Trajectories of Grain Particles from Trough-Type Grain Spreaders

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Abstract

An optical velocity sensor was used to measure the velocities of wheat, corn, and grain sorghum in a spreader trough at different incline angles and lengths. The measured velocities were similar to the theoretical sliding velocities calculated using the kinetic friction coefficient of 0.27 for corn and sorghum and 0.29 for wheat moving on a metal surface.

A model was developed to predict the trajectories and velocities of grain from spreader troughs of different lengths and incline angles. Air drag force and the size and weight of grain kernels were taken into account in the model. The trajectories project the landing locations of grain which are useful in design and adjustment of spreader troughs to improve the distribution of grain and component materials in different-sized bins.

Introduction

When a bin is filled with a stream of grain from a gravity spout, the grain pile is conical and the fine material concentrates in an area under the filling point (Van Denburg and Bauer, 1964). Because fine material has greater airflow resistance per unit depth than whole kernels (Haque et al., 1978), segregation of the fine material causes uneven distribution of airflow within the grain mass and may result in nonuniform drying, hot spots and undesirable moisture migration. Trough-type grain spreaders are commonly used to reduce the segregation of fine material and improve the distribution of grain in bins.

Designs of trough-type grain spreaders are mainly based on trial and error. Currently, practical methods for predicting spreading efficiency, grain trajectories, and grain velocities for a spreader are not available. Design of equipment by trial and error is time consuming and expensive. Frequently, several prototypes must be constructed and tested before an acceptable design is achieved. Methods for predicting spreader performance would permit rapid evaluation of spreader design and provide a quantitative method for defining spreader adjustments.

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The objectives of this study were to determine the velocities of grain in a grain spreader trough and to develop a model to predict the trajectories and velocities of grain from trough-type grain spreaders.

Materials and Methods

Velocity of Grains

Velocities of wheat, corn, and grain sorghum in a grain spreader trough were measured with an optical velocity sensor (Chang et al., 1986). Moisture contents of the grain were 12.2, 13.7, and 12.5% (w.b.) for wheat, corn, and sorghum, respectively. Figure 1 shows the experimental setup for grain velocity measurements. The sheet metal trough (0.14 m wide x 0.1 m high x 2.44 m long) was adjustable for operation at different incline angles. The hopper was connected to a spout to receive test grain. An adjustable gate located between the hopper and trough was used to control grain flow rate. Two sizes of gate opening (0.14 m x 0.06 m and 0.14 m x 0.1 m) and three trough incline angles (25°, 35°, and 45°) were tested. For each combination of gate opening and trough incline angle, fourteen replicated velocity measurements were made for grain in the trough at each of the three locations 1.22, 1.83, and 2.44 m from the gate.

In theory, the sliding velocity of grain in an inclined trough can be calculated from the equation of uniformly accelerated motion (Ohanian, 1985):

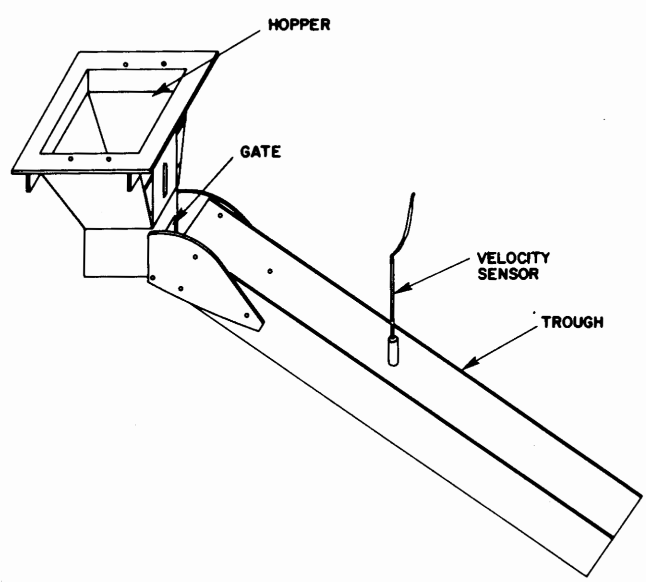

Figure 1–Schematic of a trough-type grain spreader.

$$V^2 = V_0^2 + 2Sg (\sin\theta - \mu\cos\theta) \quad (1)$$

where
- V = grain velocity at distance S from the upper-end of the trough, m/s,
- V_0 = grain velocity at the upper-end of the trough, m/s,
- S = distance from the upper-end of the trough, m,
- g = gravitational acceleration, m/S^2,
- θ = incline angle of the trough (down from the horizontal), degree,
- μ = kinetic friction coefficient.

V_0 in equation 1 was determined from the cross-sectional area of gate opening and the volume flow rate of grain through the opening. The volume flow rate was determined by measuring the time required for a given quantity of grain flowing through the opening. Kinetic friction coefficient of grain on metal surfaces from the literature ranged from 0.18 to 0.36 depending on the moisture content (Snyder et al., 1967; Stewart et al., 1969).

TRAJECTORY

If the velocity and direction of grain particles leaving the trough of a spreader are known, the trajectory of grain particles can be estimated from the following procedures:

Let V(t) be the velocity of grain particles at time t after leaving the trough and θ(t) the angle (down from the horizontal) of particles at time t after leaving the trough, then the velocity components of particles in X and Y directions at time t can be expressed as:

$$\dot{X}(t) = V(t) \cos\theta(t) \quad (2)$$

$$\dot{Y}(t) = V(t) \sin\theta(t) \quad (3)$$

$$\theta(t) = \tan^{-1} \frac{\dot{Y}(t)}{\dot{X}(t)} \quad (4)$$

The drag force F on a particle moving through air can be expressed as (Daugherty and Ingersoll, 1954):

$$F = 1/2 \rho C V^2 A \quad (5)$$

where
- C = drag coefficient,
- ρ = density of air, kg/m^3,
- V = velocity of particle, m/s,
- A = area of particle projected normal to V, m^2.

The drag coefficient C varies with Reynolds number, R, and can be estimated from equation 6, which was obtained by regression analysis using data from Garrett and Brooker (1965).

$$C = \exp(4.4 - 1.18R + 0.0708 R^2) \quad (6)$$

where
- R = Reynolds number, DV/ν,
- D = diameter of particle, m,
- ν = kinematic viscosity of air, m^2/s,

If both gravity and air drag forces are considered, the equations of motion of a particle are:

$$\ddot{X}(t) = \frac{-F}{M} \cos\theta(t) \quad (7)$$

$$\ddot{Y}(t) = g - \frac{F}{M} \sin\theta(t) \quad (8)$$

where $\ddot{X}(t)$ and $\ddot{Y}(t)$ are acceleration components of the particle at time t in X and Y directions, respectively, M is the mass of the particle, and g is the gravitational acceleration.

The velocity components and direction of the particle at time t + Δt, where Δt is a small time increment, can be expressed as:

$$\dot{X}(t + \Delta t) = \dot{X}(t) + \ddot{X}(t)\Delta t \quad (9)$$

$$\dot{Y}(t + \Delta t) = \dot{Y}(t) + \ddot{Y}(t)\Delta t \quad (10)$$

$$\theta(t + \Delta t) = \tan^{-1} \frac{\dot{Y}(t + \Delta t)}{\dot{X}(t + \Delta t)} \quad (11)$$

Velocity and position of the particle relative to the lower-end of the trough at time t + Δt can be expressed as:

$$V(t + \Delta t) = \left[\dot{X}^2(t + \Delta t) + \dot{Y}^2(t + \Delta t)\right]^{1/2} \quad (12)$$

$$X(t + \Delta t) = X(t) + 1/2\left[\dot{X}(t + \Delta t) + \dot{X}(t)\right]\Delta t \quad (13)$$

$$Y(t + \Delta t) = Y(t) + 1/2\left[\dot{Y}(t + \Delta t) + \dot{Y}(t)\right]\Delta t \quad (14)$$

Let the position of the lower-end of trough relative to the center of the bin floor be (x_l, y_l), then the position of the particle relative to the center of the bin floor at time t can be expressed as:

$$x(t) = x_l + X(t) \quad (15)$$

$$y(t) = y_l - Y(t) \quad (16)$$

For numerical calculations using equations 2 through 16, values of particle diameter, particle weight, and the projected area of a particle are needed. These values (Table 1) were determined using two, 400-kernel samples from each type of grain and assuming grain kernels were spherical. A gas pycnometer (Model SPY2, Quantachrome

TABLE 1. Kernel weight, equivalent diameter, and projected area of grain kernels

Type of grain	Kernel weight g	Spherical equivalent kernel diameter cm	Spherical equivalent projected area of a kernel cm^2
Corn	0.301	0.763	0.457
Wheat	0.046	0.394	0.122
Sorghum	0.041	0.386	0.117

Corp., Syosset, NY) was used to measure kernel volume in each 400-kernel sample. The average kernel volume was then determined and used to calculate a spherical equivalent kernel diameter and projected area.

VERIFICATION

Tests were conducted with wheat, corn, and sorghum to verify the grain trajectories predicted by the model. Figure 2 shows the experimental setup for determination of grain trajectories. The hopper was connected to a cylindrical container which held about 0.2 m^3 of test grain. A 4 m x 5 m section of linoleum which had a 11.5 cm square grid pattern, was installed about 5 cm behind the grain trajectories to provide position references. Two trough lengths (1.89 m and 2.44 m) and three trough incline angles (25°, 35°, and 45°) with a hopper-gate opening (fig. 1) of 0.14 m x 0.1 m were tested for each type of grain. Three photographs (slides) of grain trajectories were taken for each grain test and two tests were conducted for each combination of trough length and incline angle. Relative positions of the trajectories to the lower-end of the trough were determined from the photographs. The results were used to compare actual trajectories with those predicted by the model.

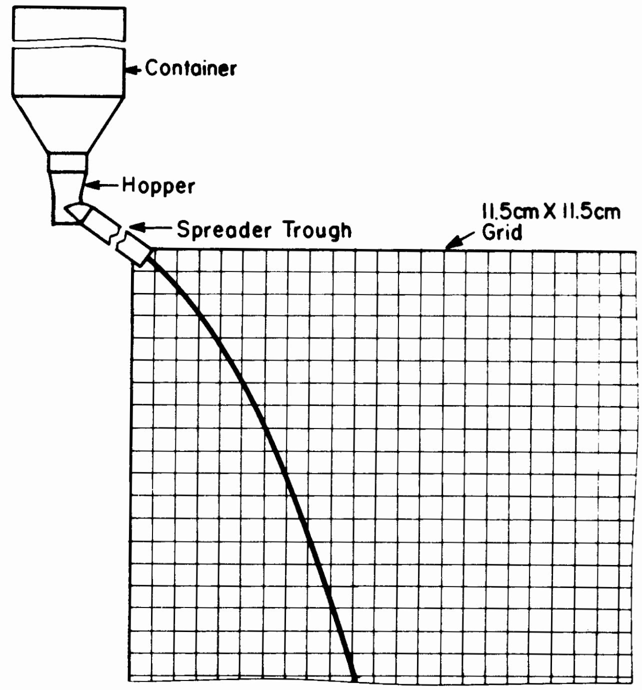

Figure 2–Schematic showing the experimental set-up for determinations of grain trajectories.

RESULTS AND DISCUSSION

The measured and predicted velocities of grain in the trough at different incline angles and lengths for gate openings of 0.14 m x 0.1 m and 0.14 m x 0.06 m are given in Tables 2 and 3, respectively. At the same trough incline angle, the grain velocities from the two different gate openings (flow rates) were not significantly different (0.05 level) for wheat and sorghum. However, the velocities of corn from the smaller opening were significantly higher than those from the larger opening at 35° and 45° incline angles. Higher velocities of corn from the smaller opening at larger incline angles were attributed to particles rolling rather than sliding in the trough. When the moving grain layer was thin or approached a single layer thickness at the steep incline angles, particle rolling was observed to occur.

At the same trough incline angle, velocities of wheat, corn and sorghum in the trough were similar except for corn from the smaller opening at steep incline angles. By trial and error using different values of kinetic friction coefficient in equation 1 to calculate the sliding velocities of grain, it was found that the deviations between calculated and measured sliding velocities of grain in the trough were minimum and were less than 6% of the calculated values when friction coefficients of 0.27 for corn and sorghum and 0.29 for wheat were used.

Velocities of grain at the gate or at the upper-end of the trough (V_0) were 0.49, 0.48, and 0.40 m/s for wheat, corn, and sorghum, respectively. The effects of trough incline

TABLE 2. Measured (Vm) and predicted (Vp) velocities of grain in a spreader trough at different incline angles and locations with a gate opening of 0.14 m x 0.1 m

Grain	Incline Angle degree	Distance from the top-end of trough, m: 1.22 Vm m/s	1.22 Vp m/s	1.83 Vm m/s	1.83 Vp m/s	2.44 Vm m/s	2.44 Vp m/s
Corn	25	2.17* (0.06)†	2.12	2.62 (0.09)	2.57	3.01 (0.08)	2.96
	35	2.95 (0.12)	2.94	3.61 (0.14)	3.59	4.12 (0.16)	4.13
	45	3.36 (0.07)	3.55	4.19 (0.09)	4.33	4.89 (0.16)	4.99
Wheat	25	2.05 (0.06)	2.01	2.45 (0.05)	2.44	2.83 (0.05)	2.81
	35	2.85 (0.06)	2.88	3.58 (0.09)	3.51	4.09 (0.08)	4.04
	45	3.49 (0.05)	3.50	4.38 (0.08)	4.27	4.90 (0.09)	4.92
Sorghum	25	2.18 (0.04)	2.10	2.65 (0.04)	2.56	3.06 (0.05)	2.94
	35	2.85 (0.08)	2.93	3.58 (0.09)	3.58	4.25 (0.06)	4.12
	45	3.61 (0.06)	3.54	4.42 (0.07)	4.32	5.08 (0.09)	4.98

* Average of 14 measurements.
† Standard deviation.

TABLE 3. Measured (Vm) and predicted (Vp) velocities of grain in a spreader trough at different incline angles and locations with a gate opening of 0.14 m x 0.6 m

Grain	Incline Angle degree	1.22 Vm m/s	1.22 Vp m/s	1.83 Vm m/s	1.83 Vp m/s	2.44 Vm m/s	2.44 Vp m/s
		Distance from the top-end of trough, m					
Corn	25	2.16* (0.07)†	2.12	2.61 (0.10)	2.57	3.05 (0.07)	2.96
	35	3.01 (0.07)	2.94	3.77 (0.13)	3.59	4.75 (0.29)	4.13
	45	3.56 (0.12)	3.55	4.85 (0.38)	4.33	5.99 (0.26)	4.99
Wheat	25	1.91 (0.08)	2.01	2.50 (0.05)	2.44	2.85 (0.09)	2.81
	35	2.80 (0.07)	2.88	3.48 (0.09)	3.51	4.06 (0.09)	4.04
	45	3.39 (0.07)	3.50	4.29 (0.10)	4.27	4.87 (0.11)	4.92
Sorghum	25	2.09 (0.08)	2.10	2.61 (0.07)	2.56	3.01 (0.04)	2.94
	35	2.74 (0.05)	2.93	3.46 (0.13)	3.58	4.04 (0.07)	4.12
	45	3.36 (0.08)	3.54	4.28 (0.09)	4.32	4.94 (0.08)	4.98

* Average of 14 measurements.
†Standard deviation

angle and gate opening on V_0 were small for the ranges tested.

Air drag force on a grain particle in a grain stream is less than that on a single moving grain particle. If 10% of the calculated air drag force on a single moving grain particle was applied to grain in a grain stream, the average deviation (over all tests) between predicted and measured trajectories was minimum and all predicted trajectories were in close agreement with measured trajectories. Figures 3 and 4 show the typical measured (two replicates) and predicted trajectories for wheat and corn, respectively, at an incline angle of 35° and trough length of 2.44 m. The mean standard deviations of the measured and predicted trajectories were 25, 28, and 23 mm for wheat, corn, and sorghum, respectively.

The predicted trajectories of corn from a spreader trough at incline angles of 25° and 45° and trough lengths of 1.22 m and 1.83 m are shown in the right of figure 5. The trajectories project the landing locations of grain in bins at different depths, which are useful in design and adjustment of spreader troughs to obtain uniform distribution of grain in different-sized bins. For the 8.0 m diameter bin (fig. 5), corn kernels from a 1.83 m trough at an incline angle of 25° impinge the bin wall at 1.9 m from the bin floor with a velocity of 10.5 m/s and an angle of 23° from vertical. Foster and Holman (1973) reported that

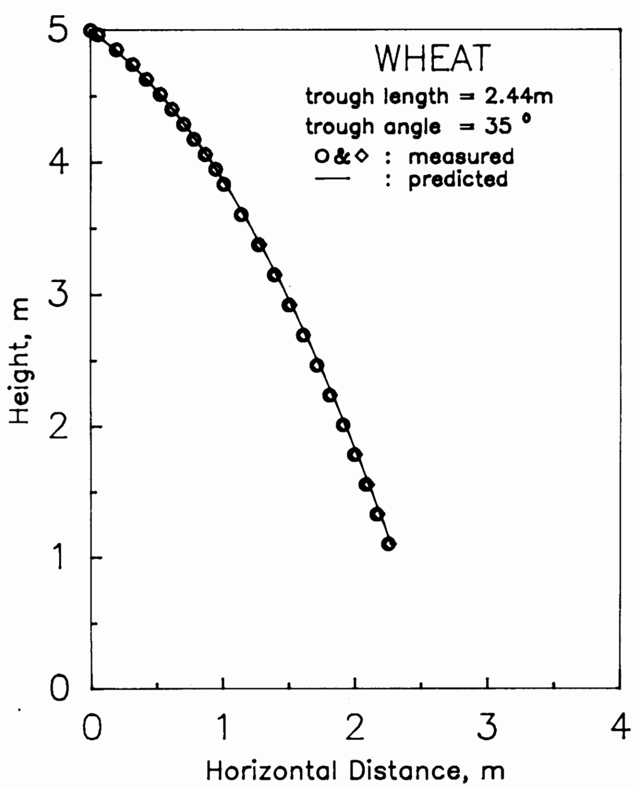

Figure 3–Measured and predicted trajectories of wheat from a 2.44 m spreader trough at an incline angle of 35 degrees.

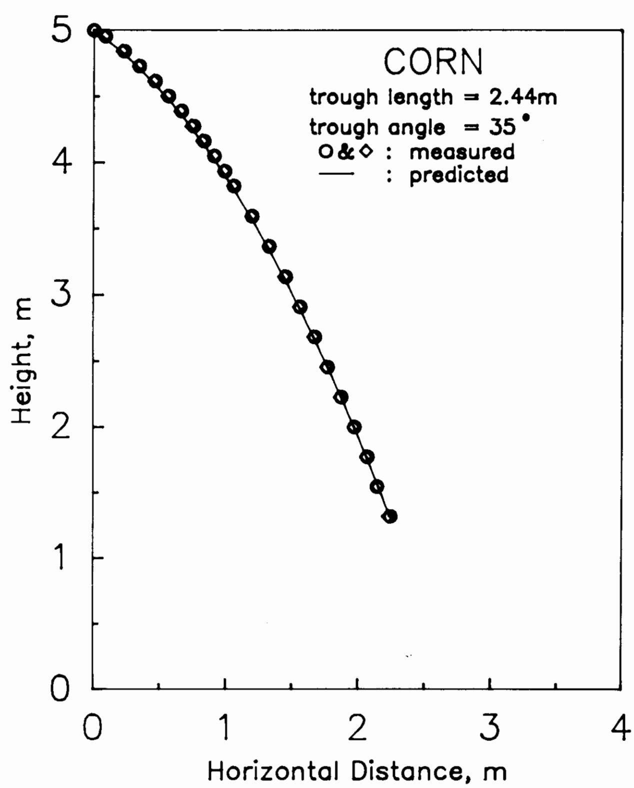

Figure 4–Measured and predicted trajectories of corn from a 2.44 m spreader trough at an incline angle of 35 degrees.

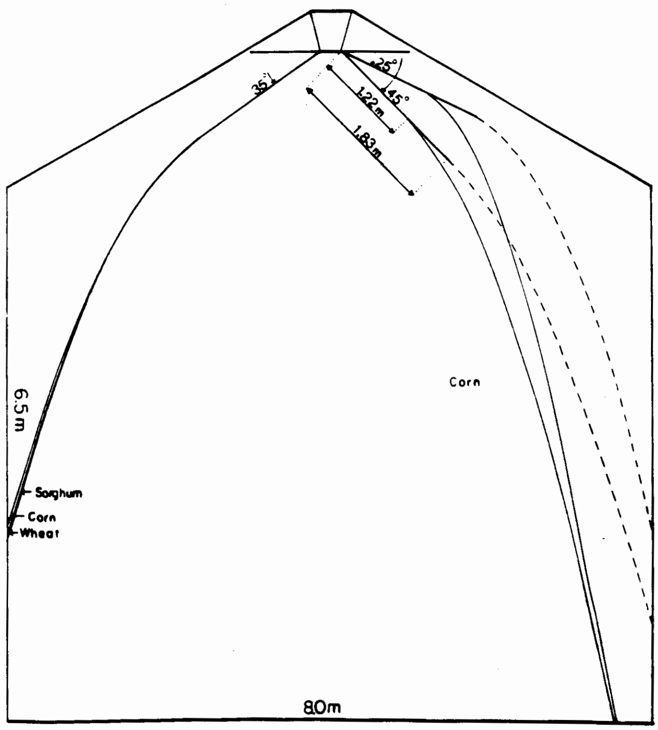

Figure 5–Schematic showing the predicted trajectories of grain from spreader troughs of different lengths and incline angles.

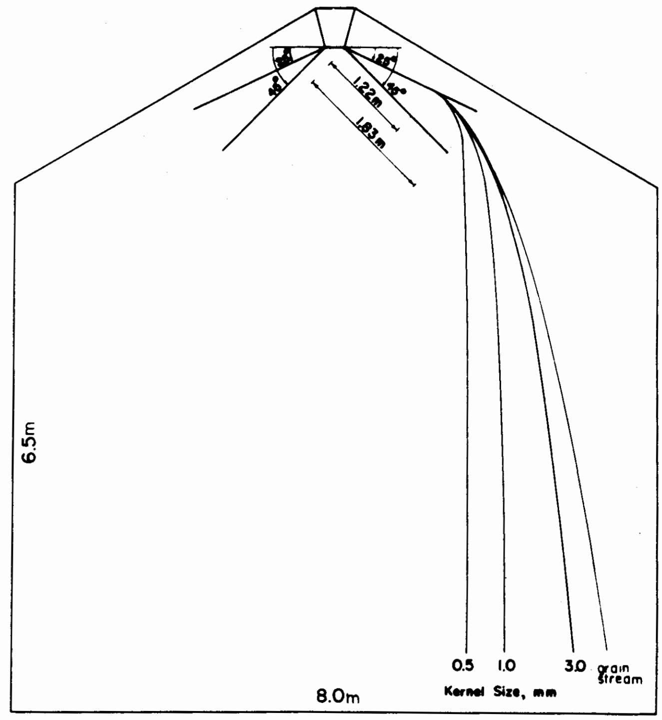

Figure 6–Schematic showing the predicted trajectories of a grain stream and corn particles of 3, 1, and 0.5 mm in equivalent diameter.

corn breakage increased rapidly when the velocities of grain impacting concrete or steel surfaces exceed 11 m/s.

The predicted trajectories of wheat, corn, and sorghum from a 1.83 m spreader trough at an incline angle of 35° are shown in the left of figure 5. The trajectories were similar for the three kinds of grain.

Figure 6 shows the predicted trajectories of a grain stream in relation to the particle sizes (3, 1, and 0.5 mm equivalent diameter) from a 1.22 m trough at an incline angle of 25°. The predicted trajectories were based on the assumption that the particles separated from the grain stream after leaving the trough and were subjected to a greater air drag force. As a result, the horizontal travel distance decreased as particle size decreased. Particles, 1 mm or smaller, fell almost straight down after leaving the trough. This is one of the causes of fine material segregation during grain bin loading and is especially relevant to broken corn vs. whole corn trajectories.

Summary and Conclusions

Velocities of wheat, corn, and grain sorghum in a spreader trough at different incline angles, lengths, and flow rates were measured with an optical velocity sensor. The measured velocities of grain were similar to the calculated theoretical sliding velocities based on the kinetic friction coefficient of 0.27 for corn and sorghum and 0.29 for wheat moving on metal surfaces. At the same trough incline angle, velocities of grain in the trough were similar for wheat, corn, and sorghum. Sliding velocity of grain in the trough was affected by the incline angle but was not affected by the flow rate. At the low flow rate and steep trough angle, corn kernels tended to roll rather than slide in the trough.

A model was developed to predict the trajectories of grain from spreader troughs of different length and incline angles. Air drag force, kernel size, and kernel weight were taken into account in the model. Air drag force on grain in a grain stream was about 10% of that on a single grain particle. The trajectories project the landing locations, angle, and velocity of grain in different-sized bins. By design or by controlled variations in lengths and incline angle of spreader troughs, more uniform distribution of grain and grain components, in different-sized bins can be obtained.

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