A MODIFIED EMPIRICAL DRAG COEFFICIENT FOR WATER DROP BALLISTICS

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ABSTRACT

Variation of the empirical drag coefficient, in Seginer's water drop ballistics equation as a function of drop size and velocity, increased the accuracy of predicted velocities when compared to measured values. The velocity exponent, n, used to calculate the drag coefficient in Seginer's equation increases not only with increasing drop size but also increases with increasing fall velocity. Using falling water drop terminal velocities from a meteorological model enables the calculation of water drop ballistics for any air density. The throw distance of drops from an impact sprinkler were simulated with the drag coefficient dependent on drop size and velocity, and show that simulated throw distances are greater for small drops than if the velocity exponent has a typically used constant value of two. **KEYWORDS.** Droplets, Velocity, Water, Simulation, Sprinkler, Aerodynamics.

INTRODUCTION

Falling water drop velocities can be measured experimentally or calculated numerically using finite difference solutions of theoretical differential equations. Velocities can be measured using photographic, electrostatic, and laser techniques. However, if velocities can be predicted accurately using numerical methods, time could be saved conducting water drop impact experiments.

A differential equation developed by Seginer (1965) has been used extensively for predicting the velocity of falling water drops. Wang and Pruppacher (1977) developed a computer model to predict falling water drop velocities. Velocities predicted by these two methods were used to prove that the velocity exponent n, used to calculate an empirical drag coefficient in Seginer's equation, increases not only with increasing drop size as Seginer has shown but also with increasing fall velocity. The objective of this article is to define n as a function of drop size and velocity, and to more accurately predict, using Seginer's equation, the subterminal velocity of freely falling water drops, as determined from comparisons to measured velocities.

DYNAMICS OF WATER DROP MOTION

A water drop traveling through air has two significant forces acting upon it, a gravitational force acting vertically downward and aerodynamic drag forces acting opposite the direction of motion. Drag forces increase with increasing velocity, and are directly related by a drag coefficient, C_D:

$$C_D = \frac{2F}{\rho V^2 A} \tag{1}$$

where
- C_D = drag coefficient (nondimensional),
- F = drag force (ML/T^2),
- ρ = air density (M/L^3),
- V = velocity (L/T), and
- A = cross-sectional area perpendicular to the direction of motion (L^2).

For rigid bodies, this drag coefficient is a function of the Reynolds number only. For water drops, the drag coefficient can be greater than that for a rigid sphere because of deformation (flattening) of the leading surface of the drop causing greater drag forces. Gunn and Kinzer (1949) showed that water drops larger than approximately 1 mm (0.04 in.) diameter falling vertically at terminal velocity have drag coefficients greater than that of rigid spheres.

EMPIRICAL DRAG COEFFICIENT

Seginer (1965) developed the following differential equation describing water drop ballistics using an empirical drag coefficient, C_n:

$$g - \frac{dV}{dt} = C_n V^n \tag{2}$$

where
- g = acceleration of gravity (L/T^2),
- dV/dt = resultant acceleration on the drop (L/T^2),
- C_n = empirical drag coefficient ($L^{1-n}T^{n-2}$),
- V = velocity (L/T),
- n = numerical exponent (nondimensional).

Equation 2 can be solved by finite difference numerical techniques to predict velocity and, subsequently, distance traveled for small time intervals. Seginer used data from Laws (1941) to determine the variation of both n and C_n with drop size. The n value increased from approximately 1.6 to 2.87 as drop diameter increased from 1.0 to 6.0 mm (0.04 to 0.24 in.). Seginer (1965) stated that a value of n = 2 should be suitable for predicting the velocity of raindrops and sprinkler drops.

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Falling Water Drop Velocity Model

Wang and Pruppacher (1977) developed a computer model to predict the velocity of freely falling water drops. The model uses equations developed by Beard (1976) that predict terminal velocity for any air temperature and atmospheric pressure and are a function of the Reynolds, Bond and Davies Numbers, and also a Physical Property Number defined as:

$$\text{Physical Property Number, } P = \frac{\sigma^3 \rho_a^2}{\mu^4 (\rho_w - \rho_a) g}$$

where

σ = surface tension of water (M/T^2),
ρ_a = density of the air (M/L^3),
μ = dynamic viscosity of air (M/LT),
ρ_w = density of water (M/L^3), and
g = acceleration of gravity (L/T^2).

Beard's equations are used to compute the Reynolds Number at terminal velocity as a function of the Davies Number for 0.019 to 1.07 mm (7.5E-4 to 0.04 in.) diameter drops, and the Bond and Physical Property Numbers for 1.07 to 7.0 mm (0.04 to 0.28 in.) drops. Terminal velocity is determined from the Reynolds number as follows:

$$Y = b_0 + b_1X + b_2X^2 + b_3X^3 + b_4X^4 + b_5X^5 + b_2X^2 \qquad (3)$$

0.019 to 1.07 mm (7.5E-4 to 0.04 in.)	1.07 to 7.0 mm (0.04 to 0.28 in.)
$= \ln(D)$	$X = \ln(BP^{1/6})$
$= -3.18657$	$b_0 = -5.00015$
$= 0.992696$	$b_1 = 5.23778$
$= -0.153196E\text{-}2$	$b_2 = -2.04914$
$= -0.987059E\text{-}3$	$b_3 = 0.475294$
$= -0.578878E\text{-}3$	$b_4 = -0.0542819$
$= 0.885176E\text{-}4$	$b_5 = 0.00238449$
$= -0.327815E\text{-}5$	$b_6 = 0$

(4)

$$R_T = \exp^Y \left[1 + \frac{2.52\, l_o \mu \rho_o}{d\, \mu_o \rho_a} (T/T_o)^{1/2} \right]$$

$$R_T = P^{1/6} e^Y \qquad (5)$$

nd subsequently, $V_T = \mu R_T / \rho_a d$ (6)

where

D = Davies Number (nondimensional),
B = Bond Number (nondimensional),
P = Physical Property Number (nondimensional),
R_T = Reynolds Number at terminal velocity (nondimensional),
l_o = 6.62E-8 m (2.6E-6 in.), the mean free path of air molecules at 20° C (68° F) and 101.3 kPa (14.7 psi),
ρ_o = 1.205 kg/m^3 (0.075 lbm/ft^3), the air density at 20° C (68° F) and 101.3 kPa (14.7 psi),
μ_o = 1.81E-5 Pa-sec (3.8E-7 lb-sec/ft^2), dynamic viscosity of air at 20° C (68° F) and 101.3 kPa (14.7 psi),
T = absolute air temperature,
T_o = 293.15° K (527.67° R), and
V_T = water drop terminal velocity (L/T).

Using these relationships, Wang and Pruppacher's model determines water drop velocities and fall distances for increasing incremental values of the Reynolds Number. A key assumption in their model is that the aerodynamic resistance on an accelerating drop is the same as the resistance on a smaller drop at terminal velocity with the same Reynolds number. They measured water drop velocities for 0.5 to 20 m (1.6 to 65.6 ft) fall heights at air conditions of 20° C and 100.0 kPa (68° F and 14.5 psi) and verified that the model was accurate except for 2 to 8 m (6.6 to 26 ft) fall heights where it may slightly (1%) underpredict velocity. Hinkle et al. (1987) compared the model to water drop velocities measured at 20° C, 84.1 kPa (68° F, 12.2 psi) and 0.5 to 5 m (1.6 to 16.4 ft) fall heights and also showed this slight underprediction for fall heights greater than 2 m (6.6 ft).

Analyses and Discussion

Comparison of Velocities Predicted Using the Empirical Coefficient with Measured Velocities

Velocities predicted by equation 2 for n = 2, n as a function of drop diameter as determined by Seginer, and n as a function of both drop diameter and velocity were compared to velocities measured by Laws (1941) near sea level and by Hinkle et al. (1987) at 1570 m (5150 ft) elevation. A fourth-order Runge-Kutta method was used to solve equation 2 numerically with finite time differences. The empirical coefficient, C_n, was determined using equation 2 and terminal velocities predicted by the Wang and Pruppacher model. At terminal velocity, dV/dt = 0 and equation 2 is solved for C_n as:

$$C_n = g / V_T^{\,n} \qquad (7)$$

The differences between velocities predicted using n = 2 and measured velocities by Laws (1941) and Hinkle et al. (1987) are shown in Tables 1 and 2, respectively. Laws' measured velocities over a range of air temperatures (20.7

TABLE 1. Differences between falling water drop velocities predicted by Seginer's (1965) differential equation (n=2) and velocities measured by Laws (1941) near sea level

Fall Height (m)	(ft)	Water drop velocity difference (%) — Drop diameter: 1 5 mm / 0 06 in.	2 0 / 0.08	3.0 / 0 12	4.0 / 0 16	5 0 / 0 20	6.0 / 0.24	
0.5	1.6	4.7	2.4	0.6	- 0.4	- 1 6	- 1.8	
1.0	3.3	4 0	3.6	0 7	- 1 0	- 2 1	- 2 9	
2.0	6.6	3 6	3 0	1 2	- 1 3	- 3 4	- 5.1	
3 0	9 8	1.2	1.9	1.5	- 1 3	- 4 0	- 6.2	
4 0	13.1	- 0 1	1 4	1 3	- 1.7	- 4 9	- 7.1	
5.0	16.4	- 1.0	0.8	0.7	- 2.1	- 5 0	- 7.1	
6 0	19.7	- 1 5	0 5	0.3	- 2 3	- 5.0	- 6.8	
8 0	26 2	- 1.6	- 1.1	- 0.4	- 2 6	- 5.0	- 6.3	
Terminal velocity from W-P model		5.44	6.54	8 09	8 87	9 14	9 19	m/s
		17.8	21.5	26.5	29 1	30 0	30.1	ft/sec

TABLE 2. Differences between falling water drop velocities predicted by Seginer's (1965) differential equation (n=2) and velocities measured by Hinkle et al. (1987) at 1570 m (5150 ft) elevation

Fall Height (m)	(ft)	Water drop velocity difference (%) — Drop diameter 2.5 mm / 0.10 in.	3.0 / 0.12	3.5 / 0.14	4.0 / 0.16	4.5 / 0.18	5.0 / 0.20	5.5 / 0.22
0.5	1.6	1.5	1.1	1.1	1.0	0.8	0.8	0.8
1.0	3.3	0.7	0.2	-0.1	-0.4	-0.4	-0.5	-0.5
2.0	6.6	0.8	0.6	0.0	-1.0	-1.9	-2.6	-2.8
3.0	9.8	0.3	0.1	-0.7	-1.7	-2.7	-3.4	-3.5
4.0	13.1	-0.5	0.1	-0.7	-1.9	-3.1	-4.1	-4.7
5.0	16.4	-1.0	-0.4	-1.0	-2.3	-3.5	-4.8	-5.7
Terminal velocity from W-P model	m/s	7.98	8.72	9.25	9.60	9.82	9.94	9.99
	ft/sec	26.2	28.6	30.3	31.5	32.2	32.6	32.8

to 25.0° C [69.3 to 77.0° F]) and atmospheric pressures [99.2 to 101.2 kPa [14.39 to 14.68 psi]). Laws drew smooth curves through these data and interpolated the curves to define velocities for specific drop sizes and fall heights. Since these data represent the average conditions of his experimental conditions, average values of 22.8° C and 100.5 kPa (73 F and 14.58 psi) were used in Wang and Pruppacher's (1977) model to determine terminal velocities. Hinkle et al's (1987) data are for 20° C and 84.1 kPa (68°F and 12.2 psi). Predicted velocities using n = 2 tended to be greater for small drops and low fall heights, and lesser for large drops and greater fall heights. For very great fall heights, the velocity differences should approach zero, since the finite difference solution approaches terminal velocity.

If n is allowed to vary with drop diameter as suggested by Seginer (1965), the differences between computed and measured velocities are less for larger drops but greater for the smaller drops than when n = 2. The differences between velocities predicted by equation 2 with n dependent on drop diameter, n = f(d) from Seginer (1965) and those measured by Hinkle et al (1987), are shown in Table 3. Velocities predicted by equation 2 and n = f(d) cannot be verified with Laws' (1941) data because those data were used to determine n = f(d).

Comparison of Velocities Predicted Using the Empirical Coefficients to Velocities Predicted by Wang and Pruppacher's Model

Velocities were calculated using Seginer's (1965) equation with n = 2 and Wang and Pruppacher's (1977) model, all at 20° C and 101.3 kPa (68° F and 14.7 psi). The velocity differences are illustrated in figure 1 and are similar to those in Tables 1 and 2. If n varies with drop diameter as defined by Seginer, the velocity differences are similar to those in figure 1 but ranging ±2% instead of ±5%. An n value <2 for the smaller drops and >2 for the larger drops reduces the velocity differences. These observations further establish that n should increase with increasing drop diameter just as Seginer (1965) had shown.

TABLE 3. Differences between falling water drop velocities predicted by Seginer's (1965) differential equation with n = f (diameter) and velocities measured by Hinkle et al. (1987) at 1570 m (5150 ft) elevation

Fall Height (m)	(ft)	Water drop velocity difference (%) — Drop diameter 2.5 mm / 0.10 in	3.0 / 0.12	3.5 / 0.14	4.0 / 0.16	4.5 / 0.18	5.0 / 0.20	5.5 / 0.22
0.5	1.6	-0.1	0.1	0.7	1.2	1.4	2.0	2.0
1.0	3.3	-1.5	-1.2	-0.6	-0.1	0.6	1.2	1.9
2.0	6.6	-1.9	-1.1	-0.8	-0.7	-0.5	-0.1	0.7
3.0	9.8	-2.4	-1.7	-1.5	-1.3	-1.1	-0.5	0.5
4.0	13.1	-3.0	-1.8	-1.5	-1.5	-1.4	-1.1	-0.4
5.0	16.4	-3.2	-2.1	-1.8	-1.9	-1.9	-1.9	-1.6
n value		1.70	1.79	1.90	2.05	2.22	2.42	2.66

The velocity differences in figure 1 suggest that n values should increase with increasing fall height (velocity). The velocity differences for 3 to 5 mm (0.12 to 0.20 in.) drops are positive initially (velocity predicted by Seginer's equation are greater than velocities predicted by Wang and Pruppacher's model), they then become negative with increasing fall velocity. Decreasing the n value causes the velocity differences to be less positive and increasing the n value causes the velocity differences to be less negative. The velocity differences can be further reduced if n is less than n = f(d) initially and increases with increasing fall velocity.

The variation of n with fall velocity is apparent in Seginer's (1965) linear regression of Laws' (1941) data from which was determined n-f(d). However, these data are not linear for the lower velocities. If smooth curves are drawn through Laws' data (Seginer, 1965), the slope (n value) increases as velocity increases for the lower fall velocities. Unfortunately, Laws (1941) would have had to measure velocity over much smaller fall height intervals at the lower velocities to quantify n as a function of velocity.

Determination of n as a Function of Drop Diameter and Velocity

The value of n was determined as a function of drop diameter and fall velocity by iterating the value of n within the finite difference solution of equation 2 to predict velocities that matched those predicted by Wang and Pruppacher's (1977) model, all at 20° C and 101.3 kPa (68° F and 14.7 psi). The n values computed from this iteration are shown in figure 2 for 1, 2, 3, 4, 5, and 6 mm (0.04, 0.08, 0.12, 0.16, 0.2, 0.24 in.) diameter drops. The n values are plotted against velocity divided by terminal velocity, v/v_T so that equations to predict n will be independent of

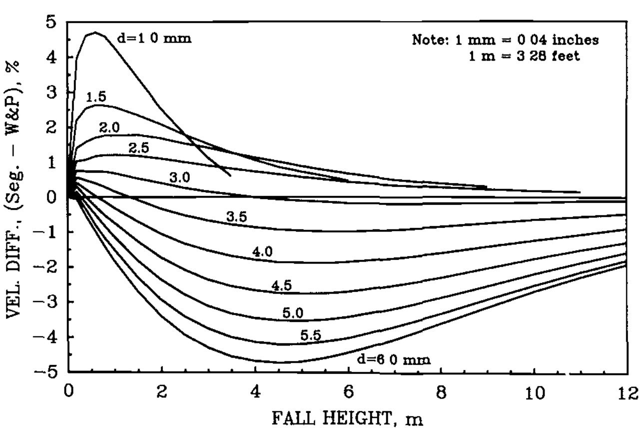

Figure 1–The differences between velocities predicted by Seginer's differential equation with n=2 and velocities predicted by Wang and Pruppacher's computer model.

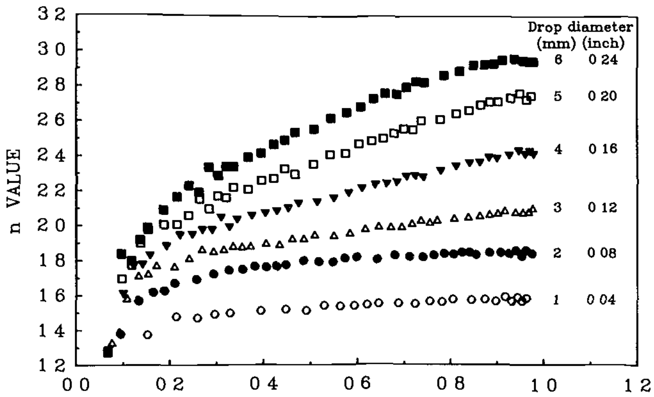

Figure 2–The variation of n with fall velocity (as determined from iteration of n in Seginer's differential equation to match Wang and Pruppacher's computer model velocities).

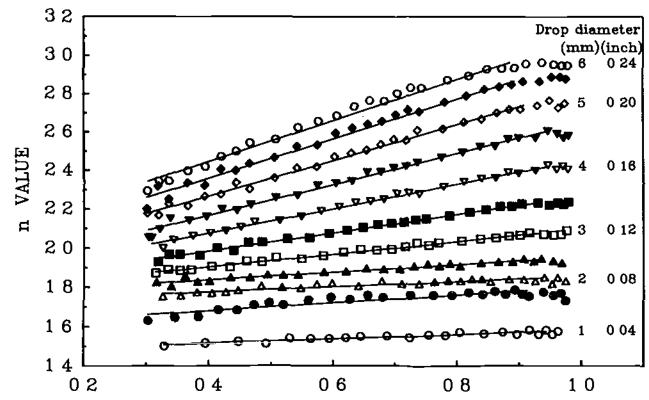

Figure 3–Linear regressions of n as a function of fall velocity for different drop diameters.

atmospheric conditions. Air temperature and atmospheric pressure are needed only to calculate B, D, P and subsequently, terminal velocity. Seginer (1965) also proposed using v/v_T to yield nondimensional velocities.

The value of n is nearly linear with v/v_T for $0.3 \leq v/v_T \leq 0.98$, which represents most of a water drop's initial fall distance. One to 6 mm (0.04 to 0.24 in.) diameter water drops attain three-tenths of their terminal velocity before falling 0.4 m (1.3 ft), and 1, 1.5, and 2 mm (0.04, 0.06, and 0.08 in.) drops reach 98% of terminal velocity after 3.5, 5.5, and 8 m (11.5, 18, and 26 ft) of fall, respectively. Water drops of 2.5 to 6 mm (0.10 to 0.24 in.) diameter reach 98% of terminal velocity after 10 to 12 m (33 to 39 ft) of fall.

Numerous curve-fitting attempts were applied to the n value and velocity data. The resulting $n = f(d, v/v_T)$ functions were used in the finite difference solution of equation 2 to predict falling water drop velocities which were compared to Laws' (1941) and Hinkle's et al. (1987) measured velocities. A linear relationship between n and v/v_T had correlation coefficients equal to or greater than polynomial, exponential or logarithmic regressions. The slope, intercept and correlation coefficients for the linear regressions of n to v/v_T for $0.3 \leq v/v_T \leq 0.98$ are shown in Table 4. The linear equations are plotted in figure 3.

The differences between velocities predicted using these linear equations with equation 2, and the measured velocities by Laws (1941) and Hinkle et al. (1987) are shown in Tables 5 and 6, respectively. The empirical drag coefficient, C_n, was calculated by equation 7 for each step of the finite difference solution of equation 2. As in previous comparisons, the velocity differences were less than if n = 2. The differences also were generally less than those computed with n as a function only of drop diameter. The differences between velocities predicted with equation 2 using $n = f(d, v/v_T)$ and Hinkle et al.'s (1987) velocity data were overall within 1.7%.

Most of the velocity differences in Table 6 were negative and occurred at the intermediate (>2m, >6.5 ft) fall distances. Since Wang and Pruppacher's (1977) model slightly underpredicts in this range of fall distances and the $n = f(d, v/v_T)$ relationships were determined using their model, the solution of equation 2 with the $n = f(d, v/v_T)$ equations will underpredict velocities for this intermediate range of fall heights, also.

TABLE 4. Linear regression results for n value as a function of fall velocity divided by terminal velocity*

n = Intercept + Slope (velocity/ terminal velocity)

Drop diameter (mm)	(in.)	Intercept	Slope	Correlation coefficient (r)
1.0	0.04	1.467	0.118	0.944
1.5	0.06	1.596	0.209	0.922
2.0	0.08	1.718	0.142	0.946
2.5	0.10	1.756	0.203	0.976
3.0	0.12	1.772	0.330	0.995
3.5	0.14	1.797	0.472	0.995
4.0	0.16	1.825	0.631	0.996
4.5	0.18	1.850	0.796	0.997
5.0	0.20	1.905	0.917	0.996
5.5	0.22	1.954	1.016	0.993
6.0	0.24	2.024	1.058	0.992

* As determined by iterating n in Seginer's equation to predicted velocities to match those from Wang and Pruppacher's model.

Simulated Sprinkler Water Drop Ballistics with N as Function of Drop Diameter and Velocity

The values of n and C_n in Seginer's (1965) equation have a significant effect on simulated throw distances of sprinkler drops. The ballistics of water drops from an agricultural impact sprinkler were simulated by finite difference solution of equation 2 separated into vertical and

TABLE 5. Differences between falling water drop velocities predicted by Seginer's (1965) differential equation with $n = f(d, v/v_T)$ and velocities measured by Laws (1941) near sea level

Fall Height (m)	(ft)	Water drop velocity difference (%) – Drop diameter 1 5 mm / 0 06 in	2 0 / 0 08	3 0 / 0 12	4 0 / 0 16	5 0 / 0.20	6 0 / 0.24
0.5	1 6	2 1	0 9	- 0 1	- 0 5	- 1.1	-0 9
1.0	3.3	1.3	1.7	0.0	- 0 7	- 0 9	- 1 1
2 0	6 6	1 3	1 3	0.7	- 0 3	- 1 1	- 1.9
3 0	9 8	- 0 3	0 4	1 3	0.2	- 1 0	- 2.2
4 0	13 1	- 1 2	0 3	1.2	0 0	- 1 6	- 2.6
5 0	16 4	- 1 6	0.0	0.8	- 0 3	- 1.6	- 2.6
6 0	19.7	- 2.0	- 0 2	0 4	- 0 5	- 1.6	- 2 5
8.0	26.2	- 1 8	--1 5	- 0.3	- 1 0	- 2 2	- 2 7

TABLE 6. Differences between falling water drop velocities predicted by Seginer's (1965) differential equation with n = f (d, v/v_T) and velocities measured by Hinkle et al. (1987) at 1570 m (5150 ft) elevation

		Water drop velocity difference (%)							
		Drop diameter							
Fall Height (m)	(ft)	2 5 0.10	3 0 0.12	3 5 0.14	4.0 0.16	4.5 0 18	5 0 0.20	5.5 0 22	mm in.
0 5	1.6	0.5	0 5	0 8	0 9	0 9	1.3	1.1	
1.0	3.3	- 0 5	- 0.5	- 0.3	-0.2	0 2	0.4	0.8	
2.0	6.6	- 0 4	0.1	0.1	-0 2	- 0 6	- 0 6	-0.4	
3.0	9 8	- 0.8	- 0.2	- 0 2	-0.4	- 0 7	- 0 7	-0 2	
4 0	13.1	- 1.4	- 0.1	0 0	-0 3	- 0.7	- 0.9	-0.9	
5.0	16.4	- 1 6	- 0 4	- 0.2	-0 5	- 0 9	- 1 4	- 1.7	

horizontal components. The acceleration on a drop is determined from:

$$\frac{dV_y}{dt} = g \pm C_n V_r^{\,n-1} V_y \tag{8}$$

$$\frac{dV_x}{dt} = C_n V_r^{\,n-1} V_x \tag{9}$$

where

V_r = resultant water drop velocity (L/T),
t = time (T),
V_y = vertical component of water drop velocity (L/T),
V_x = horizontal component of water drop velocity (L/T).

Equations 8 and 9 are similar to equations given by von Bernuth and Gilley (1984). A fourth-order Runge-Kutta technique was used to solve equations 8 and 9. Equations 3, 4, 5, and 6 were used to determine terminal velocity for any air density. Equations 7 and n = f(d,v/v_T) as given in Table 4 were used to calculate n and C_n, respectively.

The exit velocity of water from an impact sprinkler is typically far greater than free falling terminal velocities. The value of n is not defined by n = f(d,v/v_T) as shown in figure 3 for velocities greater than terminal. The value of n may not continue to increase for velocities greater than the terminal velocity. Therefore, the relative differences in horizontal travel distance of sprinkler drops were simulated for n = 2 and for n = f(d,v/v_T) but with n limited to that which would be relevant at terminal velocity. Simulated results for a typical impact sprinkler mounted on a center pivot are shown in Table 7. The initial conditions are:

- The sprinkler is mounted 4 m (13 ft) above the impact surface.
- The exit angle above horizontal of the water from the sprinkler is 23°.
- The exit velocity of the water from the sprinkler is 25 m/s (82 ft/s), typical of a sprinkler with a nozzle pressure of 330 kPa (48 psi).

Simulated throw distances with n = f(d,v/v_T) and n limited to n at terminal velocity, are greater for the smaller drops and less for the larger drops than if n = 2. Velocities of the drops range from 25 m/s (82 ft/sec) to approximately 80% of terminal velocity. The small drops have n values <2 which increases throw distances. The large drops have n values >2 which decreases throw distances. Greater n values simulate greater air resistance on the drop. However, simulated throw distances for the 5 and 6 mm (0.20 to 0.24 in.) drops are less than the throw distance for the 4 mm (0.16 in.) drop. Maximum n values are too great for the larger drops. In figure 3, the computed n values are less than the regression lines near terminal velocity and appear to be asymptotic to some finite value suggesting a maximum value less than that calculated by the linear equations of Table 4.

The larger drops are the last drops to form from the water stream exiting an impact sprinkler. This fact increases the throw radii of the larger drops, a condition not included in the ballistics of drops using only Seginer's (1965) equation. Additional techniques would be needed to simulate the break up of drops from the water jet and of the air stream flowing next to the water jet. Water emitted from spray nozzles with impingement plates typically forms drops within a short distance of the plate and with velocities much less than that of impact sprinklers. Consequently, this improved ballistics technique will work better for spray nozzles than for impact sprinklers.

Conclusions

The velocities of water drops falling freely and vertically can be calculated more accurately with Seginer's (1965) differential equation by using linear functions of n, which are dependent upon fall velocity and drop diameter, than by using a typically used constant value of 2 for n. The differences between velocities predicted with Seginer's

TABLE 7. Simulated horizontal throw distances and impact velocities of water drops from an impact sprinkler using Seginer's differential equation with n = 2 and n calculated with the linear n = f (v, v/v_T) equations

		n = 2				n = f (v/ v_T), n ≤ f (v_T)					
Drop diameter		Throw distance		Impact velocity		Throw distance		Impact velocity		Min. n	Max. n
(mm)	(in.)	(m)	(ft)	(m/s)	(ft/s)	(m)	(ft)	(m/s)	(ft/s)		
1	0.04	4.54	14.9	4.00	13.1	5.81	19.1	3.99	13.1	1.56	1.58
2	0.08	9.40	30.8	6.22	20.4	9.87	32.4	6.20	20.3	1.83	1.86
3	0.12	12.50	41.0	7.38	24.2	12.15	39.9	7.38	24.2	2.03	2.10
4	0.16	14.02	46.0	7.91	25.9	12.70	41.7	8.00	26.2	2.31	2.46
5	0.20	14.55	47.7	8.09	26.5	12.45	40.8	8.27	27.1	2.60	2.82
6	0.24	14.63	48.0	8.12	26.6	12.07	39.6	8.35	27.4	2.83	3.08

* The sprinkler height is 4 m (13.3 ft), nozzle angle is 23°, and the exit velocity of the water is 25 m/s (82 ft/sec).

(1965) equation and velocities measured at sea level and at 1570 m (5150 ft) elevation were less than if n were assumed a constant (2) or a function of drop diameter only. Using $n = f(d,v/v_T)$ should also predict more accurately the horizontal throw distances of water drop from spray nozzles and the smaller drops from impact sprinklers for any air temperature and pressure.

The results of this study have the following experimental and practical significance.

1. The value of the empirical drag coefficient is a function of water drop size and velocity, just as the physically-based drag coefficient used for rigid bodies (eq. 1) is a function of the Reynolds number and, consequently, a function of drop size and velocity.
2. Wang and Pruppacher's (1977) model can be used to directly calculate raindrop velocities for any air temperature and pressure.
3. Because the linear equations used to calculate n are a function of velocity nondimensionalized by terminal velocity, the ballistics of sprinkler drops can be predicted with Seginer's (1965) equation for any air temperature and pressure because terminal velocities can be determined with Wang and Pruppacher's (1977) model. Subsequent differences in sprinkler distribution can then be determined.
4. Potential differences in runoff for different air pressures (i.e., elevations) and temperatures can be determined with this modified ballistics procedure and an infiltration model.

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