Falling Water Drop Velocitie

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ABSTRACT

THE velocities of freely falling water drops released from various heights in still air were measured using two different techniques at an elevation of 1570 m (5150 ft) above sea level. An electrostatic technique was used to measure the time for an electrically charged water drop to pass through two metal rings of known spacing. A photographic technique, using two electronic flash units, was used to photograph a falling drop against a grid background, at the beginning and ending of a known time interval.

The measured results were compared to velocity data measured at sea level and to results from a computer model which predicts freely falling water drop velocities as a function of drop size, air pressure, and air temperature. The high elevation (low air presure and density) environment resulted in measured velocities significantly greater than those measured at sea level. The computer model predicted velocities close to the measured velocities. The computer model was also used to predict velocity differences at various elevations. Finally, the computer model was used with a finite difference computer program of the ballistics of water drops from sprinklers to show the effect of elevation on the impact velocity and radius of throw of water drops from irrigation sprinklers.

INTRODUCTION

The velocity of falling water drops has been measured by a number of researcher dating back to the early 1900's. Unfortunately, the early researchers had limited electronic instrumentation compared to today's technology and their results were of questionable accuracy. The most notable experiments in which falling water drop velocities were measured in still air were conducted by Laws (1941), Gunn and Kinzer (1949), and by Wang and Pruppacher (1977), all at elevations near sea level.

Water drop velocity measurements made by Laws (1941) were by far the most accurate up to that time. Laws used a stroboscopic photographic technique to capture multiple images of falling drops on film. Velocity was determined from the distance between images and the strobe time interval. Measurements were made for 1 to 6 mm diameter drops which were dropped from 0.5 to 20 mm fall heights.

Gunn and Kinzer (1949) measured the terminal velocity of falling water drops with diameters ranging from 0.1 to 5.8 mm. An electrically charged falling water drop passed through two metal rings of known spacing, inducing a charge on the rings which was amplified and traced on a strip chart recorder. Their measured terminal velocities are almost equal to those of Laws for the intermediate size drops but are up to 1.5% less for the smallest and largest drops. Laws' terminal velocities are generally greater due to lower air density resulting from 2.8°C warmer air temperatures during his tests compared to the conditions of Gunn and Kinzer.

Wang and Pruppacher (1977) measured the velocity of water drops for five diameters ranging from 1.67 to 6.70 mm using a technique almost identical to Gunn and Kinzer's. They measured the terminal velocity of falling water drops and the time and distance needed to reach 99% of terminal velocity. They wrote and verified a finite difference computer model based on the theory of Beard (1976) which predicts the acceleration, velocity, time and distance traveled of a falling water drop. Evaporation of the drops during their fall was not reported for any of the three experiments but relative humidity was measured and reported.

Wang and Pruppacher predicted velocities for other temperatures and air pressures which relied on the findings of Pruppacher and Pitter (1971). Pruppacher and Pitter measured the deformation of large water drops due to air resistance. They found that a particular size drop had the same deformation characteristics at 0.5 atm as it did at 1 atm or air pressure. This was true for drop sizes ranging from 1 to 7 mm diameter.

The objective of this experiment was to measure the velocities of falling water drops at an elevation of 1570 m (5150 ft) above sea level, where the air pressure is approximately 5/6 of an atomosphere. Measurements were made for drops with diameters ranging from 2.45 to 5.64 mm and for fall heights of 0.5 to 5 m. Two methods were used to measure water drop velocity. These velocity results were compared to the results of Laws (1941) for the larger drops from the lower fall heights where air resistance effects are small. The present data were also compared to computer model results for the air temperature and pressure of the present experiments at 1570 m (5150 ft) elevation.

METHODS AND MATERIALS

The experiments were conducted at the Agricultural Engineering Research Center at Colorado State University. A drop chamber was built using a 0.3 m (12 in.) diameter plastic pipe erected inside the laboratory building which has a ceiling height of 6.1 m (20 ft). A movable slider which held the dripper apparatus was

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attached to a pulley system for positioning at various heights.

The water drop velocity and drop size experiments used a microcomputer interfaced with an analog to a digital converter (ADC). The system consisted of an Apple IIe computer* and a Cyborg Isaac 91A ADC. The Isaac 91A has analog input and analog output features which were used for sensing, triggering and measurement operations. In addition, a software package, Appligration II, was used for the double ring velocity measurements because of its high speed sampling (up to 9.1 kHz), graphing, and data analysis capabilities.

Double ring velocity measurements

A double ring electronic-electrostatic technique was one method used to measure water drop velocity. It consisted of a device to charge the water drop, a metal ring receiver-amplifier, and the microcomputer-ADC. A charged metal ring around the drip tubing electrostatically induced a charge on the drop which was carried with the drop after falling from the tubing. The metal ring receiver-amplifier was used to sense the passage of the charged water drop through the two metal rings. The charged water drop induced a charge on the rings which was recorded using the computer-ADC. Both the drop charger and the receiver-amplifer were modeled after the devices used by Gunn and Kinzer (1949), with the improvement of using integrated circuits (Heath, 1983) instead of vacuum tube electronics. The dripper and receiver devices are illustrated in Fig. 1.

The amplifier circuit consisted of two LF351 linear operational amplifier (op map) integrated circuits arranged in two stages to obtain a total amplification of 1000. The LF351 linear op amps have a high input impedance which was needed because a high resistance was connected between the amplifier input and electrical ground. The op amps require ± 5 to 15 volts DC nominal voltage, therefore two nine volt batteries adequately powered the circuit. An electrical diagram of the amplifier circuit is also shown in Fig. 1.

The velocity results were determined from the receiver-amplifer output for a drop passing through the two rings spaced either 0.2 m or 0.5 m apart. The computer software displayed the output as two peaks on a scrolling graph from which the time interval was found using features of the software. Fall height was the distance from the end of the drip tubing to a point halfway between the two rings. Velocity was determined by dividing the ring spacing by the time interval.

Photographic velocity measurement

An electronic-photographic technique was also used to measure water drop velocity. The water drop was photographed against a screen with a grid pattern using two electronic flash units and a 35 mm camera. The approach of the water drops was sensed using a single ring amplifier chamber similar to the two ring setup used in the electrostatic velocity measurement. The signal of the falling drop activated two miniature relays which in turn triggered the flash units. The electronic flash units

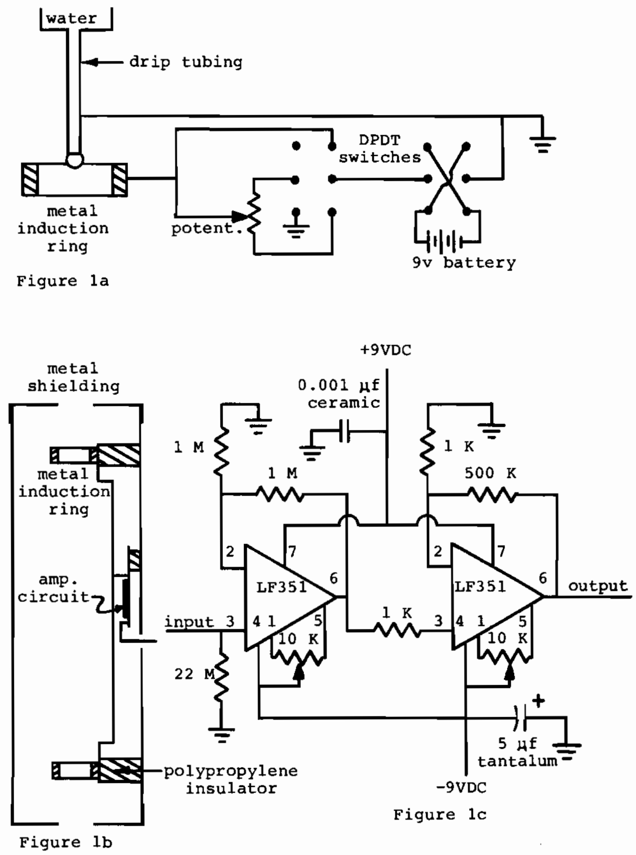

Fig. 1—(a) Water drop charging system with reversible and variable voltage features. (b) Double metal ring receiver-amplifier chamber for sensing the passage of a charged water drop. (c) Amplifier circuit used in the metal ring water drop sensing chamber.

had thyristor circuitry with a flash duration less than 1/2000 of a second, sufficient to stop the images. The pictures were taken in darkness, with the shutter opened manually until the double flash occurred.

The velocity of the drop for this method was determined by dividing the observed distance between drop images on the developed print by the known time interval between flashes with correction for parallax error. The time delay and flash interval were controlled by a computer program and were adjusted to get the appropriate flash spacing and position relative to the fall height distance. The time intervals were determined by observing the output of an 100 MHz oscilloscope.

The correction for parallax error was needed to account for the distance between the two drop images and the screen. The photographic setup and the parallax are illustrated in Fig. 2. The parallax error was computed using the following equation, which was derived geometrically using similar triangles.

$$\text{Percent error} = \frac{d(s-f)}{sl - d(s-f)} \cdot 100 \quad \ldots\ldots\ldots\ldots\ [1]$$

where
d = distance from drop to grid, L
l = distance from camera and flash to grid, L
s = distance between the two drop images, L
f = distance between flash units, L

Drop size determination

Water drops of various sizes were produced using

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*Note: The mention of trades names or commerical products does not constitute their endorsement or recommendation for used by the USDA-ARS or Colorado State University.

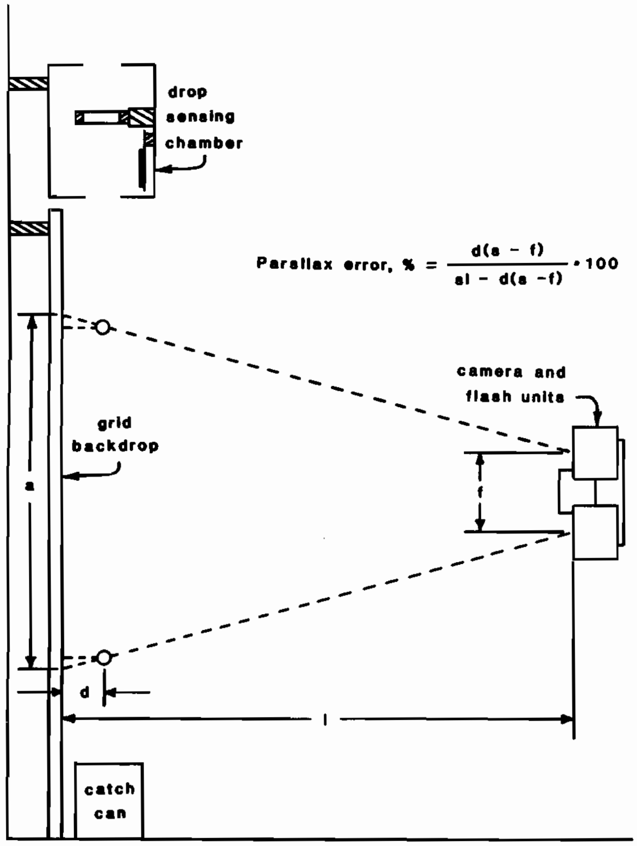

Fig. 2—Photographic water drop velocity measurement setup showing the drop sensing chamber, grid screen backdrop, camera, double flash units and the dimensions used to calculate the parallax error.

stainless steel syringe needles (14 to 27 gauges) for the smaller drops and five sizes of brass telescope tubing for the larger drops. The needles and tubing were cut at right angles to produce drops with virtually no spin when released.

Drop size was determined be collecting a fixed number of drops in a flask and weighing the flask before and after the test. The drops were counted using the computer-ADC. Evaporation was minimized by keeping the drop distance small and by placing a plastic mesh inside the flask to minimize splash. In addition, a control flask containing water and plastic mesh was also weighed before and after each test to correct for any evaporation from the flask.

Evaporation from the falling water drops was estimated by the same technique used to determine drop size but with the dripper apparatus set at various heights above the drop counter and flask. The difference in water collected among the different heights was assumed to be evaporation loss or loss due to the formation of very small secondary droplets which fell outside the collection area. These tests were conducted separately from the velocity measurements and at relative humidities near the low end of the range of relative humidities measured during the velocity measurements.

To obtain a consistent drop size, the drip rate was always set at approximately one drop per second. Threadgill et al., (1974) have observed that the drop rate can affect drop size. This relatively low drip rate also helped to keep the air essentially stagnant inside the chamber.

RESULTS AND DISCUSSION

Falling water drop velocities were measured using the double ring electronic-electrostatic technique for eleven different drop sizes from ten different fall heights. Five measurements were made for each particular drop size and fall height to average out any variations and reduce experimental error.

Falling water drop velocities were also measured using the photographic technique for five drop sizes and five fall heights. Three photographs were taken of each particular drop size and fall height to average out any variation. Fewer replications and drop-size fall-height settings were made with this method because of greater time and cost requirements.

Distilled water was used in these experiments and was allowed to equal the air temperature before use. Air temperatures ranged from 19 to 27°C for these velocity experiments. Water density differences for this temperature range have a negligible effect on falling water drop velocity.

The measurements were made over a number of days, and air temperature and atmospheric pressure were constantly changing. The calculated velocities were converted to a standard temperature of 20°C (68°F) and a standard pressure of 84.1 kPa (631 mm Hg, 24.84 in Hg) using an equation developed by Gunn and Kinzer (1949). This equation predicts a fractional change in velocity which is equal to the fractional change in air density times a coefficient that is dependent upon the Reynolds number (R) and the drag coefficient (C_d). This equation was written as,

$$\frac{\Delta V}{V} = \frac{-\Delta\rho}{\rho}\left[1 - \frac{\Delta(\log R)}{\Delta(\log R^2 C_d)}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots [2]$$

where,

V = velocity of the water drop, L/t
ρ = density of the air, m/L^3
R = Reynolds number = $\rho Vd/\mu$, dimensionless
C_d = drag coefficient = $2F_d/\rho V^2 A_p$, dimensionless
d = drop diameter, L
μ = dynamic viscosity of the air, m/Lt
F_d = drag force, mL/t^2
A_p = projected area of the drop, L^2

Equation [2] was developed from gravitational force and air drag equations and a terminal velocity equation. It was intended for correcting terminal velocity to a standard temperature and pressure assuming no drop deformation due to aerodynamic drag (i.e., the drops are spherical). Justification of its use for drops at velocities other than terminal involves using an analysis by Wang and Pruppacher (1977). They state that the C_d of an accelerating drop can be found by evaluating the C_d curve for drops at terminal velocity using the Reynolds number of the accelerating drop. This assumption is a key factor in their finite difference velocity program, and is verified by the close agreement between their program and experimental results.

Instead of using the C_d curve for rigid spheres, the C_d curve for water drops deformed by aerodynamic drag at terminal velocity, as determined by Gunn and Kinzer (1949), was used for C_d for the accelerating water drop in equation [2]. Wang and Pruppacher's (1977) velocity

TABLE 1. FALLING WATER DROP VELOCITIES MEASURED BY THE DOUBLE RING ELECTROSTATIC METHOD AND CONVERTED TO A STANDARD CONDITION OF 20 °C AND 84.1 kPa

Fall height, m	Falling water drop velocities, m/s; drop diameter, mm 2.45	2.67	2.91	3.14	3.45	3.83	4.07	4.52	4.82	5.21	5.64
5	7.11	7.25	7.40	7.53	7.70	7.88	7.98	8.14	8.23	8.33	8.41
4.5	6.92	7.06	7.20	7.32	7.47	7.63	7.72	7.87	7.95	8.03	8.10
4	6.73	6.84	6.95	7.06	7.20	7.35	7.43	7.57	7.64	7.70	7.74
3.5	6.47	6.57	6.68	6.78	6.90	7.03	7.10	7.22	7.27	7.33	7.35
3	6.16	6.26	6.36	6.46	6.55	6.67	6.73	6.82	6.87	6.90	6.91
2.5	5.80	5.86	5.94	6.01	6.10	6.20	6.26	6.35	6.40	6.44	6.45
2	5.36	5.41	5.47	5.53	5.60	5.68	5.72	5.79	5.82	5.85	5.85
1.5	4.81	4.86	4.91	4.95	5.00	5.05	5.07	5.11	5.12	5.13	5.14
1	4.07	4.11	4.14	4.16	4.19	4.21	4.22	4.23	4.23	4.24	4.24
0.5	2.97	2.98	2.99	3.00	3.01	3.02	3.03	3.03	3.03	3.04	3.04

program uses Gunn and Kinzer's data and they show that their program results agree well with their experimental results, so the C_d curve for water drops should be inherently more accurate for use in equation [2]. Use of Gunn and Kinzer's terminal velocity C_d curve, that was developed for results measured at sea level, can be justified for lower air pressures from the findings of Pruppacher and Pitter (1971). These authors showed that falling water drop deformation characteristics change insignificantly for air pressures between 0.5 and 1.0 atmosphere of air pressure. The velocity corrections for the present results were one percent or less.

Relative humidity of the air was monitored during these tests and ranged from 10 to 50%. Evaporation tests conducted at 20 to 25% relative humidity show that the change in drop size for even the most liberal estimate for evaporation of 1½% has only a small effect on the velocity results of the present experiment. The smaller water drops have more potential for greater relative evaporation losses due to their higher surface to volume ratio. However, the larger drops had similar losses, which included losses due to evaporation and losses due to the formation of tiny secondary droplets which fell outside the colleciton area therefore representing an additional water loss. This was observed as a slight wetness on the top shield of the drop sensing chamber. Using the value of 1½% for total water loss, the greatest possible change in velocity would occur for a 2.45 mm drop from a height of 5 m and the change would be less than 0.01 m/s.

Water drop velocities for the standard conditions of 20°C and 84.1 kPa of the present experiments compared quite closely for the two different measurement methods and are tabulated in Tables 1 and 2. Velocities measured by the double ring electronic method are shown in Fig. 3

TABLE 2. FALLING WATER DROP VELOCITIES MEASURED BY THE PHOTOGRAPHIC METHOD AND CONVERTED TO A STANDARD CONDITION OF 20 °C AND 84.1 kPa

Fall height, m	Falling water drop velocities, m/s; drop diameter, mm 2.67	3.14	3.83	4.52	5.64
5	7.24	7.64	7.98	8.13	8.43
4	6.74	7.07	7.31	7.44	7.72
3	6.22	6.48	6.64	6.85	6.91
2	5.35	5.56	5.66	5.78	5.83
1	4.04	4.14	4.17	4.22	4.28

for 11 different drop sizes and ten different fall heights.

The present data were compared to Laws' velocity results for the larger drops from the lower fall heights, where air resistance forces are small and any differences in air resistance would be negligible. For these conditions, the present velocity results are up to 1.9% smaller than Laws' results. Comparing the smaller drops and higher fall heights shows that present results are much greater than the velocities measured by Laws, due to a lower air pressure and density at the higher elevation. For the smallest drop (2.45 mm) and greatest fall height (5 m), the difference is 7.4%.

Computer model results

Wang and Pruppacher's (1977) computer model was used to calculate the velocity of falling water drops for the standard atmospheric conditions at sea level and for the low pressure atmospheric conditions of the present experiments. Their model uses equations developed by Beard (1976) which predict terminal velocity at any air

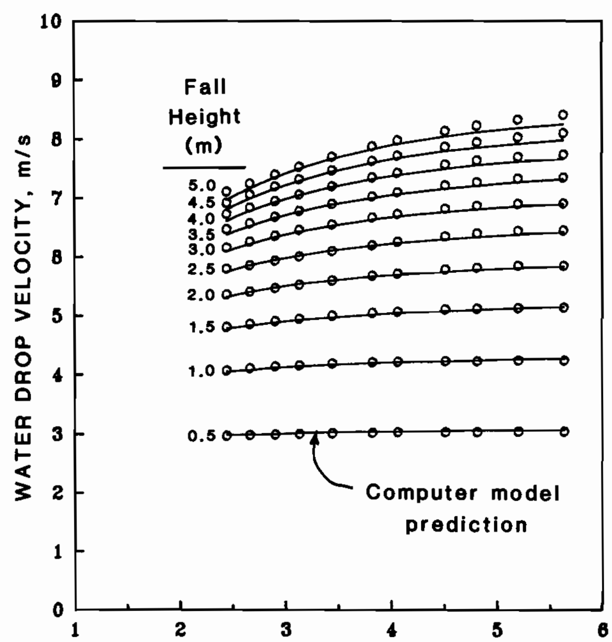

Fig. 3—Measured falling water drop velocity for the standard conditions (20°C, 84.1 kPa) of the present experiments at an elevation of 1570 m (5150 ft) above mean sea level and predicted velocity curves from Wang and Pruppacher's computer model for the same conditions.

temperature and pressure, and are a function of the following:

$$\text{Reynolds number,} \quad R = \frac{\rho_a V d}{\mu},$$

$$\text{Bond number,} \quad B = \frac{4d^2 g(\rho_w - \rho_a)}{3\sigma},$$

$$\text{and a physical property number,} \quad P = \frac{\sigma^3 \rho_a{}^2}{\mu^4(\rho_w - \rho_a)g}$$

where,

ρa = density of the air, m/L^3
V = water drop velocity, L/t
d = drop diameter, L
μ = dynamic viscosity of air, m/Lt
g = gravitational constant, L/t^2
ρ_w = density of the water, m/L^3
σ = surface tension of water, m/t^2

Beard's equations for 1.07 to 7 mm diameter drops predict R at terminal velocity as a function of B and P and terminal velocity is then determined from the definition of R, as follows:

$$X = \ln(BP^{1/6}) \quad \ldots\ldots [3]$$

$$Y = b_0 + b_1 X + b_2 X^2 + b_3 X^3 + b_4 X^4 + b_5 X^5 \quad \ldots [4]$$

$b_0 = -5.00015$

$b_1 = 5.23778$

$b_2 = -2.04914$

$b_3 = 0.475294$

$b_4 = -0.0542819$

$b_5 = 0.00238449$

$$R_T = P^{1/6} e^Y \quad \ldots\ldots [5]$$

$$V_T = \mu R_T / \rho_a d \quad \ldots\ldots [6]$$

where,

R_T = Reynolds number at terminal velocity, dimensionless
V_T = water drop terminal velocity, L/t

Wang and Pruppacher's model determines the time and distance that a water drop falls for finite differences of R. Time, distance and velocity are summed as R increases from zero to R at terminal velocity.

Wang and Pruppacher measured the velocity of falling water drops using an electronic-electrostatic technique similar to that used by Gunn and Kinzer (1949). They used a 35 m column with controlled temperature and humidity. Measurements were made for five drop diameters ranging from 1.67 to 6.70 mm, all in 20°C, 100.0 kPa saturated air to minimize evaporation.

Wange and Pruppacher's computer model accurately predicts the terminal velocity of falling water drops. This is to be expected since Beard's equations in the model were developed from terminal velocity data. For three of the five diameters, the model was equally accurate at predicting the instantaneous velocity of an accelerating drop. For the other two diameter, the model slightly underpredicted the velocity of an accelerating drop, primarily in the two to eight meter range of fall distance.

Wang and Pruppacher's model was run for the standard conditions (20°C, 84.1 kPa) of the present experiments. The predicted results are within one percent of the experimental values, except for the smallest and largest drops from the three to five meter fall heights, where the predicted results approach two percent less than the present measured results (Fig. 3). Wang (1985) states that the computer model may slightly underpredict the velocity of an accelerating drop in the 2 to 8 meter fall height range. The model assumes an accelerating drop has the same shape as a smaller drop at terminal velocity and the same Reynolds number but since velocities are less in this range implies that the drops are actually slightly more deformed (flattened). The amount of differences between the measured and the predicted results is not random but varies gradually with respect to drop size and fall height. The equations developed by Beard and used in the computer model were developed from terminal velocity data for drop sizes up to 7 mm diameter. Therefore, the model may be more accurate for the intermediate range (2.5 to 5 mm) of drop sizes.

Laws' data for an accelerating drop were compared to predictions from Wang and Pruppacher's computer model for the average conditions under which Law made his measurements. The difference were rather random with respect to both drop height and diameter, and the velocity differences ranged up to 3%. Considering the technology at the time, Laws' velocity results are fairly accurate and are much more accurate than any of the earlier published results. However, when compared to more recent results measured by more accurate methods, the accuracy of Laws' data becomes a matter of concern, especially when these data are used to predict falling water drop velocities at various atmospheric conditions.

Wang and Pruppacher's model accurately predicts terminal velocities of water drops at 20°C and air pressures near one atmosphere. Results from this model can be compared to terminal velocities measured by Laws (1941), and Gunn and Kinzer (1949). Laws made measurements in air with temperature ranging from 20.7 to 25°C, air pressure ranging from 99.2 to 101.8 kPa and relative humidities ranging from 20 to 80%. Laws did not correct his data to a standard condition. He simply fitted smooth curves to his data and interpolated the curves to find water drop velocities for specific drop diameters and fall heights. Gunn and Kinzer made drop velocity measurements over a range of temperature and pressure but converted them to standard conditions of 20°C and 101.3 kPa (1 atm) using equation [2]. Their measurements were made at approximately 50% relative humidity.

Water drop terminal velocities predicted by Wang and Pruppacher's model for Gunn and Kinzer's standard sea level conditions (20°C, 101.3 kPa) are just slightly larger than terminal velocities measured by Gunn and Kinzer,

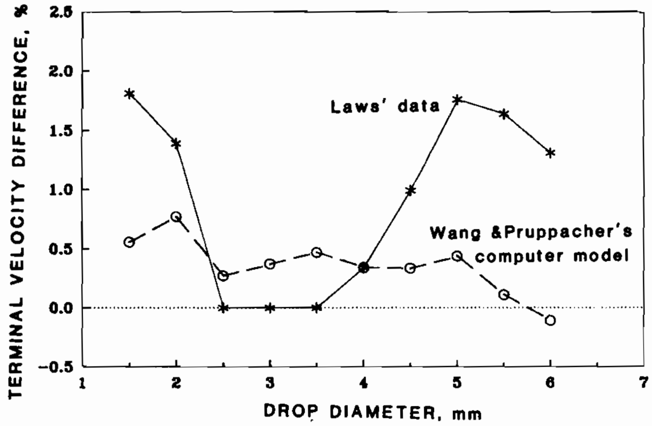

Fig. 4—Percent differences in terminal velocity of Laws' data and Wang and Pruppacher's computer model predicted values to Gunn and Kinzer's data.

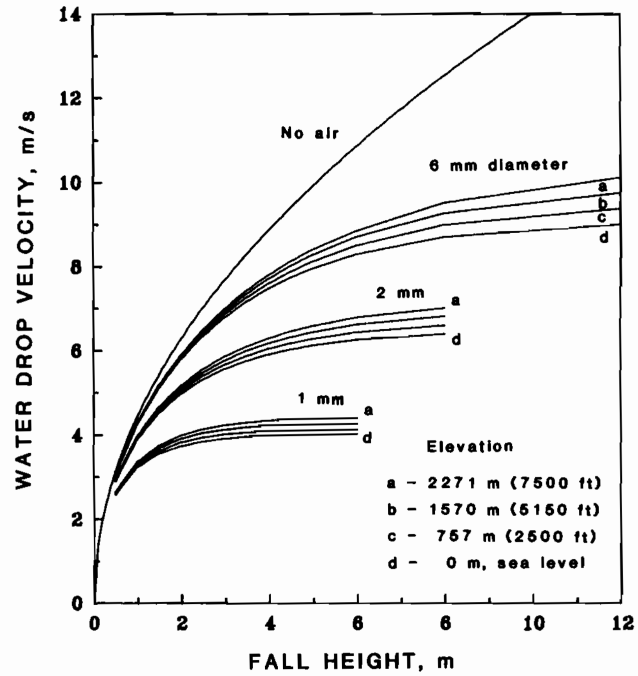

Fig. 5—Falling water drop velocities predicted by Wang and Pruppacher's model for three water drop diameters and four elevations at standard atmospheric pressure and 20°C.

as shown in Fig. 4. The difference is fairly uniform and tends to decrease with increasng drop diameter. The difference is relatively small (0 to 0.8%) and may be exculsively due to evaporation, considering that Gunn and Kinzer's measurements were made in 50% relative humidity air and the computer model results represent those for saturated air. The fact that the difference decreases with increased drop size also supports this idea, since larger drops have a lower surface to volume ratio and hence, should have less relative evaporation than similar small drops.

Laws' terminal velocities are significantly greater than those of Gunn and Kinzer for the smaller and larger drops and in general more scattered than measurements by Gunn and Kinzer or predictions by the computer model. Laws' velocities are generally greater because his velocities are representative of an average temperature of 22.8°C and an average atmospheric pressure of 100.5 kPa, which has an air density 1.2% less than the standard conditions. His data are more scattered because his measured velocities were not converted to a standard condition but instead represent a range of temperatures, pressures and relative humidities. Also, his photographic method of measurement was probably less accurate than the more recent electrostatic methods of measurements.

Because Wang and Pruppacher's computer model can be used to predict the velocity of an accelerating water drop, it was used to predict velocity differences expected for various drop sizes at different elevations, under different standard atmospheric pressures. The velocity profile curve for 1 mm, 2 mm and 6 mm diameter drops are shown in Fig. 5 for atmospheric pressures of 101.3 kPa (one atmoshpere), 92.5 kPa, 84.1 kPa and 77.0 kPa, which corresponds to approximate elevations of 0, 757 mm (2500 ft), 1570 mm (5150 ft) and 2271 m (7500 ft) above sea level, all at 20°C.

The 1 mm diameter drops approach terminal velocity after a much shorter fall distance than the larger drops. The velocity of the 6 mm drop for the first meter of fall distance is close to the velocity of a free falling body in a vacuum because during initial fall distances, aerodynamic drag forces are small compared to gravity forces.

At terminal velocity, the larger drops have the greatest percent increase in velocity with decreasing air density. Terminal velocities for the 1 mm water drops for the four respective elevations are, 4.03 m/s, 4.17 m/s (3.5%), 4.32 m/s (7.2%), and 4.46 m/s (10.7%), with the percent relative velocity increase in parentheses. Likewise, for the 6 mm water drops, the terminal velocities for the four respective elevations are 9.17 m/s, 9.59 m/s (4.6%), 10.06 m/s (9.7%), and 10.51 m/s (14.6%).

Sprinkler trajectory simulation

Wang and Pruppacher's computer model was combined with a differential equation presented by Seginer (1965), to estimate the effects of elevation on velocity and radial distribution of sprinkler water drops. Seginer's equation includes a gravity term and an aerodynamic drag term, and is as follows:

$$\frac{dV_r}{dt} = g \pm C_n V_r^{\,n} \quad \ldots\ldots\ldots\ldots\ldots\ldots [7]$$

and in vertical and horizontal directions is, respectively,

$$\frac{dV_y}{dt} = g \pm C_n V_r^{\,n-1} V_y \quad \ldots\ldots\ldots\ldots\ldots [8]$$

$$\frac{dV_x}{dt} = C_n V_r^{\,n-1} V_x \quad \ldots\ldots\ldots\ldots\ldots [9]$$

where,

V_r = resultant water drop velocity, L/t
t = time, t
g = acceleration of gravity, L/t^2
C_n = coefficient of aerodynamic drag, $t^{n-2}L^{1-n}$
n = exponent of the velocity term, dimensionless
V_y = vertical component of water drop velocity, L/t
V_y = horizontal component of water drop velocity, L/t

Equations [8] and [9] are similar to equations for the vertical and horizontal directions defined by von Bernuth and Gilley (1984). Both n and C_n varied as a function of drop size and fall height for vertically falling water drops but Seginer rationalized that for water drops from sprinklers, n should have an approximate value of two. Since $dV/dt = 0$ at terminal velocity, then C_2 is equal to the acceleration of gravity divided by terminal velocity squared and subsequently, varies with drop size only.

Ballistics calculations of sprinkler water drops using Seginer's differential equation and terminal velocity predicted by Wang and Pruppacher's computer model shows that velocity and radius of throw increase significantly as elevation increases due to the differences in air density. A finite difference computer program using Seginer's differential equation separated into vertical and horizontal directions, and solved using a fourth order Kutta-Runge numerical method, was run for typical sprinkler irrigation situations. An example of the radial distribution is shown in Fig. 6 for an impact sprinkler on a center pivot. The trajectory of 2 mm and 6 mm drops are shown for four elevations at their standard atmospheric pressures and 20°C with an exit velocity of 25 m/s (82 ft/s), typical of a sprinkler with a nozzle pressure of 330 kPa (48 psi). Radii of throw increased 5.2, 10.8, and 16.1% and 5.6, 11.7, and 17.6% at the upper three elevations for the 2 mm and 6 mm drops, respectively. Kincaid (1982) measured up to 10% greater maximum radii of throw at 1220 m (4000 ft) elevation than published data measured near sea level. Also, resultant velocities of the water drops increased 3.0, 6.2, and 9.3%, and 3.6, 7.2, and 10.8% at the upper three elevations for the 2 mm and 6 mm drops, respectively. Note that the time of flight of the water drops increased slightly with increasing elevation. This example assumes no evaporation.

CONCLUSIONS

Both the double ring electronic and the photographic methods worked quite well for measuring falling water drop velocities, and the results from both methods were in fairly close agreement. However, the double ring electronic method utilizing the computer-ADC and software package offered better accuracy and precision, easier repetition and faster results than the photographic method. The computer model predicted velocities generally within one percent of measured velocities even though the model was written to predict terminal velocity. The model may slightly underpredict the velocity of an accelerating drop.

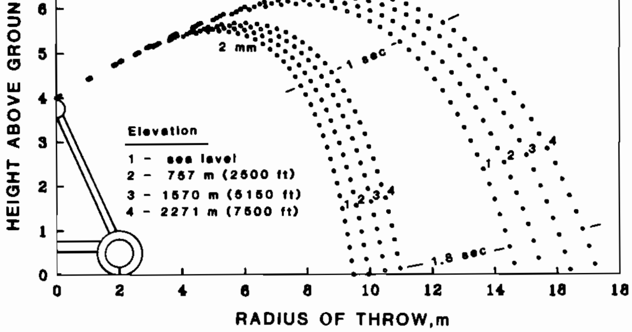

Fig. 6—Calculated water drop ballistics of 2 mm and 6 mm diameter drops from a center pivot for four elevations at standard atmospheric pressure and 20°C, using Seginer's differential equation and terminal velocities from Wang and Pruppacher's model.

A velocity correction equation for air pressure and temperature, derived by Gunn and Kinzer (1949) from equations of force and velocity at terminal velocity, can be used for accelerating drops based on the separate findings of Wang and Pruppacher (1977) and of Pruppacher and Pitter (1971). Wang and Pruppacher assumed the drag coefficient of an accelerating water drop can be found by interpolating the $C_d - R$ curve for drops at terminal velocity. This assumption was verified by the close agreement between the results of their computer model and their measured results. Pruppacher and Pitter showed that the deformation characteristics of water drops at low air pressures are similar to those at one atmosphere pressure.

The lower air pressure and density at the experimental location did result in greater water drop velocities than at sea level. For the smallest drops from the highest fall heights where air density differences are most significant, the present results were up to 7.4% greater than those measured by Laws (1941) at sea level. However, for the largest drops from the lower fall heights, where air resistance differences are small, the present results are close to the results of Laws and the computer model.

Significant velocity differences were found using Wang and Pruppacher's model for vertically falling water drops for four representative elevations, at their standard pressures and at a constant temperature. Modeled terminal velocity and a finite difference solution of the differential equations for the ballistics of sprinkler water drops showed almost equally significant water drop velocity differences among the four elevations. In addition, radii of throw of the water drops from the sprinkler increased even more significantly among the four elevations. Therefore, elevation differences may have a significant influence on the distribution patterns of sprinkler irrigation systems and the dynamic parameters of water drops striking a soil surface.

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