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Analyses of Variation of Fine Material and Broken Kernels in Grain

F. S. LAI*

A method of analyzing the variation in the distribution of fine material and broken kernels in grain is proposed. Since a periodic distribution is highly unlikely, a systematic sampling plan is recommended. The application of χ^2, degree of mixing, and the discrete Fourier transform for the analyses of the variation of grain mixtures has been tested. It has been found that the discrete Fourier transform power spectrum possesses more significant physical meaning than the variance itself.

1. Introduction

If grain is graded on the basis of the results of sampling for the purpose of marketing, a simple analysis of the variation of the percentage of fine material and broken kernels should be available. Analysis of variation would not be difficult if no significant variation existed within the lot. If the lot is uniformly mixed throughout, a single sample from any place within the lot would be representative of the entire lot. Generally, however, a grain lot contains fine material and broken kernels of different sizes, and uniformity is difficult to maintain within the lot. The fine material and broken kernels tend to segregate during handling and loading. In fact, it is virtually impossible to uniformly load a freight car, barge or truck with grain without mechanical spreaders.

Recently, Stephens and Foster[1] used mechanical grain spreaders to fill bins with grain. In evaluating the variation of distribution of fine material and broken kernels in the bin, they encountered difficulties in analyzing the samples, because the fine material and broken kernels tended to segregate. One of the difficulties is finding a single measure of the variation of the percentage of fine material and broken kernels.

The most widely used criterion for measuring the variation of a grain sample is based on the variance of spot samples. The variance, however, reveals little as to the size distribution of particles in the mixture because it is a statistical quantity which depends largely on the number as well as the size of samples taken from the mixture. Extensive studies [2–7] on the analysis of variation of solids mixture have been carried out. Most of these investigations have emphasized the analysis of variance of spot samples. However, the use of the analysis of variance to evaluate the uniformity of distribution of fines and broken kernels in a bin filled with grain has several deficiencies. There are empty cells in the spot sampling due to the fluctuation of the height of the grain pile along the radial direction. Consequently, it is impossible to calculate a standard analysis of variance.

Recently, the orthogonal transforms have been used to interpret the data from image analysis.[8–10] The Fourier transform (FT) is one of the orthogonal transforms that can be applied to characterize the mixing pattern of grain. The concentration of fine materials and broken kernels obtained from a sample can be digitalized and the FT can be performed in a discrete manner. Since the Discrete Fourier Transform (DFT) algorithms are available,[8, 11] rapid treatment of the collected data is possible. This paper proposes a new method to determine the relationship between the homogeneity of a grain lot and its DFT power spectrum.

* U.S. Grain Marketing Research Center, Agricultural Research Service, U.S. Department of Agriculture, Manhattan, Kansas 66502, U.S.A.

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2. The problem

Initially, a batch of grain contains an amount, measured by weight, of fine material and broken kernels. The fine material and broken kernels are distributed according to an arbitrary unknown function of the co-ordinates in the mixture inside the container or the lot. The grain lot is transferred to another container, and during transfer, an additional amount of fine materials is generated because of handling (breakage and abrasion). The geometric configurations of the grain piles at the beginning and the end of transfer will be different if different spreaders are employed. Questions arise as to the proper sampling procedure and a meaningful way for expressing the homogeneity of the mixture.

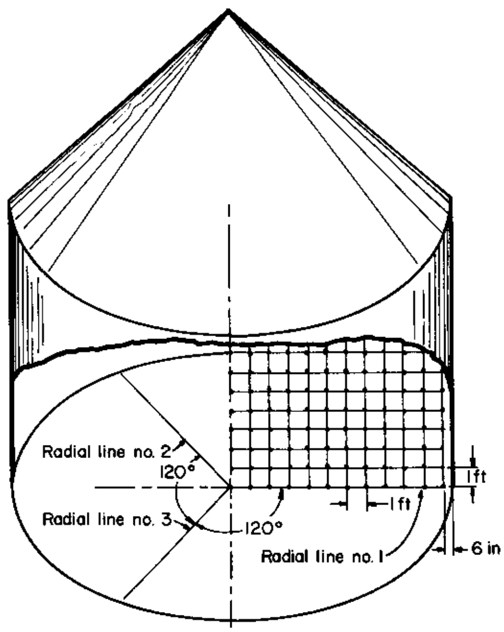

Fig. 1. Sampling scheme for measuring fine material distribution

3. Solutions

Stephens and Foster[1] used a standard grain probe 10 ft long divided into compartments to sample the bin along 3 radial directions, in a vertical plane, at 1-ft intervals starting at the centre of the bin (*Fig. 1*). Samples from each two adjacent compartments in the probe were combined and then screened with a 12/64-in round-hole sieve. The fine material and broken kernels of corn are defined as kernels and pieces of kernels of corn which will pass readily through a 12/64-in round hole sieve.[12]

The percentage by weight of fine material and broken kernels that passed through the sieve in each sample was recorded. Typical results are given in *Figs 2–5* for filling with the use of an auger spreader (test 2). In general, each method of filling is not symmetrical about the axis of the bin. Hence 3 radial lines were used.

Figs 2–4 show the distributions of fine material and broken kernels along the three radial directions for test 1, and *Fig. 5* shows the distributions of the mean of the fine material and broken kernels along the three radial directions. *Figs 6–9* show the results for test 2. These plots exemplify two

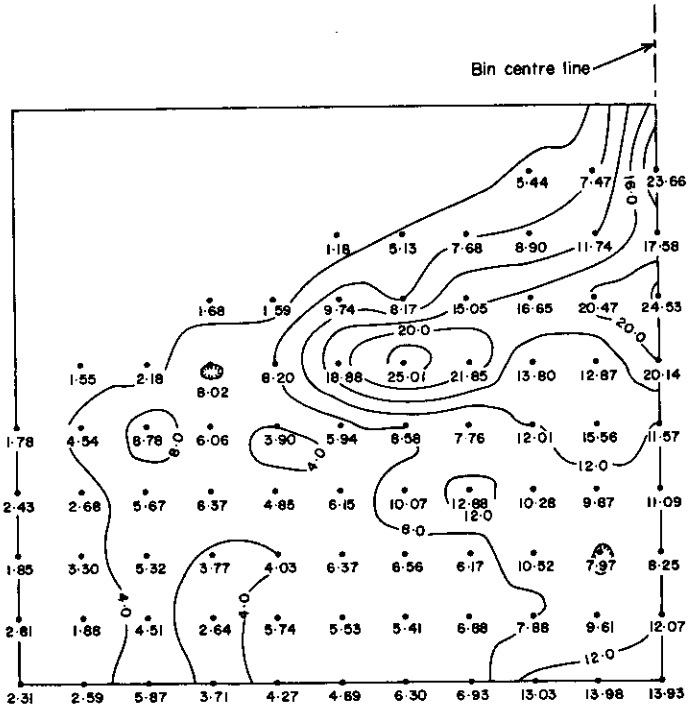

Fig. 2. Fine material distribution in a bin, when filled without use of a mechanical spreader (along radial line no. 1, see Fig. 1). *Units are in % by weight. Test 1*

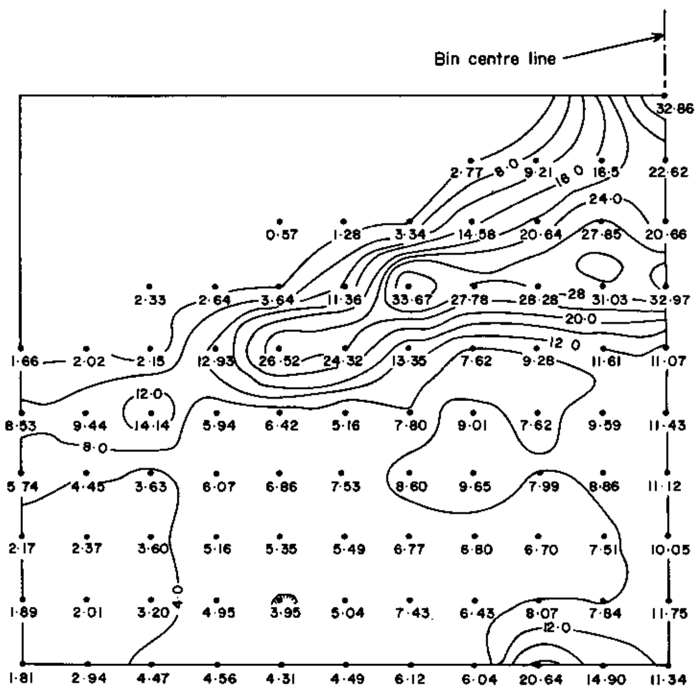

Fig. 3. Fine material distribution in a bin, when filled without use of a mechanical spreader (along radial line no. 2). Units are in % by weight. Test 1

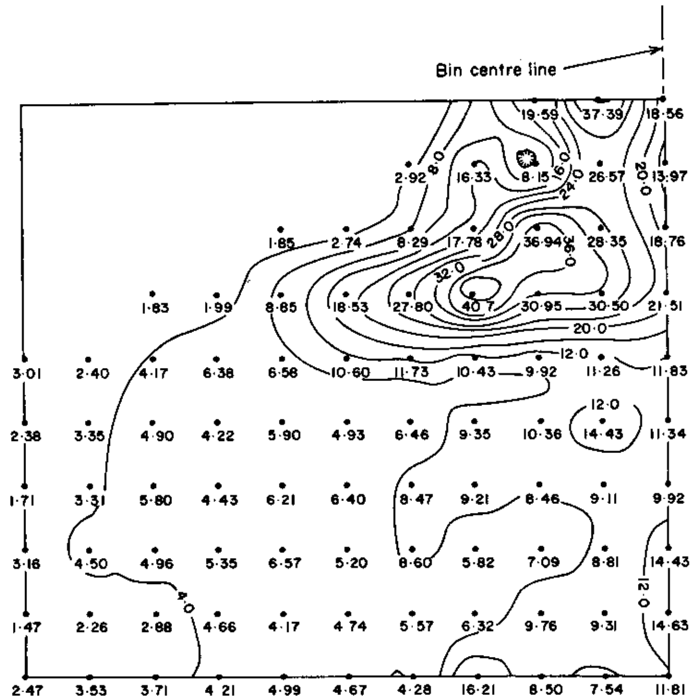

Fig. 4. Fine material distribution in a bin, when filled without use of a mechanical spreader (along radial line no. 3). Units are in % by weight. Test 1

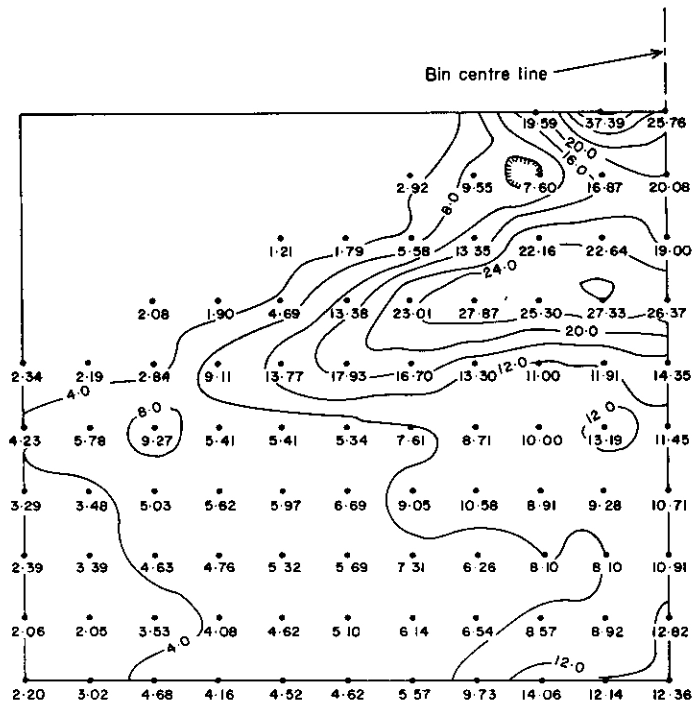

Fig. 5. Fine material distribution in a bin, when filled without use of a mechanical spreader (mean of the 3 radial lines). Units are in % by weight. Test 1

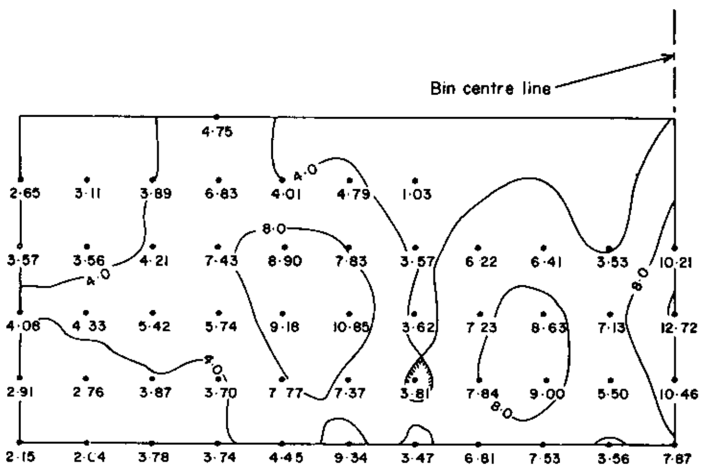

Fig. 6. Fine material distribution in a bin, when filled with use of an auger spreader (along radial line no. 1). Units are in % by weight. Test 2

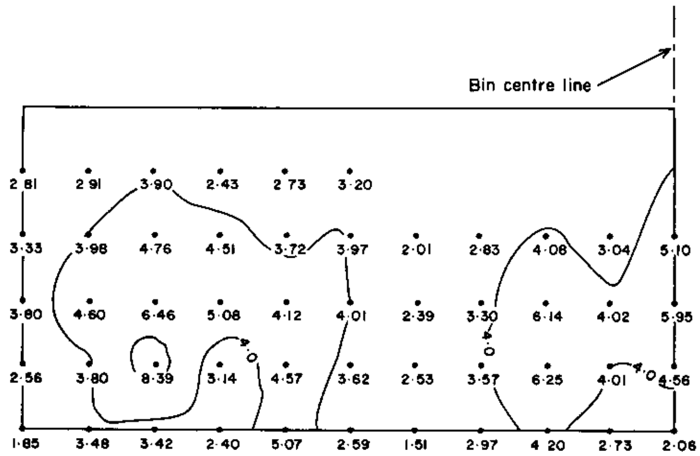

Fig. 7. Fine material distribution in a bin, when filled with use of an auger spreader (along radial line no. 2). Units are in % by weight. Test 2

properties of the data sets. There is a well defined trend from outside to centre and a less well defined trend from bottom to the top for test 1. However, no trends exist for test 2. This implies that the segregation of fine material and broken kernels is distinguishable in the radial direction but not quite so distinguishable in the vertical direction.

Stephens and Foster[1] used chi-square as a statistic representing the variability in fine material and broken kernels. By averaging over three radial lines, they found

$$\chi_1^2 = 2{\cdot}52 \text{ d.f.} = 253$$

and

$$\chi_2^2 = 0{\cdot}33 \text{ d.f.} = 153$$

for test 1 and test 2, respectively. In principle, a lower value of χ^2 indicates more uniformity in the distribution of the fine material and broken kernels. The use of the mechanical spreader

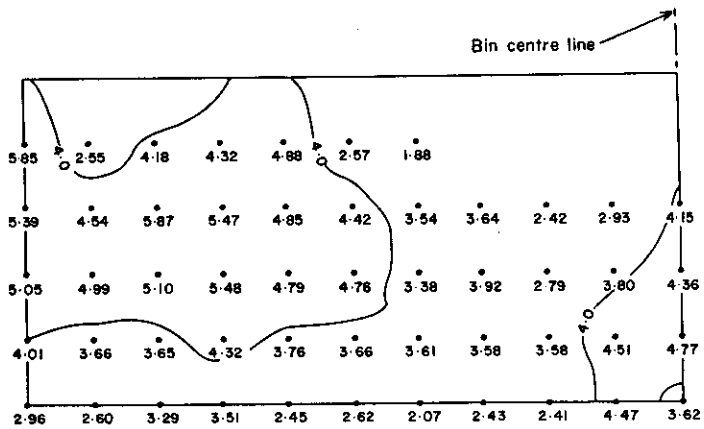

Fig. 8. Fine material distribution in a bin, when filled with use of an auger spreader (along radial line no. 3). Units are in % by weight. Test 2

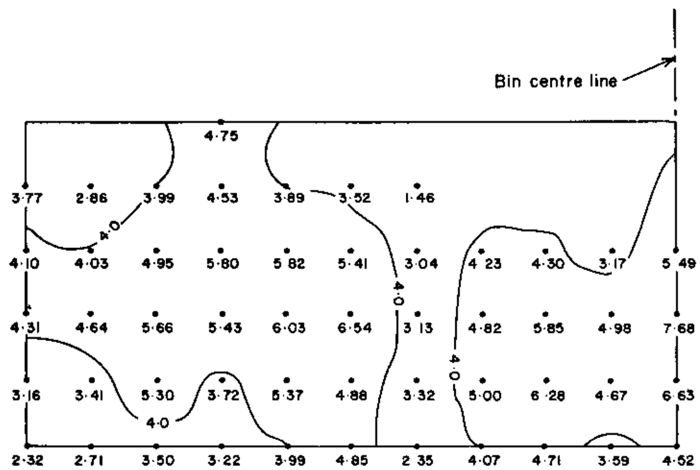

Fig. 9. Fine material distribution in a bin, when filled with use of an auger spreader (mean of the 3 radial lines). Units are in % by weight. Test 2

(test 2) resulted in a significantly more even distribution of the fine material and broken kernels inside the storage bin than when no spreader was used (test 1).

An alternative to the use of chi-square for expressing the distribution of fine material and broken kernels, the quantitative expression of the degree of mixing can also be used to represent unambiguously the variation of fine material and broken kernels in the bin. The degree of mixing, M, can be defined as

$$M = 1 - \frac{s}{s_0}$$

where M = a numerical index which indicates the degree of mixing (or segregation of the mixture)

s_0 = theoretical standard deviation for the grain bed in the completely segregated state = $\sqrt{X(1-X)}$

s = standard deviation among samples of size n
n = total number of particles in a spot sample
X = population fine material content expressed in decimal form.

When $M = 0$, the distribution of the fine material and broken kernels is in the completely segregated state, and when $M \to 1$, the fine material and broken kernels distribution is approaching the completely mixed state. We have

$$M_1 = 0{\cdot}748$$

and

$$M_2 = 0{\cdot}869$$

for test 1 and test 2, respectively. Again, the degree of mixing indicated the overall improvement in the distribution of fine material and broken kernels from test 1 to test 2.

Along each vertical radial plane, the sampling was performed horizontally and vertically. At each position, there is a corresponding value of the concentration

$$\{X(m_1, m_2)\},\ m_1 = 0,1,\ldots, N_1-1$$
$$m_2 = 0,1,\ldots, N_2-1$$

where N_1 and N_2 are the number of sampling points in the horizontal and vertical directions, respectively. $\{X(m_1, m_2)\}$ is called the data matrix. The DFT is defined as

$$C_{xx}(k_1, k_2) = \frac{1}{N_1 N_2} \sum_{m_2=0}^{N_2-1} \sum_{m_1=0}^{N_1-1} X(m_1, m_2) W_1^{k_1 m_1} W_2^{k_2 m_2},$$
$$k_1 = 0,1,\ldots, N_1-1,$$
$$k_2 = 0,1,\ldots, N_2-1,$$

where

$$W_l = \exp(-2\pi \mathrm{i}/N_l)\ l = 1, 2$$
$$\mathrm{i} = \sqrt{-1}.$$

The data are in the form of a $N_1 \times N_2$ matrix, i.e.

$X(0,1)$
$X(1,0)$
$\vdots$
$X(N_1-1,0)$

$$[X(m_1, m_2)] = \begin{bmatrix} X(0,0) & X(0,1) & \ldots X(0,N_2-1) \\ X(1,0) & X(1,1) & \ldots X(1,N_2-1) \\ \vdots & \vdots & \vdots \\ X(N_1-1,0) & X(N_1-1,1) \ldots & X(N_1-1, N_2-1) \end{bmatrix}.$$

The DFT power spectrum is defined as

$$P(k_1, k_2) = |C_{xx}(k_1,k_2)|^2\ \ k_1 = 0,1,\ldots, N_1-1,$$
$$k_2 = 0,1,\ldots, N_2-1.$$

The physical meaning of the DFT power spectrum of a grain mixture can be explained by considering a grain mixture in the completely mixed state. It can be shown that the component of the power spectrum when $k_1 = k_2 = 0$ is the square of the average concentration, since

$$C_{xx}(0,0) = \frac{1}{N_1 N_2} \sum_{m_2=0}^{N_2-1} \sum_{m_1=0}^{N_1-1} X(m_1,m_2) = \bar{X}.$$

Hence

$$P(0,0) = [\bar{X}]^2.$$

It can also be shown that the power spectrum of a completely uniform mixture is the delta function. Since $P(0,0)$ is independent of the mixture homogeneity, it can be conveniently used to normalize the power spectrum. The deviation of the normalized spectrum from the delta function was used as the measure of the variation of the distribution of fine material and broken kernels. Since the power spectrum of a uniform distribution is the delta function, i.e.,

$$P(k_1,k_2) = \begin{cases} 1\cdot0 & k_1 = k_2 = 0, \\ 0\cdot0 & \text{otherwise}, \end{cases}$$

the following index, u^2, defined by the equation:

$$u^2 = \sum_{k_2=0}^{N_2-1} \sum_{k_1=0}^{N_1-1} [P(k_1, k_2)-0]^2 - 1\cdot0$$

was used as an index of variation. For a completely uniform mixture, u^2 equals 0. The difference between the index of any mixture and the value 0 is a measure of variation. The results of the calculations are given in Table I. It can be seen that such an index of uniformity is independent of the mean concentration and the number of samples, since the power spectrum obtained by DFT was normalized with respect to the mean. In the same tables, 5 artificial fine material and broken kernels distributions are also given. The distribution as shown in case C is for the

TABLE I

Comparison of the DFT power spectrum uniformity index, u^2, mixing index, M, and χ^2

Fine material distribution	u^2	M	χ^2 (*d. f.*)
A. With mechanical spreader			
(1) along radial line 1	0·0321	0·886	0·6276 (51)
(2) along radial line 2	0·00464	0·930	0·2294 (49)
(3) along radial line 3	0·00184	0·792	2·058 (50)
B. Without mechanical spreader			
(1) along radial line 1	0·0575	0·794	3·16 (81)
(2) along radial line 2	0·0540	0·737	5·13 (82)
(3) along radial line 3	0·106	0·713	6·44 (87)
C. 0 0 0 0 0 1 1 1 1 1 0 0 0 0 0 1 1 1 1 1 0 0 0 0 0 1 1 1 1 1 0 0 0 0 0 1 1 1 1 1 0 0 0 0 0 1 1 1 1 1 0 0 0 0 0 1 1 1 1 1 0 0 0 0 0 1 1 1 1 1 0 0 0 0 0 1 1 1 1 1 0 0 0 0 0 1 1 1 1 1 0 0 0 0 0 1 1 1 1 1	0·36	0·0	50·0 (99)
D. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	0·36	0·0	50·0 (99)

TABLE I—*continued*

Fine material distribution	u^2	M	χ^2 (d.f.)
E. 1 0 1 0 1 0 1 0 1 0 0 1 0 1 0 1 0 1 0 1 1 0 1 0 1 0 1 0 1 0 0 1 0 1 0 1 0 1 0 1 1 0 1 0 1 0 1 0 1 0 0 1 0 1 0 1 0 1 0 1 1 0 1 0 1 0 1 0 1 0 0 1 0 1 0 1 0 1 0 1 1 0 1 0 1 0 1 0 1 0 0 1 0 1 0 1 0 1 0 1	1·0	0·0	50·0 (99)
F. 1	0·0	1·0	0·0 (99)
G. ½	0·0	1·0	0·0 (99)

completely segregated state in the radial direction and that in case D is for the vertical direction. In both cases u^2 equals 0·36. In case E, the contents of fine materials and broken kernels alternate between 100% and 0%, $u^2 = 1{\cdot}0$. In the last two cases, F and G, the mixture is in the completely uniform state, $u^2 = 0{\cdot}0$.

The statistic u^2 ranks cases C and D between E and F (or G). Even though case E appears to be more regular than either case C or D, it actually is still far from homogeneous, since the concentration fluctuates between the two extremes of 0% and 100%.

In Table I, we also listed the degree of mixing and chi-square values for all cases. In using either degree of mixing or χ^2, we can not distinguish among the three cases C, D and E. Surely, the DFT power spectrum variation index, u^2, provides a better understanding of the distribution of the percentage of fine material and broken kernels.

4. Conclusions

The use of a grain probe as a sampler is an acceptable practice in the grain industry when sampling is confined to bulk materials at rest, in trucks, and in bins[2]. The systematic sampling plan is recommended in sampling the distributions of fine material and broken kernels in a bin. Samples were selected from a lot at a specific regular interval in terms of quantity, time and space. A major reason for use of this sampling plan is that samples so collected are more evenly distributed over the whole population. The DFT technique was applied satisfactorily to characterize the variation of the distribution of fine materials and broken kernels. There are obvious differences between radial lines 1, 2 and 3. The wide variation in values of u^2 reflects the difference. It seems that sampling in one radial line only is inadequate for estimating the mean amount of fines, or for estimating the variability within the bulk. For the case without a mechanical spreader, 3 radial lines may not be sufficient. It seems, therefore, that any sampling scheme must include several radial lines and the method of filling must also be specified.

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