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ADAPTING A GRAIN STORAGE MODEL IN A 2-D GENERALIZED COORDINATE SYSTEM TO A 2-D CYLINDRICAL SYSTEM

by

Mark E. Casada
Agricultural Engineer
Engineering Research Unit
USDA–ARS, Grain Marketing and Production Research Center
1515 College Avenue
Manhattan, KS 66502

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Summary:

An existing model was adapted to study grain storage in cylindrical bins. A simple mathematical transformation allowed the governing equations, as well as many of the boundary conditions, to be adapted from a generalized coordinate system to a cylindrical coordinate system. Heat transfer due to diffusion and convection in and above a bed of stored granular products were modeled. The interaction of the grain bed with the solar heated headspace was included. The new model was tested to evaluate conservation of mass by the fluid flow equations—this conservation is an issue because the finite difference solution is not a closed system, but exchanges both dry air and moisture with the headspace system. The error in conservation of air (exchanged with the headspace) was generally less than 1% with a mesh of 65 x 65 nodes. The effect of grid refinement on these and other accuracy issues was also investigated.

Keywords:

Bins, Grain Storage, Finite Difference, Free Convection, Modeling, Temperature

ADAPTING A GRAIN STORAGE MODEL IN A 2-D GENERALIZED COORDINATE SYSTEM TO A 2-D CYLINDRICAL SYSTEM

Mark E. Casada

INTRODUCTION

Cereal grains, like many agricultural products, are at their highest quality level in most regards immediately after harvest and cleaning. During subsequent processing, storage, and transportation the products may lose significant marketable value due to quality and other losses. Grain storage models offer an inexpensive means to predict the conditions in stored grain. These predictions in turn can reveal needed improvements in grain storage design and management procedures, which will reduce the losses of quality and value of stored grain.

The temperature and moisture content of grain are generally considered to be the most important factors in controlling quality during storage (Loewer et al., 1994). Long-term (at least several weeks) moisture migration from natural convection currents induced by temperature gradients in stored grain is one well-known problem resulting from adverse temperatures during storage (Foster and Tuite, 1992). Safe relative humidity levels required to minimize deterioration during storage of biological products such as grains, seeds, and nuts are generally known and adhered to by producers and processors of these products. However, moisture migration during storage and transportation of these products may lead to localized areas with unsafe moisture levels causing unacceptable amounts of deterioration, even though the average moisture level in the lot is considered safe.

A complete model of the heat and moisture transfer processes during storage of grains requires accounting for the natural convection currents in addition to heat and moisture diffusion. Wooding (1957) and Combarnous and Bories (1975) gave the governing equations for natural convection in homogeneous porous media with Darcy flow: conservation of mass (air), momentum, and energy combined with a fluid equation of state. Adding an equation for conservation of moisture (Smith and Sokhansanj (1990a and 1990b), its latent heating effect to the energy equation, and a thin-layer drying equation permit a complete analysis of the heat and moisture transfer due to diffusion and convection in a grain storage bin (Casada and Young, 1994a; Khankari et al., 1995a)

There have been several numerical studies on natural convection heat transfer in porous media for recirculating flows in enclosures. The majority of these studies were based on the assumption of Darcy flow. Prasad and Kulacki (1984a, b) presented a stream function formulation from Wooding's (1957) equations for the natural convection problem in a rectangular porous medium assuming Darcy flow. This method has been used to solve a variety of problems in rectangular enclosures with different boundary conditions (Prasad, 1987; Prasad and Kulacki, 1986, 1987; El-Khatib and Prasad, 1987). The Darcy flow (Darcy, 1856; Bear, 1972) used by those authors was based on the assumption that the velocity was linearly related to the hydraulic gradient and required a value of less than one for the particle Reynolds number, Re_p.

The stream function formulation was used by Stewart and Dona (1988) to predict the transient natural convection currents in grain storage bins. They used the modified Ergun equation from Patterson et al. (1971) to model the flow and found that the inertia term was only significant for particle Reynolds numbers greater than one. Vafai and Tein (1981) analytically developed a set of equations similar to those developed by Wooding (1957) using the local volume averaging technique.

In addition to Stewart and Dona (1988), mentioned above, there have been a number of models developed for heat transfer in stored grain in cylindrical bins, neglecting the interaction with moisture transfer (e.g., Jayas et al., 1992; who also listed most of the earlier works). Recently, a few models have been developed that include moisture transfer for stored grain in cylindrical bins (Tanka and Yoshida, 1984; Nguyen, 1986; Khankari et al., 1994; Obaldo et al., 1991; Abbouda et al., 1992; Khankari et al., 1995a, b). Singh et al. (1993) presented a three-dimensional finite difference model for stored grain in a rectangular enclosure.

One of the earliest attempts to model moisture movement in stored grain was by Lo et al. (1975), using a one-dimensional equation developed by Chen and Clayton (1971). A number of models have been developed for the simplified case of heat conduction in stored grain with natural convection neglected (e.g., Converse et al., 1973 and Sarker and Kunze, 1991). Obaldo et al. (1991) used Fick's law and developed a finite element model to describe the diffusion of moisture in stored grain with heat transfer and natural convection neglected.

Tanka and Yoshida (1984) modeled the temperature and moisture profiles in deep narrow silos. Nguyen (1986) used a numerical model to determine changes of temperature and moisture in grain storages with rectangular geometries. Khankari et al., (1994) also modeled heat and moisture diffusion in stored grain with natural convection effects neglected. Smith and Sokhansanj (1990a and 1990b) modeled the simultaneous heat and moisture diffusion and convection in grain storages with the effect of the headspace above the grain neglected. There are also recent numerical studies by Abouda et al. (1992) and Khankari et al. (1995a, b) that include both heat and moisture transfer in stored grain bulks. None of these works address certain of the boundary interactions that are important in grain storage situations; namely, air and moisture exchange with the headspace and the resulting solar heating interaction, variable resistance to heat transfer at the boundary through a moderate Biot number, and temperature difference between the grain and air.

A computer model for a personal computer was developed and evaluated by Casada and Young (1994a, b) and used to study short-term storage (during transportation) of peanuts. The model predicted heat and moisture transfer due to natural convection currents and diffusion in two-dimensional granular beds. This model used a diurnally varying ambient temperature for boundary conditions and accounted for the resistance to heat transfer of the walls and air films. Moisture movement and solar heating in the headspace was accounted for in the model, and found to give a significant contribution to moisture migration. The model was designed for use with arbitrarily shaped two-dimensional geometries, but did not have the capability of being used directly with cylindrical shaped storage bins. The governing equations, coded in a generalized coordinate system, may be transformed algebraically to a cylindrical coordinate system.

OBJECTIVES

The objectives of this research were to use the generalized coordinate system model (Casada and Young, 1994a) and:

1. transform the governing equations of the original model to apply directly to a two-dimensional cylindrical geometry, thus preserving the usefulness of the original finite difference model for application to the cylindrical geometry,
2. modify the original boundary conditions to apply to a two-dimensional cylindrical geometry, and
3. update the finite difference solution to work with the transformed equations and boundary conditions and evaluate the modified cylindrical model as an aid to grain storage design and management.

MODEL DEVELOPMENT

The governing equations for natural convection heat and moisture transfer in homogeneous porous media with Darcy flow were (Wooding, 1957; Combarnous and Bories, 1975; Casada and Young, 1994a)[1],

Conservation of Mass (Air)

$$\phi \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho V) = 0 \tag{1}$$

Conservation of Momentum

$$\frac{\rho}{\phi} \frac{\partial V}{\partial t} = -\nabla P + \rho g - \frac{\mu}{K} V \tag{2}$$

Conservation of Energy

$$\nabla \cdot (k_f \nabla T_f) - \nabla \cdot \left[(\rho c)_f V T_f \right] = \frac{\partial}{\partial t} \left[\phi (\rho c)_f T_f \right] + h_p A_{s/v} (T_f - T_s) \tag{3}$$

$$\nabla \cdot (k_s \nabla T_s) = \frac{\partial}{\partial t} \left[(1-\phi)(\rho c)_s T_s \right] + h_p A_{s/v} (T_s - T_f) + h^*_{fg} \dot{M}_w \tag{4}$$

Conservation of Moisture

$$\nabla \cdot (D'_m \nabla \gamma_m) - \nabla \cdot (V \gamma_m) = \frac{\partial \gamma_m}{\partial t} + h_m A_{s/v} (\gamma_m - \gamma_{m,s}) \tag{5}$$

Fluid Equation of State

$$\rho = \rho_0 \left[1 - \beta_e (T_f - T_0) \right] \tag{6}$$

Using the stream function formulation in a generalized coordinate system, these became (Casada and Young, 1994a),

$$\alpha \frac{\partial^2 \psi}{\partial \xi^2} - 2\beta \frac{\partial^2 \psi}{\partial \eta \partial \xi} + \gamma \frac{\partial^2 \psi}{\partial \eta^2} + \delta \frac{\partial \psi}{\partial \xi} + \varepsilon \frac{\partial \psi}{\partial \eta} = \mathrm{Ra}^* \cdot \mathrm{J} \left(y_\eta \frac{\partial \theta}{\partial \xi} + y_\varepsilon \frac{\partial \theta}{\partial \eta} \right) \tag{7}$$

$$\begin{aligned} \phi \cdot \mathrm{J}^2 \left[\left(\frac{\partial \theta_f}{\partial \tau} + u \frac{\partial \theta_f}{\partial \xi} + v \frac{\partial \theta_f}{\partial \eta} \right) + St_f (\theta_f - \theta_s) \right] = \\ \alpha \frac{\partial^2 \theta_f}{\partial \xi^2} - 2\beta \frac{\partial^2 \theta_f}{\partial \eta \partial \xi} + \gamma \frac{\partial^2 \theta_f}{\partial \eta^2} + \delta \frac{\partial \theta_f}{\partial \xi} + \varepsilon \frac{\partial \theta_f}{\partial \eta} \end{aligned} \tag{8}$$

$$\begin{aligned} \mathrm{J}^2 \left[\frac{\partial \theta_s}{\partial \tau} + St_s (\theta_s - \theta_f) \right] = \\ \frac{\alpha_s}{\alpha_f (1-\phi)} \left(\alpha \frac{\partial^2 \theta_s}{\partial \xi^2} - 2\beta \frac{\partial^2 \theta_s}{\partial \eta \partial \xi} + \gamma \frac{\partial^2 \theta_s}{\partial \eta^2} + \delta \frac{\partial \theta_s}{\partial \xi} + \varepsilon \frac{\partial \theta_s}{\partial \eta} \right) + \mathrm{J}^2 H_{fg} \frac{\partial \Omega}{\partial \tau} \end{aligned} \tag{9}$$

[1] See Nomenclature at the end for definitions of terms.

$$\phi \cdot \mathrm{J}^2\left[\left(\frac{\partial \Gamma}{\partial \tau}+u\frac{\partial \Gamma}{\partial \xi}+v\frac{\partial \Gamma}{\partial \eta}\right)+St_m(\Gamma-\Gamma_s)\right]= Le_f\left(\alpha\frac{\partial^2\Gamma}{\partial \xi^2}-2\beta\frac{\partial^2\Gamma}{\partial\eta\,\partial\xi}+\gamma\frac{\partial^2\Gamma}{\partial\eta^2}+\delta\frac{\partial\Gamma}{\partial\xi}+\varepsilon\frac{\partial\Gamma}{\partial\eta}\right) \tag{10}$$

Where Ra* indicates the driving force for the natural convection. The equivalent to equations (7) through (10) in a two dimensional cylindrical coordinate system, is,

$$A^2\frac{\partial^2\psi}{\partial z^2}+\frac{\partial^2\psi}{\partial r^2}+\frac{1}{r}\frac{\partial\psi}{\partial r}=r\cdot\mathrm{Ra}^*\frac{\partial\theta}{\partial r} \tag{11}$$

$$\phi\left[\frac{\partial\theta_f}{\partial\tau}+u\frac{\partial\theta_f}{\partial z}+v\frac{\partial\theta_f}{\partial r}+St_f(\theta_f-\theta_s)\right]=A^2\frac{\partial^2\theta_f}{\partial z^2}+\frac{\partial^2\theta_f}{\partial r^2}+\frac{1}{r}\frac{\partial\theta_f}{\partial r} \tag{12}$$

$$\frac{\partial\theta_s}{\partial\tau}+St_s(\theta_s-\theta_f)=\frac{\alpha_s}{\alpha_f(1-\phi)}\left[A^2\frac{\partial^2\theta_s}{\partial z^2}+\frac{\partial^2\theta_s}{\partial r^2}+\frac{1}{r}\frac{\partial\theta_s}{\partial r}\right]+H_{fg}\frac{\partial\Omega}{\partial\tau} \tag{13}$$

$$\phi\left[\frac{\partial\Gamma}{\partial\tau}+u\frac{\partial\Gamma}{\partial z}+v\frac{\partial\Gamma}{\partial r}+St_m(\Gamma-\Gamma_s)\right]=Le_f\left(A^2\frac{\partial^2\Gamma}{\partial z^2}+\frac{\partial^2\Gamma}{\partial r^2}+\frac{1}{r}\frac{\partial\Gamma}{\partial r}\right) \tag{14}$$

Comparing equations (7) and (11) reveals that all of the terms in the cylindrical stream function equation (11) appear in the generalized equation (7), plus a few extra terms. Equation (7) may be transformed to equation (11) by proper definition of the parameters (formerly transformation metrics—definitions are in table 1) multiplying each differential term. Table 1 defines the appropriate transform values based on the comparison of these equations. With these definitions replacing the former metrics in the computer model, the model simulates the desired cylindrical geometry. Comparison of equations (8) through (10) to equations (12) through (14), reveals that the same transformation constants are appropriate to transform equations (8) through (10) to their two-dimensional cylindrical counterparts, equations (12) through (14).

The original equations were solved on a regular square mesh with grid spacing of unity in both directions (a choice made during the transformation for simplicity). Thus, the code did not include the possibility of using different mesh spacing in the two coordinate directions. Such variation in mesh spacing is desirable and may be included in the definitions of the transformation constants as shown in the "Including Mesh" column of table 1 to avoid rewriting any code.

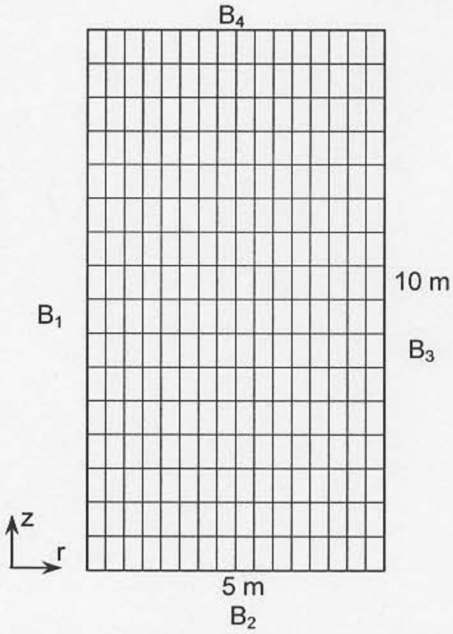

Figure 1. 17 x 17 mesh with boundaries identified.

BOUNDARY CONDITIONS

The boundary conditions on the original model were directly applicable only to the generalized coordinate system. The same transformations from table 1 were applied to some of the boundary conditions to transform them to cylindrical coordinate system as well. Some of the boundary conditions from the original model were more complicated than necessary for this cylindrical grain storage model and were replaced. The energy boundary condition with finite resistance to heat transfer from the wall was not tested because it is not applicable to storage in steel grain bins. That boundary condition would be useful for application to concrete storage silos and may be investigated later.

Table 1. Transformation constants.

Original Metrics		Cylindrical Transforms	
		Fundamental	Including Mesh
$J = x_\xi y_\eta - y_\xi x_\eta$	$\rightarrow$	$J = 1$	$J = 1$
$\alpha = x_\eta^2 + y_\eta^2$	$\rightarrow$	$\alpha = 1$	$\alpha = 1/(\Delta r)^2$
$\beta = x_\xi x_\eta - y_\xi y_\eta$	$\rightarrow$	$\beta = 0$	$\beta = 0$
$\gamma = x_\xi^2 + y_\eta^2$	$\rightarrow$	$\gamma = A^2$	$\gamma = A^2/(\Delta z)^2$
$\delta = f_1\{\alpha, \beta, \gamma\}$	$\rightarrow$	$\delta = 1/r$	$\delta = 1/(r \cdot \Delta r)$
$\varepsilon = f_2\{\alpha, \beta, \gamma\}$	$\rightarrow$	$\varepsilon = 0$	$\varepsilon = 0$
$y_\eta = \text{metric}_1$	$\rightarrow$	$y_\eta = r$	$y_\eta = r/\Delta r$
$y_\xi = \text{metric}_2$	$\rightarrow$	$y_\xi = 0$	$y_\xi = 0$
Coordinates:			
ξ = x-transform	$\rightarrow$	$\xi = r$	
η = y-transform	$\rightarrow$	$\eta = z$	

The boundary conditions required for implementation of the finite-difference model are summarized in table 2 and the boundary domains are illustrated in figure 1. Other appropriate boundary conditions may be substituted as necessary with minimum impact on the rest of the model, as noted below. Those listed as "alternative" are useful for applying the model to other situations.

Table 2. Boundary conditions used in the standard test simulation and alternative boundary conditions

Equation	(No.)	Standard Boundary Condition	Alternative Boundary Conditions
Stream function	11	$\psi = 0$ on B_1, B_2, and B_3 free porous surface on B_4	$\psi = 0$ on all boundaries
Fluid energy	12	T = f{t} on B_1, B_2, and B_4 Symmetry on B_3	Conduction through wall with moderate Bi on B_1, B_2, B_4
Solid energy	13	T = f{t} on B_1, B_2, and B_4 Symmetry on B_3	Conduction through wall with moderate Bi on B_1, B_2, B_4
Moisture transport	14	Impermeable wall on B_1 and B_2 Symmetry on B_3 Moderate Bi_m on B_4	Impermeable wall on all boundaries

Stream Function Equation

In addition to the common boundary condition on the stream function, equation (11), of constant stream function ($\psi = 0$) on impermeable boundaries and the line of symmetry, the necessary boundary condition for the top boundary, which allows flow across the boundary was the condition,

$$\frac{\partial^2 \psi}{\partial \bar{z}\, \partial \bar{r}} = 0 \ , \tag{15}$$

which enforces both:

$$\frac{\partial u}{\partial \bar{r}} = 0 \tag{16}$$

$$\frac{\partial v}{\partial \bar{z}} = 0 \tag{17}$$

This condition requires that the velocity is not changing at the free surface. It is necessary for a headspace above the porous medium. With constant stream function specified on the other three boundaries, this gives the four conditions that are required mathematically and satisfies the physical condition of no flow through the walls or across the line of symmetry.

Energy Equations

The energy boundary conditions on the wall of the bin (the left side boundary) and on the top of the grain was based on negligible resistance to heat transfer, i.e., a large Biot number for heat transfer. This was similar to a constant temperature boundary condition except that the temperature was diurnally varying with the ambient conditions. A symmetry boundary condition was applied to the right side boundary along the centerline of the bin. An adiabatic boundary condition, no heat loss, was specified on the bottom boundary, in contact with the ground. This could cause errors in temperature near that boundary (Chang et al., 1993) and the effect of heat transfer through the bottom may be investigated in the future.

Moisture Transport Equation

For moisture transport, equation (14), no diffusion was specified through the bottom, top, and left sidewalls, and across the right hand line of symmetry. The top boundary condition on this equation may also be expected to differ, depending on the application. As with the energy equations, when the flow boundary condition allowed fluid flow through the top boundary, convective mass transfer was used as the boundary condition on the top surface.

FINITE DIFFERENCE SOLUTION

The solution technique for the simulations was:

1. Solve the two energy equations simultaneously for the grain and air temperatures at each node for the next time step using the modified Crank-Nicolson scheme, described below, with successive over-relaxation (SOR).
2. Solve the flow equation, equation (11), for the stream function at each interior node for the current time step using point Guass-Seidel iteration with SOR.
3. Calculate the velocity components from the stream function using finite difference approximations to the derivatives in the definition of stream function.
4. Solve the moisture transport equation for the humidity ratio at each node for the current time step using modified Crank-Nicolson with SOR, and simultaneously calculate the grain moisture content at each node using a thin-layer drying equation.
5. Update the boundary conditions.
6. Return to step 1 until the desired total time has been reached.

A standard test was defined for evaluating the flow (stream function) and temperature portions of the model. As noted above, the same algebraic transformations (table 1) were applied to the moisture transport equation; however, this transformation of the moisture portion of the model has not yet been tested. The standard test was a simulation in a cylindrical bin with a solid floor, 10-m deep (level grain surface) and 10-m diameter filled with shelled corn, and with uniform resistance to

Table 3. Properties used for the standard test.

Property, Units	Air[1]	Shelled Corn[2]
c_p, J/kg·K	2135.	2026.
k, W/m^2·K	0.0261	0.159
ρ, kg.m^3	1.177	625.
α, m^2/s	2.21×10^{-5}	1.26×10^{-7}
ν, m^2/s	1.575×10^{-5}	–
K, m^2	–	2.26×10^{-8}
ϕ, m^3/m^3	–	0.40

[1]Kays and Crawford (1980)
[2]ASAE (1990)

airflow. Simulations were run for 90 days using a diurnally varying ambient temperature with a maximum of 5°C and a minimum of –12°C. The initial grain temperature and moisture content were 29°C and 14.7% w.b., respectively. Data used for grain and air properties are shown in Table 3.

The model was tested for a range of mesh sizes from 17 x 17 nodes up to 65 x 65 nodes. It was also tested for time steps up to 4 hours—the maximum practical while incorporating the effects of the diurnally varying ambient temperature.

RESULTS AND DISCUSSION

The use of the cylindrical transformation constants from table 1 allowed the original model to be used for a cylindrical geometry. Changes to the computer code were needed to redefine the old metrics for the cylindrical transformation—very simple code changes. Other code changes were also necessary for the boundary conditions and in places where inconsistencies arose because the original code had not been written to allow for this transformation. After re-coding the problem portions with both systems in view, the resulting code can now be used relatively easily with either coordinate system.

For the stream function equation, values of the SOR parameter from 1.3 to 1.5 were best for speeding up convergence of the Gauss-Seidel iteration, which is similar to the original railcar model (Casada and Young, 1994a). The best value for convergence varied slightly with mesh size. The smallest mesh (17 x 17) was faster with the larger SOR of 1.5 and the largest mesh (65 x 65) was faster with the smaller SOR of 1.3. For the energy equations, an SOR parameter of 1.4 was best for speeding up convergence, with no noticeable variation over the range of mesh sizes tested. This was the same as with the original railcar model.

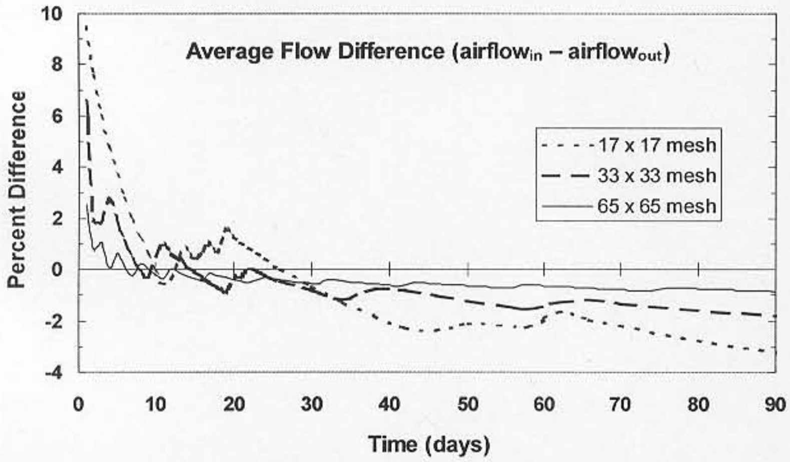

Figure 2. Conservation of mass (air) appraisal for equation (15).

Figure 2 shows that the free surface boundary condition (equation 15) was effective with the refined mesh for the stream function solution in the cylindrical coordinate system. Figures 3 and 4 show the predicted streamlines for the standard test in the grain bed after 30 and 90 days, respectively. An important measure of the effectiveness of this boundary condition was its ability to enforce the conservation of mass at the top surface. Physically, all of the mass flow (of air) out through this boundary must reenter through another part of the boundary because in the model the headspace is closed except for this boundary. The mass flows in and out of the headspace through the top surface of the grain, compared in figure 2 for the standard test simulation (averaged over each day). The flow difference followed a diurnal cycle. With the most refined mesh tested (65 x 65), it was within 1% of exact conservation of mass except at the initial time steps. This is better conservation than was obtained with the original model in a generalized coordinate system (Casada and Young, 1994a), probably because of the more refined mesh at the transition between flow into and flow out of the headspace—the original model mesh was refined only on the boundary.

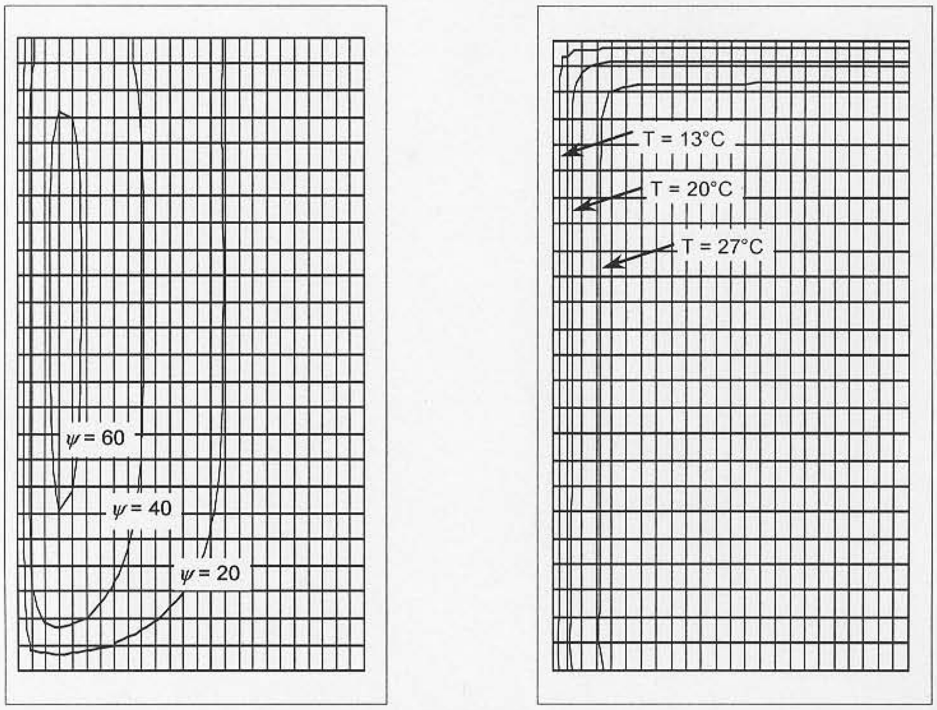

Figure 3. Streamlines and temperature contours at 30 days, 25 x 25 mesh, standard test.

The standard Crank-Nicolson method (Crank and Nicolson, 1947) is essentially an arithmetic average of the fully explicit and fully implicit methods. Fifty percent of each term is carried at the current time step and fifty percent is lagged at the previous time step. While this method is unconditionally stable for many cases, it was not unconditionally stable when used with equations 12, 13, and 14 mainly because of the source terms often dominated the equations when there was rapid heat and moisture transfer. This rapid heat and moisture transfer occurred twice a day with the diurnally varying temperature boundary condition. With the modified Crank-Nicolson solution the entire source term, e.g. the St_f term in the fluid energy equation, was carried at the current time step, instead of lagging half at the previous time step. This placed all of that large term on the main diagonal of the solution matrix and insured diagonal dominance of the matrix.

As mentioned, the source terms in the equations, as well as the convective terms, prevent the Crank-Nicolson method from being unconditionally stable, but the modified technique worked well and was stable with the energy equations for time steps of 4 h at a maximum Rayleigh number of 1100. The solution of the moisture transport equation was not tested, although it tended to be less stable than the energy equations in the original model (Casada and Young 1994a). A 0.5 h time step was used with all model equations for calculations in the standard test simulation.

The energy equations had greater stability problems at the initial time steps because of the large step change in the boundary conditions initially combined with the ever-present source terms. To mitigate these problems, smaller time steps were used for the first 20 time steps, starting at 1% of the base time step size and increasing in two-step changes to the base time step after the first 20 time steps. This allowed larger time steps to be stable than would otherwise be effective.

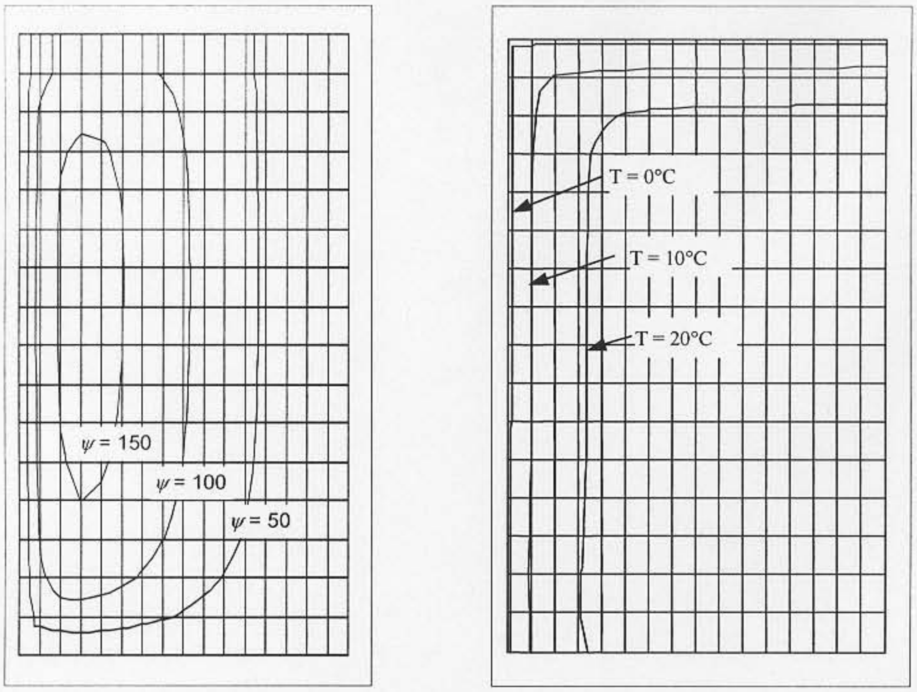

Figure 4. Streamlines and temperature contours at 90 days, 17 x 17 mesh, standard test.

The predicted temperature contours at 1 and 3 months of storage are also shown in figures 3 and 4, for the standard test. Only regular mesh spacing arrangements were tested. The gradients of temperature and velocities at the boundaries are relatively large—only the most refined mesh, 65 x 65, appears to be approaching sufficient refinement where the solution is not changing with mesh spacing. This is illustrated by the temperature gradients predicted near the wall in figure 5. It is expected that refining the mesh at the boundaries will allow the use of fewer nodes overall with equal accuracy. As noted above, refining the mesh only at the boundaries is likely to reduce the accuracy of the conservation of air flowing through the headspace. However, the small reduction in accuracy of that conservation is not likely to impair the accuracy of the rest of the model (Casada

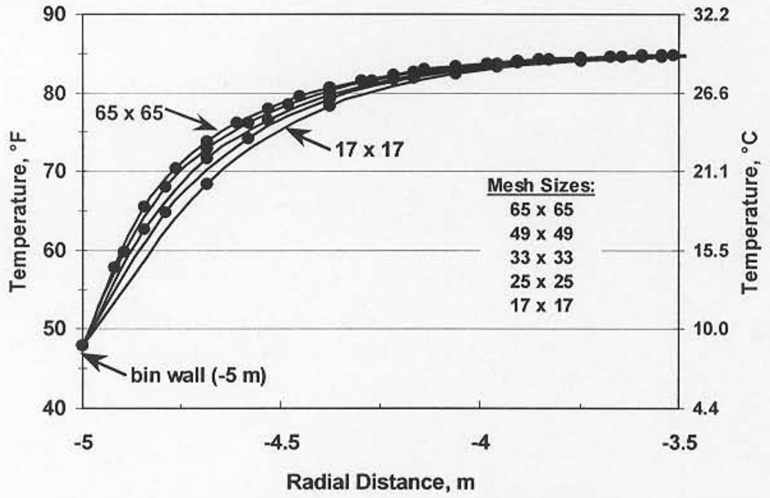

Figure 5. Temperature gradients near the wall for various mesh sizes for the standard test.

and Young, 1994a), making a refined mesh at only the boundaries to be the preferable arrangement. This will be further tested in the future.

The new cylindrical model will allow study of a range of conditions and variables for their effect on grain storage and grain storage management using computer simulations. Potential factors to be investigated include: 1) grain bin size, 2) air flow rate, 3) grain packing condition, 4) type of grain, 5) aeration control scheme, 6) sealed versus ventilated headspace, and 7) weather conditions. Each of these storage factors will affect grain temperature and/or moisture content, which will ultimately affect the survival of storage insects and fungi.

In addition, the type of grain and bin loading methods will have small effects on a number of factors. These additional factors will be added to the model. The packing condition is one such factor, which will result in different cooling rates for different grains and loading rates. The anisotropic nature of the packing factors for grain may also have an effect on the natural convection flows. Additionally, the different equilibrium moisture relationship for different grains will result in different equilibrium relative humidities in the grain mass to achieve the same desirable grain moisture content.

The influence of weather conditions may be studied to see the impact of typical year to year variations. The typical variations will be defined as temperatures that are one and two standard deviations above and below the normal data. Field tests are currently being conducted to evaluate the simulation model.

CONCLUSIONS

A model for heat, momentum, and moisture transfer in stored grain in a two-dimensional generalized coordinate system was adapted to a two-dimensional cylindrical coordinate system. A simple mathematical transformation allowed the existing finite difference model to be transformed to the cylindrical coordinate system. The modified (cylindrical) model appears to be effective at modeling heat and momentum transfer in stored grain. The moisture transfer solution has not been tested yet. The model responded to various inputs and physical parameters as expected from physics, but it has not yet been validated with experimental data.

The following specific conclusions were based on the development and evaluation of the new cylindrical model:

1. The transformation using the redefined metrics enabled the existing model to be used for a cylindrical geometry, although the computer code had to be modified substantially in places since it had not been written originally to allow for this transformation.

2. The special boundary conditions from the original model—the free surface boundary condition on stream function, the diurnally varying ambient temperature, and the headspace interaction boundary conditions on the energy equations—were also effective at representing the physical processes with the cylindrical model.

3. The special solution techniques from the original model—the modified Crank-Nicolson method and the variable time step—were effective at promoting convergence with the cylindrical model.

4. The cylindrical energy model was tested at larger Rayleigh numbers, Ra^*, than the original model—up to $Ra^* = 1100$ compared to $Ra^* = 185$—and exhibited stable solutions for these cases and mesh sizes from 17 x 17 to 65 x 65.

5. The current regular mesh spacing in the model requires a grid refinement of 65 x 65 nodes to obtain good accuracy for the large gradients at the boundaries, although changing to a refined mesh only at the boundaries is expected to allow fewer total nodes for the same accuracy.

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NOMENCLATURE

c = specific heat
g = acceleration due to gravity (9.8 m/s^2)
h^*_{fg} = latent heat of vaporization of water from solid particles (J/kg)
h_m = convective mass transfer coefficient (m/s)
h_p = convective heat transfer coefficient at grain kernel surface (W/m^2K)
k_f = equivalent thermal conductivity in the fluid path ($W/m \cdot K$)
k_s = equivalent thermal conductivity in the solid matrix ($W/m \cdot K$)
u = r-component (horizontal) of velocity (m/s)
v = z-component (vertical) of velocity (m/s)
A = R/H = aspect ratio
$A_{s/v}$ = surface area per unit volume of solid, m^2/m^3
D'_m = modified mass diffusion coefficient for porous media (m^2/s)
H = height of grain bin (m)
K = permeability of porous medium (m^2)
M = dry basis moisture content (decimal)
M_0 = initial particle moisture content (decimal)
M_w = rate of evaporation or condensation per unit volume ($kg/s \cdot m^3$)
R = radius of bin (m)
T = temperature (K)
T_0 = reference temperature for equation of state (K)
V = Darcian velocity vector (m/s)

Greek Symbols

α_f, α_s = fluid and solid phase thermal diffusivity, respectively (m^2/s)
β_e = coefficient of thermal expansion of air (1/K)
γ_m = humidity ratio ($kg_{H2O}/kg_{dry\ air}$)
μ = dynamic viscosity ($Pa \cdot s$)
ρ = density (kg/m^3)
ρ_0 = reference density of air in equation of state (kg/m^3)
ϕ = void fraction of porous medium = porosity (decimal)

Dimensionless Numbers

r,z = coordinates
H_{fg} = $(h^*_{fg} \cdot M_0)/[c_s(T_w - T_0)]$
Le_f = D'_m/α_f = fluid phase Lewis number
Ra^* = $(\rho g \beta_e K L \Delta T)/(\mu \alpha_t)$ = Rayleigh number
Re_p = $D_p V_p g/\mu$ = particle Reynolds number
St_m = $[6h_m L^2(1-\phi)]/[D_p \alpha_t \phi]$ = modified moisture Stanton number
St_f = $[6h_p L^2(1-\phi)]/[D_p k_f \phi]$ = modified fluid Stanton number
St_s = $[6h_p L^2]/[D_p(\rho c)_s \alpha_f]$ = modified solid Stanton number
θ = $(T - T_0)/(T_w - T_0)$
ψ = stream function
τ = $(t/\alpha t)/D^2$ = dimensionless time
Γ = $\gamma_m/\gamma_{m,0}$
Ω = M/M_0

Subscripts

a = air; f = fluid, s = solid; 0 = initial time; t = total