New Model for Determining Thermal Diffusivity with the Thermal Probe

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ABSTRACT

A new model for the temperature rise of an infinite medium with a thermal probe is developed which accounts for the finite diameter and specific heat of the probe. The new model is more accurate than the old model as demonstrated during the determination of thermal diffusivity of burley tobacco in bales. Multiple thermocouples in the medium and multiple probings are suggested for the determination of thermal diffusivity. Reduction of moisture diffusion to a negligible level is discussed.

INTRODUCTION

One of the most common transient methods of determining thermal conductivity of agricultural materials is the thermal probe (Mohsenin, 1980). The method involves electrically heating a high, thermally conductive (compared to the sample) probe of large length to diameter ratio and measuring the probe's temperature rise. The thermal conductivity can be calculated from the slope of the plot of temperature rise against the natural log of time. In general, small diameter probes have been used to better approximate the line heat source but Woodside (1959) showed that drying near the probe could be significant for very small probes and large power inputs. He suggested using larger probe diameters and lower power inputs.

Nix et al. (1967) determined thermal diffusivity with the line heat source using an additional thermocouple in the sample at a known distance from the line heat source. This was the original use of the line heat source for determining thermal diffusivity with the use of a second thermocouple. This technique was applied to grain dust by Chang et al. (1980). In these uses of the line heat source method for determining thermal diffusivity, the solution of the heat diffusion equation for an infinite line heat source was used; and then, necessarily, a small diameter (compared to the radial distance to the second thermocouple) wire was the heat source. No example was found in the literature of using this second thermocouple with a larger diameter probe and a proper solution to the heat diffusion equation that accounts for the finite diameter and specific heat of the probe.

Specific objectives of the research described in this paper were:

1. To develop a solution for the transient temperature distribution in the medium that accounts for the finite diameter and specific heat of the probe.
2. To determine the validity of the solution to the determination of thermal diffusivity in burley tobacco bales.
3. To reduce moisture diffusion surrounding the probe to a negligible level.

MATHEMATICAL DEVELOPMENT

Part of the attractiveness of the thermal probe is the simplicity of requiring only the temperature of the probe for calculation of thermal conductivity. Determination of thermal diffusivity with the probe requires a temperature reading in the medium and a trial and error solution for the thermal diffusivity from the following equation (Chang et al., 1980):

$$\theta = \frac{Q}{2\pi k}\left[\frac{-E}{2} - \ln X - \sum_{n=1}^{\infty} \frac{(-1)^n\,(X^2)^n}{(2n)\,(n!)}\right] \quad \ldots\ldots[1]$$

For equation [1] to be valid for calculations of thermal diffusivity, it is assumed that the probe has negligible diameter and very high thermal properties. We propose to derive an equation that accounts for the finite diameter and specific heat of the probe and then to compare the two equations using experimental data from baled burley tobacco.

Assuming that the probe is sufficiently long for one-dimensional radial heat flow, the governing equation was the one-dimensional heat diffusion equation in cylindrical coordinates:

$$\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r} = \frac{1}{\alpha}\frac{\partial\theta}{\partial t}\ ;\ r > R \quad \ldots\ldots\ldots\ldots[2]$$

The following assumptions were made: 1) the probe has radius, R, and infinite thermal conductivity compared to the medium, 2) the heat generation in the probe starts at time zero and is thereafter constant; and 3) the medium is infinite in the radial direction. Thermal conductivity has been determined in anisotropic material with the thermal probe (Woodside, 1959); therefore, a

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solution to equation [2] will be applicable to anisotropic medium. The boundary and initial conditions were:

$$-2\pi Rk\,(\partial\theta/\partial r) = Q - S\,(\partial\theta/\partial t)\ ;\ r = R, t > 0 \quad \ldots\ldots [3]$$

$$\theta \text{ finite as } r \to \infty \quad \ldots\ldots\ldots\ldots [4]$$

$$\theta = 0;\ t = 0 \quad \ldots\ldots\ldots\ldots [5]$$

The solution to equation [2] subject to the initial and boundary conditions was accomplished using Carslaw and Jaeger's (1959) technique. Equation [2] was solved by Laplace transform using the inversion methods of Carslaw and Jaeger. The details of solution are found in Appendix A of Casada (1985). The first three terms of the solution of equation [2] were:

$$\begin{aligned}
\theta = [Q/4\pi k)] \Big\{ & L + R^2 L(1 - 2/\beta)/(2\alpha t) \\
& + [1/(4\alpha t)]\,[R^2 + r^2 - 2R^2\,(1 - 2/\beta)\ln(R/r)] \\
& + [R^4 L/(16\alpha^2 t^2)]\,[1 - 12/\beta + 24/\beta^2 \\
& \quad + 8(1 - 4/\beta + 4/\beta^2)\ln(R/r) - (3 - 12/\beta + 12/\beta^2)L] \\
& + [R^4/(16\alpha^2 t^2)]\,[11 + 2\pi^2)/4 - (8 + 2\pi^2)/\beta \\
& \quad + 2\pi^2/\beta^2 - (3 - 20/\beta + 32/\beta^2)\ln(R/r) \\
& \quad - (4 - 16/\beta + 16/\beta^2)(\ln(R/r))^2] \\
& - [R^2/(16\alpha^2 t^2)]\big\{r^4/(4R^2) \\
& \quad - 2r^2[(1 - 2/\beta)(1 + \ln(R/r)) - 1/2]\big\} \\
& + R^2 r^2 L(1 - 2/\beta)/(8\alpha^2 t^2) \\
& \quad - R^2 r^2(1 - 2/\beta)/(8\alpha^2 t^2) + \ldots \Big\} \quad \ldots\ldots [6]
\end{aligned}$$

where

$L = \ln[4\alpha t/(Cr^2)]$

$C =$ exp (Euler's constant)

$\beta = 2\pi R^2 k/\alpha S$.

Terms of order $1/t^3$ and higher were neglected, which necessitated the use of times large enough to make those terms small. We found that when only the first two terms of the solution of equation [2] were retained, the model would not describe the temperature rise in the medium well.

When all terms of order $1/t$ and higher were neglected, equation [6] reduced to the same equation for the probe temperature as is normally used for thermal conductivity determination. Thus, the determination of thermal conductivity was not changed but the determination of thermal diffusivity by equation [6] would be expected to be superior to its determination by equation [1]. The validity of equation [6] was evaluated by comparing the standard error of regression of equation [1] and [6] when applied to temperature-time data of a point at a given distance from the probe in burley tobacco bales.

PROBE DESIGN AND ERROR CONSIDERATIONS

A 0.29 m long x 2.38 mm diameter probe described by Casada and Walton (1989) was used in this research. The 0.254-mm constantan heating wire was coated with a high thermally conductive silicone paste before being inserted in the four hole ceramic tube. The ceramic tube was coated in the same manner and was then inserted in the stainless steel tube. The heating wire was connected to a constant voltage power supply that was adjusted to 1.9 V. During a test, this voltage was scanned every second with a digital voltmeter and microcomputer. The heating wire loop at the bottom of the probe was protected with a drop of silicone rubber caulk.

Two-0.254 mm diameter iron/constantan thermocouple probes were inserted in the remaining two holes in the ceramic tube. One thermocouple was 3.2 mm above, and the other 3.2 mm below the center of the probe. These thermocouples were scanned every second using the digital voltmeter.

Probe diameter was a compromise between ease of construction, magnitude of heat flux at the probe surface, and maintenance of axial flow as affected by length over diameter ratio. The first two considerations made a large diameter desirable and the last consideration favored a small diameter probe.

The effect of axial heat flow and finite sample size were negligible as determined by calculations made from the techniques given by Mohsenin (1980).

Moisture Diffusion Caused by Temperature Gradient Near Probe

We were interested in the change in moisture content of an annulus or volume element around the probe with an outer radius twice the probe radius because we intended to place the thermocouple near that point in the tobacco. The change in moisture content, Δm, for the volume element was an adaption of Woodside's analysis (1959):

$$\Delta m = [(\partial P/\partial T)R' - \rho_w]\ DQt/(3\pi R^2 \rho_s kT) \quad \ldots\ldots [7]$$

Equation [7] can be used to predict the change in sample moisture content near the probe during a proposed test. If significant amounts of drying are predicted, the probe design may be changed by either increasing the probe diameter or decreasing the power input until the drying around the probe is reduced to an acceptable level. This method was used to reduce moisture diffusion to an insignificant level.

EXPERIMENTAL PROCEDURE

The thermal diffusivity of burley tobacco bales was determined with the thermal probe by placing two, 36-gauge iron/constantan thermocouples in burley tobacco bales at a distance equal to approximately one probe diameter (0.3 cm) from the surface of the probe and located near the midpoint of the probe, axially. The distance from the probe was measured by a dial caliper. The temperature rise of the points was monitored as a function of time after initiation of heating. Each test was initiated by the microcomputer which activated the heater and, thereafter, scanned the thermocouples and heater voltage at one second intervals for 300 s.

Tests were run on two samples from each of nine bales of burley tobacco from the bottom, middle and top stalk positions at low, normal and high moisture contents. The two replicated samples were extracted from each bale by cutting 0.15 m of length from each end of the bale and cutting the bale vertically in half. The two samples, thus obtained from each bale, were 0.3 x 0.3 m in cross-section and 0.9 m high. The samples were first tested at their naturally occurring density, which was dependent on sample moisture content, and then compressed to a density of 320 kg/m^3 if the density was less than this value. Some of the high moisture samples did not require further compression.

To determine the validity of the model developed as equation [6] for the determination of thermal diffusivity in burley tobacco bales, both equation [1], which assumes negligible probe diameter, and equation [6], which accounts for finite probe diameter, were fit to the temperature rise of the monitored points by minimizing the sum of squares of the deviation from regression. The accuracy of the two equations was compared via standard error of regression defined as the square root of the sum of the squared differences between observed and predicted temperature rise divided by the number of observations less one. The data for the diffusivity determination were from time intervals varying from 80-200s to 150-200s. The initial times varied to obtain an initial dimensionless time, $4\alpha t/r^2$, sufficiently large so that equation [6] did not require the truncated higher order terms to describe the temperature rise at the additional thermocouple.

TABLE 1. Standard Errors of Regression and Thermal Diffusivities of Equations [1] and [6] as a Function of Moisture Level, Stalk Position, Replication, Thermocouple and Density

	Standard Error of Regression				Thermal Diffusivity, $m^2/s \times 10^8$			
	Normal Density		High Density		Normal Density		High Density	
Test Code	Eqn. [1]	Eqn. [6]	Eqn. [1]	Eqn. [6]	Eqn. [1]	Eqn. [6]	Eqn. [1]	Eqn [6]
LB11	.0489	.0375	--	--	7.37	9.58	--	--
LB12	.0361	.0154	--	--	5.12	5.83	--	--
LB21	--	--	--	--	--	--	--	--
LB22	.0300	.0192	--	--	6.32	7.36	--	--
LM11	.0322	.0107	.0293	.0233	5.42	5.44	5.79	5.75
LM12	.0746	.0410	.0476	.0283	7.37	8.37	7.94	9.38
LM21	.0323	.0166	.0230	.0138	7.67	9.52	7.32	9.44
LM22	.0333	.0180	.0141	.0143	5.39	6.04	5.76	6.40
LT11	.0428	.0119	.0250	.0159	5.16	5.63	5.89	6.21
LT12	.0302	.0127	.0191	.0203	5.14	5.39	6.06	6.25
LT21	.0694	.0239	.0379	.0230	5.10	5.79	6.35	7.21
LT22	.0381	.0169	.0217	.0116	5.76	6.44	7.20	8.04
NB11	.0349	.0130	.0260	.0111	6.58	7.96	6.47	7.24
NB12	.0239	.0257	.0373	.0214	3.57	3.37	3.58	3.52
NB21	.0392	.0149	.0236	.0132	3.87	4.02	4.84	5.08
NB22	.0300	.0280	.0278	.0237	5.72	6.98	6.67	7.82
NM11	.0627	.0251	.0542	.0243	6.98	8.63	6.64	7.64
NM12	--	--	--	--	--	--	--	--
NM21	.0299	.0134	.0297	.0158	8.26	9.80	8.04	9.28
NM22	.0241	.0140	.0217	.0178	7.20	8.06	7.63	8.50
NT11	.0350	0179	.0268	.0182	6.66	7.53	7.84	8.76
NT12	.0294	.0138	.0259	.0126	5.36	5.60	5.57	5.64
NT21	--	--	--	--	--	--	--	--
NT22	.0398	.0234	.0355	.0156	6.49	7.30	6.47	7.08
HB11	.0319	.0187	.0294	.0179	8.14	9.70	7.99	9.36
HB12	--	--	--	--	--	--	--	--
HB21	.0415	.0285	.0320	.0217	8.79	10.29	9.22	10.64
HB22	.0287	.0136	.0225	.0119	6.77	7.02	7.19	7.45
HM11	.0109	.0195	--	--	9.15	11.19	--	--
HM12	.0111	.0183	--	--	4.76	4.67	--	--
HM21	.0322	.0204	--	--	7.14	8.17	--	--
HM22	.0249	.0154	--	--	6.69	7.40	--	--
HT11	--	--	.0167	.0160	--	--	6.80	7.19
HT12	--	--	.0207	.0154	--	--	6.77	7.14
HT21	.0248	.0121	--	--	5.24	5.26	--	--
HT22	.0280	.0225	--	--	8.19	9.43	--	--
Mean	.0353	.0194	.0282	.0177	6.38	7.26	6.70	7.39

Test Code: First character = moisture level (L=low, N=normal, H=high); second character = stalk position (B=bottom, M=middle, T=top; Third character = Replication (1 or 2), Fourth character = thermocouple (1 or 2).

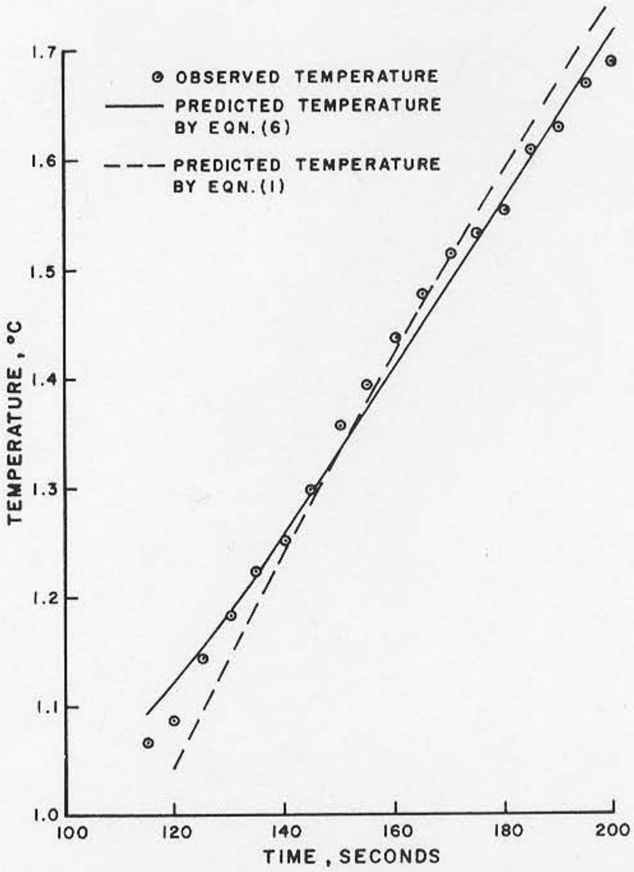

Fig. 1—Observed and predicted temperature rise by equations [1] and [6] of a point 3.84 mm from the probe in the burley tobacco bale.

RESULTS AND DISCUSSION

Standard errors of regression and thermal diffusivities for equations [1] and [6] are shown in Table 1. Equation [6] was superior to equation [1] in 49 of the 53 values of standard errors of regression. The average S.E. for equation [6] was 0.0194 and 0.0177 compared to 0.0353 and 0.0282 for equation [1] at the normal and high density, respectively. Typical observed and predicted temperature rise at the second thermocouple (low moisture, middle stalk position, replication 2) is shown in Fig. 1. Equation [1] consistently underpredicted early in the time period and overpredicted late in the time period. Equation [6] typically overpredicted early in the time period and fit well thereafter. The average standard error as a percent of the average test temperature rise for all tests was 1.33% and 2.29% for equations [6] and [1], respectively. The new model (equation [6]) for temperature rise near the probe is a better model for its current application based on its more accurate prediction of temperature rise compared to the old model (equation [1]), which was attributable to the new model's accounting for probe diameter and thermal properties. The average thermal diffusivity for all tests was 7.33 x 10^{-8} m^2/s by equation [6], a value that is 12.4% higher than the 6.52 x 10^{-8} m^2/s calculated by equation [1]. The magnitude of the difference calculations would result if equation [6] were not used for the thermal probe constructed for this study.

The results from the two thermocouples and the two replications in Table 1 give an indication of the variability of thermal diffusivity measurements in burley tobacco bales. The variability within these four values was attributed to biological variation and error in measuring the radial distance to the thermocouple. An error analysis by Casada (1985) showed that the uncertainty in movement of the thermocouple placement (assumed equal to the thermocouple bead diameter, 0.35 mm) caused an average uncertainty of $\pm 16\%$ in the thermal diffusivity as calculated by equation [6]. The average coefficient of variation (standard deviation x 100/mean) for these four determinations in the thermal diffusivity calculated from equation [6] was $\pm 21.1\%$. Thus, a substantial portion of the uncertainty in thermal diffusivity may be attributable to the difficulties in the measurement of the radial distance from the probe to the thermocouple. With the potential for relatively large errors, it is clear that care must be exercised in the placement of the thermocouples in the medium. A natural inclination would be to move the thermocouples further from the probe, but this would increase the required temperature rise at the probe which would thereby increase moisture diffusion.

The pathway for heat to diffuse from the probe to the thermocouple was a parallel combination of interconnected air space and tobacco leaves which may vary greatly from high density variation within the bale. A local air pocket could create considerable variation in the data. Some natural biological variation would also be expected. Therefore, we attribute the variation seen in Table 1 to an error in measurement of the radial distance from probe to thermocouple, local air-pathway variations and natural biological variation. More thermocouples, more replications and multiple probings should be used to determine reliable average thermal diffusivities.

CONCLUSIONS

Conclusions based on the research were: 1) the new model (equation [6]) for determining thermal diffusivity is superior to the old model (equation [1]) because it accounts for probe diameter and thermal properties and is, thereby, more accurate than the old model; 2) determination of the radial distance from the probe to the thermocouple imbedded in the medium is a serious potential source of error. Great care must be exercised in this measurement; 3) multiple thermocouples and probings are suggested with adequate replications to define mean thermal diffusivity and the standard deviation about the mean that is caused by density and biological variation; and 4) in moist porous media, power may be reduced and probe diameter increased to reduce moisture diffusion to a negligible level.

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NOMENCLATURE

D	diffusion coefficient of medium, m^2/s
E	Euler's constant = 0.5772157
k	thermal conductivity, W/(m°C)
Δm	change in moisture content, dry basis
n	positive integers
P	vapor pressure, N/m^2
Q	line source strength per unit length, W/m
r	distance from line source, m
R	probe radius, m
R'	gas constant for water vapor, N·m/(kg · °K)
S	probe specific heat per unit length, J/kg · °C · m
$T(t,r)$	temperature of medium, °C
T_i	initial temperature of system, °C
T	temperature of annulus, °K
t	time from start of heat flow, s
X	$r/2\sqrt{\alpha t}$), dimensionless parameter
α	thermal diffusivity, m^2/s
ρ_s	bulk dry density of medium, kg/m^3
ρ_w	density (concentration) of vapor, $kg/m^2/s$
Θ	$\Theta(t,r) = T(t,r) - T_i$