ESTIMATING A PROCTOR DENSITY CURVE FROM INTRINSIC SOIL PROPERTIES

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ABSTRACT. *Tillage studies have shown that maximum, tillage-induced, aggregate breakdown occurs near the optimum water content on a Proctor compaction curve for the soil. Many soil databases do not contain Proctor data; therefore, prediction equations were developed to estimate the optimum water content, peak Proctor dry bulk density, and the Proctor compaction curve for a soil based on common intrinsic properties (sand, silt, clay, and organic matter) from 39 soil samples. A good relationship was obtained between soil intrinsic properties and the optimum water content with an adjusted R^2 value of 0.86. The shape of the Proctor compaction curve was estimated using two lines intersecting at the Proctor optimum water content/peak density point. An overall adjusted R^2 value of 0.72 was obtained for the predicted versus measured values of the Proctor dry density for the entire Proctor compaction curve.*
Keywords. *Soil water content, Soil-tillage interactions, Proctor density curve, Soil compaction.*

The U.S. Department of Agriculture (USDA) appointed a team of scientists to take a lead role in developing the Wind Erosion Prediction System (WEPS) (Hagen, 1991). This process-based wind erosion model is to replace the wind erosion equation (WEQ) currently used by the USDA Soil Conservation Service (Argabright, 1991). Because soil surface conditions greatly influence a soil's susceptibility to wind erosion, WEPS will need to predict accurately the changes in a soil's surface aggregate size distribution from tillage operations.

Tillage experiments conducted by Tangie et al. (1990), Wagner et al. (1991), and Ambe (1991) show that maximum, tillage-induced, aggregate breakdown occurs near the Proctor optimum water content. These studies also indicate an inverse relationship between the amount of large aggregates remaining after tillage and the Proctor compaction or density curve (PDC) of the soils, as shown in figure 1. Overall, results show that post-tillage, aggregate size distributions are strongly influenced by the soil water contents at which tillage operations are performed. Therefore, the effects of soil water content on the tillage-induced aggregate breakdown process needs to be estimated in models such as WEPS. However, many soil databases, including those maintained by the USDA Soil

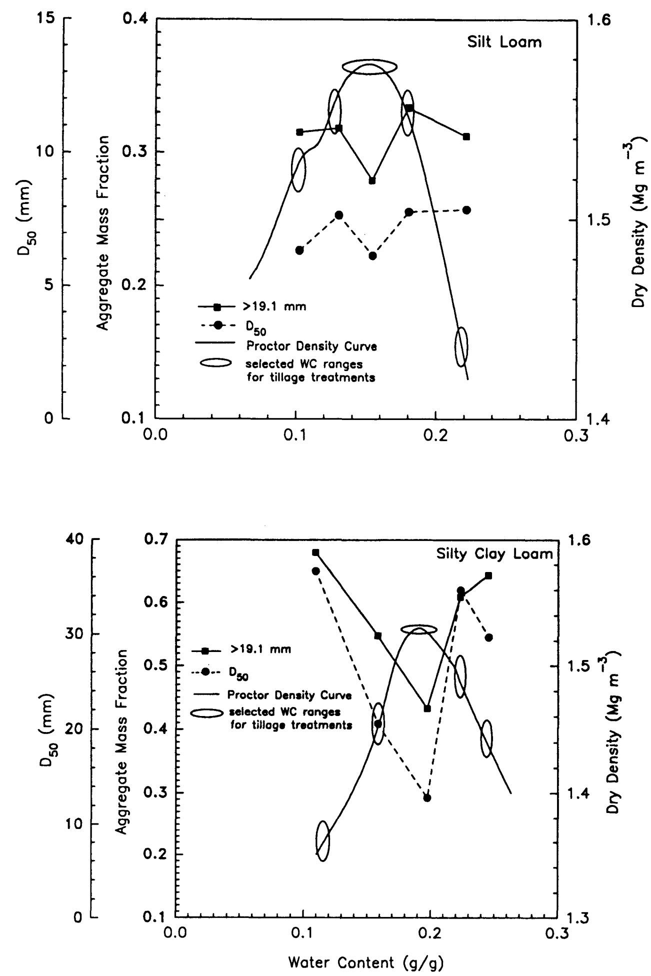

Figure 1–Tillage-induced ASD vs. soil PDC. This figure is reproduced from Ambe (1991) and shows the relationships between the Proctor density curve for two soils and the post-tillage mean aggregate size distribution (D_{50}) and aggregate mass fraction greater than 19.1 mm diameter. The pre-tillage aggregate size distributions were considered the same for all tillage treatments, which were performed within the designated water content ranges, on each soil.

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The intrinsic properties chosen for this study were based solely on general availability and does not imply that the inclusion of other intrinsic properties would not significantly enhance the ability to estimate Proctor data.

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Conservation Service (SCS), do not currently contain extensive Proctor compaction data. Thus, a need exists to estimate PDC data from other, more commonly measured, soil classification data, if WEPS is to be successfully implemented by SCS.

The Proctor compaction test provides a standardized method of determining a soil's resistance to compaction over a range of soil water contents under a constant value of compaction energy. The optimum water content (OWC) is the amount of water required to produce a maximum dry density (MDD) from the test procedure. Currently, the Proctor test is used primarily by civil engineers to determine the OWC for foundation and highway construction purposes and is not routinely conducted on agricultural soil samples. However, there is a body of literature that attempts to correlate the two soil properties, OWC and MDD, and the Proctor compaction curve to various soil classification data.

Wang et al. (1984) reviewed several studies correlating OWC with: liquid limit and plasticity index (Jumikis, 1946); gradation (Turball, 1948); specific gravity, gradation, and grain size distribution (Rowan and Graham, 1948); and gradation, grain size distribution, and plasticity index (Davidson and Gardiner, 1949). Hamdani (1983) developed a one-point method for estimating OWC and MDD, which required only the determination of a soil's dry density at $9\%_{(g/g)}$ water content.

Ring et al. (1962), using artificially mixed soils, examined simple relationships between OWC and MDD with plastic limit, liquid limit, fineness average, average particle size, and particles finer than 0.001 mm. They found good correlations of OWC with the Atterberg limits and of MDD with OWC and plastic limit. Ramaih et al. (1970) found a linear relationship between MDD-liquid limit and OWC-liquid limit but no "definite correlation" with either plastic limit or plasticity index, whereas Hamdani (1983) developed an exponential equation relating OWC to MDD. Equations developed by Wang et al. (1984) were too complex (>20 independent variables), although they recommended "specified forms for practical applications".

To fit the PDC, Amir et al. (1976) derived a logarithmic equation, but studies by Raghavan et al. (1977) later showed that the PDC could be better fitted by using two coefficients on the wet and dry sides of the curve (i.e., above and below the OWC). Recently, a quasi-theoretical model that estimates the density-moisture-stress function was developed by McBride (1989).

The original compaction test proposed by Proctor (1933) did not outline a reliable method of providing the specified compactive effort to the soil sample. To correct this deficiency the Proctor test was modified over time, resulting in a lower compactive force, 600 kN-m/m^3, being specified in the ASTM Standard Effort test (ASTM D698), than originally suggested by Proctor of approximately 700-1200 kN-m/m^3. Since the Proctory dry density (PDD) is strongly influenced by the compactive effort applied (Guo and Schuler, 1991), a quantitate evaluation of and comparison between the methods reviewed are nearly impossible.

An additional problem is that most of the equations reviewed in the literature have classification data as independent variables, like plastic and liquid limit, that are difficult to acquire accurately and may even be undefined for certain soil classes, such as plastic limit for cohesionless soils. Estimating the PDC, OWC, and MDD preferably should involve soil properties that are readily obtainable and repeatable for the WEPS model.

Therefore, the primary objectives of this study were to:

- Estimate the optimum water content and maximum dry density for a Proctor compaction curve using readily obtainable intrinsic soil properties, namely the percentages of sand, silt, clay, and organic matter
- Estimate the entire Proctor compaction curve as a function of the soil's particle size distribution and organic matter content.

Procedures

Proctor density data for 39 soil samples from 19 U.S. states were used in the study. Twenty-six of the soil samples were collected from cropland Water Erosion Prediction Program (WEPP) sites and tested by the USDA-SCS National Soil Survey Laboratory (NSSL) with standard Proctor tests (ASTM D-698, method A) and particle size analyses (Gee and Bauder, 1986). The remaining 13 soil samples were from Kansas sites used in the development of the WEPS model and were also tested by the NSSL using the same procedures. The textural composition of the soils ranged from 3.6 to 49.4% clay, 4.4 to 79.7% silt, and 2.4 to 91.9% sand. Organic matter content ranged from 0.14 to 3.13%. Optimum water content and maximum dry density values ranged from 8 to $26\%_{(g/g)}$ and 1.46 to 1.99 Mg/m^3, respectively. Table 1 lists particle size analysis, organic matter content, OWC, and MDD values for each soil.

Because OWC prediction was the most important factor in the effects of soil water content on tillage-induced aggregate breakage, a model was developed first to predict OWC using soil property data. This model was assumed to have the following mathematical form:

$$OWC = \alpha_0 + \alpha_1 x_1 + \ldots + \alpha_k x_k + \varepsilon_k \qquad (1)$$

where

OWC = optimum water content

$\alpha_0, \alpha_1, \ldots, \alpha_k$ = population parameters

ε_k = a random error representing the contribution of nonmeasured variables to OWC

$x_1, x_2, \ldots, x_k$ = predictor variables selected from a list of soil properties

The criterion for predictor variables was having values that were measurable, expected to influence the response variable, and reasonably uncorrelated. Predictor variables with these characteristics are listed in table 2.

Sample estimation of (a_i) of the population parameters (α_i) was obtained by applying the method of multiple regression to data from a sample of 39 soils. The resulting empirical model (eq. 2) was an estimate of the OWC for a given soil defined by a given configuration of values for ($x_1, x_2, \ldots, x_k$).

$$\underline{OWC} = a_0 + a_1 x_1 + a_2 x_2 + \ldots + a_k x_k \qquad (2)$$

Table 1. Soil classification data

Soil Series Name	NSSL ID	Clay (%)	Sand (%)	OM (%)	OWC $(\%)_{g/g}$	MDD (Mg/m^3)	K_l	K_h
Mexico	89p1135s	22.1	4.6	1.55	19.5	1.58	0.0156	–0.0240
Tifton	89p1136s	4.7	86.5	0.43	8.0	1.91	0.0100	–0.0270
Bonifay	89p1137s	3.7	91.9	0.25	10.0	1.77	0.0023	–0.0165
Cecil	89p1139s	33.6	51.8	0.66	17.5	1.73	0.0393	–0.0249
Opequon	89p1141s	32.9	12.1	1.50	21.0	1.55	0.0201	–0.0163
Fredrick	89p1142s	16.8	22.0	1.23	18.5	1.59	0.0196	–0.0140
Manor	89p1143s	24.6	44.2	0.97	19.5	1.65	0.0216	–0.0140
Caribou	89p1144s	14.2	46.0	1.84	20.0	1.60	0.0142	–0.0280
Collamer	89p1145s	17.0	4.8	1.06	18.0	1.65	0.0194	–0.0228
Miamian	89p1146s	30.5	30.4	2.01	19.0	1.65	0.0065	–0.0220
Miami	89p1148s	15.9	4.4	0.79	18.0	1.68	0.0108	–0.0261
Grenada	89p1149s	20.4	3.5	0.99	17.5	1.65	0.0093	–0.0282
Acadamy	89p962s	13.6	63.0	0.34	13.0	1.93	0.0381	–0.0308
Los Banos	89p964s	49.4	15.7	1.45	23.5	1.53	0.0046	–0.0147
Whitney	89p966s	6.7	75.0	0.27	10.0	1.99	0.0175	–0.0401
Sverdrup	89p970s	22.6	46.9	1.54	15.5	1.67	0.0145	–0.0093
Amarillo	89p972s	7.5	86.5	0.14	9.0	1.92	0.0190	–0.0240
Barnes	89p974s	25.3	42.4	2.52	20.5	1.59	0.0155	–0.0218
Williams	89p976s	26.9	41.8	1.61	15.0	1.79	0.0166	–0.0240
Pierre	89p978s	48.7	11.5	1.35	26.0	1.47	0.0129	–0.0171
Palouse	89p980s	22.1	8.3	1.35	18.0	1.65	0.0116	–0.0207
Woodward	89p984s	12.0	48.5	0.75	12.5	1.79	0.0140	–0.0289
Zahl	89p986s	29.8	46.4	1.70	17.0	1.67	0.0204	–0.0166
Sharpsburg	89p990s	41.0	2.4	1.70	23.5	1.48	0.0040	–0.0127
Portneuf	89p994s	9.7	16.1	0.77	20.5	1.53	0.0072	–0.0280
Keith	89p996s	17.8	47.3	0.94	18.5	1.63	0.0144	–0.0447
Inavale	91z349s	5.0	85.1	0.42	10.0	1.92	0.0162	–0.0401
Harney	91z350s	30.5	12.3	1.00	18.5	1.64	0.0237	–0.0219
Fargo	91z351s	47.4	12.1	3.13	26.0	1.46	0.0057	–0.0147
Smolan	91z352s	32.3	9.1	1.35	19.0	1.65	0.0117	–0.0221
Richfield	91z353s	26.2	28.9	0.97	18.0	1.68	0.0176	–0.0280
Lincoln	91z354s	15.8	58.1	1.04	12.5	1.82	0.0237	–0.0240
Dalhart	91z355s	7.5	75.1	0.68	9.0	1.83	0.0301	–0.0312
Reading	91z356s	25.1	7.2	1.50	18.5	1.63	0.0087	–0.0223
New Cambria	91z357s	42.4	12.7	1.91	22.0	1.57	0.0120	–0.0223
Santanta	91z358s	8.5	71.5	0.77	11.0	1.90	0.0176	–0.0401
Carr	91z359s	3.6	74.7	0.47	16.0	1.63	0.0074	–0.0325
Wymore	91z360s	25.2	10.5	1.31	17.0	1.67	0.0122	–0.0210
Hanie	91z361s	5.9	61.4	0.64	15.5	1.64	0.0141	–0.0243

Identical procedures were applied to find a relation between MDD and OWC with a linear model, as shown in equation 3:

$$MDD = \beta_0 + \beta_1 * OWC + \varepsilon_1 \qquad (3)$$

where

MDD and OWC = mean values of the maximum dry density and the optimum water content from Proctor compaction curves

β_0 and β_1 = population parameters

ε_1 = error term

According to Hillel (1980), the line connecting the peaks of all the bulk density versus wetness curves corresponds approximately to the 80% degree-of-saturation line. Therefore, another way to determine MDD based on OWC is using equation 4.

$$\underline{MDD} = \frac{\rho_s}{1.0 + \frac{\rho_s}{\theta} \times OWC} \qquad (4)$$

where θ is the degree-of-saturation (%) and ρ_s is the average particle density (Mg/m^3).

To estimate the entire PDC, two approaches were taken. The first was to fit a given PDC with a quadratic function. After the three parameters of the quadratic function were obtained, regression procedures, as discussed above, were applied along with visual inspection of data plots to relate them with soil properties. However, the results did not suggest any strong correlations between the parameters and the soil properties. Therefore, a second approach was taken that assumed that the PDC water content values less than and greater than the OWC could be fitted with two straight lines. The general model is given in equation 5.

$$PDD = \gamma_0 + \gamma_1 * WC + \varepsilon \qquad (5)$$

where PDD is the Proctor dry density and WC is the water content value associated with that density.

Results and Discussion

Data from 39 soil samples were used to calculate the a_i in equation 2. The estimates (a_i) and their standard errors are given in table 3. The adjusted R^2 value was 0.86, and the root-mean-square-error (RMSE) as an estimate of the standard error of $\underline{OWC}$ values for a configuration of x_i(i = 1, 2, 3, 4) values was $1.75\%_{(g/g)}$. The resulting prediction equation is:

$$\underline{OWC} = a_0 + a_1(cl) + a_2(cl*cl) + a_3(sa*sa) + a_4(om*om) \qquad (6)$$

A strong correlation between OWC and MDD was found (fig. 2), as suggested by other researchers (Wang et al., 1984; Hamdani, 1983; and Ring et al., 1962). Although Hamdani (1983) obtained an exponential equation for this relationship, also shown in figure 2, it did not fit as well as the linear model obtained for this data which yielded an adjusted R^2 value of 0.87 and an RMSE

Table 2. Description of predictor variables

Variables	Description and Units of Measure	Min	Mean	Max
cl	Clay content (%)	3.6	21.7	49.4
si	Silt content (%)	4.4	40.6	79.7
sa	Sand content (%)	2.4	37.8	91.9
om	Organic matter content (%)	0.14	1.15	3.13

Table 3. Estimates of parameters (coefficient of variables in eq. 2) and their standard errors

i	Variable (x_i)	Parameter Estimates (a_i)	Standard Errors of Estimates
0	1	21.09	1.8086
1	cl	–0.2892	0.1197
2	cl*cl	–0.007075	0.001962
3	sa*sa	–0.001375	0.0002000
4	om*om	0.4234	0.1905

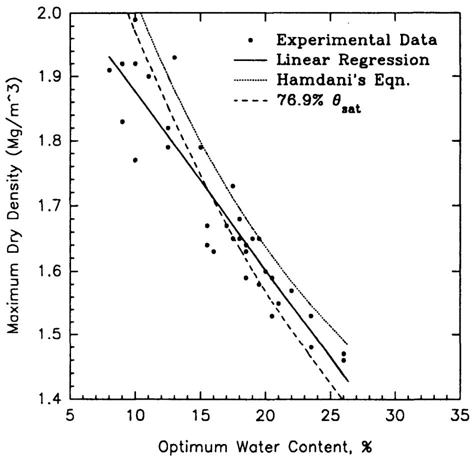

Figure 2–Proctor OWC, MDD relationship.

as an estimate of the standard error of MDD of 1.68 Mg/m^3. The resulting linear prediction equation is given in equation 7 where the b_0 and b_1 values are presented in table 4.

$$\underline{MDD} = b_0 + b_1(OWC) \quad (7)$$

The MDD-OWC relationship was also determined using equation 4 with a θ value near 80% as suggested by Hillel (1980). Using a nonlinear optimization method, a degree-of-saturation value of 76.9% was obtained for the model, yielding an R^2 value of 0.86. The 95% confidence interval of θ was 73.9 and 79.8%. This curve is also represented in figure 2. Although both equation 7 and equation 4 gave fairly accurate relationships between MDD and OWC, equation 4 may be more appropriate than equation 7 for estimating MDD, because it is a physical-based model which relates a specific void ratio (100-θ) to the MDD obtained from the Standard Proctor test (ASTM D698).

The effort to describe the PDC with a quadratic model was not successful. However, the two straight line approach (eq. 5), revealed that estimation of the population parameters γ_i was hardly affected by soil properties. Because the standard deviations of the two mean slope values were very low (table 5), the entire PDC can be predicted with two straight lines and their intersection point (OWC, MDD). The average slopes of the two lines are 0.0154 and –0.0241 for water contents less than and greater than the OWC, respectively. The resulting prediction model is given in equation 8.

Table 4. Estimates of parameters (coefficient of variables in eq. 3) and their standard errors

i	Variable (x_i)	Parameter Estimates (b_i)	Standard Errors of Estimates
0	1	2.151	0.03088
1	OWC	–0.02748	0.001757

Table 5. Estimates of parameters (coefficent of variables in eq. 8) and their standard errors

Variable (x)	Parameter Estimates (k)	Standard Errors Estimates
$k = k_l$ (WC≤OWC)	0.0154	0.008
$k = k_h$ (WC>OWC)	–0.0241	0.008

$$\underline{PDD} = k(WC - OWC) + MDD$$

$$k = \begin{cases} k_l = 0.0154 \pm 0.008 & WC \le OWC \\ k_h = -0.0241 \pm 0.008 & WC > OWC \end{cases} \quad (8)$$

Using the actual (OWC, MDD) values (table 1), the model yielded adjusted R-square values of 0.96 and 0.98 for water contents less than and greater than the OWCs, respectively. When the predicted OWC (eq. 6) and the predicted MDD (eq. 7) were used, the model yielded an adjusted R-square value of 0.67. If equation 4 was used instead of equation 7, an adjusted R^2 of 0.72 was achieved.

The applicable range of water contents, upper and lower limits, of predicted PDCs needs to be fully defined. Most PDCs exhibit an asymptotic behavior, if extended to very high and very low water contents. The literature implies that the absolute upper limit of a PDC is the saturated water content. The lower water content limit is not often discussed. However, Faure (1981) proposed a "characteristic water content", w_c, be used to represent that the lower water content limit at which the PDD values level off. This inflection point for a given compaction effort is not only dependent upon the content, but also the type of clay (montmorillonite/kaolinite) present. The PDC data sets studied here did not consistently extend into these regions of the Proctor compaction curve, so no analysis was attempted at determining these upper and lower water limits.

Summary and Conclusions

A model was developed to predict the optimum water content (OWC) of a Proctor density curve for a range of soils based on contents of sand, clay, and organic matter. The model, fitted to 39 soil samples, had an adjusted R^2 value of 0.86 and an RMSE value of $1.75\%_{(g/g)}$. Measured values of OWC ranged from $8.0\%_{(g/g)}$ to $26\%_{(g/g)}$.

The relationship between MDD and OWC can be represented by a linear model with an adjusted R^2 value of 0.87 and an RMSE value of 1.68 Mg/m^3. Measured values of MDD ranged from 1.46 to 1.99 Mg/m^3. This linear model, equation 7, showed no significant improvement over the relationship, equation 4, mentioned by Hillel (1980), therefore, either equation may be selected for estimating MDD from OWC.

The entire Proctor density curve can be characterized by one point (OWC, MDD) and two straight lines intersecting at that point. The slopes for those lines are 0.0154 and –0.0241, respectively, for water contents less than and greater than the OWC. An overall adjusted R^2 value of 0.72 was obtained for the predicted versus measured values of PDD for the entire Proctor compaction curve.

These equations can be used to provide an estimate of Proctor data for soils databases lacking this information and having limited intrinsic properties with which to estimate it from. This will expand the number of soils databases that can be used by tillage models, such as the one being developed for WEPS, that predict tillage-induced aggregates as a function of pre-tillage water content.

This study and its results have revealed several areas deserving future work:

- A relatively broad range of agricultural soils was encompassed in this study, but additional soil data sets need to be obtained to verify the validity of the prediction equations obtained.
- A data set with more complete Proctor compaction curve data that extend into very dry, less than –1500 J/kg WC, and near saturated WC ranges is needed to determine the wet and dry "limits" of the PDC and compare them with Faure's characteristic water content relationships.
- Because percent OM has a significant influence on the OWC values for these agricultural soils, additional data sets with a wider range of OM values should be studied to see if OM's influence on a PDC remains the same outside the range used in this study.
- If additional soil properties are assumed available for use as independent variables, such as clay type and/or cation exchange capacity, better prediction equations can undoubtedly be obtained.

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