Wind Profile Parameters and Turbulence Intensity Over Several Roughness Element Geometries

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ABSTRACT

Estimating equations were developed for the mean velocity profile parameters ($Z_0$, $u_*$, and $D$) in the logarithmic law and for longitudinal turbulence intensity ($\sigma_u/\bar{u}_z$). The estimates were based on wind tunnel measurements over several roughness element shapes, sizes, heights, and geometrical patterns. All the prediction equations had correlation coefficients $> 0.90$. These equations have application in wind erosion of soil particles, water evaporation, and transport of gases. They only apply to a particular set of conditions, which indicates a need for more universally applicable equations for the profile parameters and turbulence intensity.

INTRODUCTION

Knowledge concerning mean and turbulent wind flow properties near the surface is essential because they influence the exchange of mass, heat, and momentum between the boundary and the environment, like erosion of soil particles, evaporation of water, and transport of gases.

Much information has accumulated about the wind profile in the boundary layer, both in the atmosphere and in wind tunnels.

For aerodynamically rough atmospheric flows in the "constant stress" layer, the following form of the logarithmic law is most often used to describe the mean velocity profile:

$$\bar{u}_z = \frac{u_*}{k} \left[ \ln \left( \frac{Z - D}{Z_0} \right) + \Phi(z) \right] \quad \quad \quad \quad [1]$$

where $\bar{u}_z$ is the mean velocity at height $z$ from some reference plane; $u_*$ is the friction velocity defined as $(\tau_0/\rho)^{1/2}$, where $\tau_0$ is the shear stress at the surface and $\rho$ is air density; $k$ is von Karman’s constant $(0.4)$; $D$ is an “effective” height of roughness; $Z_0$ is a roughness parameter, and $\Phi(z)$ is the integral diabatic influence function. The function, $\Phi(z)$ is zero for adiabatic conditions, which include most wind tunnel flows. We are primarily interested in soil erosion by wind, which generally occurs in favor-
windspeed measurements taken above the roughness elements in the lower 10 to 20 percent of the boundary layer using the adiabatic form of equation [1]. This generally involved six to ten heights and, in some cases, three to five replications at each height. We are defining the boundary layer depth, \(d\), as the height \(Z\) where \(u' = 0.99 u_\infty\) and \(u_\infty\) is the free-stream mean velocity.

**EXPERIMENTAL DATA AND OBSERVATIONS**

Many “independent” variables and combinations of variables that characterize the roughness elements (number, height, diameter, etc.) were correlated with the dependent variables \(Z_0\), \(u_\ast\), \(D\), and \(\sigma_{u'}/\bar{u}_Z\) using selected variables and stepwise multiple regression where variables were entered in the order of their greatest contribution to variance.

### Roughness Parameter, \(Z_0\)

Several published equations for the roughness parameter, \(Z_0\), are of the form (Cowan, 1968; Szeicz, 1969; Thom, 1971):

\[
Z_0 = \lambda h^n
\]

where \(h\) is average roughness height, and \(\lambda\) and \(n\) are constants with different values for particular cases. For equilibrium flows, Couinian (1971) included a term for the proportion of surface area occupied by the roughness elements:

\[
Z_0 = (1.08 A_T/A - 0.08)h, \quad 0.10 < A_T/A < 0.25
\]

where \(A_T\) is the plan surface area of roughness elements, and \(A\) is the total plan surface area. In equation [4], \(\lambda = 1.08 A_T/A - 0.08\) and \(n = 1\).

Lettau (1969) suggested this equation for \(Z_0\):

\[
Z_0 = 0.5 h A_N/A
\]

where \(A_N\) is the average silhouette (frontal) area of the roughness elements, and \(N/A\) is the number of roughness elements per unit area. Equation [5] expressed in terms of equation [3] would have \(\lambda = 0.5 A_N/A\) and \(n = 1\). For standing circular cylinders, equation [5] becomes:

\[
Z_0 = 0.5 \frac{h^2 d_s}{L_x L_y}
\]

where \(d_s\) is average cylinder diameter (silhouette); \(L_x\) is the center-to-center distance between cylinders in the flow direction; and \(L_y\) the corresponding distance normal to flow direction.

To obtain high correlation coefficients, we separated the cylinder data into two height categories to generate these equations:

\[
\hat{Z}_0 = 0.32 \frac{h^{0.56} d_s^{0.23}}{L_x^{0.4} L_y^{0.34}}, \quad 0 < h < 4 \text{ cm},\ R = 0.94\text{, and}
\]

\[
\hat{Z}_0 = 971 \frac{h^{1.06} d_s^{1.82}}{L_x^{2.1} L_y^{1.86}}, \quad 4 < h < 43 \text{ cm},\ R = 0.96\text{.} \quad [8]
\]

In equation [7], \(d_s\) ranged between 0.28 and 1.59 cm and \(L_x\) and \(L_y\) between 2.54 and 20.32 cm; in equation [8], \(d_s\) ranged between 0.28 and 2.54 cm and \(L_x\) between 10.16 and 60.96 cm, and \(L_y\) between 10.16 and 30.48 cm.

For closely packed sand grains and closely packed smooth spheres, we found the equation of Nikuradse (1950) was adequate:

\[
Z_0 = 0.033 d_p
\]

where \(d_p\) is particle diameter.

### Friction Velocity, \(u_\ast\)

Because \(u_\ast\) varies with mean velocity, we chose 894 cm/s freestream velocity (\(u_\infty\)) as a standard and denoted \(u_\ast_0\) as the \(u_\ast\) associated with that standard. Again, separating the cylinder data in two height categories, we obtained:

\[
\hat{u}_\ast_0 = 68.66 \frac{h^{0.13} d_s^{0.05}}{L_x^{0.07} L_y^{0.12}}, \quad 0 < h < 4 \text{ cm},\ R = 0.92,\text{ and}
\]

\[
\hat{u}_\ast_0 = 165.5 \frac{h^{0.08} d_s^{0.21}}{L_x^{0.28} L_y^{0.22}}, \quad 4 < h < 43 \text{ cm},\ R = 0.96\text{.} \quad [10]\n\]

with the same restrictions on \(d_s\), \(L_x\), and \(L_y\) as for \(Z_0\).

### Effective Height, \(D\)

Using all the cylinder data, we obtained this equation for effective height \(D\):

\[
\hat{D} = 0.839 h - 1.182, \quad 0.4 < h < 43 \text{ cm},\ r = 0.99\quad [12]\n\]

Although equation [12] has a high simple correlation coefficient, values of \(h > 1.4 \text{ cm}\) give negative values for \(D\), which is mathematically correct but is physically impossible, if \(D + Z_0\) is negative (a finite windspeed below the surface). Other combinations of \(h\), \(d_s\), \(L_x\), and \(L_y\) did not improve the equation.

### Turbulence Intensity, \(\sigma_{u'}/\bar{u}_Z\)

Regression equations for the local longitudinal turbulence intensity at a reference height equal to \((Z - D)/d = 0.05\) \((d\) is the boundary layer depth) were:

\[
\hat{\sigma}_{u'}/\bar{u}_Z = 42.7 \frac{h^{0.21} d_s^{0.05}}{(L_x L_y)^{0.17}}, \quad 0 < h < 4 \text{ cm},\ R = 0.91,\text{ and}
\]

\[
\hat{\sigma}_{u'}/\bar{u}_Z = 227.5 \frac{h^{0.20} d_s^{0.50}}{L_x^{0.45} L_y^{0.42}}, \quad 4 < h < 43 \text{ cm},\ R = 0.94\text{.} \quad [13]\n\]

\[
\hat{\sigma}_{u'}/\bar{u}_Z = 227.5 \frac{h^{0.50} d_s^{0.50}}{L_x^{0.45} L_y^{0.42}}, \quad 4 < h < 43 \text{ cm},\ R = 0.94\text{.} \quad [14]\n\]
with the same restrictions on $d_S$, $L_X$, and $L_Y$ as for the other parameters.

**INTERPRETATION AND DISCUSSION**

**Roughness Parameters, $Z_0$**

Values of $Z_0$ calculated with equation [7] or [8] for several selected height-size-spacing combinations occasionally agreed with Lettau’s equation [5] but seldom agreed with Counihan’s equation [4] (Table 1). Neither Lettau’s nor Counihan’s equation discriminated among geometrical patterns of roughness elements with the same number of elements per area. For example, cases 2, 3, and 4 in Table 1 all have equal values for height, size, and number of elements per area but different geometrical patterns or distributions—case 2, uniform spacing; case 3 in rows parallel to flow direction; and case 4 in rows normal to flow direction. With equation [7], different values for $Z_0$ were calculated for the three cases, whereas with Lettau’s and Counihan’s equations, the same $Z_0$ value was calculated for all cases. Except for two cases, Counihan’s equation gave negative $Z_0$ values (Table 1). Most of the examples, however, had $A_F/A$ values far greater than the limitations specified for his equation. The only cases with positive $Z_0$ values were for 11 with an $A_F/A$ value of $0.077$, slightly lower than the minimum limit (0.10), and 15, with $A_F/A$ value of $0.307$, slightly larger than the maximum limit (0.25). Of course, equation [7] or [8] is also restricted to the range of experimental data used in developing them and it would be difficult to apply to roughness elements of nonuniform cross-section (both frontal and plan views). Unfortunately, regression equations can seldom be extrapolated and almost never include all the range of independent variables that may influence the dependent variable in question—in our case, all the possible shapes, heights, concentrations, and patterns of roughness elements that influence the roughness parameter, $Z_0$. Consequently, published equations for $Z_0$ apply only to particular cases, and we are only adding to that number and must wait future development of more universally applicable equations.

Roughness parameters computed with Nikuradse’s equation [9] for closely packed elements showed fair to good agreement ($r = 0.93$) with those determined from mean velocity profile measurements (Table 2). The higher values from velocity profiles over gravel suggest that shape irregularity and smoothness


<table>
<thead>
<tr>
<th>Case no.</th>
<th>$h$, cm</th>
<th>$d_S$, cm</th>
<th>$L_X$, cm</th>
<th>$L_Y$, cm</th>
<th>Eq. [6] (Lettau), cm</th>
<th>Eq. [7] or [8] (Counihan), cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
<td>0.655</td>
<td>5.08</td>
<td>5.08</td>
<td>0.00317</td>
<td>-0.03295</td>
</tr>
<tr>
<td>2</td>
<td>2.5</td>
<td>0.655</td>
<td>5.08</td>
<td>5.08</td>
<td>0.07392</td>
<td>-0.16475</td>
</tr>
<tr>
<td>3</td>
<td>2.5</td>
<td>0.655</td>
<td>5.08</td>
<td>5.08</td>
<td>0.07392</td>
<td>-0.16475</td>
</tr>
<tr>
<td>4</td>
<td>2.5</td>
<td>0.655</td>
<td>5.08</td>
<td>5.08</td>
<td>0.07392</td>
<td>-0.16475</td>
</tr>
<tr>
<td>5</td>
<td>2.5</td>
<td>0.278</td>
<td>10.16</td>
<td>2.54</td>
<td>0.03366</td>
<td>0.07987</td>
</tr>
<tr>
<td>6</td>
<td>5.0</td>
<td>0.655</td>
<td>5.08</td>
<td>5.08</td>
<td>0.07392</td>
<td>-0.19365</td>
</tr>
<tr>
<td>7</td>
<td>10.0</td>
<td>0.278</td>
<td>20.32</td>
<td>20.32</td>
<td>0.03366</td>
<td>-0.38237</td>
</tr>
<tr>
<td>8</td>
<td>20.0</td>
<td>1.589</td>
<td>50.96</td>
<td>15.24</td>
<td>0.34208</td>
<td>-1.55386</td>
</tr>
<tr>
<td>9</td>
<td>43.0</td>
<td>2.54</td>
<td>30.48</td>
<td>30.48</td>
<td>2.52761</td>
<td>-3.18671</td>
</tr>
<tr>
<td>10</td>
<td>1.0</td>
<td>0.655</td>
<td>2.54</td>
<td>2.54</td>
<td>0.05076</td>
<td>-0.02359</td>
</tr>
<tr>
<td>11</td>
<td>1.0</td>
<td>1.589</td>
<td>5.08</td>
<td>5.08</td>
<td>0.03079</td>
<td>0.00299</td>
</tr>
<tr>
<td>12</td>
<td>0.1</td>
<td>0.28</td>
<td>20.32</td>
<td>20.32</td>
<td>0.000903</td>
<td>-0.00798</td>
</tr>
<tr>
<td>13</td>
<td>25.4</td>
<td>0.28</td>
<td>15.24</td>
<td>2.54</td>
<td>2.33333</td>
<td>-0.98836</td>
</tr>
<tr>
<td>14</td>
<td>43.0</td>
<td>2.54</td>
<td>76.2</td>
<td>30.48</td>
<td>1.01105</td>
<td>-3.33868</td>
</tr>
<tr>
<td>15</td>
<td>2.5</td>
<td>1.589</td>
<td>2.54</td>
<td>2.54</td>
<td>0.76967</td>
<td>0.62992</td>
</tr>
</tbody>
</table>

*Outside range of experimental data.

**TABLE 2. ROUGHNESS PARAMETERS ($Z_0$) DERIVED FROM MEAN VELOCITY PROFILES AND COMPUTED FROM NIHURADSE’S (1950) EQUATION OVER CLOSELY PACKED PARTICLES**

<table>
<thead>
<tr>
<th>Kind of particle</th>
<th>$d_p$, cm</th>
<th>Eq. [9] (Nikuradse), cm</th>
<th>Profile, cm</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spheres</td>
<td>0.41</td>
<td>0.0127</td>
<td>0.0080</td>
<td>Schlichting (1960)</td>
</tr>
<tr>
<td>Spheres (tapioca)</td>
<td>0.61</td>
<td>0.0202</td>
<td>0.0189</td>
<td>Lyles (1971)</td>
</tr>
<tr>
<td>Spheres (glass)</td>
<td>1.64</td>
<td>0.0547</td>
<td>0.0499</td>
<td>Lyles (1971)</td>
</tr>
<tr>
<td>Spheres (glass)</td>
<td>2.45</td>
<td>0.0818</td>
<td>0.0988</td>
<td>Lyles (1971)</td>
</tr>
<tr>
<td>Gravel</td>
<td>0.238-0.283</td>
<td>0.0087</td>
<td>0.0328</td>
<td>Chowdhury (1966)</td>
</tr>
<tr>
<td>Gravel</td>
<td>0.200-0.332</td>
<td>0.0089</td>
<td>0.0293</td>
<td>Lyles</td>
</tr>
<tr>
<td>Gravel</td>
<td>0.635-0.952</td>
<td>0.0265</td>
<td>0.0543</td>
<td>Lyles</td>
</tr>
<tr>
<td>Sand (smoothed)</td>
<td>0.015-0.029</td>
<td>0.0007</td>
<td>0.009 + 0.00004*</td>
<td>Lyles</td>
</tr>
<tr>
<td>Sand (smoothed)</td>
<td>0.042-0.059</td>
<td>0.0017</td>
<td>0.009 + 0.00040*</td>
<td>Lyles</td>
</tr>
<tr>
<td>Sand</td>
<td>0.015-0.025</td>
<td>0.0007</td>
<td>0.00046</td>
<td>Zingg (1953)</td>
</tr>
<tr>
<td>Sand</td>
<td>0.025-0.030</td>
<td>0.0009</td>
<td>0.00137</td>
<td>Zingg (1953)</td>
</tr>
<tr>
<td>Sand</td>
<td>0.030-0.042</td>
<td>0.0012</td>
<td>0.00274</td>
<td>Zingg (1953)</td>
</tr>
<tr>
<td>Sand</td>
<td>0.042-0.059</td>
<td>0.0017</td>
<td>0.00066</td>
<td>Zingg (1953)</td>
</tr>
<tr>
<td>Sand</td>
<td>0.059-0.084</td>
<td>0.0024</td>
<td>0.00487</td>
<td>Zingg (1953)</td>
</tr>
</tbody>
</table>

*Average of 15 profiles.
OVER CLOSELY PACKED PARTICLES. PROFILE DATA, EXCEPT Sand (smoothed)

Sand
Sand
Sand
Sand
Sand
Kind of particle

* Calculated from Zingg's (1953) velocity profile data.

|| Kind of particle | d_p, cm | Profile, cm/s | Eq. [15], cm/s |
|------------------|---------|--------------|--------------|
| Sand             | 0.010-0.012 | 28.68        | 29.43        |
| Sand (smoothed)  | 0.015-0.030 | 28.46        | 31.61        |
| Sand             | 0.021-0.025 | 31.27        | 31.68        |
| Sand             | 0.042-0.047 | 34.30        | 33.84        |
| Sand (smoothed)  | 0.042-0.059 | 29.73        | 34.27        |
| Sand             | 0.078-0.084 | 35.60        | 35.83        |
| Gravel           | 0.200-0.332 | 41.30        | 40.47        |
| Gravel           | 0.635-0.952 | 44.70        | 45.14        |
| Spheres (tapioca)| 0.066     | 44.50        | 43.94        |
| Spheres (glass)  | 1.641     | 47.50        | 48.55        |
| Spheres (glass)  | 2.463     | 50.70        | 50.54        |
| Sand             | 0.015-0.025 | 25.63*       | 31.24        |
| Sand             | 0.025-0.030 | 28.60*       | 32.25        |
| Sand             | 0.030-0.042 | 32.65*       | 33.13        |
| Sand             | 0.042-0.059 | 31.85*       | 34.27        |
| Sand             | 0.059-0.084 | 36.41*       | 35.49        |

*Calculated from Zingg's (1953) velocity profile data.

besides average diameter affect Z_Q. In determining
his velocity profile Z_Q values, Zingg (1953) used a
graphical technique, which is subject to large errors
and which could account for his larger profile values
for four of five cases.

Although we used current measuring and recording
instrumentation, Z_Q values computed from velocity
profiles over the same surface varied greatly as indi-
cated by the data over the smoothed sand surfaces
in Table 2. There was a 5- to 6-fold difference be-
tween minimum and maximum values of Z_Q obtained
from the 15 profiles over the same surfaces. Such
variability complicates interpreting data and identifi-
ing factors affecting Z_Q. Effects of upwind distance
(fetch) on any of the parameters have not been con-
sidered. For the shorter elements, mean velocity
and turbulence were measured more than 360 times
the height of elements downwind; for the taller ele-
ments, these parameters were measured between 34
and 285 times the height of the elements downwind.

Friction Velocity, u_\alpha

Perhaps the most interesting profile parameter
is the friction velocity, u_\alpha, which indicates the wind's
capacity to transport soil and characterizes the tur-
be extrapolated much beyond the height-spacing-size
data used in their development. Equation [10] cannot
be used for closely packed spheres because predicted
values increase as particle diameter decreases. Logi-
tically, one would look for an equation of the Nikuradse
form to predict u_\alpha over closely packed particles.
Based on profile-derived values of u_\alpha over sand,
gravel, and spheres, this equation

\[ u_\alpha = 46.2(d_p)^{1/6} \quad 0 < d_p < 2.54 \text{ cm} \]  \[ 15 \]

has a very high simple correlation coefficient (0.99)
(Table 3), where d_p is the average particle diameter.
Two cases in Table 3 indicated mechanical smoothing
of sand grains results in lower profile values of u_\alpha.
Comparisons with Zingg's profile data indicated
predicted values of u_\alpha were within 2 to 20 percent
in Table 3. Friction velocity values derived from
measured mean velocity profiles over identical sur-
faces may vary considerably, depending on number
of replications, number of vertical data points con-
sidered and their relation to boundary layer depth,
and precision of flow generating, measuring, and
recording equipment.

Also, remember that u_\infty is associated with a free-
stream velocity (u_\infty) of 894 cm/s. If friction velocities
are desired for other u_\infty’s, this simple equation may
be used:

\[ u_\alpha = u_\infty \frac{\bar{u}_\alpha}{894} \]  \[ 16 \]

where (\bar{u}_\alpha) is the freestream velocity in question.

Effective Height, D

A simple regression equation [12] correlates 124 D
values, obtained from mean velocity profiles, with
a coefficient of 0.99. The distribution of the data
may partially explain this. About 80 percent of the
D values used in the regression analysis were < 3 cm;
mot were < 1 cm. About 66 percent of the D values
< 3 cm were underestimated by equation [12], but
the deviations in magnitude are generally small and
could explain the high correlation coefficient (Fig. 1).

For our short cylinder data (h < 4 cm), \epsilon/h aver-
aged 0.65, which compared favorably with Perry's
(1968) results, but obviously \epsilon/h is not a constant
when considering taller roughness elements—decreasing
as height increases (Fig. 2). In our case, however,
the displacement height is an artifact of applying
the log law to velocity measurements obtained above
roughness elements and the interheight velocities
(those below the point where the mean velocity
profile extrapolates to zero) are, in fact, > 0.
Turbulence Intensity, $a_U/\bar{U}_z$

Equation [13] does not apply for closely packed spheres, predicting larger values of $a_U/\bar{U}_z$ as sphere diameter ($d_p$) decreases, but actual (measured) values, as expected, increased with sphere diameter. From limited data, we obtained this equation for closely packed particles:

$$a_U/\bar{U}_z = 25.0 d_p^{-0.144}, \quad 0 < d_p < 2.54 \text{ cm}, \quad r = 0.99 \ldots \ldots \text{[17]}$$

where $d_p$ is particle diameter, and the reference height is $(Z - D)/\delta = 0.05$. Table 4 indicates the "goodness of fit" for equation [17] (first 5 rows). Equation [17] underpredicted the turbulence intensity over gravel, suggesting that particle shape and smoothness influence turbulence.

**General**

One could use equations [7], [10], and [12] in the adiabatic form of equation [1] to generate a mean velocity profile in the "constant stress" layer for selected height-size-pattern combinations of roughness elements (Fig. 3). Using the roughness element data of Fig. 3 in equation [13] gave $a_U/\bar{U}_z = 27.8$ percent for $(Z - D)/\delta = 0.05$. For our wind tunnel, $\delta \approx 44$ cm for the data in question, which gave $Z = 2.7$ cm and $\bar{U}_z \approx 467$ cm/s from Fig. 3. Then, $a_U/\bar{u}_{40} = 2.52$, which agreed closely with the mean value reported by Counihan (1975) and was only slightly larger than an average value we reported earlier (Lyles et al., 1971). Such agreement supports the prediction equations reported here and mean velocity profiles computed from combining them in the log law.

(Continued on page 343)
References
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