning's n value with depth for cross-section 24 on the Little Washita River, Oklahoma (see Fig. 1).

POWER FUNCTION REPRESENTATION OF CROSS-SECTION HYDRAULIC PROPERTIES

Experience has shown that for simple and concave cross-sections without flood plains a plot of HP vs. flow depth generally produces a straight line on log-log scales. Fitting a least squared regression through this data and transforming back into the linear domain yields the traditional power function representation for hydraulic properties of a cross-section (Brown, 1982):

$$\mathrm{HP} \quad = \quad m D^{p} \tag{9}$$

where HP is the hydraulic property, D is flow depth and m and p are coefficients of the power function.

In the presence of significant flood plains, eqn. (9) may perform poorly, because a discontinuity in HP values generally occurs at overbank elevation. This is illustrated in Fig. 3 by the dashed line for flow area, wetted perimeter and conveyance factor for cross-section 24 displayed in Fig. 1. The poor performance of the simple power function is especially noticeable for those HP that

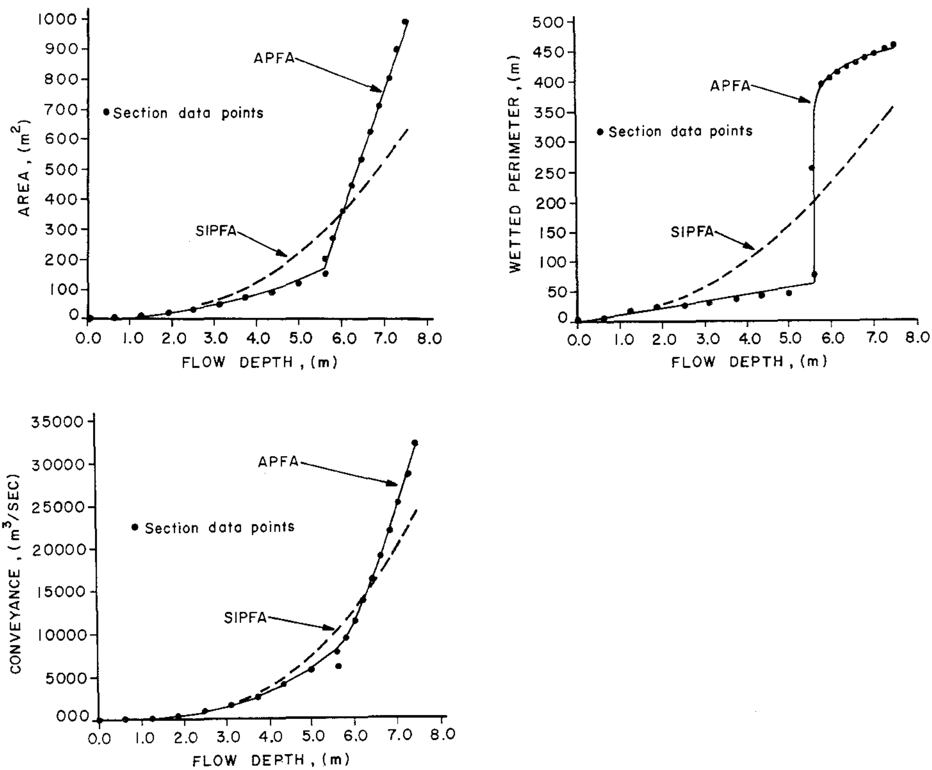

Fig. 3. Power function approximation of cross-section properties for cross-section 24 of the Little Washita River, Oklahoma. Solid line is alternative power function approximation (APFA), dashed line is simple power function approximation (SIPFA).

exhibit a marked discontinuity at overbank elevation, such as wetted perimeter.

An alternative power function for compound cross-sections approximates the HP by two power functions: one power function for main channel flow conditions and a separate power function for overbank flow conditions. The power function for main channel flow conditions is identical to the one given by eqn. (9). The coefficients are derived from actual HP data for main channel flow conditions only:

$$\mathrm{HP} = m_1 D^{p_1} \quad \text{for } D < D_o \tag{10}$$

The second power function, applicable for overbank flow only, is derived in a translated coordinate system that has its origin defined by flow depth at overbank elevation and corresponding main channel HP value, as illustrated in Fig. 4. A simple power function is derived using HP data for overbank flow conditions in the translated coordinate system. Subsequent back transformation into the original coordinate system results in a power function with two additional constants: D_o the overbank elevation and HP_o the value of the hydraulic property at D_o.

$$\mathrm{HP} = m_2(D - D_o)^{p_2} + \mathrm{HP}_o \quad \text{for } D > D_o \tag{11}$$

Use of a translated coordinate system places the origin to the HP discontinuity at overbank elevation. As a result the new approximation reproduces the abrupt change in trend of the section property for flow depths at and above overbank elevation. Applying this alternative power function to the cross-

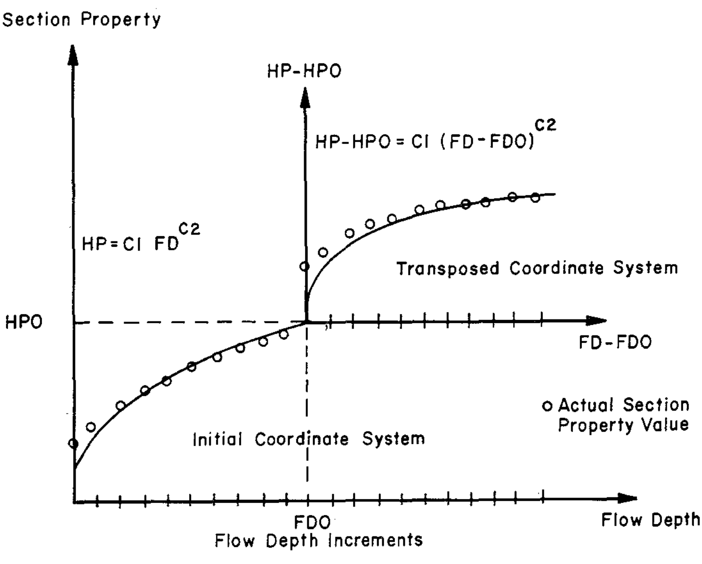

Fig. 4. Schematic diagram of the derivation of the power function for main channel and overbank flow conditions.

section depicted in Fig. 1 results in a good approximation of the section data as shown by the solid line in Fig. 3.

The alternative power function representation given by eqns. (10) and (11) is more accurate than the traditional simple power function approximation because:

(1) main channel and overbank flow conditions are represented by separate and independent relations;

(2) each relation retained all three degrees of freedom for an optimal fit of the respective data;

(3) continuity in the approximated HP values is preserved at overbank elevation while the abrupt change in data trend is reproduced.

The above listed features of the alternative power function representation make it more accurate and applicable to a large number of compound cross-section shapes as illustrated by three case applications presented in the next section.

CASE APPLICATIONS

The performance of the alternative power function for HP of compound sections is tested on three sections of the Little Washita River, Oklahoma. The first section, Section 24, was presented previously (Fig. 1). The second section, Section J1, is characterized by two large overbank channels, high natural levees and a pronounced sand bar in the main channel (Fig. 5). Flow takes place in the main channel until the free water surface exceeds the natural levee elevation at 342.7 m a.s.l. At higher water surface elevations both main

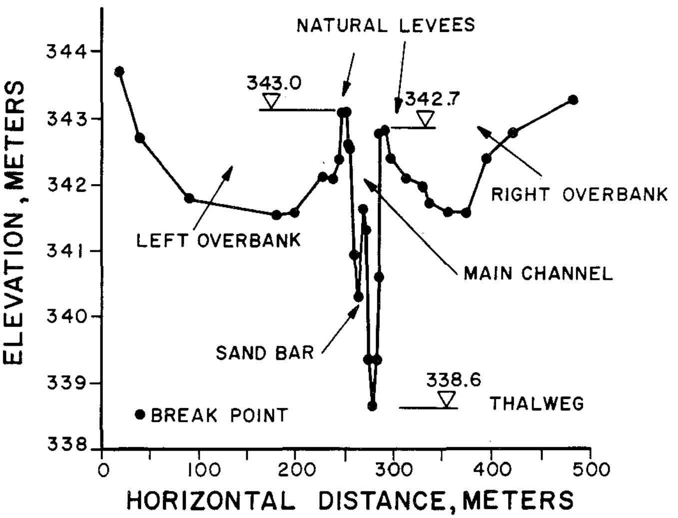

Fig. 5. Cross-section J1, Little Washita River, Oklahoma. Horizontal to vertical distortion is 83 to 1.

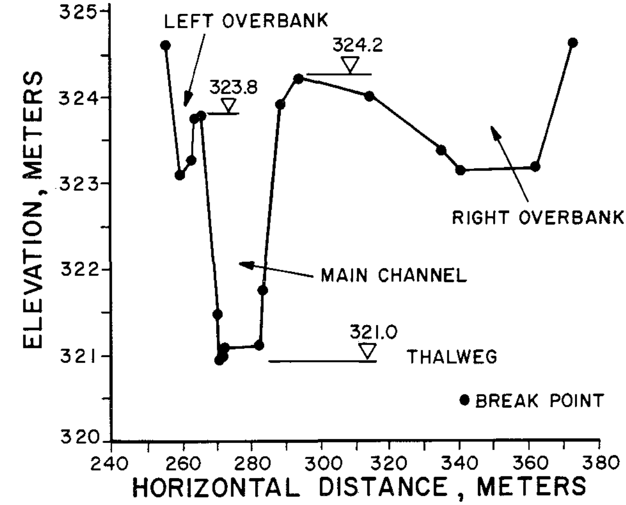

Fig. 6. Cross-section F1, Little Washita River, Oklahoma. Horizontal to vertical distortion is 24 to 1.

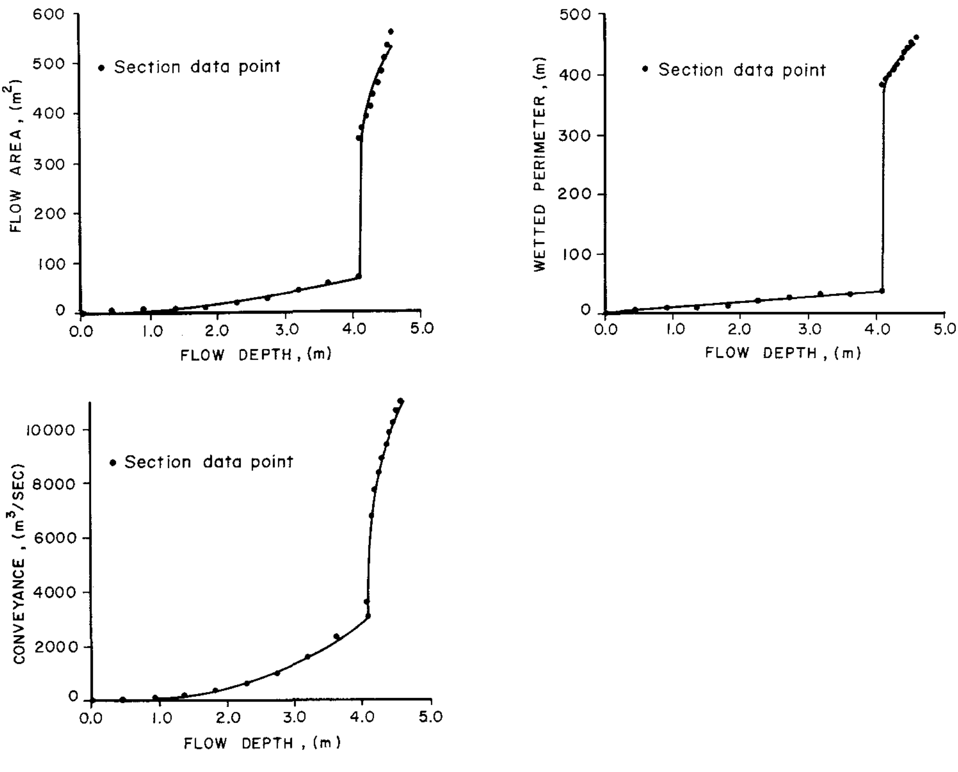

Fig. 7. Hydraulic property approximation by the alternative power function for cross-section J1 of the Little Washita River, Oklahoma.

channels and overbank channel are actively conveying flow. The third section, Section F1, has a rectangular shaped main channel, and two overbank channels of different size and shape (Fig. 6). Overbank flow establishes as the free water surface exceeds 323.8 m a.s.l. For all three sections Manning's roughness value is estimated at about 0.04 for the main channel and about 0.06 for the overbank channel. The breakpoint values for Sections J1 and F1 are given in Table 2.

The selected sections are inappropriate for evaluation by the simple power function approximation and its adaptations because of the sharp discontinuity in HP as main channel flow transits to overbank flow conditions. But the sections are well suited for the proposed alternative power functions (eqns. (10) and (11)) which can follow the abrupt change in data trend at overbank elevation. The results of the section evaluation with the alternative power function representation are given in Table 3. This table provides the coeffi-

TABLE 2

Break points for cross-sections J1 and F1, Little Washita River, Oklahoma

Break point number	Horizontal reference x (m)	Elevation y, a.s.l. (m)	Break point number	Horizontal reference x (m)	Elevation y, a.s.l. (m)
Cross-section J1					
1	21.0	343.6	17	276.1	399.3
2	41.1	342.6	18	279.8	338.6
3	91.6	341.7	19	283.1	339.3
4	182.5	341.5	20	286.5	340.5
5	199.6	341.5	21	287.1	342.7
6	279.8	342.1	22	292.9	342.7
7	239.2	342.0	23	298.7	342.3
8	245.9	342.3	24	314.5	342.0
9	250.8	343.0	25	332.2	341.9
10	252.9	343.0	26	339.2	341.7
11	255.1	342.5	27	358.1	341.5
12	257.5	342.5	28	375.8	341.5
13	261.8	340.9	29	396.2	342.3
14	266.7	340.2	30	423.6	342.7
15	270.6	341.6	31	486.3	343.2
16	273.1	341.3			
Cross-section F1					
1	256.0	324.6	10	282.5	321.1
2	259.7	323.1	11	283.5	321.8
3	263.0	323.0	12	289.0	323.9
4	264.3	323.8	13	294.1	324.2
5	265.8	323.8	14	314.6	324.0
6	270.4	321.5	15	335.3	323.4
7	270.7	321.0	16	340.8	323.1
8	271.6	321.0	17	362.7	323.2
9	271.9	321.1	18	373.4	324.6

TABLE 3

Coefficients of power functions and corresponding standard errors for sections J1, F1 and 24

	Power function	Coefficient	Power	S.E.
Section J1				
Main channel	A vs. FD	4.57	1.903	4.4
	WP vs. FD	9.03	0.970	9.2
	C vs. FD	83.66	2.525	7.1
Overbank channel	A vs. FD	539.67	0.222	20.5
	WP vs. FD	438.23	0.081	11.8
	C vs. FD	10013.29	0.339	0.7
Section F1				
Main channel	A vs. FD	11.09	1.362	2.5
	WP vs. FD	16.33	0.314	6.3
	C vs. FD	232.36	2.061	7.9
Overbank channel	A vs. FD	111.09	0.575	12.8
	WP vs. FD	105.87	0.249	19.7
	C vs. FD	3113.27	0.834	0.9
Section 24				
Main channel	A vs. FD	4.11	2.148	11.2
	WP vs. FD	9.12	1.107	17.8
	D vs. FD	60.23	2.843	17.0
Overbank channel	A vs. FD	440.16	0.936	6.7
	WP vs. FD	370.43	0.081	1.3
	C vs. FD	10180.06	1.337	0.3

A, flow area; WP, wetter perimeter; C, conveyance factor; FD, flow depth.

cients of the power function and corresponding standard error for the main and overbank channels, respectively. The standard error ranges from 0.3 to 20% with an average for all HP and all three sections of under 8%. A plot of the HP data and the fit by the alternative power function representation is shown in Figs. 3, 7 and 8 for visual interpretation. In all three cases the power functions approximate the data relatively well.

SUMMARY AND CONCLUSIONS

The representation of hydraulic properties of channel sections by power functions has been an effective and popular approach. However, traditional power functions have generally failed to adequately represent compound sections because they are unable to follow abrupt changes in the trend of HP values as flow depth increases through overbank elevation and overbank flow establishes. The proposed alternative power function bypasses this problem by defining two connected power functions: one for main channel and the other for overbank flow conditions. The first power function for main channel flow conditions originates at the zero HP flow depth location and it is applicable until overbank elevation is reached. The second power function for overbank

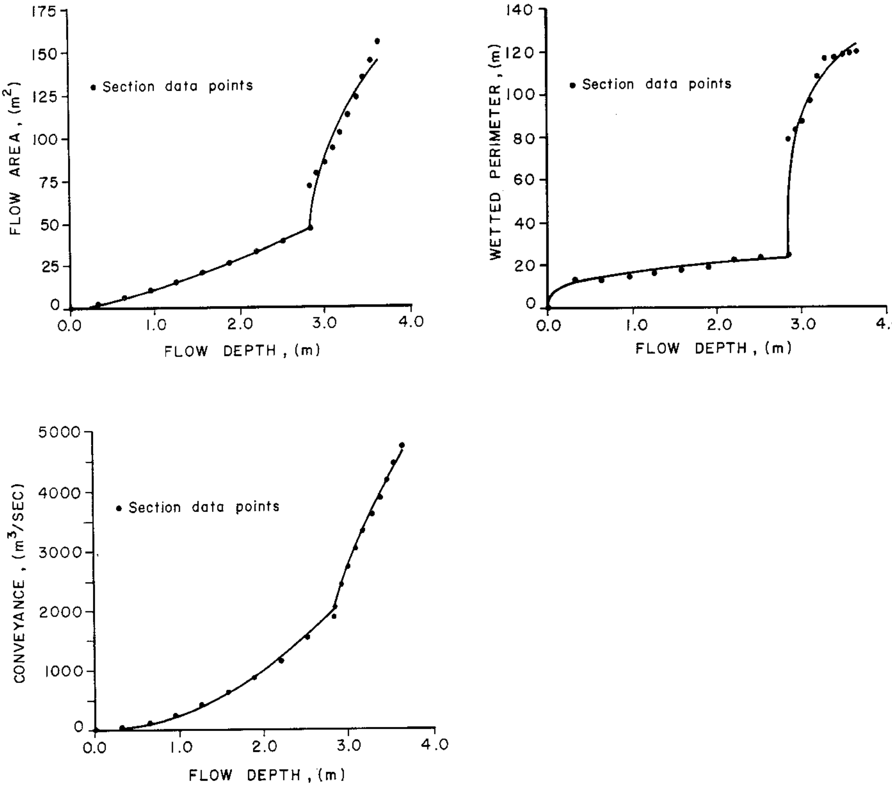

Fig. 8. Hydraulic property approximation by the alternative power function for cross-section F1 of the Little Washita River, Oklahoma.

flow conditions is derived in a translated coordinate system that has its origin defined by flow depth at overbank elevation and corresponding main channel HP value. Thus, this second power function retains all degrees of freedom to fully approximate the changing trend of data for overbank flow conditions and it is not subjected to any restrictions imposed by the first power function.

The alternative power function representation reproduces the abrupt change in trend of the HP values as flow depth increases through overbank elevation. It also maintains continuity in HP values for the full range of flow conditions and therefore makes it attractive for incorporation into numerical and analytical flow routing methods.

Application of the alternative power function representation to actual compound section where traditional simple power functions have failed have resulted in good approximations. For the tested conditions the alternative power function representation is more accurate than traditional power functions. It is also applicable to a larger range of complex sections including natural levees and minor channel irregularities such as sand bars and small terraces. The method is best suited for natural channels which generally display smooth changes in top width with depth as opposed to man-made rectangular or trapezoidal canals. The alternative power function representation is simple and highly efficient from the computational point of view. Its efficiency and extended capabilities make it a very attractive procedure for one-dimensional channel flow routing in drainage networks where a large number of cross-sections need repeated evaluation.

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