Wheat Yield Models with L

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AVERAGE crop yields are now below the economic optimum because the best water and fertilizer use and other cultural practices, singly and in combination, have either not yet been developed by agricultural research or are only used by a minority of growers possessing the knowledge and skill to apply them. Quantitatively defining the physical response of crops to water is essential for improving farming on sound economic basis. Water shortages would be significantly alleviated by improving the cultural practices of crop production and irrigation efficiency.

Increased research attention has been given to developing crop yield models. Economic crops have been studied mostly by considering their dry matter (total above ground production including straw and grain) and grain components of yield. Very little attention has been given to the root biomass except for root crops since it is very difficult to sample.

Objective

This study was made to formulate and test wheat yield models for limited soil water and climatic variation. Dry matter and grain components of wheat were studied.

YIELD MODELS

Dry Matter Production

Briggs and Shantz (1913a, 1913b, 1914) grew 55 species and crop varieties in pots under various soil water levels. Their results and those of later researchers, Shantz and Piemeisel (1927), showed close relationships between dry matter production and transpiration. Since these studies were not conducted under field conditions, Briggs and Shantz warned "the water requirement measurements must be considered relative rather than absolute." This basic data qualification was often overlooked or disregarded by its users for the next half century.

de Wit (1958) and later Arkley (1963) analyzed data, including that of Briggs and Shantz (1913a), from many parts of the world and attempted to reconcile climatic differences. For areas of high radiation, the semiarid Great Plains, de Wit related cumulative dry matter (DM) to cumulative transpiration (W_t) scaled by the seasonal average free water evaporation rate E_o as follows:

$$DM = m \frac{W_t}{E_o} \quad \text{.................} [1]$$

where m is a constant which depends only on the plant. For areas of low radiation, the Netherlands, de Wit's equation did not apply. Arkley attempted to reconcile the discrepancy in the results of the two climatic regions by replacing E_o by mean daily relative atmospheric saturation deficit (100-h) during the period of most active growth. The mean daily relative atmospheric humidity (h) is in percent.

For de Wit and Arkley's analysis, most of the data analyzed were not from field studies. Hence, their results should be considered relative rather than absolute. Their models assume a zero intercept contrary to some of the statistical findings.

Leggett (1959), Powers et al. (1961), Arkley (1963), Whittlesly and Colyar (1968), and Hanks et al. (1969) have shown that dry matter is related linearly to evapotranspiration (ET) with a high degree of correlation. In all these studies no expression is given for grain yield which is the component of major economic importance.

Grain Yield

Various models have been used to estimate grain yield as a function of soil water levels during the crop season. The most common is a statistical regression of seasonal water use. Other models include asymptotic (Mitscherlich and Spillman (Heady and Dillon 1961)), additive (Moore 1961, Flinn and Musgrave 1967, and Dudley, Howel and Musgrave 1971), and multiplicative (Jensen 1968, Hall and Butcher 1968, Anderson and Maas 1971) models.

Jensen (1968) suggested that if plant nutrients are not limiting, the relative yield of a determinate crop that flowers could be related to available water by the multiplicative model

$$\frac{G}{G_o} = \prod_{i=1}^{n} \left[\frac{W_{et}}{W_{oc}}\right]_i^{\lambda_i} \quad \text{............} [2]$$

where

$\frac{G}{G_o}$ = represents the relative yield of a marketable product,

G = is the actual yield,

G_o is the yield when soil water is not limiting,

$\frac{W_{et}}{W_{oc}}$ = represents the relative total evapotranspiration during the given stage of physiological development,

λ_i = represents the relative sensitivity of the crop to water stress during the i^{th} (i=1, 2 . . .,n) stage of growth.

The right side of Equation [2] is a product. Jensen evaluated and obtained the following values of λ for grain sorghum:

λ_1 = 0.5 for emergence to boot stage.

λ_2 = 1.5 for boot to milk stage, and

λ_3 = 0.5 for milk to maturity stage.

These may be compared with 0.7, 1.0

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and 0.5 obtained by statistical reanalysis of the same data by Neghassi (1974). Graphical comparison of observed versus predicted yield indicated little sensitivity since the relative interstage ET had many observations near 1.0. An experiment in which some plants are subjected to a greater stress to give a larger range in the relative ET $\left[\frac{W_{et}}{W_{oc}}\right]$ may be necessary to validate the model.

Another multiplicative model which gives crop yield under limited water conditions is suggested by Hall and Butcher (1968):

$$G = \prod_{i=1}^{n} a_i G_o \quad \ldots\ldots\ldots\ldots\ldots\ldots [3]$$

where $a_i = a_i(w_i)$ is a coefficient which is a function of soil water content at the end of the i^{th} ($i = 1, 2 \ldots, n$) irrigation period. Since no data were presented to support the model, experimentally or statistically verifying it is difficult.

PRELIMINARY ANALYSIS

Yield Data

Hard, red winter wheat, *Triticum aestivum L.*, was grown at the University of Nebraska Agricultural Experiment Station, North Platte, Nebraska, for 7 yr (Table 1). The 1968 data were unavailable because the crop was destroyed by green bugs in the fall of 1967. The crop was grown under four field treatments of bare and mulched soil with and without addition of nitrogen fertilizer. Stubble mulch tillage was used on the mulched plots during the 14½ month fallow period. The wheat straw mulch averaged 5160 kg ha^{-1} at seeding time and was used to increase soil water storage efficiency during fallow. Bare soil was obtained by incorporation of the wheat straw residue with a one-way disk at the start of fallow. Subsequent tillage operations were the same as used for stubble mulch treatments. The fertilized plots received 57 kg ha^{-1} N broadcast prior to last tillage before seeding. The nitrogen fertility of the unfertilized soils was about 20 kg ha^{-1} N each year of the study which is above the critical level for wheat production under conditions of limited soil water. No irrigation water was applied and there was no surface runoff from the bare or mulch treatments each year of the study.

The soil was Holdredge silt loam, very uniform and representative of much of the land used for wheat production in the Central Great Plains (Smika and Ellis 1971). Wheat was seeded in 30.5 cm rows in 0.033 ha plots. Each treatment was replicated three times. Plant population was kept constant at 97 plants m^{-2} yearly by thinning.

Four stages of phenological development, representing boot, heading, soft dough, and maturity were identified. Boot stage of wheat is characterized by the appearance of sheathlike structures on the upper end of the grain plants which enclosed the inflorescence prior to its emergence. Heading is represented by the appearance of a tightly formed fruit cluster. Soft dough is the stage at which the kernel, when broken open, shows a soft or cheesy condition. Maturity is the state of full or complete development. As these definitions indicate, each stage is not reached simultaneously by all plants. The stages occur over a range of time and caused great variation in the time between stages. Further, the sampling process is destructuve. Different plants are taken at each measurement, increasing the variability.

To reduce the variation due to the time between stages and small dry matter samples, the dry matter yields were interpolated to mean number of days from spring growth initiation. The mean number of days from spring growth initiation to boot, heading, soft dough, and maturity were 60, 73, 98, and 113 days, respectively. Dry matter samples, including entire above ground portions, were taken from a 0.84 m^2 of each plot at each stage of plant development and also in late fall of the preceding year. The number of tillers at each growth stage were counted. Grain yield was measured at harvest.

Individual annual analyses of variance had significant difference in grain yield at the 5 percent probability level for nitrogen, replication and mulch nitrogen interaction for 1 yr of 7 yr (Neghassi 1974). Mulch had significant effects for 2 of the 7 yr. The combined analysis of variance of the yield data (Table 2) indicated a significant variation from year to year

TABLE 1. TILLERS, FINAL DRY MATTER, GRAIN YIELDS, AND CUMULATIVE SPRING EVAPOTRANSPIRATION FOR WINTER WHEAT, NORTH PLATTE, NEBRASKA, 1963-1970, 1968 EXCLUDED

Year	Treatment	Tillers per m^2	Final dry matter kg ha^{-1}	Grain yield kg ha^{-1}	Ratio of grain to final dry matter	Cumulative spring evapotranspiration cm
1963	bare-O-N	960	6610	1630	0.25	35.1
	bare-N	1100	7450	1760	0.24	34.6
	mulch-O-N	880	6220	1630	0.26	35.4
	mulch-N	1100	7390	1760	0.24	34.7
1964	bare-O-N	1000	7060	3410	0.48	34.2
	bare-N	1070	8480	3540	0.42	34.7
	mulch-O-N	860	5900	2660	0.45	36.2
	mulch-N	1050	6990	2980	0.43	35.8
1965	bare-O-N	1070	6380	3270	0.51	36.8
	bare-N	980	6670	3190	0.48	36.8
	mulch-O-N	700	6500	2350	0.36	36.8
	mulch-N	1060	6800	2820	0.42	36.8
1966	bare-O-N	1320	7690	3670	0.48	36.1
	bare-N	1360	8140	3800	0.47	36.0
	mulch-O-N	1100	7320	3310	0.45	35.8
	mulch-N	1160	8030	3500	0.44	35.2
1967	bare-O-N	820	7240	2370	0.34	32.4
	bare-N	870	7110	2480	0.35	32.7
	mulch-O-N	870	7170	2420	0.34	32.3
	mulch-N	840	7020	2630	0.38	32.6
1969	bare-O-N	1630	8140	3640	0.45	30.9
	bare-N	1350	7540	3610	0.48	32.0
	mulch-O-N	1460	7800	3620	0.46	32.8
	mulch-N	1240	8020	4070	0.51	32.8
1970	bare-O-N	980	7170	2650	0.37	37.7
	bare-N	810	7860	2980	0.38	32.7
	mulch-O-N	1310	7170	2800	0.39	38.2
	mulch-N	1250	6940	2820	0.41	38.6
7-year averages	bare-O-N	1110	7180	2950	0.41	34.7
	bare-N	1080	7560	3070	0.39	34.9
	mulch-O-N	1020	6860	2730	0.40	35.4
	mulch-N	1100	7310	2940	0.40	35.2
Overall average		1080	7230	2910	0.40	35.1

TABLE 2. ANALYSIS OF VARIANCE FOR FINAL DRY MATTER AND GRAIN YIELDS PRESENTED IN TABLE 1

Source of variation	Degrees of freedom	Sums of squares	Mean squares	F
		Final dry matter		
Treatments	3	1,979,981	659,993	3.15
Nitrogen	1	1,317,625	1,317,625	6.34*
Mulch	1	657,903	657,903	3.17
Mulch x nitrogen	1	4,453	4,453	0.02
Years	6	5,075,949	845,991	4.17*
Error	18	3,743,429	207,968	
Total	27	10,799,357		
		Grain yield		
Treatments	3	311,485	103,828	2.57
Nitrogen	1	94,540	94,540	2,34
Mulch	1	107,756	107,756	2.66
Mulch x nitrogen	1	109,189	109,480	2.70
Years	6	11,444,883	1,907,480	47.13†
Error	18	728,597	40,476	
Total	24	12,484,947		

* = Significant at the 5 percent level.
† = Significant at the 0.1 percent level.

for both final dry matter and grain yields. This difference indicates that climatic variation did affect yield and the data could be used for testing yield models. Nitrogen had a significant effect on dry matter yields but was not a significant factor in grain yields.

The overall average grain yield of 2906 kg ha^{-1} is significantly higher than the average of 1770 kg ha^{-1} for the United States and comparable with the average 2840 kg ha^{-1} for Mexico (Framji 1972).

Evapotranspiration

Soil water was measured with a neutron probe to a 180-cm depth at spring growth initiation and at each of the four succeeding stages. Daily climatic data including temperature, precipitation, pan evaporation, percent sunshine, and wind speed were recorded.

The combination method modified by Penman (1963) with crop coefficients specified by Jensen (1968) and regional calibration of constants by Heermann (1973) was used to estimate ET. Estimated ET accumulated from spring growth initiation to dates of boot, heading, soft dough, and maturity (W_{et}) was compared with corresponding cumulative water use calculated from rainfall and measured soil water (W_m) assuming no deep percolation or surface runoff. A linear regression indicates a one-to-one correspondence between W_{et} and W_m, with a small intercept value and a high corelation (Fig. 1). Since there was a high correlation, W_{et}, which can be accumulated to any desired time from estimated daily ET, was used for the subsequent analysis. Cumulative relative crop ET (W_r) has been defined as

$$W_r = \int_{t_o}^{t_i} \left[\frac{ET_c}{ET_o}\right] dt \quad \ldots\ldots [4]$$

where

ET_c/ET_o = the ratio of actual crop evapotranspiration (ET_c) to potential (ET_c) for the specific crop

ET_p = potential evapotranspiration for the well-watered reference crop,

K_{co} = crop coefficient, water nonlimiting,

t = time (days),

t_o = date of spring growth initiation, and

t_i = date of stage of growth i. in which $ET_c = K_{co} ET_p$

RESULTS AND DISCUSSION

Dry Matter Yields

Dry matter yield (DM) of winter wheat was related to cumulative ET (W_{et}) and cumulative relative ET (W_r). The general relationships in quadratic form may be given by

$$DM = k_o + k_1 W_{et} + k_2 W_{et}^2 \quad \ldots\ldots [5]$$

$$DM = r_o + r_1 W_r + r_2 W_r^2 \quad \ldots\ldots\ldots [6]$$

The units are DM in kg ha^{-1}; W_{et} in cm; W_r in cm cm^{-1}; k_1 in kg ha^{-1}cm^{-1}; k_2 in kg ha^{-1}cm^{-2}; and $k_o r_o$, r_1, and r_2 in kg ha^{-1}.

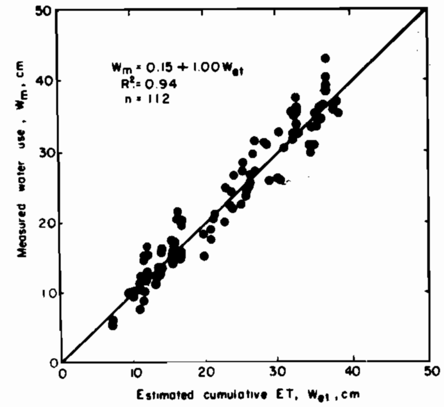

FIG. 1 Comparison of estimated cumulative evapotranspiration and measured water use after spring growth initiation for hard, red winter wheat, North Platte, Nebr., 1963-1970, 1968 excluded.

Linear and quadratic forms of the above equations were tested. Positive intercepts are not common in the linear form of Equation [5]. The positive intercept may have resulted from the accumulation of W_{et} from date of spring growth initiation and not from date of planting.

The linear and quadratic forms of both models have high correlation and significant regression (α = 0.001) coefficients. However, the quadratic models explained very little additional variation and have only a small effect on predicting yields as compared to the linear models. The linear models had a high correlation (Figs. 2 and 3). Linear models may be sufficient for predicting dry matter yields when soil water was limiting. Yield response per unit of water for linear models estimated from data gathered by various researchers for wheat are comparable with the 195 kg ha^{-1}cm^{-1} obtained in this study. Some examples

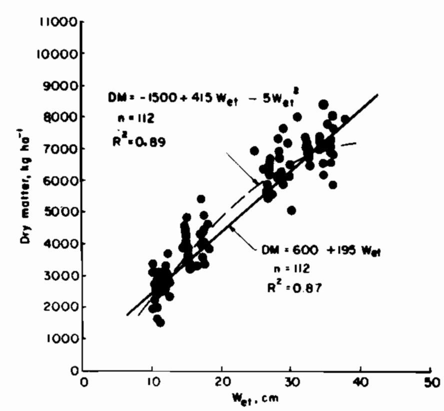

FIG. 2 Dry matter as a function of cumulative evapotranspiration, W_{et}, for hard red winter wheat, North Platte, Nebr., 1963-1970, 1968 excluded.

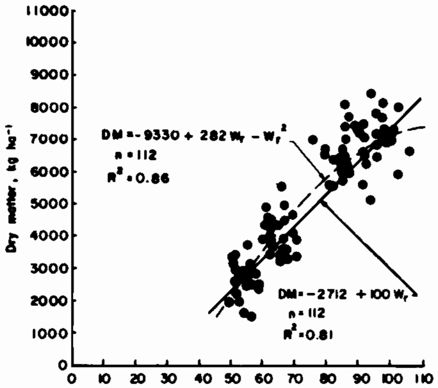

FIG. 3 Dry matter as a function of relative evapotranspiration or relative transpiration, W_r, for hard winter wheat, North Platte, Nebr., 1963-1970, 1968 excluded.

are 186 kg $ha^{-1}cm^{-1}$ for spring wheat and 197 kg $ha^{-1}cm^{-1}$ for wheat varieties analyzed by Arkley (1963), and 210 kg $ha^{-1}cm^{-1}$ estimated from the winter wheat studies by Hanks et al. (1969). The data did not allow investigation of models when soil water was maintained near optimum throughout the crop season. The average spring cumulative ET of 35.1 cm was 66 percent of the cumulative potential ET of 52.86 cm for the same period.

The main reason quadratic and sigmoid production functions are used in applied agriculture is that they display certain characteristics basic to marginal economic analysis. Sigmoid functions allow for increasing, decreasing, and negative marginal products, ~~for example~~ $[(\frac{dDM}{dW_{et}})]$.

These properties enable the three classical stages of production to be identified (Heady 1961, pp. 91-96), of which the second is the rational production stage. ~~Quadratic~~ Linear models allow for none of these; they have constant marginal products, k_1 and r_1.

W_r attempts to reconcile yield models over varied climatic years and regions. ET_0 incorporates many climatic variables including the atmospheric saturation deficit which Arkley (1963) selected as scaling factor. Thus, ET_0 may be preferable. Scaling crop ET renders the independent variables dimensionless and the coefficients of Equation [6] have dimensions of DM.

Grain Yield Models

Grain yield can be measured only after crop maturity which poses a major problem in studies attempting to trace the effects of water stress during the various stages of growth and development, and, in part, accounts for the diversity of grain yield models. To approximate the effect of stress on grain yield, the effect of stress on dry matter yields may be traced and then grain yield related to dry matter. Other relevant yield indicators, such as tillers, and dry matter production the preceding fall, may be included.

Extreme high temperatures and high winds were more prevalent in 1963, mostly during the last part of June, and reduced the grain yields (Table 1). Maximum temperatures were 37 to 41 C and wind speed was 10 to 20 km per hr from June 23 to 30. The amount of dry matter production of the preceding fall was very low for 1963 and 1967 which also limited the grain yields potential for these 2 yr. However, the final dry matter was not similarly affected. The final dry matter for 1963 and 1967 was comparable with the other years. This discrepancy may partly account for the lowest grain yield to final dry matter ratios for the 2 yr.

The following linear and nonlinear models for winter wheat were formulated on the basis of the above hypothesis:

$$G = b_0 + b_1 DM_p + b_2 N_t + b_3 DM_f \ldots [7]$$

$$G = c_0 + c_1 DM_p + c_2 N_t + C_3 DM_f + c_4 M \ldots [8]$$

where

DM_p = the dry matter produced during the preceding fall ($kg\ ha^{-1}$);
DM_f = the final dry matter ($kg\ ha^{-1}$);
N_t tillers per m^2;
M denotes the number of days $T_{max} \geq 27$ C, wind $U_a \geq 10$ kph; and b and c are parameters.

In addition to these, a linear model that relates grain yield to interstage dry matter was tested and found to be in apparent violation of the underlying physical principles (Neghassi 1974).

Least squares estimates of the coefficients in the linear Equations [7] and [8] are summarized in Table 3. Equation [7] relates grain yield to dry matter produced during the preceding fall (DM_p), tillers (N_t), and final dry matter (DM_f). Equation [8] adds the term M, which includes the climatic extreme effect of temperature and wind. The determination coefficients ($R^2 = 0.53$ for Equation [7] and 0.75 for Equation [8]) are significant at the 1 percent probability level. The regression coefficients are all significantly different from zero.

The two linear models seem to have physical basis. DM_p may be considered a proxy for the factors that affect growth and development of the root system and above ground parts after planting and before the plant goes into winter dormancy. The number of tillers (N_t) are affected by the soil and climatic conditions, such as crown temperature (Smika 1973). Final dry matter (DM_f) is directly related to W_{et} or W_r in the dry matter models.

The lowest annual grain yield in 1963 was associated with the largest M, which significantly decreased grain yields. The negative coefficient of -114 kg ha^{-1} day^{-1} in which $T_{max} \geq 27$ C and $U_a \geq 10$ kph (Table 3) indicates how much grain yield decreases as exposure to such climatic extremes increases. The addition of M increased R^2 from 0.53 in Equation [7] to 0.75 in Equation [8], which is a significant addition to the explained variation. Climatic extremes and yearly variations in climate are important factors for winter wheat grain production.

Jensen (1968) suggested a nonlinear model Equation [2] analogous to the

TABLE 3. LEAST SQUARES ESTIMATE OF COEFFICIENTS IN LINEAR GRAIN YIELD MODELS (6) AND (7)

Model	Coefficients								Sample size n
	Subscripts							Determination R^2	
	Symbol	0	1	2	3	4	5		
Eq. 7	b	-1314	0.98 (0.36)‡	0.85 (0.51)	0.40 (0.18)			0.53†	28
Eq. 8	c	232	1.27 (0.27)	0.31 (0.38)	0.31 (0.13)	-114.12 (24.64)		0.75†	28

*Excludes 1963.
†R^2 is significant at 1 percent probability level.
‡Numbers in parentheses are standard errors of regression coefficients.

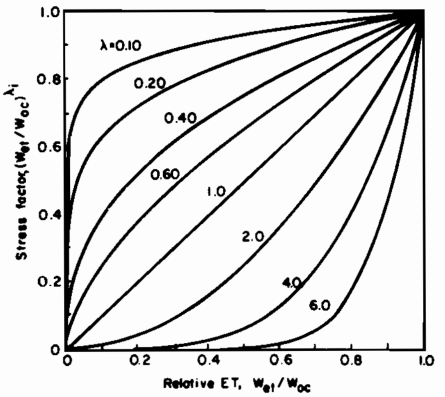

FIG. 4 Effect of varying values of stress exponent [elasticity] λ_i on crop yield response, Jensen Model, when $\lambda_i \geqslant 0$.

Cobb-Douglas Equation [14] generally used in aggregate industrial economic analysis. The model is nonlinear in the parameters λ_i. It may be classified as intrinsically linear or intrinsically nonlinear depending on whether the error term is additive or multiplicative. It is difficult to assess the nature of the error term. However, statistical algorithms for estimating parameters of nonlinear models are available (Marquardt 1963).

The physical significance of $\lambda_i \geqslant 0$ can be interpreted from Fig. 4. Since

$$\left[\frac{W_{et}}{W_{oc}}\right]_i \lessgtr 1.0 \text{ for all i,}$$

$$\left[\frac{W_{et}}{W_{oc}}\right]_i^{\lambda_i} < \left[\frac{W_{et}}{W_{oc}}\right]_i \text{ if } \lambda_i > 1.0$$

$$= \left[\frac{W_{et}}{W_{oc}}\right]_i \text{ if } \lambda_i = 1.0$$

$$> \left[\frac{W_{et}}{W_{oc}}\right]_i \text{ if } \lambda_i < 1.0$$

A further interpretation of the significance of λ_i can be made by the method of elasticities popularly employed in economic analysis. Equation [2] is homogenous of degree K

$$(K = \sum_{i=1}^{n} \lambda_i)$$

which implies return to scale increases if K>1.0, is constant if K = 1.0, and decreases if K<1.0. Each λ is the elasticity of production with respect to the i^{th} relative ET or $\left[\frac{W_{et}}{W_{oc}}\right]_i$ Elasticity if defined as the percentage change in quantity produced attributable to a percent change in an independent variable.

That is,

$$\lambda_i = \frac{\partial (G/G_o)}{\partial \left[\frac{W_{et}}{W_{oc}}\right]_i} \cdot \frac{\left[\frac{W_{et}}{W_{oc}}\right]}{G/G_o} \quad \ldots\ldots\ [9]$$

$$\lambda_i = \frac{\partial \ln(G/G_o)}{\partial \ln \left[\frac{W_{et}}{W_{oc}}\right]_i}$$

Accordingly, the λ_i values for the interval from spring growth initiation to boot, boot to heading, heading to soft dough and soft dough to maturity are -0.490, 2.709, -5.451, and 4.580, respectively. A 1 percent change in

$$\left[\frac{W_{et}}{W_{oc}}\right]_i$$

during the second and fourth stages have yield changes of 2.71 and 4.58 percent. The λ_i for winter wheat involve both positive and negative values. Negative values imply that yield increases as water stress increases, which is physically unrealistic. Negative λ_i cannot have any physical relevance, particularly under conditions of limiting soil water. A reanalysis by requiring $\lambda_i \geqslant 0$ resulted in $\lambda_1 = \lambda_2 = \lambda_3 = 0$, and only $\lambda_4 \geqslant 0$, which again is not realistic.

The multiplicative model Equation [2] does not seem to be satisfactory for wheat. The model is commonly used with aggregative inputs in which the λ_i are replaced by discrete and unique variables, such as amounts of seasonal water, nitrogen, phosphorus, etc. However, further testing may be necessary before the model can be accepted or rejected.

CONCLUSIONS

Wheat dry matter yield was highly correlated to cumulative ET and cumulative daily relative ET. The linear models (Equations [5] and [6]) with approximate marginal productivities of 200 kg ha^{-1} cm^{-1} and 100 kg ha^{-1} $cmcm^{-1}$ may be sufficient for predicting dry matter yields when soil water is limiting. The scaling of ET by corresponding daily ET_o is believed to reconcile climatic differences. Further tests and comparisons are necessary to establish ranges for the coefficients and applicabilitv of the scaling.

Grain yield models are extremely difficult to formulate. Winter wheat grain yields may be predicted using dry matter production the preceding fall, tillers, final dry matter, and number of days that $T_{max} \geqslant 27$ C and $U_a \geqslant 10$ kph:

$$G = 232 + 1.27\ DM_p + 0.91\ N_t + 0.31\ DM_f - 114\ M. \quad \ldots\ldots\ [10]$$

The coefficient of determination R^2 = 0.75. DM_p may be considered a proxy for the factors affecting growth during the fall. DM_f may be replaced by one of the variables in the dry matter yield models. Yield increases as DM_p, N_t, and DM_f (W_{et}) increases and decreases as exposure to temperature and extreme wind velocities increase, M. Yield models developed without climatic variation may be of limited value and experimental studies must include climatic variation for yield model development.

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