Simulation as a Tool for Predicting Errors of Feed Meters

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FEED meters are increasingly important on the farm as well as in feed-manufacturing and food-processing plants. Filling containers and blending ingredients are important jobs for feed meters. On-the-farm applications have multiplied with automatic blending and grinding equipment and with controlled feeding of livestock. Controlled feeding is meant to include the concept of limited feeding as well as that of feeding a metered amount equal to or slightly less than the animal's wants.

Farm-feeding systems usually require a metering device at each pen so costs of equipment must be low, yet metering accuracy is required. Animals must not be underfed, thereby reducing their daily gains; they must not be overfed, resulting in unnecessary waste; they must not be thrown "off feed" due to erratic feeding.

Feed meters may be classified as volumetric or weight types. Volumetric metering, as referred to here, means metering by timing the operation of a device that conveys feed at an assumed constant average rate of flow. Weight-metering devices sense the weight of the material and stop the inflow from the input conveyor when the desired weight is obtained. In general, volumetric metering has the advantages of low initial cost and simplicity of design, while weight metering is preferred for greater accuracy and dependability.

This paper covers the development of and the results from simulation models which predict the effects of nonuniform flow rates, timer and control inaccuracies, and other factors on the performance of volumetric and weight-type feed meters (11)*.

Good descriptions of batching weight meters and continuous-flow weight meters used in the feed industry are given by Sanders (8) and Richardson (6, 7). Schneider (9, 10), White (12), and Larson and Hodges (3) have developed weight meters for farm applications. The Behlen Mfg. Co. (1) markets a controlled feeder for swine that is fed by an auger and has an adjustable weight meter at each pen.

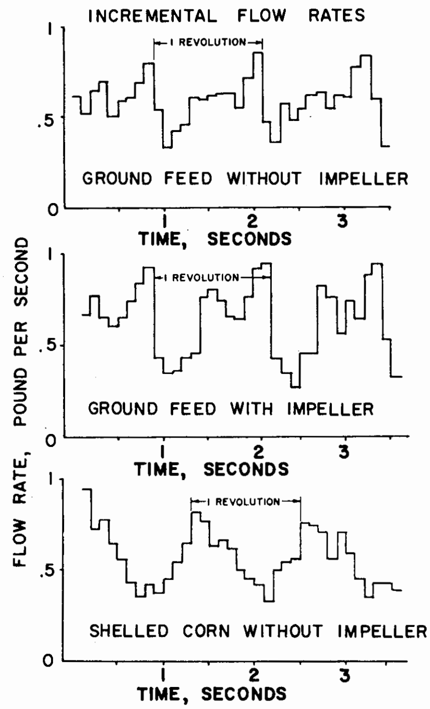

FIG. 1 Measured incremental flow rates for the test auger.

Puckett and Peart (5) reported on six types of volumetric meters and their operating characteristics. Butt (2) tested the ratchet-driven volumetric auger meter used on a blender-grinder. Several companies manufacture volumetric meters using moving belts or augers. These meters are used for blending into a grinder or for metering grain and supplement onto silage in a bunk feeder for cattle.

Most commercial literature indicates that the accuracy of the metering mechanism is subject to uniform feed density and flow rates. Myers (4) reports that non-uniform flow rates have a definite adverse effect upon weight-metering accuracies. The practice of reducing flow rates during the final stage of weighing is helpful, but demands for higher weighing rates cause other accuracy problems. He lists low-frequency vibrations, variations in feed density and flow characteristics, and method of infeed as other factors affecting accuracy. Richardson (6) reports that the materials-handling equipment, rather than the scale itself, is usually the cause of inaccurate weighing.

The approach to the problem of predicting the performance of volumetric and weight meters was to separate and individually study the main variables that affect the accuracy of meters. Mathematical models representing volumetric and weight meters were formulated and programmed on a digital computer to determine how the meters would perform with nonuniform flow rates and with timer inaccuracies. Incremental flow-rate data collected in the laboratory was used as input data to the simulation models. With predictions on the performance of both types of meters, comparisons were made between the different causes of errors and between the two types of meters.

Incremental Flow Rates

The variables, such as variations in product-flow characteristics and bulk densities and variations in the output of the materials-handling equipment, were combined and determined experimentally to obtain some nonuniform flow rate data to use in the simulation models. The output of a nominal 4-in. auger operating at a slow speed was measured to determine typical magnitudes of variation in flow rate.

A test facility capable of measuring the output of a conveyor every 0.1 sec for short periods of time was constructed. The test auger consisted of a nominal 4-in. standard pitch auger powered by a variable-speed motor and fed by a V-shaped intake hopper. After the test auger was started, the output was measured by passing a sled under the auger discharge spout. The sled was loaded with 22 individual trays, each 4 in. wide in the direction of travel and fastened together so that all of the conveyed material was collected while the sled was under the auger. The sled was pulled at a speed such that each tray was in the loading position for 0.1 sec. After each test run, the contents of each tray was weighed with an accuracy of 0.1 g or about 0.5 percent at the lower flow rate.

A series of tests were conducted to investigate experimentally the performance of the auger under different operating conditions. Tests were performed with a 13 percent protein feed mix of ground corn and soybean oil meal and with shelled corn (12 percent wb) at

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* Numbers in parentheses refer to the appended references.

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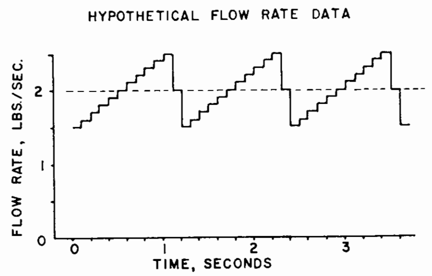

FIG. 2 Hypothetical flow-rate data illustrating the volumetric metering model.

auger speeds of 50 and 100 rpm. Tests were also performed to determine the effect of an impeller† on the incremental flow rates.

Fig. 1 shows the incremental flow rates that were measured for three of the 50-rpm tests. Except for the change in frequency, the 100-rpm results were the same. These results show how the flow rates vary as the auger rotates. Comparisons between the shelled corn and the ground-feed data show that the flow characteristics of the material do have an effect on the incremental flow rates. The reason for these differences is probably due to the angle of repose of the material. As the auger pushes the feed toward the discharge spout, the shelled corn probably flows away from the flighting more uniformly than the ground feed. These results show that the impeller does not have very much effect at the slow speeds used in these tests.

Volumetric Metering

Errors Due to Nonuniform Flow Rates

Theoretically a volumetric meter can be considered as a device that displaces equal volumes of material per unit of time. Within variations in the bulk density of the feed and variations in the output of the materials-handling equipment, the feed metered for each unit of time has a fixed weight. A meter is calibrated by determining the average flow rate and setting the meter to operate for a period of time equal to the desired weight divided by the average flow rate.

The metering operation of a volumetric meter was represented mathematically by

$$WT = \int_{t_s}^{t_c} Q(t)\ dt$$

where

WT = the weight that is actually metered

$Q(t)$ = the instantaneous flow rate (a function of t)

t_s = an arbitrary starting time

and

t_c = the cutoff time such that

$$t_c - t_s = \frac{\text{Desired weight}}{\text{Average flow rate}}$$

† Normally an impeller is used to force material out of the discharge spout.

The performance of a volumetric meter subjected to non-uniform flow rates can be investigated by studying combinations of the hypothetical data shown in Fig. 2. For any desired weight, the meter expects the flow rate to be equal to the calibrated average flow rate (dotted line). In practice, a volumetric meter starts at random within a cycle of the flow-rate data; thus the effect of the starting location has to be investigated. Considering the flow rates above the average as positive and those below as negative, the metering error for any desired weight is equal to the algebraic sum of the positive and negative areas between the arbitrary starting time and the cutoff time. Fig. 3 shows how the errors vary for different starting times as the desired weight increases. Note that for this hypothetical data, if the desired weight is a multiple of the weight metered per cycle of the data, there is no error.

The volumetric metering mathematical model described above simulated the possible errors that would result if the incremental flow rate data obtained

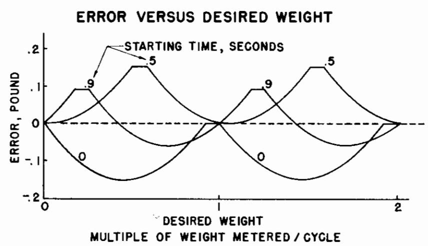

FIG. 3 Plots of error from desired weight versus the desired weight for three different starting times within a cycle of the hypothetical data.

in the laboratory were metered with a volumetric meter. This model determined the possible errors that could result due to non-uniform flow rates as the desired weight increased. Maximum errors as determined by this model for the 50-rpm data are shown in Table 1. Fig. 4 shows the effect of the desired weight versus the average weight metered per cycle on the maximum possible errors. If the desired weight is a multiple of the average weight metered per cycle, most of the errors due to non-uniform flow rates cancel each other.

TABLE 1. VOLUMETRIC METERING RESULTS
(Maximum errors due to non-uniform flow rates)

Flow rate data 50 rpm	Maximum positive errors	Maximum negative errors
Ground feed, without impeller	0.055 lb	−0.057 lb
Ground feed, with impeller	0.114	−0.122
Shelled corn, without impeller	0.118	−0.117

These combinations are known as minimum-error desired weights. It should be noted that, except for variations between minimum-error desired weights, the desired weight does not affect the maximum errors.

Errors Due to Inaccurate Controls

Inaccuracies of the timer and controls also cause errors in the amount of material metered. Improper timer settings due to either errors in the calibrated average flow rate or errors in setting the timer will result in errors equal to

$$(t+\Delta t)(\bar{Q}+\Delta\bar{Q}) - t(\bar{Q})$$

where

t = theoretical time the meter was set to operate with a calibrated average flow rate $\bar{Q}$,

Δt = actual deviation in time from the theoretical time of operation t, and

$\Delta\bar{Q}$ = actual deviation in average flow rate from the calibrated average flow rate $\bar{Q}$.

Variation within the timer and controls of a volumetric meter for a given timer setting also cause errors in the amount of material metered. The meter performance varies in the amount metered from

$$t_{\min}\ (\bar{Q}) \text{ to } t_{\max}\ (\bar{Q})$$

where

$t_{\min}$ = minimum time of all of the timer performances

$t_{\max}$ = maximum time of all of the timer performances

and

$\bar{Q}$ = known average flow rate.

Assuming that the error of the timer and controls follows a normal distribution and that the flow rate and standard deviation of the controls are known and constant, 99 percent of the errors from the desired weight will be between $-2.58(STD)\bar{Q}$ and $+2.58(STD)\bar{Q}$, where STD is the standard deviation of the timer and controls.

To conclude, the possible error due to either errors in setting the timer or to errors within the timer for a given setting are directly proportional to the average flow rate.

Errors Due to Nonuniform Flow Rates and Inaccurate Controls

Errors due to combinations of nonuniform flow rates and inaccuracies

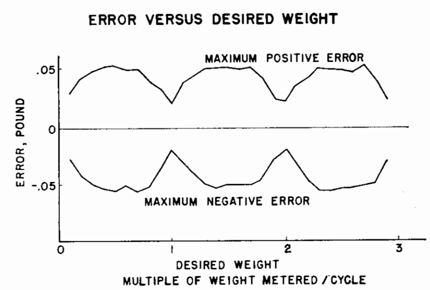

FIG. 4 Effect of desired weight on maximum possible error from a volumetric meter. Flow-rate data: 50 rpm ground feed without an impeller.

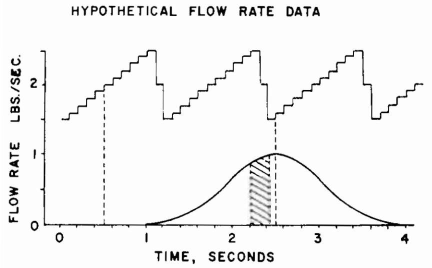

FIG. 5 Hypothetical data and assumed distribution function of the timer for the hypothetical problem.

within the timer and controls can be obtained by allowing the cutoff time t_c to vary from the desired and using the volumetric metering model described above to calculate the weight metered. With this method of approaching the problem, it would be difficult to consider all combinations of timer deviations and non-uniform flow rates.

All of these combinations of variables were analyzed by using a probability density approach. In order to use this density approach, the sequence of operations used in the original volumetric metering model was reversed. This new mathematical simulation model is described and illustrated with the solution of the following hypothetical problem:

Problem: Using the flow-rate data presented in Fig. 5 and a timer standard deviation, *STD*, of 0.5 sec, determine the probability of obtaining between 3.5 and 4.0 lb of feed when the meter is set for a desired weight, *DWT*, of 4.0 lb and the average flow rate, $\bar{Q}$, is 2.0 lb per sec.

To determine all combinations of possible errors for this desired weight, this procedure was repeated for other weight increments (0.0 to 0.5, 0.5 to 1.0, 1.0 to 1.5, etc) and for all starting times in 0.1-sec intervals. The probabilities of each weight increment for each t_s were added together and the resulting total distribution normalized to determine the probability distribution of possible errors.

This mathematical model simulated the combinations of errors that could result due to non-uniform flow rates and to an inaccurate timer.

The results of this simulation for one set of incremental flow-rate data are shown in Fig. 6. This figure shows the distribution of possible weights metered with timer standard deviations of 0, 0.01, and 0.04 sec. Comparison of the distributions show that as the standard deviation increases, the timer error acts to provide a more normal distribution of weighing errors.

The over-all error of the simulated volumetric meter can be analyzed by examining limits on the normalized frequency distributions. Fig. 7 shows a plot of 99 percent upper and lower limits of error versus the standard deviation of the timer. The dotted lines on the same figure show the 99 percent limits of error due to only the variability of the timer (metering with a uniform flow rate).

The possible error from the desired weight increases very rapidly as the timer standard deviation increases. Comparison of the limits of error for

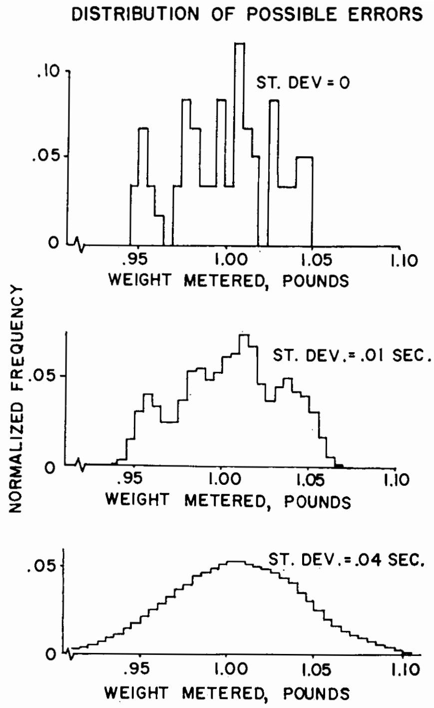

FIG. 6 Distributions of possible errors with different timer standards deviations.

the results with non-uniform flow rates versus the results with a uniform flow rate shows that for practical timer standard deviations the additional error due to the non-uniform flow rates is small compared with the total possible error. The results of this simulation show that the possible error of a volumetric meter is approximately equal to a multiple of the average flow rate times the standard deviation of the timer plus the error due to the non-uniform flow rates.

Solution:

Step	Example
1 Arbitrarily select starting time, t_s = 0.5 sec, and determine desired cutoff time, t_c, from desired weight, *DWT*, and average flow rate, $\bar{Q}$.	$t_c = t_s + (DWT/\bar{Q})$ $t_c = 0.5 + (4.0/2.0) = 2.5$ sec
2 From given incremental flow rate data, determine time, t_i, when lower limit, 3.5 lb, has been metered.	$t_i = 2.20$ sec (beginning of shaded area on Fig. 5)
3 Determine probability of timer stopping meter at t_i or sooner, $P(t \leq 2.20$ sec). Normalize the deviation to obtain a Z value and find the probability in normal tables.	$Z = \frac{t_i - t_c}{STD} = \frac{2.20 - 2.50}{0.5} = -0.60$ From tables, $P(t \leq 2.20) = 0.2743$
4 Determine time, t_o, when 4.0 lb would be metered, as in step 2.	$t_o = 2.43$ sec (Average flow rate for this time period is not equal to the overall average, $\bar{Q}$)
5 Determine probability that timer will stop meter at t_o or sooner, $P(t \leq 2.43$ sec), as in step 3.	From tables, $P(t \leq 2.43) = 0.4483$
6 Compute the probability of obtaining more than 3.5 lb but less than 4.0 lb as the difference between the two probabilities.	$P(3.5 \leq W \leq 4.0) =$ $P(t \leq 2.43) - P(t \leq 2.20) =$ $0.4483 - 0.2743 = 0.1740$ (shaded area in Fig. 5)

Laboratory Tests

The test auger that was used for the incremental flow-rate investigation was equipped with controls so that it could be used as a volumetric meter. The controls consisted of a "Y" spout attached to the discharge of the auger and a solenoid to control the position of the directional diverter on the spout. The solenoid was controlled with a holding circuit operated by a control timer, with an electronic timer wired

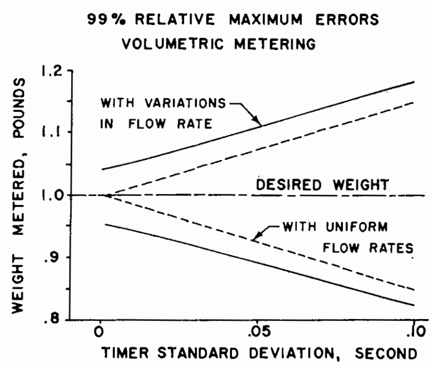

FIG. 7 Effect of timer standard deviation with and without variations in flow rate.

in to measure the time the solenoid was energized.

The experimental volumetric metering tests were performed with the same metering conditions as were used in the simulation model. The experiment was performed with one setting of the control timer by collecting the material that was diverted to the metering discharge spout while the solenoid was energized. The weight of material that was collected and the time that the solenoid was energized were recorded. This procedure was repeated 270 times to determine the distribution of weights metered and the measured distribution of the control timer. The amount of material collected for each metering observation was weighed on a dial scale accurate to ±1 g, or less than 0.2 percent of the smallest weight.

Fig. 8 shows the frequency distribution of the times observed during this test. Examination of the plot of the frequency of observation versus time shows that the timer errors approximately follow a normal distribution as was assumed for the simulation. The distribution of the weights metered for this test is shown in Fig. 9. The actual weights metered varied from 1.3 to 2.0 lb.

Comparison of the results shows a predicted maximum error (at the measured timer standard deviation) of 0.33 lb and an observed maximum error of 0.70 lb. Note that these variations would be the same for any desired weight if the average flow rate was constant, so the percent error would decrease for greater desired weights. The authors feel that comparison between the observed and the predicted results shows that some of the variables in the laboratory setup were not accounted for in the simulation model. Some of these variables were variations in the response time of the solenoid and directional diverter, the average flow rate, and the product-flow characteristics of the material.

Weight Metering

The process of metering by weight can be considered as a continuous flow of particles each of mass m_i being dropped a distance h onto a scale. When the force on the scale (material weight plus impact force) equals the desired weight, the meter stops the flow of particles.

The performance of such a meter can be developed by simulating the operations of this type of meter. This simulation can be achieved by developing a method of calculating the force on the weighing mechanism and stopping the inflow of material when the scale registers the desired weight.

Force is defined as time rate of

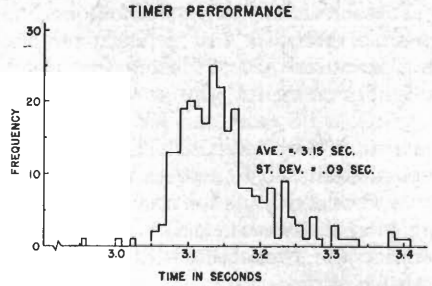

FIG. 8 Frequency distribution of the timer performance for the volumetric metering tests.

change of momentum or

$$F = d(mv)/dt = v\ dm/dt + m\ dv/dt$$

where

m = mass of the particle
t = time
v = velocity of fall at impact

Therefore, dm/dt is the flow rate of the material and dv/dt is the acceleration due to gravity, g, (assuming free fall), and $m\ dv/dt$ is, by Newton's second law of motion, the weight of a particle of mass m subjected to a constant acceleration (gravity). Thus the force of a single particle falling and hitting a flat surface is equal to the weight of the particle plus $v\ dm/dt$.

But $v = \sqrt{2gh}$

where h equals the distance the particle falls with a constant acceleration, g. Thus an additional force of $\sqrt{2gh}\ dm/dt$ is subjected to the surface of the scale.

If a continuous flow of material is being dropped onto the scale, some of the particles are in the air and do not affect the total force on the scale. The weight of these particles is equal to $gt\ dm/dt$ where t is the time each particle is in the air. However, $t = \sqrt{2h/g}$; or the weight of material in the air is $\sqrt{2gh}\ dm/dt$ which is exactly the same as the impact force on the scale. Therefore, the weight of the falling material for a uniform flow rate (dm/dt is constant), is exactly compensated for by its own impact on the scale.

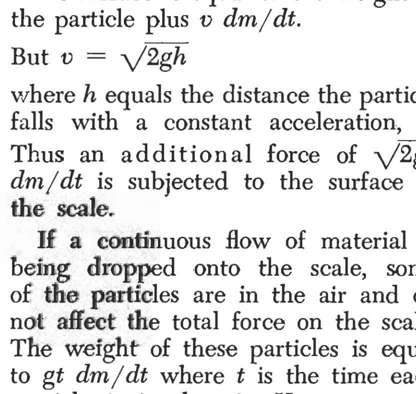

FIG. 9 Frequency distribution of weights metered for the volumetric metering tests.

In mathematical form, the force on the scale at any time t is equal to

$$F_t = g \sum_{i=1}^{n} m_i + \sqrt{2gh} \left. \frac{dm}{dt} \right|_t$$

where n is the number of particles that have accumulated on the scale at the time F is measured.

If the inflow of particles is stopped when F equals the desired weight, the amount of material that was actually metered is equal to the weight of all the particles of feed collected on the scale plus the weight of the particles that were in the air when the inflow of material was stopped.

The performance of a weight meter can be predicted by studying how the meter will perform when it is metering non-uniform inflow rates. The operations of the weight-metering simulation model are shown in Fig. 10. The model calculates the force on the scale by summing up the weights of the material collected plus the impact force. When the force on the scale equals the desired weight, the inflow of material is stopped and the material that is in the air falls to the scale. On the figure, the inflow of material is stopped at 2.27 sec and the weight of the material that is in the air is added to the weight of material collected on the scale to determine the amount that would be metered. The weight of material that is in the air is equal to

$$\int_{t_c}^{t_c + \sqrt{2h/g}} Q(t)\ dt$$

The meter expects the flow rate to be equal to that which the scale registered when the inflow of material was stopped. Thus the error in a weight-metering application is equal to the difference between the expected and the actual flow rate integrated over the time it takes a particle to fall to the scale. In Fig. 10 this difference is shown as the shaded area. In other words, a weight meter is a "self-calibrating" volumetric meter set for a period of time ($\sqrt{2h/g}$) depending on the height of fall.

The results of the weight-metering simulation which analyzed the effects due to non-uniform flow rates are shown in Fig. 11. This figure shows the effect of height of fall on the maximum positive and the maximum negative errors. The average values of all the simulated weights metered are also shown in the figure.

The plot of maximum error versus height of fall for the ground-feed data shows that as the height of fall increases there is a greater chance that the actual weight metered will be below the desired weight. The minimum possible weight metered decreases rapidly as the height of fall increases. The reason for this effect is that the flow rates for the ground-feed data decreased faster than they increased. Thus the difference between the expected flow rate and the actual flow rate was greatest while the flow rate was decreasing, causing greater errors below the desired weight.

The plot of maximum positive and maximum negative errors versus height of fall for the shelled corn data shows that the possible errors are symmetric around the zero error line. The shelled-corn flow-rate data increased and decreased at approximately the same rate so the greatest change was the same whether the flow rate was increasing or decreasing. The rate of change of flow rate is the characteristic of the non-uniform flow-rate data that determines what the maximum possible error of a weight meter could be.

The weight-metering mathematical model was not expanded to include other variables in the metering mechanism.

Volumetric Metering Versus Weight Metering

Fig. 11 shows the maximum errors that the weight-metering results indicated as the height of fall increases, with the maximum error predicted for the volumetric meter superimposed on the figures (dotted lines). In general, from these results a volumetric meter performs just about as well as a weight meter when the two meters are subjected to the same non-uniform flow rates. The maximum possible errors from the weight meter increase as the height of fall increases. For small heights, the magnitude of these errors are less than those obtained from a volumetric meter; but as the height increases, the errors associated with the weight meter rise above those from the other meter.

The assumptions made in this analysis should be reviewed before any conclusions are made that a volumetric meter performs as well as a weight meter. For both models the assumption was made that the metering mechanism performs perfectly. Therefore, these results compare only the errors caused by the non-uniform flow rates. For the volumetric-metering analysis, the assumption was made that the average flow rate was constant and was known for calibration. This assumption was not necessary for the weight meter because it was self-calibrating. In practice, this assumption for the volumetric meter would be valid if the handling equipment and the flow characteristics of the material resulted in a dependable constant average flow rate. These requirements would be obtained if the meter had a positive volumetric displacement and the flow characteristics were such that the material would always flow into the metering conveyor (i.e., would not bridge in the holding bin).

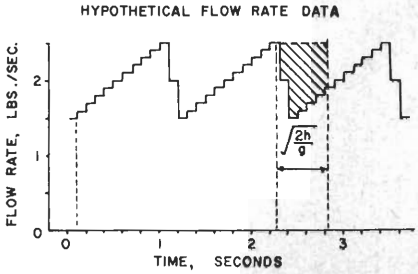

FIG. 10 Hypothetical flow-rate data illustrating the weight-metering model.

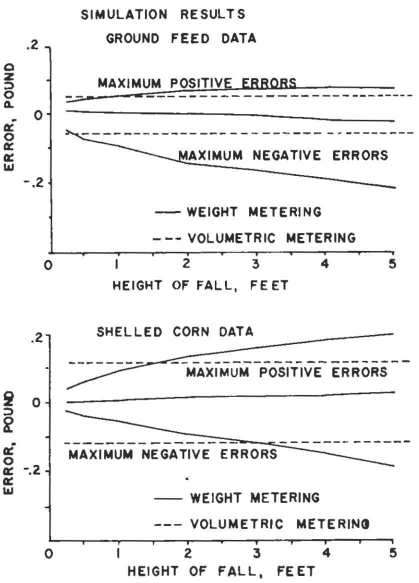

FIG. 11 Effect of height of fall on the performance of a weight meter in comparison to the maximum errors from a volumetric meter.

If for a particular metering application, the constant flow rate assumption could not be made, the weight meter would be much more dependable than the volumetric meter. The weight meter would sense when the flow rate changed, whereas the volumetric meter would not.

The performance of meters in practice depends on the value of the parameters that describe its performance and on the uniformity of product inflow rates. The application of these results should be used only to predict the performance of a metering system with the parameters and characteristics of the particular system carefully considered.

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