Transient Heat Transfer Within Wheat Stored in a Cylindrical Bin

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MOISTURE content and grain temperature are considered the most critical factors in maintaining grain quality. Because temperature differentials in stored grains may cause moisture migration and subsequent deterioration, many investigators have studied grain temperature changes in various types of storage bins, especially flat storage and round steel bins (Kline and Converse, 1961; Sorenson et al., 1957; Schmidt, 1955; and Williamson, 1964).

Temperature changes in stored wheat may result from internal and external sources of heat. Internal sources are respiration, micro-organisms, and insect activity. External heat comes largely from the atmospheric environment around the storage bin. A theoretical treatment of heat transfer in wheat stored in a flat storage bin was attempted by Babbitt (1945). In his study the effect of external environmental changes on wheat temperatures was estimated by an analytical solution for semi-infinite solid (Carslow and Jaeger, 1959 and Ingersoll et al., 1954). Numerous problems on heat conduction in circular cylinders were treated by Carslow and Jaeger (1959).

This investigation examined the effect of changes in external air temperature on the temperature of wheat stored in a concrete upright bin for 2½ years. In addition, a transient heat transfer equation was developed to describe the grain temperature distributions in the concrete bin.

Symbols, Nomenclature, and Unit

A = amplitude in a periodic function for outside ambient air temperature, deg F

C_p = specific heat of grain, Btu per lb per deg F

C_1 = a constant

h = a surface conductance or coefficient of surface heat transfer, Btu per hr per sq ft per deg F

i = the imaginary unit

I_o = a modified Bessel function of the first kind of order zero

J_o = a Bessel function of the first kind of order zero

J_1 = a Bessel function of the first kind of order one

k = thermal conductivity of grain, Btu per hr per sq ft per unit temperature gradient, deg F per ft

K = thermal diffusivity of grain, sq ft per day

L^{-1} = inverse Laplace operator

M_o = modulus of a modified Bessel function

P = a period of periodic function

R = the radius of a bin, ft

r = distance from the center of a bin, ft

S = a parameter in Laplace transformation

S_n = isolated singular points (poles) in the integrand of the equation of the inversion integral

t = time, days

T = grain temperature at a given r for a time, deg F

T_a = outside ambient air temperature, deg F

T_o = initial grain temperature, deg F

T_A = a constant in a periodic function for outside ambient air temperature, deg F

T_S = surface grain temperature at the wall of a bin, deg F

V = dimensionless temperature function, defined as $(T-T_o)/(T_A-T_o)$

$\bar{V}$ = Laplace transformation of V

Ξ = the height of a cylindrical bin

α_n = roots of $J_o(R\alpha_n)=0$

γ = a portion of the limit for the inversion integral

ϵ = a phase angle, degree, or radian

Θ_o = phase of a modified Bessel function

λ = a variable for replacing a parameter S in the inversion integral

ρ_n = the residues of the integrand in the inversion integral at $\lambda=S_n$ for any fixed t

w = a frequency of periodic function, radian per time unit

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* Refer to the Nomenclature section for explanation of symbols.

MATERIALS AND METHODS

Hard red winter wheat was used. The wheat was grown near Cheney, Kansas and delivered to an elevator there by truck in June 1963. The moisture content ranged from 10.3 to 18.0 percent for the samples taken from each truck. The grade and other quality factors based on a sample representing the full bin were: Grade, No. 1 Hard Winter; test weight, 61 lb per bushel; moisture, 12.8 percent; protein, 12.5 percent; germination, 96 percent.

The wheat was stored in one of the 22,000-bu, circular, 18-ft-diameter, 110-ft high concrete bins. The bin was located in the southern outside line of storage annex bins, with nearly 40 percent of the bin's circumference exposed to outdoor air (Fig. 1). Thermocouple cables were positioned in both the radial and axial directions in the bin. Grain temperatures were measured periodically with an electronic potentiometer.

Before starting static storage observations on grain temperatures, the wheat was turned and mixed three different times to establish uniform grain temperature and moisture content throughout the test bin. Each turning operation involved moving the wheat to another bin and then returning it to the test bin No. 102 (double turn).

The first double mixing-turning operation came July, 1 month after harvest. The wheat was turned again in August and a liquid fumigant was applied. In November, about one-half of the wheat was transferred to another bin to permit installation of two horizontal thermocouple cables; then the wheat was returned to fill the test bin. The final mixing-turning operation was January 1964.

The grain temperature in June, when the wheat was placed in the test bin, was about 100 F. The wheat had cooled an average of 8 F after the first mixing-turn and about 6 F by the second turn. Before the final turns in January, wheat temperatures ranged from 30 F near the wall to about 80 F at the center of the bin. After the turn-

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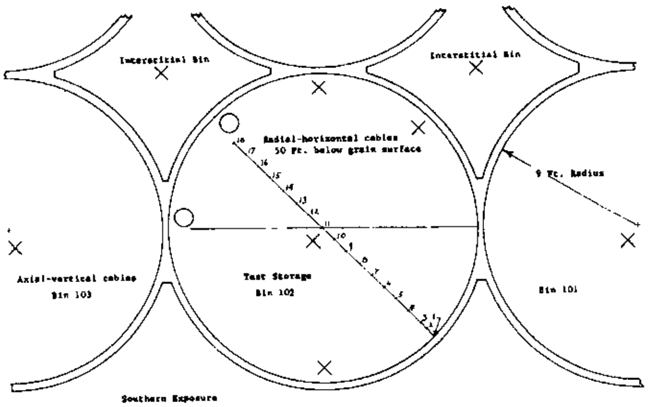

FIG. 1 Schematic diagram of test bin.

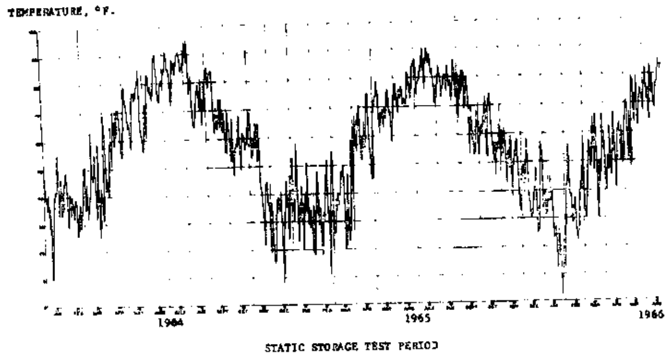

FIG. 2 Outside daily mean air temperatures during the wheat storage investigation at Cheney, Kans.

ing operations, all wheat temperatures in the bin were between 35 and 50 F. The grain temperatures stabilized to about 45 F throughout the test bin before the test period began in February 1964. The wheat was then held in storage without turning or aeration for 2½ years.

Samples taken during each turn indicated little change in quality except for a reduction in germination that occurred during the first few months in storage (Table 1) before the winter mixing turns.

The Analytical Procedure

An analytical solution describing the heat transfer was developed to predict the temperature distribution in a bin of wheat. The results of the analytical approach were compared to temperature data collected from the test bin. For purposes of this analysis, the grain temperature changes associated with external temperature changes were divided into three classifications. The *diurnal* changes reflected the daily day-to-night temperature variation, whereas the *seasonal* change reflected the slower by greater variation in temperature on an annual basis.

A third change, the *secular* change, reflected the normalizing of the grain temperature with the mean temperature of the surroundings. Internal heat generation from respiration, micro-organisms, and insect activity was not considered; the wheat temperature changes were attributed to external conditions only. The analytical procedure used is described as:

A. Transient Heat Transfer Through Grain Stored in Cyl'ndrical Bin with Periodic Change of External Temperature

Assumptions

The following assumptions were used in obtaining an analytical solution which describes grain temperature distribution in a cylindrical bin:

1. Dimensional ratio, height (Ξ) over radius (R) is large.

2. A negligible surface resistance present at the vertical concrete wall (coefficient of heat transfer, h, is considerably larger than thermal conductivity of grain, k).

3. Properties of grain remain constant.

4. Initial grain temperature, T_o, is at a specified temperature.

5. Grain temperature at the center of bin (r=0) is a finite.

6. Ambient temperature, T_a, or grain temperature at the wall, (r=R), is represented by the following periodic function:

$$T_R = T(R,t) = T_A + A \sin (wt-\epsilon) \quad \ldots \ldots \ldots \ldots \ldots [1]$$

7. No heat source is available in stored grain.

Fourier Heat Conduction Equation

A phenomenological equation describing the transient heat transfer through a solid in a cylindrical system with the assumptions 1 and 3 can be written in terms of dimensionless temperature function V as:

$$\frac{\partial V}{\partial t} = K\left(\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r}\right) \quad \ldots [2]$$

where V is defined as

$$V(r,t) = \frac{T(r,t) - T_o}{T_A - T_o} \quad \ldots \ldots [3]$$

and K is thermal diffusivity.

Initial and Boundary Conditions

From assumptions 4, 5, and 6, the following initial and boundary conditions can be written for the system:

$$V(r,0) = 0, \text{ at } t = 0,\ 0 \leq r < R \quad \ldots \ldots \ldots [4]$$

$$V(0,t) = \text{finite, at } t > 0,\ r = 0 \quad \ldots \ldots \ldots [5]$$

$$V(R,t) = 1 + \left(\frac{A}{T_A - T_o}\right) \sin (wt-\epsilon), \text{ at } t > 0,\ r = R \quad \ldots \ldots \ldots [6]$$

Derivation of Solution

1. Laplace transformation

Laplace transformation of equation [2] with the initial condition,

TABLE 1. GRADE AND OTHER QUALITY MEASURES OF THE WHEAT USED IN THE TESTS

Date	Sample description	Official grade*	Moisture content	Test weight	Dockage and foreign material	Shrunken and broken kernels	Defects total†	Protein	Germination
		Number	Percent	Lb per bu.	Percent	Percent	Percent	Percent	Percent
6/10/63	New wheat—filling bin composite—Bin 102	1HW	12.8	61.0	....	1.5	1.50	12.50	96
7/ 8/63	Mixing-turn-composite	1HW	12.8	61.4	0.35	2.0	2.35	12.65	94
8/20/63	Mixing-turn-composite	1HW	12.6	62.1	0.45	2.4	2.85	12.35	97
1/14/64	Mixing-turn-composite	1HW	12.8	61.8	0.65	1.0	1.65	13.00	63
7/25/66	Static storage ended								
7/25/66	Sampled from first ⅓-bin—loadout	1DHW	12.8	61.8	0.30	2.2	2.50	12.60	36
7/26/66	Sampled from second ⅓-bin—loadout	1DHW	13.0	61.8	0.44	2.7	3.14	12.15	26
	Sampled from third ⅓-bin—loadout	1DHW	13.4	61.7	0.57	2.9	3.47	12.60	19
7/27/66	Mixed sample from all grain transferred	1DHW	13.0	61.8	0.45	2.4	2.85	12.60	No test
9/ 2/66	Average of the grade certificates from 11 boxcars	1HW 1DHW 1HDHW 1HHW‡	13.3	61.8	0.90	1.6	2.50	12.38	No test

* Numerical grade designation with subclass, HW=Hard Winter; DHW=Dark Hard Winter; and the special grades, HHW=Heavy Hard Winter; HDHW=Heavy Dark Hard Winter.
† Total defects here include dockage and damaged kernels.
‡ Represents the range in official grades from the 11 certificates.

$V(r,0) = 0$, yields:

$$\frac{d^2\bar{V}}{dr^2} + \frac{1}{r}\frac{d\bar{V}}{dr} - \alpha^2\bar{V} = 0 \quad . . . [7]$$

where $\alpha^2 = S/K$

Note that equation [7] is Bessel's equation (order zero) of the first kind. A solution of equation 7 which satisfies the boundary condition (equation [5]) is

$$\bar{V}(r,S) = C_1 I_o(r\alpha) \quad . . . . . . [8]$$

where I_o is a modified Bessel function of the first kind and C_1 is a constant.

C_1 is evaluated by applying Laplace transformation of the boundary condition (equation [6]) into equation [8]. Then equation [8] becomes:

$$\bar{V}(r,S) = \frac{1}{S}\frac{I_o(r\alpha)}{I_o(R\alpha)} - \frac{A}{T_A - T_o}\left(\frac{w\cos\epsilon - S\sin\epsilon}{S^2 + w^2}\right)\frac{I_o(r\alpha)}{I_o(R\alpha)} \quad . . . . [9]$$

2. Inverse Laplace Transformation

The inverse of equation [9] is obtained by applying the inversion theorem and residue theorem (Ingersoll et al. 1954).

$$L^{-1}[\bar{V}(r,S)] = V(r,t) = \frac{1}{2\pi i}\int_{\gamma - i\infty}^{\gamma + i\infty} e^{\lambda t}\,\bar{V}(\lambda)d\lambda \quad . [10]$$

$$= \sum_{n=1}^{N} \rho_n(t)$$

where $\rho_n(t)$ is the residue of the integrand, $e^{\lambda t}\,\bar{V}(\lambda)$ at $\lambda = S_n$, for any fixed t, and S_n are isolated singular points (poles).

The inverse of the first term in equation [9] is the sum of residues at poles of the integrand in equation [10]. Poles are at y=0 and $\lambda = -K\alpha^2_n$. Then, the inverse of the first term is found to be

$$V_1(r,t) = 1 - \frac{2}{R}\sum_{n=1}^{\infty}\frac{J_o(r\alpha_n)}{\alpha_n J_1(R\alpha_n)} e^{-K\alpha^2_n t} \quad . . . . [11]$$

Similarly, for the inverse of the second term in equation [9], poles of the integrand are at $\lambda = -K\alpha^2_n$ and $\lambda = \pm iw$. After adding residues at $\lambda = -K\alpha^2_n$ and $\lambda = \pm iw$, and rearranging the equation in terms of modulus, M_o, and phase Θ_o of the modified Bessel function, the inverse of the second term in equation 9 is found to be:

$$V_2(r,t) = \frac{2KA}{R(T_A - T_o)}\sum_{n=1}^{\infty}\left(\frac{w\cos\epsilon + K\alpha^2_n\sin\epsilon}{K^2\alpha^4_n + w^2}\right)\frac{\alpha_n J_o(r\alpha_n)}{J_1(R\alpha_n)} \cdot e^{-K\alpha^2_n t} + \left(\frac{A}{T_A - T_o}\right)$$

$$\frac{M_o\left(r\sqrt{\frac{w}{K}}\right)}{M_o\left(R\sqrt{\frac{w}{K}}\right)}\sin\left\{(wt - \epsilon) + \Theta_o\left(r\sqrt{\frac{w}{K}}\right) - \Theta_o\left(R\sqrt{\frac{w}{K}}\right)\right\} . . [12]$$

Finally, the solution of equation [2], which satisfies the given initial and boundary conditions, is the inverse Laplace transformation of equation [9]. Therefore, the sum of equations [11] and [12] is the solution of equation [2]. Rearranging the solution in terms of T(r,t) yields the following final solution:

$$T(r,t) = T_A - \frac{2(T_A - T_o)}{R}\sum_{n=1}^{\infty}\frac{J_o(r\alpha_n)}{\alpha_n J_1(R\alpha_n)} e^{-K\alpha^2_n t} + \frac{2KA}{R}\sum_{n=1}^{\infty}\left(\frac{W\cos\epsilon + K\alpha^2_n\sin\epsilon}{K^2\alpha^4_n + w^2}\right)$$

$$\frac{\alpha_n J_o(r\alpha_n)}{J_1(R\alpha_n)} e^{-K\alpha^2_n t} + \frac{AM_o\left(r\sqrt{\frac{w}{K}}\right)}{M_o\left(R\sqrt{\frac{w}{K}}\right)}$$

$$\sin\left\{(wt - \epsilon) + \Theta_o\left(r\sqrt{\frac{w}{K}}\right) - \Theta_o\left(R\sqrt{\frac{w}{K}}\right)\right\} . . . . [13]$$

It should be noted that a solution, similar to equation [13], for the case when $T_o=0$ and $T_s = A\sin(wt - \epsilon)$, is given in Carlslow and Jaeger (1959).

B. Transient Heat Transfer Through Grain Stored in a Cylindrical Bin with a Constant External Temperature

Initial and Boundary Conditions

Assumptions involved in the analysis are the same except for assumption 6. The initial condition and the first boundary condition are identical to equations [4] and [5]. However, the second boundary condition becomes:

$$V(R,t) = 1, \text{ at } t > 0, r = R \quad . [14]$$

Solution

The analytical solution of equation [2], which satisfies the above initial boundary conditions, expressed in terms of T(r,t), is

$$T(r,t) = T_A - \frac{2(T_A - T_o)}{R}\sum_{n=1}^{\infty}\frac{J_o(r\alpha_n)}{\alpha_n J_1(R\alpha_n)} e^{-K\alpha^2_n t} \quad . . . . [15]$$

Equation [15] can be obtained from Carlslow and Jaeger (1959).

To analyze temperature changes of wheat in a concrete cylindrical bin that result from periodic changes of outside air temperature, the mass of wheat, initially at uniform temperature, was considered as inanimate material extending to infinity in the direction of the axis of the bin (assumptions 1, 2, 7). The implication of assumption 1 is that heat transfer takes place in only the horizontal path.

In practice, temperature changes are never confined to one direction; but in a storage bin, where the radial dimension is small compared with the axial, the temperature gradients in the axial direction are small, except near the exposed top surface and at the bottom (floor level). Our experimental data confirmed this assumption.

It was assumed that the external temperature changes could be represented by a periodic function of time. The external temperature changes recorded near Cheney, Kansas, for the period January 1964 to June 1966, are shown in Fig. 2. Fig. 3 shows that the simulated external temperature changes generated by a periodic function agree with the actual temperature change.

It was also assumed that the daily external temperature could be expressed by a periodic function of time in which the period is equal to one day even though the daily external temperature changes are irregular.

Another assumption was that properties of grain remain constant. Thermal properties of wheat used in this investigation, where conductivity (k) = 0.0783 Btu per hr per sq ft per unit temperature gradient (deg F) per ft, specific heat (C_p) = 0.39 Btu per lb per deg F, and thermal diffusivity (K) = 0.00417 sq ft per hr or 0.1 sq ft per day, were interpolated from Babbitt (1945) and Kazarian and Hall (1965).

Equation [13] was used for examining the diurnal temperature change and seasonal temperature change; equation [15] was used for examining the secular temperature change. The values for corresponding arguments of functions J_o, J_1, M_o, Θ_o, and those of α_n in equations [13] and [15] were obtained from the tables in references (McLachlan, 1934 and NBS, 1966).

Discussion of Results

Diurnal Temperature Changes

To evaluate diurnal changes, temperature data from February 1964 were considered. The average daily external temperature was 36 F with a difference of 24 F between day and night. The analysis by equation [13] and the daily wheat temperatures observed showed practically no change even at 0.5 ft from the wall. The diurnal temperature variation had very little effect on the temperature of wheat when the average ambient air temperatures were close to average grain temperature near the wall.

Seasonal Temperature Changes

The periodic function used to simulate the seasonal outside temperature changes was based on a mean annual temperature of 57.5 F established by weather data for the location. The amplitude or maximum deviation from the mean was 27.5 F. Fig. 3 shows the observed outside ambient changes with a line for the simulated periodic function.

The observed actual wheat temperatures at different distances from the exposed bin wall are shown with the predicted or simulated temperatures in Figs. 3 to 7. The simulated temperatures were a close representation of the actual or observed temperatures.

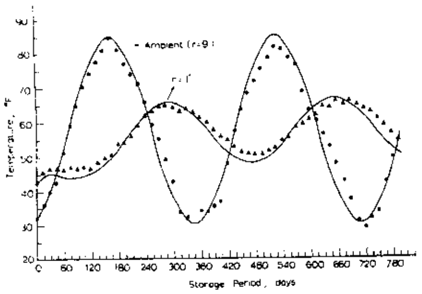

FIG. 3 Ambient and simulated outside air temperature shown with actual and simulated wheat temperature at 8-ft radial distance from exposed bin wall during static storage test period.

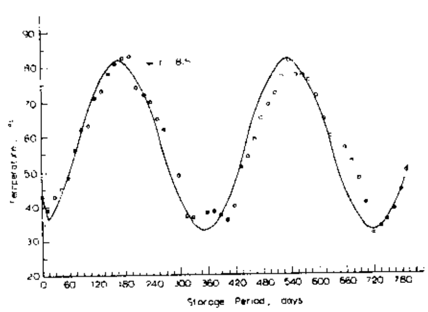

FIG. 4 Simulated wheat temperature shown with the actual readings for (r=8.5 ft), the 0.5 ft distance from the exposed bin wall during the static storage test period.

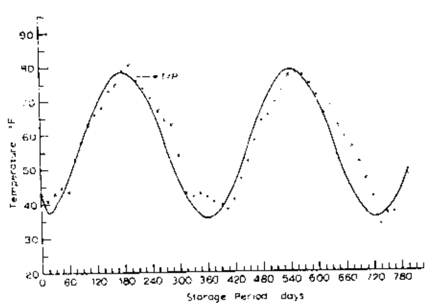

FIG. 5 Simulated wheat temperature shown with the actual readings for (r=8 ft), the 1-ft distance from the exposed bin wall during the static storage test period.

The mean absolute temperature difference (disregarding sign) and mean temperature difference, between the simulated and the observed temperature at various positions (bin radii) were calculated (Table 2). The simulated grain temperatures were 1.0 to 1.9 F lower than the observed grain temperatures except at the wall.

The close agreement indicates equation [13] may be used to approximate grain temperature distributions at various positions in a radial direction in a concrete upright bin if the initial grain temperature and external temperature changes surrounding a bin are known.

TABLE 2. MEAN DIFFERENCE BETWEEN SIMULATED AND OBSERVED TEMPERATURES

Distance from center,	Mean of absolute difference,	Mean difference,
ft	deg F	deg F
9	3.1	0.3
8.5	4.4	−1.0
8	4.0	−1.5
7	3.2	−1.9
5	1.9	−1.9
1	2.5	−1.1

Annual amplitude differences between the external temperature changes and those in the grain at various positions in the bin are shown in Fig. 8. At a distance of 2 ft in from the side wall the wheat temperature changes about 60 percent as much as the change in average outdoor temperatures. At 4 ft the wheat temperature change is about 45 percent of that at the wall, and at 8 ft, 30 percent of that at the wall.

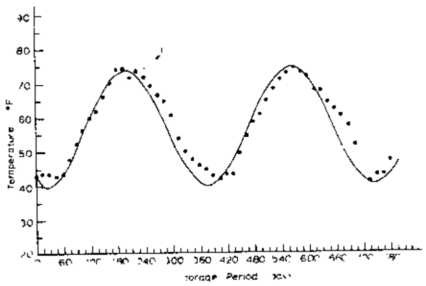

FIG. 6 Simulated wheat temperature shown with the actual readings for (r=7 ft), the 2-ft radial distance from the exposed bin wall during the static storage test period.

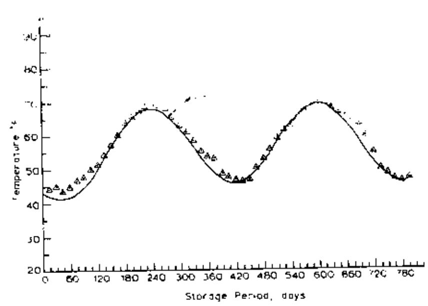

FIG. 7 Simulated wheat temperature shown with the actual readings for (r=5 ft), the 4-ft radial distance from the exposed bin wall during the static storage test period.

The lag between the grain temperature change and the external temperature change increased almost linearly as the distance from the wall increased (Fig. 9). The average lag was 2 weeks on the grain 1 ft from the wall and nearly 20 weeks on the grain 8 ft from the wall. The effect of maximum external summer temperature on the stored wheat may reach 0.5 ft from the wall after 1 week, and 8 ft after 19 weeks (the middle of December).

Similar results were obtained by examining the average rates of temperature change (heating and cooling) for 1 year. The average rates estimated were ±0.31, ±0.24, ±0.18, ±0.13, and ±0.11 F per day for a distance from the wall of 0.5, 1, 2, 4, and 8 ft respectively. The rate of temperature change at one-half foot from the wall was almost three times that at 8 ft. The rate of temperature change decreased rapidly from 0 to 4 ft from the wall, but beyond 4 ft little change in the rate was observed.

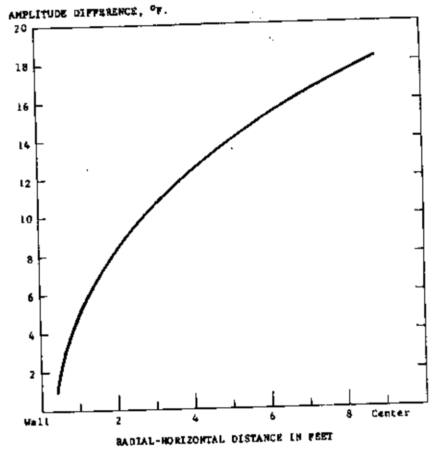

FIG. 8 Temperature amplitude difference between the outside ambient air variations and the wheat temperatures at various distances from the vertical exposed bin wall.

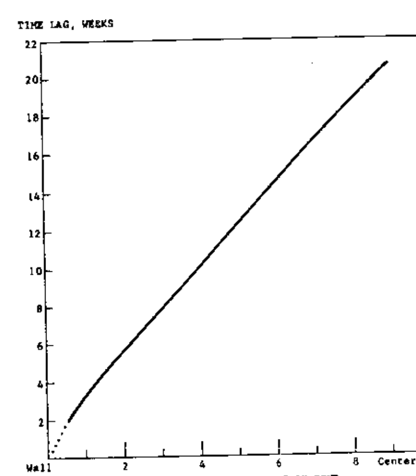

FIG. 9 Time lag period between outside seasonal temperature variations and wheat variations at different distances from the exposed bin wall.

Mean External Temperature Effect

Using equation 15 for a constant external temperature, the secular temperature change was examined. The dimensionless grain temperature distributions at various positions in a cylindrical bin, simulated by equation 15, are shown in Fig. 10. It should be noted that temperature distribution at any given constant external temperature T_A and initial grain temperature T_o can be estimated by Fig. 10. For example, the test bin was filled with wheat at an initial temperature of 45 F and exposed to an average temperature of 57.5 F. Grain temperature increased 12.5, 10.5, 7, 2.5, and 1.5 F after 1 month at 0.5 ft and 1, 2, 4, and 8 ft from the wall.

A sharp increase in stored wheat temperature within 2 ft from the wall took place for a month; grain temperature then increased slowly to the mean temperature of the surroundings. Temperature of wheat stored beyond 4 ft increased gradually to the mean temperature of the surroundings. The first two terms in equation [13] used for the seasonal temperature variation were equivalent to equation [15] used for the analysis of the secular temperature variations. Thus, the combination of two temperature effects, seasonal and secular, was considered in equation [13].

Planned Management to Control Grain Temperatures

The quality of grain used in these tests was maintained reasonably well without ventilation or aeration. Examination of the wheat samples at the end of the storage period showed that the grade was No. 1 Dark Hard Winter; moisture, 13.0 percent; test weight, 61.8 lb per bu; protein, 12.6 percent; and germination, 27 percent (Table 1). The only significant change was the 36 percent reduction in germination.

The key factors in the successful storage for 2½ years without turning or aeration were considered (a) the three mixing-turning operations performed prior to the test period to assure uniform moisture and to provide an average initial grain temperature of 45 F and (b) the moisture content at a level not conducive to internal heating from respiration, micro-organisms, or insect activity.

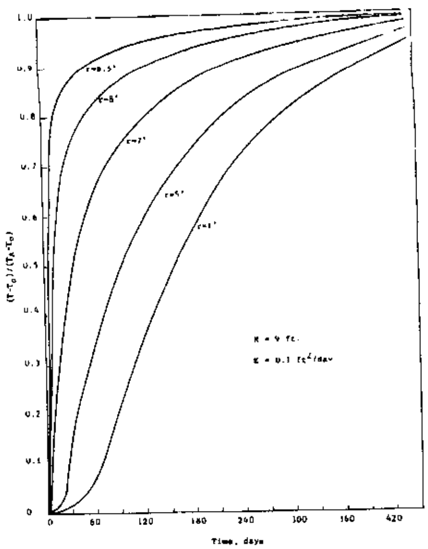

FIG. 10 Simulated grain temperature variations at various locations in a concrete upright bin (9 ft radius) for the case of a constant external temperature.

In elevator bins having exposed walls, new wheat can be expected to lose heat, which will result in temperature reduction during the fall and winter. The turning for mixing and for establishing uniform grain temperatures should be scheduled before the last of March.

For concrete upright bins where only limited or no thermocouple cables are installed, the analytical solution given here should be useful in describing temperature changes in the stored grain and in scheduling turning operations.

Conclusions

Temperature variation in the axial (vertical) direction in a concrete upright bin was small compared to that in the radial (horizontal) direction (verification of assumption 1).

The seasonal temperature variations of the ambient air may be represented by a periodic function of time.

The diurnal temperature variation had very little effect on the wheat temperature in a concrete upright bin.

The temperature distributions predicted by an analytical procedure agreed with the temperature distributions observed. The analytical solution given should be useful in predicting grain temperature distributions in a concrete upright bin and in scheduling turning operations.

The lag between the grain temperature and the external temperature increased almost linearly with the distance from the exposed wall.

The amplitude of the seasonal temperature change in the grain decreased rapidly as the distance of the grain location from the exposed wall increased. The effect of external temperatures on the seasonal change in temperature of wheat stored in a concrete upright bin is confined largely to wheat within 4 ft of the wall.

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