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Optimal Harvest Policies for Corn and Soybeans*

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Dynamic programming models which can aid the decision maker in scheduling his harvesting operations are presented. Both 1 and 2 crop models are developed. The models are illustrated with several example problems. The important feedback property, which makes dynamic programming a useful and practical decision tool, is illustrated.

Although the examples presented involve harvesting of corn and soybeans, the optimization technique can be applied to other situations where the influence of weather and some form of yield or other biological response based on timeliness, exist.

1. Introduction

In recent years, interest has developed in the simulation of the field harvesting of grain. Morey *et al.*,[1] Holtman *et al.*[2] and Carpenter & Brooker[3] developed models of corn harvesting systems. These models included the interactions of the field harvesting rate, weather, drying rate and in some instances marketing alternatives. Donaldson[4] presented a model for simulation of cereal grain harvest.

These models have been primarily developed as aids in machinery selection. This type of decision is a long-term decision, since any machine or group of machines will be used over a period of years. Also of importance are the short-term decisions concerning the operation of an existing system in a particular year.

This paper is concerned with the development of decision models to aid in the optimization of the corn and soybean harvesting operation. It will be assumed that: (1) the crops have been planted, have grown and are approaching the maturity point; (2) the decision maker has a fixed set of equipment for both harvesting and drying; (3) depending on the particular situation, a custom operator may be available for hire. Stated from the decision maker's (farm manager's) point of view the problem becomes: what is the best policy for harvesting each crop during the remainder of the season in order to maximize profit?

Several factors enter into the optimum harvest policy problem to make it important.

(1) The recoverable yield of the grain changes throughout the season, generally increasing to some maximum and then decreasing. This factor can be a function of both the moisture content of the grain and the date, as well as other factors.

(2) The average moisture content of the grain generally decreases throughout the season, possibly increasing near the latter part. The combine performance, the drying cost and the dryer capacity are all functions of the grain moisture content.

(3) Weather provides a stochastic input to the system. The probability of being able to work in the field varies throughout the season. This factor is a function of both rainfall and soil drying conditions.

(4) The price of grain varies through the season, generally moving lower as the season progresses and then possibly increasing near the end. If all or part of the grain is to be sold during the season, this factor is important.

(5) In most cases, corn and soybeans compete for the same equipment and labour during the harvest season. Each crop has its own moisture content, recoverable yield and price relationship which must be considered in order to develop a policy which maximizes profit.

The above problem has been treated by Morey *et al.*[5] for the situation when only one crop is considered. In this paper the one crop model is extended to include a stochastic harvest rate. In

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addition, a model to determine the optimal policy when both corn and soybeans are harvested is developed.

2. One-crop case

It is useful to think of the problem as a stock or inventory depletion problem. In this case, the inventory is the number of acres which remain to be harvested and the objective is to reduce this inventory in a manner which maximizes total net profit.

The determination of an optimal policy requires that the optimum decision, which may change with time, be specified for the entire season. It is realistic to consider the season to be broken into several periods or "stages" during which a particular policy is followed. A week is a convenient length for this period. The problem can then be thought of as a multistage process. This type of process lends itself to a dynamic programming formulation.

An introduction to dynamic programming along with some applications of the technique to various kinds of problems can be found in Bellman & Dreyfus[6] and Nemhauser.[7] The notation of Nemhauser will be used in the model development.

2.1. *One-crop model*

The following definitions can be made for the one crop dynamic programming formulation.

The stage, n, is the point at which a decision is made. During the period following this decision until the next decision point, the system operates according to the decision made at stage n. The stages are indexed backwards in time for convenience.

The state of the system, X_n, is defined as the number of acres yet to be harvested at stage n. This is the stock or inventory that is to be depleted in an optimal manner.

The decision variable, d_n, is defined as the number of hours of combine operation/day at stage n. This is the control variable which is manipulated by the decision maker.

The return function at each stage includes only the costs and returns which affect the selection of the decision variable. Therefore, the net return function will not include, for example, the fixed costs for any of the equipment in the system. This function can be written as:

$$r_n(X_n,d_n,k_n)=\bar{k}_n[y_n\, h_n\, d_n\, (C_{on}-C_1(M_n-14))-C_2d_n-C_3(d_n-8)\, H(d_n-8)] \quad \ldots(1)$$

where

$$\bar{k}_n=\min\left[k_n, \frac{X_n}{h_n d_n}, \frac{G_n\, t_d\, i}{y_n\, h_n\, d_n}\right] \quad \ldots(2)$$

i —number of days between stages n and $n-1$ (stages are numbered backwards from N to 1).

k_n —number of days available for field work between stages n and $n-1$, stochastic variable, $0\leqslant k_n \leqslant i$.

$\bar{k}_n$ —the number of available days which can be utilized between stages n and $n-1$.

M_n —moisture content at stage n, per cent wet basis (14% is the safe storage limit for corn).

y_n —recoverable yield at stage n which is a function of moisture content and thus a function of time, bu/acre.

h_n —harvest rate, acres/h (stochastic variable).

C_{on} —price received for grain at stage n (\$/bu).

C_1 —drying cost, \$ (% m.c.)$^{-1}$bu^{-1}

C_2 —base labour cost (\$/h).

C_3 —overtime cost, reflects the additional hourly cost for working more than 8 h/day, may include overtime labour cost or increased losses or inconvenience due to night-time work, etc. (\$/h).

$H(z)$—unit step function.

$$H(z)=0,\ z<0$$
$$H(z)=1,\ z\geqslant 0$$

t_d —maximum number of h the grain dryer can work/day.

G_n —capacity of the grain dryer which is a function of the moisture content of the grain (bu/h).

Eqn 1 specifies the return as a function of the state, X_n, the decision, d_n, and the stochastic inputs, k_n and h_n. Since the number of good days, k_n, is a random variable, it is impossible to know beforehand how many days will be available for harvest during any period. Eqn 2 results from a constraint on the number of usable days for harvest during any period. The first term within the minimization bracket is the number of good harvest days available due to weather. The second term is the number of days required to harvest the acreage of grain remaining. The third term is the number of days needed to harvest the weekly throughput of the grain dryer. The capacity of the grain dryer which is a function of the moisture content of the grain is an important consideration in many harvest systems. Therefore, $\bar{k}_n$ is the number of good harvest days between periods n and $n-1$ which can be utilized for harvesting grain due to a limitation on the number of good days, the number of acres available or the capacity of the grain dryer.

The state variable transformation equation ties together the results from stage to stage by subtracting the acres harvested in stage n from the acres yet to be harvested at the beginning of stage n:

$$X_{n-1}=X_n-h_n\,\bar{k}_n\,d_n$$

The distribution for the harvest rate, h_n, is assumed to be normal. This choice is based in part on the work of Donaldson,[4] who found in empirical studies of combines harvesting cereal grains that the distribution of harvest capacities was approximately normal. Let μ_h be the mean harvest rate in acres/h and σ_h be the standard deviation in the same units.

The stage by stage returns and the transformation equation can now be combined in the dynamic programming recursive equations. First, $f_n(X_n)$ is defined as the maximum expected return with n stages remaining and X_n acres yet to harvest. With one stage remaining we integrate over the normally distributed harvest rate to obtain the expected value and maximize over the range of d_1, the daily h of combine operation.

$$f_1(X_1)=\max_{\substack{d_1\\0\leqslant d_1\leqslant t_h}}\left[(2\pi)^{-\frac{1}{2}}\sigma_h^{-1}\int_{-\infty}^{\infty}\sum_{k_1=0}^{i}p_{k_1}\{r_1(X_1,d_1,h_1)\}\,e^{-\frac{1}{2}(h_1-\mu_h)^2/\sigma_h^2}\,dh_1\right] \qquad \ldots(4)$$

where t_h is the maximum number of combine h available/day.

With n stages remaining f_n is found by maximizing the sum of f_{n-1} and the expected return for stage n.

$$f_n(X_n)=\max_{\substack{d_n\\0\leqslant d_n\leqslant t_h\\n=2,\ldots,N}}\left[(2\pi)^{-\frac{1}{2}}\sigma_h^{-1}\int_{-\infty}^{\infty}\sum_{k_n=0}^{i}p_{k_n}\{r_n(X_n,d_n,k_n,h_n)+f_{n-1}(X_n-\bar{k}_n\,h_n\,d_n)\}e^{-\frac{1}{2}(h_n-\mu_h)^2/\sigma_h^2}\,dh_n\right] \qquad \ldots(5)$$

where p_{k_n}=probability of having k_n suitable work days between stages n and $n-1$.

The complexity of the formulation rules out the possibility of an analytical solution. In order to obtain a numerical solution to Eqns (4) and (5) the integral must be evaluated numerically. Hermite-Gauss quadrature is particularly suited for numerical integration in which the integrand contains a form of e^{-x^2} and the interval of integration is $(-\infty, \infty)$, which is the case with an expected value over the normal distribution. Hildebrand[8] gives a good development of these results. Even though the range of h_n from $-\infty$ to 0 is unrealistic, if the standard deviation, σ_h, is less than $\frac{1}{3}$ of the mean, μ_h, (very likely) the area under the portion of the normal curve for $h_n<0$ is very small.

Using the results from Hermite-Gauss quadrature the n stage return equation can be written as:

$$f_n(X_n) = \max_{d_n} \left[\pi^{-1} \sum_{j=1}^{J} w_j \sum_{k_n=0}^{i} p_{k_n} \{r_n(X_n, d_n, k_n, h_n) + f_{n-1}(X_n - \bar{k}_n \bar{h}\, d_n)\}\right] \qquad \ldots(6)$$

where $\bar{h} = 2\sigma_h\, a_j + \mu_h$

a_j, w_j—Hermite-Gauss absissas and weighing factors, respectively.

J=total number of points used in the quadrature (8 in this work).

The values for the absissas and corresponding weights are given in Greenwood & Miller.[9]

Before the computations can proceed, the moisture dry-down and recoverable yield functions must be specified as well as the probability mass function for each stage. The recursive equations can be solved by starting with Eqn (4) and finding the optimal policy for several values of X_1. These results can be used in Eqn (5) for finding optimum results for several values of X_2. This procedure can be repeated for $n=3, \ldots, N$.

2.2. *One-crop results*

The model will be illustrated with an example problem. In order to solve the model, the important system data must be specified. The required data can be classified into 4 areas: (1) soil trafficability; (2) grain moisture content; (3) recoverable yield; (4) dryer capacity. Data and relationships used to represent these factors are presented in Morey *et al.*[5]

Since the models are situation specific—that is in order to make the optimal decision, detailed information about many variables must be specified—care must be taken in making generalizations about these results.

The example problem is based on 300 acres of corn, harvested with a combine having a capacity of 2·5 acres/h. Assuming a 3 mile/h travel speed during harvest and a 70% field efficiency, this would require a machine width of approximately 10 ft which corresponds to a 4 row-30 in or 3 row-40 in machine. A complete list of the input parameters is specified in Table I.

The decision variable, d_n, at each stage is the number of h that the combine should be operated each working day during the period to maximize profit.

In the example this quantity is limited to a maximum of 16 h/day. The first 8 h are charged at \$4/h, C_2. Any time over 8 h is charged an additional \$4/h or a total of \$8/h.

TABLE I

Input parameters for the one-crop example

Total acreage	300
Acreage increment	20
Harvest rate (mean)	2·5 acres/h
Maximum harvest hours	16 h/d
Dryer capacity	not limiting
Dryer hours	16/d
Maximum yield	160 bu/acre
Stage increment	7 days
Initial conditions	
Date	September 1
Moisture content	40% w.b.
Total stages through November 30	13
Corn price	\$1·20/bu
Drying cost	\$0·005 (% m.c.)$^{-1}$bu^{-1}
Labour charge	\$4·00/h
Overtime or custom charge	\$4·00/h

A portion of the solution for the input data of Table I is displayed in Table II. This table displays the feedback characteristics of a dynamic programming solution. This feedback information is available because of the solution procedure, which requires that the solution to a whole series of problems be found in order to find the solution to the specific problem (in this case, optimum harvesting of 300 acres of corn which has a 40% moisture content on September 1). Since the optimum harvesting problem has stochastic inputs, this feedback property is useful. For example, looking at Table II, assume the decision maker is at the beginning of week four with 200 acres of corn remaining. The optimum decision calls for operating the combine 16 h/day. At the end of the week the actual weather conditions and more importantly, the actual results are known. If, due to weather or other conditions (for example, machinery breakdown), no harvesting is accomplished, then the optimum solution for 200 acres remaining at the beginning of week 5 is still 16 h/day. However, if harvesting progresses rapidly during week 4 there may be only 120 acres remaining at week 5, in which case the optimum solution is to run the combine 8 h/day. This feedback property makes it possible to calculate an optimum decision matrix at the beginning of the season. This decision matrix is valid as long as the actual grain moisture content and recoverable yield throughout the season remain close to the predicted values.

TABLE II

Optimal expected return (for the acres remaining) and optimal policy (h/day of combine operation) for the example one-crop problem of Table I

Week	*2*	*3*	*4*	*5*	*6*
Moisture content (%)	28·4	24·0	21·0	19·3	18·3
Recoverable yield (bu/acre)	146	144	137	131	127
Acres remaining					
200	$32,476 (9·1) h/day	32,093 (16·0)	31,146 (16·0)	30,236 (16·0)	29,260 (16·0)
160	26,023 (8·0)	25,749 (16·0)	24,979 (16·0)	24,219 (12·9)	23,438 (12·9)
120	19,533 (8·0)	19,361 (16·0)	18,774 (16·0)	18,203 (8·0)	17,615 (8·0)

The results in Table II were obtained with the standard deviation of the harvest rate, σ_h, equal to zero (deterministic harvest rate). The effect of the stochastic harvest rate was investigated by making runs using standard deviations ranging from 0 (deterministic case) to 0·75 acres/h. These results are presented for selected acreages and times in Table III. In all instances the expected return decreases as the standard deviation increases. This implies that a widely fluctuating harvest rate will tend to cause a drop in expected return. However, the effect of the stochastic harvest rate is small, and it appears that a model using a deterministic harvest rate is satisfactory.

Additional results from the one-crop model are presented elsewhere.[5, 10]

3. Two-crop case

There are 2 inventories to deplete in the 2 crop problem. These are the acres of corn and the acres of soybeans remaining. The problem is to determine which crop, if any, is to be harvested.

3.1. *Two-crop model*

The following assumptions are made in formulating the 2-crop model.

(1) The minimum time unit considered is a day. Either a day is suitable for harvest for the entire day or no harvesting can take place.

TABLE III

Comparison of expected return and optimum harvest h/day for several standard deviations of the harvest rate ($\mu_h = 2\cdot5$)

Acres remaining	*Week 2, 12 stages remaining*		
	300	*200*	*100*
σ_h			
0	\$48,575 (15·0) h/day	32,476 (9·1)	16,281 (8·0)
0·25	48,564 (15·2)	32,496 (8·8)	16,279 (8·0)
0·50	48,539 (15·4)	32,455 (8·9)	16,274 (8·0)
0·75	48,496 (16·0)	32,430 (8·9)	16,266 (8·0)
	Week 5, 9 stages remaining		
0	45,088 (16·0)	30,236 (16·0)	15,195 (8·0)
0·25	45,078 (16·0)	30,224 (16·0)	15,189 (8·0)
0·50	45,045 (16·0)	30,196 (16·0)	15,175 (8·0)
0·75	44,996 (16·0)	30,155 (16·0)	15,156 (8·0)
	Week 11, 3 stages remaining		
0	38,578 (16·0)	26,624 (16·0)	13,533 (16·0)
0·25	38,522 (16·0)	26,590 (16·0)	13,530 (16·0)
0·50	38,340 (16·0)	26,528 (16·0)	13,517 (16·0)
0·75	38,011 (16·0)	26,408 (16·0)	13,490 (16·0)

(2) At any point in time, work can progress on one crop or the other or neither, but not both (this is obviously necessary if there is only one harvesting machine available).

(3) The available harvest time for each day suitable for harvest is of fixed length and is the same for both crops.

(4) The harvest time lost in making the equipment alterations, required when changing crops, is sufficient to preclude a change-over during the day, except in the case where the harvest of one crop is completed.

Dynamic programming can again be used to formulate the problem. Based on the above assumptions, a stage length of 1 day will be used. There are 2 state variables; the acres of corn yet to be harvested at day n, $X_n{}^C$, and the acres of soybeans yet to be harvested at day n, $X_n{}^B$.

Each day the decision is to determine which crop, if any, will be harvested. Two decision variables can be defined.

$$d_n{}^C = \begin{cases} 0 \text{ if corn is not harvested} \\ 1 \text{ if corn is harvested} \end{cases}$$

$$d_n{}^B = \begin{cases} 0 \text{ if soybeans are not harvested} \\ 1 \text{ if soybeans are harvested} \end{cases}$$

Three possible policies exist for each day

$$C\text{—}d_n{}^C = 1,\ d_n{}^B = 0 \text{ harvest corn}$$

$$B\text{—}d_n{}^C = 0,\ d_n{}^B = 1 \text{ harvest soybeans}$$

$$N\text{—}d_n{}^C = 0,\ d_n{}^B = 0 \text{ harvest neither}$$

Therefore, at each stage, only 3 alternatives need to be evaluated. In the following pages these policies will be referred to as the (C,B,N) policies.

A return function must be specified which contains the combined returns and costs for both crops. If $r_n(X_n{}^C, X_n{}^B, d_n{}^C, d_n{}^B, k_n)$ is defined as the return at day n, then

$$r_n(X_n{}^C, X_n{}^B, d_n{}^C, d_n{}^B, k_n) = r_n{}^C + r_n{}^B, \qquad \ldots(7)$$

where

$$r_n{}^C = t_h\, k_n{}^C\, y_n{}^C\, h^C\, d_n{}^C\, (C_{on}{}^C - C_1{}^C\, (M_n{}^C - 14)), \qquad \ldots(8)$$

$$r_n{}^B = t_h\, k_n{}^B\, y_n{}^B\, h^B\, d_n{}^B\, (C_{on}{}^B - C_1{}^B\, |\, M_n - 13\, |), \qquad \ldots(9)$$

$$k_n{}^C = \min\left[k_n, \frac{X_n{}^C}{t_h h^C d_n{}^C}, \frac{G_n\, t_d}{y_n{}^C t_h h^C d_n{}^C}\right] \qquad \ldots(10)$$

$$k_n{}^B = \min\left[k_n, \frac{X_n{}^B}{t_h\, h^B d_n{}^C}\right] \qquad \ldots(11)$$

k_n —random variable indicating a good or bad day ($k_n = 0$ or 1)

$k_n{}^C, k_n{}^B$ —portion of a day which can be utilized at day n for harvesting corn and soybeans, respectively

$M_n{}^C, M_n{}^B$ —moisture content at day n for corn and soybeans respectively, % w.b. (14% is the safe storage moisture content for corn and 13% is the sale base for soybeans)

$y_n{}^C, y_n{}^B$ —recoverable yield at day n for corn and soybeans respectively (bu/acre)

h^C, h^B —harvesting rate for corn and soybeans respectively (acre/h)

$C_{on}{}^C, C_{on}{}^B$ —price received for corn and soybeans respectively at day n (\$/bu)

$C_1{}^C, C_1{}^B$ —drying cost or moisture dockage charge for corn and soybeans respectively [\$(% m.c.)$^{-1}$bu^{-1}]

t_h —h available for combining/day

t_d —h available for drying corn/day.

Eqns (7) to (9) specify the return as a function of the pertinent variables and parameters. Note that in this model labour cost is not included. This is because a fixed length of day is specified, t_h. A labour charge function which would reflect the cost of overtime could be easily added where necessary. The absolute value of the difference between the soybean moisture content and 13% is used, since an indirect cost is also incurred for the sale of overdry grain.

Eqns (10) and (11) represent the constraints on the portion of a day which can be used for harvesting either corn or soybeans. In Eqn (10), the first term in the minization bracket represents the weather constraint, the second term the acreage constraint and the third term the grain dryer constraint. The same definitions apply to Eqn (11) with the exception that artificial drying of soybeans is assumed not to be a constraint on system performance.

The state variable transformation equations can now be written. These equations describe the remaining acreages of corn and soybeans from day to day. For corn,

$$X_{n-1}{}^C = X_n{}^C - h^C\, k_n{}^C\, d_n{}^C\, t_h, \qquad \ldots(12)$$

and for soybeans

$$X_{n-1}{}^B = X_n{}^B - h^B k_n{}^B\, d_n{}^B\, t_h \qquad \ldots(13)$$

The return function and the transformation equations can now be combined in the dynamic programming recursive equations. First $f_n(X_n{}^C, X_n{}^B)$ is defined as the expected return with n days remaining, $X_n{}^C$ acres of corn yet to harvest and $X_n{}^B$ acres of soybeans yet to harvest when following an optimal policy. Then with one day remaining the return is found by taking the maximum over the 3 policies C, B or N.

$$f_1(X_1{}^C, X_1{}^B)=\max_{(C,B,N)} \Big[\sum_{k_1=0}^{1} p_{k_1}\{r_1(X_1{}^C, X_1{}^B, d_1{}^C, d_1{}^B, k_1)\}\Big] \quad \ldots(14)$$

Now with n stages remaining the result becomes the maximum of the sum of f_{n-1} and the return for the nth day.

$$f_n(X_n{}^C,X_n{}^B)=\max_{\substack{(C,B,N)\\ n=2,\ldots,N}} \Big[\sum_{k_n=0}^{1} p_{k_n}\{r_n(X_n{}^C,X_n{}^B,d_n{}^C,d_n{}^B,k_n)+f_{n-1}(X_{n-1}{}^C,X_{n-1}{}^B)\}\Big] \quad \ldots(15)$$

subject to Eqns (12) and (13), and where p_{k_n} is the probability that day n is suitable for field work.

There are now 2 state variables, requiring increased computer storage. The solution starts by solving Eqn (14) for many combinations of the state variables $X_1{}^C$ and $X_1{}^B$ This information is then used for solution of Eqn (15) for the second stage. This procedure is repeated for $n=3$ to N. The solution result at each stage is a table which contains the optimum decision choice for various combinations of the state variables.

3.2. *Two-crop results*

The 2-crop model is illustrated with an example using the input data of Table IV. Note that there is a charge assessed for soybeans which are too dry as well as too wet. Relationships for field moisture content and recoverable yield of soybeans are presented by Morey.[10]

TABLE IV

Input parameters for the 2-crop example

	Corn	*Soybeans*
Total acreage	200	200
Acreage increment	20	20
Harvest rate	2·5 acre/h	2·5 acre/h
Harvest h/day	10 h/d	10 h/d
Dryer capacity	not limiting	does not apply
Dryer h	16 h/d	does not apply
Maximum yield	160 bu/acre	50 bu/acre
Stage increment	1 day	1 day
Initial conditions		
Date	September 1	September 1
Moisture content	40% w.b.	22% w.b.
Total stages to November 30	91	91
Price	\$1·20/bu	\$2·50/bu
Drying or moisture dockage charge	\$0·005 (% m.c.)$^{-1}$bu^{-1}	\$0·021 (% m.c.)$^{-1}$bu^{-1}
Over drying charge	does not apply	\$0·029 (% m.c.)$^{-1}$bu^{-1}

A portion of the results for the example problem with 79 days remaining (September 13) is given in Table V. In this case the decision is which crop, if any, is to be harvested during the period. The maximum expected return along with the corresponding optimum decision is listed as a function of the acreages of corn and soybeans remaining.

TABLE V

Expected optimal return and decision with 79 days remaining (*C*—harvest corn, *B*—harvest soybeans, *N*—neither)

		79 days remaining, September 13					
Acres of beans		200	160	120	80	40	0
of corn	200	$55,414 *C*	50,877 *C*	46,336 *C*	41,792 *C*	37,241 *C*	32,679 *C*
	160	49,061 *C*	44,521 *C*	39,976 *C*	35,425 *C*	30,864 *C*	26,288 *C*
	120	42,640 *C*	38,096 *C*	33,545 *C*	28,984 *C*	24,407 *C*	19,809 *C*
	80	38,187 *B*	31,636 *B*	27,075 *B*	22,498 *B*	17,900 *B*	13,250 *C*
	40	29,702 *B*	25,141 *B*	20,564 *B*	15,965 *B*	11,331 *B*	6,633 *C*
	0	23,195 *B*	18,617 *B*	74,016 *B*	9385 *B*	4715 *B*	0 *N*

More insight into the results from this model can be obtained by displaying the optimum policy for fixed combinations of acreages remaining on the time axes (*Fig. 1, top*). The plots display the manner in which the optimum policy changes early in the season. The grain moisture content and recoverable yield functions for the corresponding time period are also displayed in *Fig. 1*. The fact that the recoverable yield of soybeans remains essentially constant over this period does not seem realistic. However, this is the result obtained from the limited available data.

Observing the time axes of *Fig. 1* for the 2 cases in which acreages of both crops remain, it is apparent that the optimum policy varies with the time of the season. It is optimum to work on soybeans until the corn reaches its peak recoverable yield point then the optimum policy specifies that corn should be harvested. This result is certainly influenced by the essentially constant recoverable yield of soybeans. In all cases the soybeans dried to less than 13% moisture content in the field before it was optimum to start harvesting.

Additional results for the 2-crop case can be found in Morey.[10]

4. Conclusions

The feedback property of a dynamic programming solution provides a very useful and practical decision tool. The decision maker (farm manager) can consult the solution table as the harvest season progresses.

The applications of the model indicate the sensitivity of the decisions to the input data. The results also provide an indication of the areas where future data collection is most needed. These include: (1) the effect of weather variables, such as air temperature and relative humidity, and rainfall on the field drying rates of the grain; (2) improved data on available field working days; (3) more complete data which would provide a statistical distribution rather than just the mean for such variables as the field drying rate and the recoverable yield of the grain.

The optimization technique which has been presented should be applicable to other operations where the influence of weather and some type of yield or biological response based on timeliness, exist.

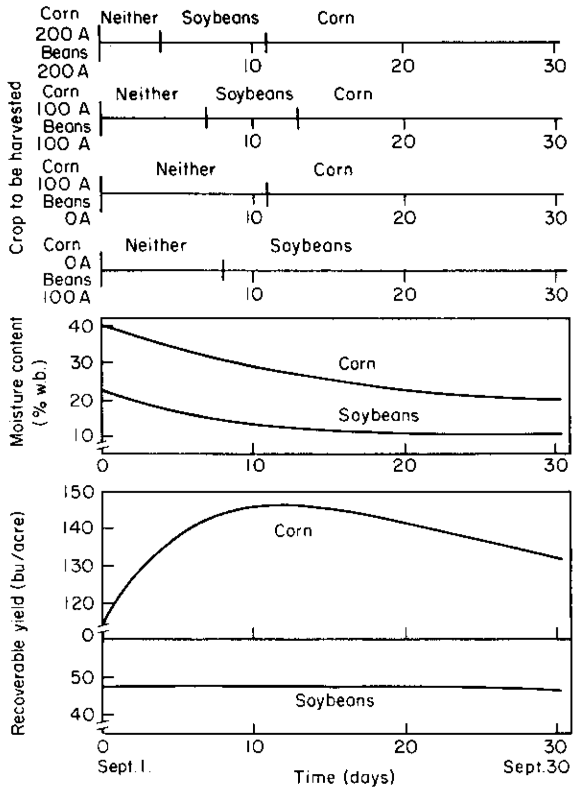

Fig. 1. Optimum policies, moisture contents and recoverable yields for the two crop example

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