A New Diffusion Model for Drying Burley Tobacco

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ABSTRACT

DIFFUSION models were developed for the lamina and midrib which model the geometry of the internal cells and which divide the total resistance to moisture diffusion between the internal resistance of the interior cells and an external resistance as provided by the epidermal-stomatal layer. The new diffusion models had a lower standard error than previous models as compared to experimental data and were thereby concluded to be superior to them.

INTRODUCTION

Curing of burley tobacco is the process by which the yellowish-green high moisture leaf found at harvest is converted to the brown aromatic low moisture leaf ready for market. The most obvious results of the physical and biochemical processes that take place during curing are drying and color change, respectively. The color change is indicative of the myriad enzymatic reactions taking place within the leaf. The leaf components (lamina and midrib) lose moisture during curing to the ambient air until they are in equilibrium with the relative humidity of the ambient air. During drying, most of the original weight of the leaf is lost because the lamina and midrib have such high moisture contents, 420 and 991% (db) respectively, at harvest (Walton et al., 1984).

Bunn and Henson (1968), Chang and Johnson (1971), and Bunn et al. (1972) determined the drying rate of whole primed leaves. Walton et al. (1982) found a differential in drying rate between the lamina and midrib during curing and showed that a significant amount of midrib moisture is tranferred through the lamina to the ambient air. Walton et al. (1984) determined the drying rates of the three burley plant components (lamina, midrib, and stalk) with sampling from intact plants.

Humphries (1964) proposed a drying model based on the physiology of the lamina. Noting that a large portion of the interior of the lamina is intercellular pore space, he reasoned that the moisture moves from the vacuole through the cytoplasm to the pore space and then out through the epidermal layers to the atmosphere. He deduced that the moisture moved through two sources of resistance in series which are the cytoplasm and the epidermal layers. Humphries then determined the moisture conductivity of the epidermal layers.

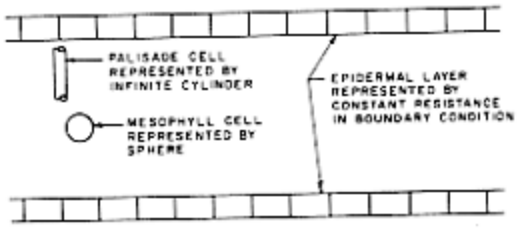

Fig. 1—Geometry of the burley tobacco lamina used in the development of the new lamina model.

A deep layer drying model (Bridges et al., 1981) has been developed which showed potential for predicting temperature and relative humidity in a burley curing facility (Bridges et al., 1982). A better model of individual leaf drying would enhance accuracy of the deep layer model. The objective of the research described in this paper was to develop and evaluate the accuracy of a cellular drying model based on leaf physiology.

MATHEMATICAL MODEL

The geometrical model under consideration for the lamina (Fig. 1) shows a spherical cell and a cylindrical cell within the boundaries of the upper and lower epidermis. The spherical cell represents the spongy mesophyll and the cylinder represents the palisade parenchyma. The model was developed using geometry of the cell and a series resistance for the boundary condition consisting of the resistances of the pore space, epidermal layer, and the boundary layer. Considering the transfer of moisture contained in the cells to the ambient side of the lower and upper epidermis (i.e. drying of the leaf), the resistance to the moisture transfer within the cell was taken into account by a diffusion coefficient. The resistance to moisture transfer from the cell surface through the epidermal layer to the ambient air is the sum of three resistances in series which are the resistance of the pore space between the cell surface and the epidermal layers, the resistance of the epidermal layers, and the resistance of the boundary layer. A typical epidermal resistance (Humphries, 1964) is 174 s/cm. Using the diffusion coefficient of 0.25 cm/s as given by Krieth (1964) and a path length of 80% of the leaf thickness which was taken as 235 microns, a calculated resistance for the pore space was 0.08 s/cm. The resistance of the boundary layer (the inverse of the convective mass transfer coefficient) was calculated to be 6.2 s/cm for an air velocity of 15 cm/s. Based on these values, we assumed the resistance of the pore space within the leaf and the boundary layer to be negligible compared to the resistance of the epidermal layer. The

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epidermal resistance was considered to be constant throughout the curing cycle. With these assumptions, transfer of moisture across the epidermal layer became the boundary condition for transfer from the mesophyll and palisade cells as will be shown in the following analysis.

The geometrical shapes chosen to represent the mesophyll and the palisade cells were the sphere and the infinite cylinder, respectively. Anderson (1937) indicated that the cells forming the cortex of the midrib were approximately cylindrical in shape. Therefore, the geometric shape chosen to represent the midrib cells was the infinite cylinder.

The governing equation was the vapor diffusion equation (Walton et al., 1976)

$$D' \nabla^2 \Theta = \frac{\partial \Theta}{\partial t} \quad \ldots\ldots [1]$$

where D' = modified mass diffusion coefficient based on mass of water per unit mass of solid, cm²/h
Θ = $(M - M_e)/(M_o - M_e)$
t = time, h
M = moisture content at time t, % d.b.
M_e = equilibrium moisture content, % d.b.
M_o = initial moisture content, % d.b.

A uniform moisture content, M_o was assumed for the initial condition. One boundary condition was that of symmetry which assumed that the moisture gradient at the center of the cell was zero. The other boundary condition was developed by writing a moisture balance which equated the moisture flow to the surface of the cell and the moisture flow through the epidermal layer. The moisture balance at the surface which provides the other boundary condition reduced to

$$-D \frac{\partial M}{\partial r}(R, t) = \frac{1}{R_{es}} [M(R, t) - M_e] \quad \ldots\ldots [2]$$

where r = radial coordinate, cm
R = cell radius, cm
D = mass diffusion coefficient based on concentration gradient, cm²/h
D = $D' p_s \beta$
p_s = dry weight density of tobacco tissue, kg/cm³
β = constant from M_e isothermal, cm³/kg
R_{es} = resistance of epidermal-stomatal system.

Equation [2] was also applied to the midrib. The approximate cylindrical shape of the cortex cells leaves considerable intercellular pore space. Resistance to moisture diffusion through the surface of the midrib is provided by the stomates in parallel with the cuticle, epidermal cells, and chlorenchyma cells. Anderson (1937) reported that the midrib has stomates but not nearly so numerous as in the lamina. He also reported that the chlorenchyma consists of two or three layers of tightly packed cells below the epidermus. The cuticle comprises the tough outer surface of the midrib.

Equation [1] was solved by separation of variables (Arpaci, 1966) subject to the boundary condition of equation [2] and the other conditions discussed earlier. The solutions for the sphere and cylinder were in terms of r and t. We integrated with respect to r to obtain the average moisture content as a function of t alone. The solution for the sphere was

$$\Theta(t) = 6 \sum_{n=1}^{\infty} \frac{(\sin \lambda_n R - \lambda_n R \cos \lambda_n R)^2}{(\lambda_n R)^3 (\lambda_n R - \sin \lambda_n R \cos \lambda_n R)} e^{-(D'/R^2)(\lambda_n R)^2 t} \quad \ldots\ldots [3]$$

where the $\lambda_n R$ values were solutions of

$$\lambda_n R \cos \lambda_n R = \left(1 - \frac{[R/D' \rho_s \beta]}{R_{es}}\right) \sin \lambda_n R \quad \ldots\ldots [4]$$

The term, $(R/D' p_s \beta / R_{es})$, was designated as a modified Biot number for mass transfer (denoted by Bi_m) so that equation [4] became

$$\lambda_n R \cos \lambda_n R = (1 - Bi_m) \sin \lambda_n R \quad \ldots\ldots [5]$$

which was identical to the classical characteristic equation found in Arpaci (1966).

The solution for the infinite cylinder was

$$\Theta(t) = 4\, Bi_m \sum_{n=1}^{\infty} \frac{J_1(\lambda_n R)}{\lambda_n R (\lambda_n^2 R^2 + Bi_m^2) J_o(\lambda_n R)} e^{-(D'/R^2)(\lambda_n R)^2 t} \quad \ldots\ldots [6]$$

where $J_o(\lambda_n R)$ = Bessel function of first kind of order zero
$J_1(\lambda_n R)$ = Bessel function of first kind of order one.

The $\lambda_n R$ values for equation [6] were solutions of

$$\lambda_n R J_1(\lambda_n R) = Bi_m J_o(\lambda_n R) \quad \ldots\ldots [7]$$

with Bi_m as previously defined.

Equation [6] was used as the model for the midrib. The model for the lamina was a combination of equations [3] and [6] based on the relative volume of mesophyll parenchyma and palisade parenchyma to total cell volume.

The cellular models for the lamina and midribs were compared to the previous models used for the lamina and midrib. The model previously used to represent the lamina was the solution of equation [1] for the geometry of the infinite thin sheet with an infinite Bi_m (negligible surface resistance), which was developed assuming that

the lamina is homogeneous

$$\Theta(t) = \sum_{n=0}^{\infty} \frac{2}{(\lambda_n L)^2} e^{-(D'/L^2)(\lambda L)^2 t} \quad \ldots\ldots\ldots\ldots [8]$$

where L = half thickness of the lamina, cm
$\lambda_n L = (2n+1)\pi/2$
n = 0, 1, 2, . . .

The model previously used for the midrib was equation [6] with infinite Bi_m (negligible surface resistance), which included the assumption of homogeneity of the midrib. Since the exponential equation was used as a standard of comparison in earlier work (Walton et al., 1982), we have included the exponential equation for comparison

$$\Theta(t) = e^{-kt} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [9]$$

where k = drying constant, h

The models compared in this paper were:

1. New lamina model - a combination of equations [3] and [6] which model the mesophyll and palisade cells as spheres and cylinders, respectively, with a non-zero surface resistance provided by the upper and lower epidermal cells in parallel with the stomata.
2. Old lamina model - equation [8] which models the lamina as a homogeneous infinite thin sheet with negligible surface resistance.
3. New midrib model - equation [6] which models the cortex cells as infinite cylinders with a non-zero surface resistance provided by the cuticle, epidermal cells, and chlorenchyma cells in parallel with the stomata.
4. Old midrib model - equation [6] which modeled the midrib as a homogeneous infinite cylinder with negligible surface resistance.
5. Exponential model - equation [9].

EXPERIMENTAL PROCEDURE

Drying data for the lamina and midrib were required to determine the accuracy of the mathematical models. Data, sufficient for the purposes of this research, was obtained by Walton et al. (1982) to evaluate the old lamina and midrib models. A brief description is included here for the convenience of the reader.

Eight plants of burley variety KY-14 were harvested at optimum maturity and separated into two groups of four plants for curing at temperature and relative humidity combinations of 29 °C, 43% and 24 °C, 65%. Two leaves were removed from the middle stalk position of each plant and separated into their lamina and midrib components. The wounded edge of both the lamina and midrib were coated with paraffin to prevent moisture loss from the severed edge. The sample size was thus two leaves separated into their leaf components (lamina and midrib). There were four replications of each sample.

Equations [3], [6], [8], and [9] were fitted to the desorption data using the method of Marquardt (1966) of minimizing the sum of squares of the differences between observed and predicted values of moisture ratio, $\Theta(t)$, through an iterative process. The parameters generated by fitting the new diffusion models to the data were D'/R^2 and Bi_m. By determining D'/R^2 as a single term, we eliminated the need to know the cell radius, R. The fitting of the midrib model, equation [6], to the data was straightforward, but the fitting of the lamina model was more involved. The model for the lamina was a combination of equations [3] and [6] based on the relative volume of mesophyll or palisade parenchyma to total volume of palisade and mesophyll cells. Since the ratio of mesophyll or palisade parenchyma to the mesophyll and palisade parenchyma was unknown, we tried values of 0.3, 0.4, 0.5, 0.6, and 0.7 to determine the ratio that gave the best fit to the data. The standard error of regression was used to compare the accuracy of the various models. The standard error was defined as the square root of the sum of the deviations from regression squared divided by the number of data points less one.

RESULTS AND DISCUSSION

The standard error of regression of the new and old lamina model, new and old midrib model, and exponential model, when applied to the separated lamina and midrib of the leaf, is shown in Table 1. The new lamina model (ratio of cylinder and sphere to total cells was 0.6 and 0.4, respectively) was 72% and 26% lower in standard error than the old lamina model and the exponential model, respectively, in describing the drying of the lamina when separated from the midrib. The new midrib model was 70% and 73% lower in standard error than the old midrib model and exponential model, respectively, in describing the drying of the midrib when separated from the lamina. Therefore, we concluded that the new lamina and midrib models based on approximate cellular geometry and a non-zero external resistance provided by the epidermal layer were superior to the other models that were evaluated.

The modified mass diffusion coefficients and modified Biot numbers for the separated lamina and midrib of the leaf are shown in Table 2. A comparison of the lamina

TABLE 1. STANDARD ERROR OF REGRESSION OF THE NEW AND OLD LAMINA MODEL, NEW AND OLD MIDRIB MODEL, AND EXPONENTIAL MODEL WHEN APPLIED TO THE SEPARATED LAMINA AND MIDRIB OF THE LEAF

Environmental conditions			Standard error of $\Theta(t) \times 10^2$					
			Lamina			Midrib		
Temp., °C	R.H., %	Plant no.	New* model	Old model	Exponential model	New model	Old model	Exponential model
29	43	1	0.71	1.39	1.32	1.13	3.24	4.12
		2	0.52	1.98	0.62	0.90	1.78	3.40
		3	0.42	1.51	0.86	0.90	2.92	3.21
		4	0.53	1.89	0.58	0.69	3.61	1.87
24	64	1	0.71	2.88	0.68	1.15	3.13	3.69
		2	0.65	3.31	0.64	0.69	2.59	2.70
		3	0.83	2.35	1.31	0.85	2.90	2.70
		4	0.75	3.22	0.92	0.57	2.95	3.72

TABLE 2. MODIFIED MASS DIFFUSION COEFFICIENTS AND MODIFIED BIOT NUMBERS FOR THE SEPARATED LAMINA AND MIDRIB OF THE LEAF

Environmental conditions			Modified mass diffusion coefficients, h^{-1}			Modified Biot Number		
			Lamina		Midrib	Lamina		Midrib
Temp., °C	R.H., %	Plant no.	Cylinder ($D'/R^2 \times 10^2$)	Sphere ($D'/R^2 \times 10^2$)	($D'/R^2 \times 10^3$)	Cylinder	Sphere	
29	43	1	1.228	0.773	1.330	3.91	3.75	9.17
		2	2.084	1.315	0.375	1.70	1.75	11.40
		3	1.348	0.815	0.751	2.88	2.95	7.04
		4	2.407	1.577	0.581	1.55	1.48	6.26
24	65	1	2.261	0.902	0.541	1.08	1.35	10.79
		2	2.674	1.176	0.332	0.74	1.05	10.13
		3	2.272	0.635	0.455	1.35	1.81	7.56
		4	1.084	0.746	0.619	1.63	1.43	10.34

and midrib coefficients showed that the midrib coefficients were much smaller than the lamina coefficients which signified the slower drying of the midrib as compared to the lamina as shown in Fig. 2. The half life for moisture desorption [$\Theta/(t) = 0.5$)] of the midrib was 186 h as compared to 20 h for the lamina. The midrib data along with the prediction of the new and old midrib models showed the problem of the old diffusion model and the corresponding superiority of the new diffusion model. The old diffusion model considered all resistance to moisture diffusion to be internal with negligible surface resistance which resulted in an over-prediction of moisture loss in the early stages of drying. The new diffusion model considered the resistance to moisture diffusion to be divided between internal and external resistance which resulted in a much better prediction of moisture loss in the early stages of drying.

In fitting the lamina model to the data, it was apparent that the concept of a series resistance to moisture movement was more important than the geometric shape assumed for the interior cells. The lamina model that was a combination of the cylindrical model (0.6) and spherical model (0.4) showed only a 3% and 6% improvement in standard error over a totally cylindrical or totally spherical model, respectively. Therefore, it was the concept of a non-zero external resistance that resulted in most of the improvement in standard error of the lamina while the proper selection of geometric models for the interior cells resulted in only small improvement in the standard error.

The modified Biot number represented the ratio of internal to external resistance. Clearly the total resistance of the midrib was higher than the total resistance of the lamina thus causing the midrib to dry slower than the lamina. The standard errors of Table 1 show that the external resistance contributed significantly to the superior "fit" of the new diffusion models to the data, yet the modified Biot numbers showed the internal resistance was much larger than the external resistance probably because the stomata improved the permeability of the epidermal-stomatal layer. The ratio of internal to total resistance was higher for the midrib than the lamina as indicated by the higher modified Biot numbers for the midrib. The external resistance of the midrib should be higher than that of the lamina because of fewer stomata in the midrib. The internal resistance of the midrib, which was proportional to R/D, was much larger than the internal resistance of lamina because the diameter of the cortex cells were nearly equal to the total thickness of the lamina and the diffusion coefficient of the midrib was smaller than that of the lamina. Thus, application of the new diffusion models to the data resulted in higher modified Biot numbers for the midrib than the lamina primarily because of the size of the cortex cells.

Unfortunately, our lack of data for some of the parameters comprising the modified Biot number prevented the calculations of the external resistances. The parameters p_s and β need to be determined for the curing lamina and midrib.

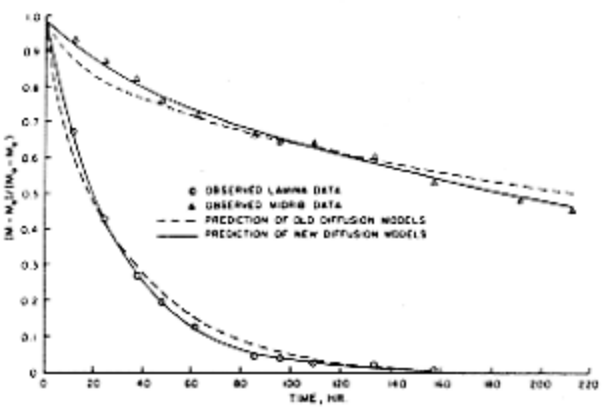

Fig. 2—Observed lamina and midrib moisture ratio during curing with prediction of new and old diffusion models.

CONCLUSIONS

The conclusions formulated from the results of this research were

1. The new diffusion models based on cellular geometry and a non-zero external resistance were superior to previous diffusion models that assumed the lamina and midrib to be homogeneous materials with zero boundary resistance.
2. The ratio of internal and external resistances was higher for the midrib than the lamina because the interior cells of the midrib are much larger than that of the lamina.
3. The concept of a non-zero external resistance was much more important in reducing the standard error than was the selection of geometric shapes for the interior cells.

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