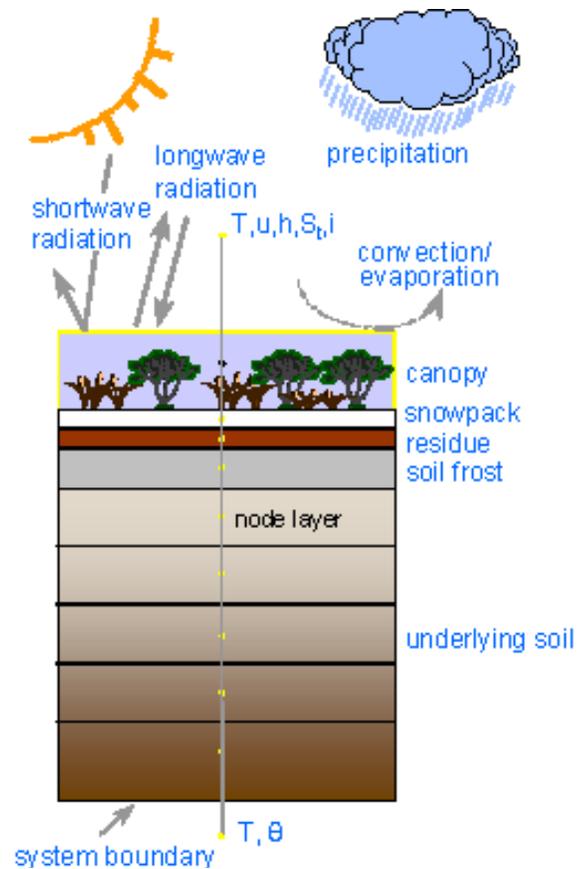


The Simultaneous Heat and Water (SHAW) Model: Technical Documentation



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The SHAW Model

The Simultaneous Heat and Water (SHAW) model, originally developed to simulate soil freezing and thawing (Flerchinger and Saxton, 1989), simulates heat, water and solute transfer within a one-dimensional profile which includes the effects of plant cover, dead plant residue, and snow. The model's ability to simulate heat and water movement through plant cover, snow, residue and soil for predicting climate and management effects on soil freezing, snowmelt, runoff, soil temperature, water, evaporation, and transpiration has been demonstrated. Unique features of the model include: simultaneous solution of heat, water and solute fluxes; detailed provisions for soil freezing and thawing; and a sophisticated approach to simulating transpiration and water vapor transfer through a multi-species plant canopy. Information from the model can be used to assess management and climate effects on biological and hydrological processes, including seedling germination, plant establishment, insect populations, soil freezing, infiltration, runoff, and ground-water seepage.

The physical system described by the SHAW model consists of a vertical, one-dimensional profile extending from the vegetation canopy, snow, residue or soil surface to a specified depth within the soil (Figure 1). The system is represented by integrating detailed physics of a plant canopy, snow, residue and soil into one simultaneous solution.

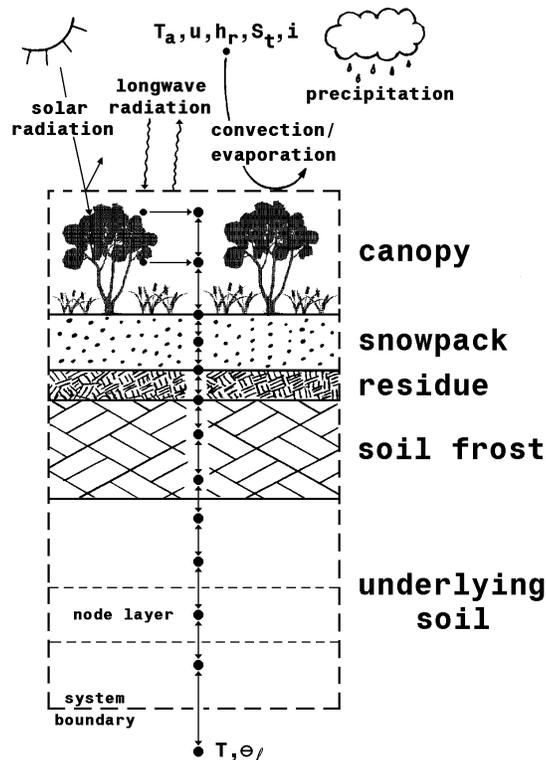


Figure 1. Physical system described by the SHAW model. (T_a is temperature, u is windspeed, h_r is relative humidity, S_t is solar radiation, i is precipitation, T is soil temperature, and θ_l is water content.)

Daily or hourly weather conditions of air temperature, wind speed, humidity, solar radiation, and precipitation above the upper boundary and soil conditions at the lower boundary are used to define heat and water fluxes into the system. A layered system is established through the plant canopy, snow, residue and soil and each layer is represented by an individual node. Energy, moisture and solute fluxes are computed between nodes for each time step, and balance equations for each node are written in implicit finite-difference form.

After solving the energy, water and solute balance for the time step, adjustments are made for precipitation, snowmelt, settling of the snowpack, interception, and infiltration at the end of each time step. The model then optionally outputs a summary of the water balance, surface energy transfer, snow depth, and frost depth as well as temperature, moisture, and solute profiles.

The following sections describe in detail each major component of the SHAW model. These include radiation and convective transfer at the surface boundary, energy and moisture balance of the plant, snow, residue and soil layers, solute transport in the soil, and precipitation-infiltration processes.

Surface Energy and Water Fluxes

The interrelated energy and water fluxes at the surface boundary are computed from weather observations of air temperature, wind speed, relative humidity and solar radiation. The surface energy balance may be written as

$$R_n + H + L_v E + G = 0 \quad (1)$$

where R_n is net all-wave radiation (W m^{-2}), H is sensible heat flux (W m^{-2}), $L_v E$ is latent heat flux (W m^{-2}), G is soil or ground heat flux (W m^{-2}), L_v is latent heat of evaporation (J kg^{-1}), and E is total evapotranspiration from the soil surface and plant canopy ($\text{kg m}^{-2} \text{s}^{-1}$).

Net Radiation

Solar radiation absorbed within the system is computed from the observed total incoming solar radiation (S_t), which consists of direct, or beam (S_b), and diffuse (S_d) components. Because direct and diffuse are absorbed and transmitted differently, total solar radiation is separated into the two components by the following equation developed by Bristow et al. (1985):

$$\tau_d = \tau_t \left[1 - \exp\left(\frac{0.6(1 - B/\tau_t)}{B - 0.4}\right) \right] \quad (2)$$

where τ_d is the atmospheric diffuse transmission coefficient ($S_d/S_{b,o}$), τ_t is the atmospheric total transmission coefficient ($S_t/S_{b,o}$), B is the maximum clear-sky transmissivity of the atmosphere (taken as 0.76), and $S_{b,o}$ is total solar radiation incident on a horizontal surface at the outer edge of the atmosphere (W m^{-2}). Hourly values for $S_{b,o}$ are calculated from the solar constant, S_o ($\sim 1360 \text{ W m}^{-2}$), and the sun's altitude above the horizon, ϕ_s . Direct solar radiation incident on a sloping

surface is related to that on a horizontal surface by

$$S_s = S_b \sin\beta/\sin\phi_s \quad (3)$$

where β is the angle which the sun's rays make with the sloping surface and ϕ_s is computed based on the latitude of the site, the time of year, and the hour of the day.

The net radiation absorbed for each layer depends not only on the incoming radiation from above, but on the reflected, scattered and emitted radiation from other layers within the plant canopy, snow, residue, soil system. Therefore a radiation balance similar to that described by Norman (1979) and Bristow et al. (1986) is performed by computing the direct, and upward and downward diffuse radiation fluxes above and below each layer. Because the upward and downward diffuse radiation are functions of each other, particularly in the case of the canopy and residue layers, an iterative technique is required to obtain the solar and long-wave radiation absorbed by each layer. Transmission and reflectance within each layer is described in the following subsections.

Solar radiation within the canopy

Solar and long-wave radiation exchange between canopy layers, residue layers and the snow or soil surface are computed by considering direct, and upward and downward diffuse radiation being transmitted, reflected and absorbed. Transmissivity to direct radiation for each canopy layer is calculated from (Goudriaan, 1988):

$$\tau_{b,i} = \exp \left(- \sum_{j=1}^{NP} K_j L_{i,j} \right) \quad (4)$$

Here, $\tau_{b,i}$ is the transmissivity to direct (beam) radiation for canopy layer i , $L_{i,j}$ and K_j are leaf area index and extinction coefficient for plant species j of the canopy layer and NP is the number of plant species in the canopy layer. The extinction coefficient is dependent on the orientation of the plant leaves and the angle of incident radiation. For leaves with random orientation, the expression is:

$$K_j = \frac{1}{2\sin\beta} \quad (5)$$

and for vertically oriented leaves (such as standing stubble),

$$K_j = \frac{1}{1.571 \tan\phi_s} \quad (6)$$

Eqn. (4) may also be used as an approximate expression (error < 0.05 for $L_{i,j} < 2$) for diffuse transmissivity ($\tau_{d,i}$) with K_j equal to 0.78 for randomly inclined leaves and 0.68 for vertically inclined leaves. Effective albedo of canopy layer i is calculated from a weighted average of albedo, leaf area and extinction coefficient for each plant species within the layer by

$$\alpha_i = \frac{\sum_{j=1}^{NP} \alpha_j K_j L_{ij}}{\sum_{j=1}^{NP} K_j L_{ij}} \quad (7)$$

where α_j is the albedo of plant species j .

Solar radiation within the residue

Transmission to direct radiation within the residue layer is calculated from

$$\tau_{b,r} = (1 - F_r) \sin \beta \quad (8)$$

where F_r is the fraction of surface area covered by the residue layer ($\text{m}^2 \text{m}^{-2}$) and β is the angle which the sun's rays make with the surface. Transmission to diffuse radiation within the residue layer is computed as

$$\tau_{d,r} = 0.667(1 - F_r) \quad (9)$$

Radiation reflected and scattered by each residue layer may be absorbed by adjacent canopy layers, residue layers and the soil surface, or lost to the atmosphere.

Solar radiation at the snow surface

Albedo of the snow, which is assumed to be independent of sun angle, is computed from (Anderson, 1976):

$$\alpha_{sp} = 1 - 0.206 C_v d_s^{1/2} \quad (10)$$

where C_v is an empirical coefficient used to calculate the extinction coefficient and d_s is grain-size diameter of ice crystals (mm). Grain-size is calculated from (Anderson, 1976):

$$d_s = G_1 + G_2(\rho_{sp}/\rho_l)^2 + G_3(\rho_{sp}/\rho_l)^4 \quad (11)$$

where G_1 , G_2 and G_3 are empirical coefficients, ρ_{sp} is density of the snow at the surface and ρ_l is density of liquid water. Albedo of shallow snowpacks (less than 4 cm) is adjusted based on the albedo of the underlying material.

Solar radiation at the soil surface

Soil albedo varies with soil water content and is calculated from (Idso et al., 1975)

$$\alpha_s = \alpha_d \exp[-a_a \theta_l] \quad (12)$$

where α_d is albedo of dry soil, θ_l is surface volumetric water content and a_a is an empirical coefficient.

Long-wave radiation

Atmospheric long-wave radiation incident on the surface, L_i (W m^{-2}), is estimated from

$$L_i = \varepsilon_{ac} \sigma T_K^4 \quad (13)$$

where ε_{ac} is atmospheric emissivity, σ is the Stefan-Boltzman constant ($5.6697 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$), and T_K is air temperature (K). Clear-sky atmospheric emissivity, ε_a , is estimated using :

$$\varepsilon_a = 1 - a_e \exp[-b_e T_a^2] \quad (14)$$

where T_a is air temperature (C) and a_e and b_e are empirical coefficients with suggested values of 0.261 and $7.77 \times 10^{-4} \text{ C}^{-1}$ (Idso and Jackson, 1969). Clear-sky emissivity is adjusted for cloudy skies from (Campbell, 1985)

$$\varepsilon_{ac} = (1 - 0.84C)\varepsilon_a + 0.84C \quad (15)$$

Fraction of cloud cover, C , is assumed constant for the day and is estimated from (Bristow et al., 1986)

$$C = 2.4 - 4\tau_t \quad (16)$$

where τ_t , the atmospheric transmissivity, is the ratio of measured solar radiation, S_p , to that incident on the outer edge of the atmosphere ($S_{b,o}$). Assumptions inherent in this expression are complete cloud cover for $\tau_t < 0.35$ and clear skies for $\tau_t > 0.6$. Emitted long-wave radiation, L_o , is computed from the Stefan-Boltzman Law assuming a surface emissivity of 1.0 and using a surface temperature computed from a detailed energy balance for the system profile.

Transmission and absorption of long-wave radiation is similar to solar radiation, with the exceptions that scattering of long-wave radiation can be ignored and long-wave emittance must be considered. A long-wave radiation balance is calculated for the snow or soil surface and each residue and canopy layer based on the fluxes incident on and emitted by each side of the layer. For simplicity, long-wave emittance by a canopy layer is calculated using a leaf temperature for all plant species equal to air temperature within the layer. Thus, no long-wave radiation exchange between plant species within a canopy layer is considered, and emitted long-wave radiation is biased by the difference between canopy air temperature and leaf temperature. However, these simplifications are not significant for most situations.

Sensible and Latent Heat Fluxes

Sensible and latent heat flux components of the surface energy balance are computed from temperature and vapor gradients between the canopy-residue-soil surface and the atmosphere. Sensible heat flux is calculated from (Campbell, 1977):

$$\mathbf{H} = - \rho_a c_a \frac{(T - T_a)}{r_H} \quad (17)$$

where ρ_a , c_a and T_a are the density (kg m^{-3}), specific heat ($\text{J kg}^{-1} \text{C}^{-1}$) and temperature (C) of air at the measurement reference height z_{ref} , T is the temperature (C) of the exchange surface, and r_H is the resistance to surface heat transfer (s m^{-1}) corrected for atmospheric stability. Here, the exchange surface is either the top of the canopy, the residue layer, the snow surface or the soil surface depending on the system profile. Latent heat flux is associated with transfer of water vapor from the exchange surface to the atmosphere, which is given by

$$\mathbf{E} = \frac{(\rho_{vs} - \rho_{va})}{r_v} \quad (18)$$

where ρ_{vs} and ρ_{va} are vapor density (kg m^{-3}) of the exchange surface and at the reference height z_{ref} and the resistance value for vapor transfer, r_v , is taken to be equal to r_H . The resistance to convective heat transfer, r_H , is computed from

$$r_H = \frac{1}{u_* k} \left[\ln \left(\frac{z_{ref} - d + z_H}{z_H} \right) + \Psi_H \right] \quad (19)$$

where u_* is the friction velocity (m s^{-1}) computed from

$$u_* = uk \left[\ln \left(\frac{z_{ref} - d + z_m}{z_m} \right) + \Psi_m \right]^{-1}, \quad (20)$$

k is von Karman's constant, d is the zero plane displacement, z_H and z_m are the surface roughness parameters for the temperature and momentum profiles, and Ψ_H and Ψ_m are diabatic correction factors for heat and momentum, computed as a function of atmospheric stability. Atmospheric stability is calculated as a ratio of thermally induced to mechanically induced turbulence (Campbell, 1977):

$$s = \frac{kz_{ref} g\mathbf{H}}{\rho_a c_a T u_*^3} \quad (21)$$

where g is gravitational acceleration. Under stable conditions ($s > 0$),

$$\psi_H = \psi_m = 4.7s . \quad (22)$$

For unstable conditions, ($s < 0$), ψ_m is approximately $0.6 \psi_H$ (Norman, 1979) and

$$\psi_H = -2 \ln \left(\frac{1 + \sqrt{1 - 16s}}{2} \right) . \quad (23)$$

If a plant canopy is present, the surface roughness parameter for the momentum profile, z_m , is taken as 0.13 times the plant canopy height and the zero plane displacement, d , is 0.77 times canopy height. Otherwise, the user-supplied value for z_m is used and d is set to zero. Surface roughness parameter for the temperature profile, z_H is assumed to be $0.2 z_m$.

Ground Heat Flux

Ground heat flux, computed from the residual of the energy balance, must satisfy the solution of the heat flux equations for the entire residue/soil profile, which is solved simultaneously and iteratively with the surface energy balance. Details of heat and water flux equations for the plant canopy, snow, residue and soil are described in the following sections.

Energy Fluxes Within the System

Heat Flux Through the Canopy

Heat and vapor fluxes within the canopy are determined by computing transfer between layers of the canopy and considering the source terms for heat and transpiration from the canopy leaves for each layer within the canopy. Gradient-driven transport, or K-theory, is used for transfer within the canopy. Lagrangian approaches for modeling turbulent transport processes within the canopy have been successfully applied by some (e.g., Raupach, 1989) as an alternative to K-theory since studies indicate that K-theory is not applicable in the canopy air space (Denmead and Bradley, 1985). However, Lagrangian approaches are not yet ready for general application as further details need to be worked out, e.g. limitations in simulating nighttime fluxes. Nevertheless, many have found K-theory particularly useful for simulating transport processes within the canopy (van de Griend and van Boxel, 1989; Goudriann, 1989; Nichols, 1992; and Huntingford et al., 1995) while other studies indicate relatively small differences in flux predictions between Lagrangian and K-theory (e.g., Dolman and Wallace, 1991). Thus, heat flux and temperature within the air space of the canopy are described by

$$\rho_a c_a \frac{\partial T}{\partial t} = \frac{\partial}{\partial z} \left(\rho_a c_a k_e \frac{\partial T}{\partial z} \right) + H_l \quad (24)$$

where the terms (W m^{-3}) represent: an energy storage term, which is negligible and not considered in the model; net heat transfer into a layer within a canopy; and a heat source term for heat transfer from the canopy elements (leaves) to the air space within the canopy. In this equation, ρ_a , c_a and T are density (kg m^{-3}), specific heat capacity ($\text{J kg}^{-1} \text{C}^{-1}$) and temperature (C) of the air within the canopy, t is time (s), z is height from the top of the canopy (m), k_e is a transfer coefficient within the canopy ($\text{m}^2 \text{s}^{-1}$), and H_l is heat transferred from the vegetation elements (leaves) to the air space within the canopy (W m^{-3}).

Transfer within the canopy

Transfer of heat and vapor within the canopy is dependent upon location within the canopy, and several approaches for computing the transfer coefficient k_e have been developed. Flerchinger and Pierson (1991) used the following expression above the zero plane displacement, d :

$$k_e = ku_*(z - d + z_H) / \phi_H, \quad (25)$$

and for heights less than d ,

$$k_e = ku_*z_H / \phi_H \quad (26)$$

where k is von Karman's constant, u_* is the friction velocity (m s^{-1}), z is height above the residue or soil surface (m), d is the height of the zero plane displacement (m), z_H is the thermal surface roughness parameter (m), and ϕ_H is a diabatic correction factor dependent on the Richardson number computed from H . Flerchinger et al. (1998) found that these relations gave better results for observed conditions than alternative expressions employed by van de Griend and van Boxel (1989), Nichols (1992), and Huntingford et al. (1995).

Heat transfer from the canopy elements

Heat transfer from the vegetation elements (leaves) to the air space within a canopy layer for a given plant species (W m^{-2}) is computed from:

$$H_{l,i,j} = - \rho_a c_a L_{i,j} \frac{(T_{l,i,j} - T_i)}{r_{h,i,j}}. \quad (27)$$

Here, $L_{i,j}$, and $T_{l,i,j}$, are leaf area index and leaf temperature of plant species j within canopy layer i , T_i is air temperature within canopy layer i , and resistance to convective transfer from the canopy leaves per unit leaf area index, $r_{h,i,j}$ (s m^{-1}), is computed from (Campbell, 1977)

$$r_{h,i,j} = 307 \left(\frac{d_l}{u_i} \right)^{1/2} \quad (28)$$

where d_l is leaf dimension (m), u_i is wind speed within the canopy layer (m s^{-1}), and 307 is a coefficient ($\text{s}^{1/2} \text{m}^{-1}$) for the thermal diffusivity and viscosity of air. Leaf temperature for each layer

within the canopy ($T_{l,i,j}$) is determined from a leaf energy balance of the canopy layer assuming the leaves within the canopy have negligible heat capacity:

$$R_{n,i,j} + H_{l,i,j} + L_v E_{l,i,j} = 0 \quad . \quad (29)$$

Here, $R_{n,i,j}$ is net all-wave radiation (W m^{-2}) for the leaf surfaces within canopy layer i for plant species j , L_v is the latent heat of vaporization, and $E_{l,i,j}$ is vapor flux ($\text{kg s}^{-1} \text{m}^{-2}$) from the leaf surfaces. Water uptake, transpiration and leaf temperature are coupled through the energy balance of the leaf, which is calculated for each plant species within each canopy layer. The leaf energy balance is computed iteratively with heat and water vapor transfer equations (Eqns. 24 and 46) and transpiration within the canopy (Eqns. 47 and 49).

Heat Flux within the Snow

The energy balance for each layer within the snowpack is written as follows:

$$\rho_{sp} c_i \frac{\partial T}{\partial t} + \rho_l L_f \frac{\partial w_{sp}}{\partial t} = \frac{\partial}{\partial z} \left[k_{sp} \frac{\partial T}{\partial z} \right] + \frac{\partial R_n}{\partial z} - L_s \left(\frac{\partial q_v}{\partial z} + \frac{\partial \rho_v}{\partial t} \right) \quad (30)$$

where the terms (W m^{-3}) represent, respectively: specific heat term for change in energy stored due to a temperature increase; latent heat required to melt snow; net thermal conduction into a layer; net radiation absorbed with a layer; and net latent heat of sublimation. Heat transferred by liquid movement in the snowpack is not considered in the energy balance equation; at the end of each time step a mass balance of the snowpack is computed to adjust the snowpack for melt, water percolation, and thermal advection. Symbols in the above equation are as follows: ρ_{sp} , w_{sp} , and k_{sp} are density (kg m^{-3}), volumetric liquid water content ($\text{m}^3 \text{m}^{-3}$), and thermal conductivity and the snow ($\text{W m}^{-1} \text{C}^{-1}$); c_i is specific heat capacity of ice ($\text{J kg}^{-1} \text{C}^{-1}$); θ_a is the volumetric air space within the snow ($\text{m}^3 \text{m}^{-3}$); ρ_l is density of water (kg m^{-3}); R_n is net downward radiation flux within the snow (W m^{-2}); L_f and L_s are latent heat of fusion and sublimation (J kg^{-1}); q_v is vapor flux ($\text{kg s}^{-1} \text{m}^{-2}$); and ρ_v is vapor density (kg m^{-3}) within the snow.

Specific heat

At temperatures below 0 °C, net energy absorbed by the snow results in a change in temperature. The volumetric specific heat of snow is computed from the density of the snow, ρ_{sp} and the specific heat of ice, which is a function of temperature (Anderson, 1976):

$$c_i = 92.96 + 7.37T_K \quad (31)$$

where T_K is temperature of the snow in Kelvin.

Latent heat of fusion

At 0 °C, the net energy absorbed by the snowpack results in melting of ice. Ice content of the snowpack is assumed constant over the hour time-step and is adjusted for any melt at the end of the time step.

Thermal conduction

The primary mechanism for energy transfer within a snowpack is thermal conduction between and within ice crystals. Thermal conductivity of snow has been empirically related to density by many researchers, although geometry of the snow crystals is important as well. An expression of the form

$$k_{sp} = a_{sp} + b_{sp}(\rho_{sp}/\rho_l)^{c_{sp}} \quad (32)$$

is suggested by Anderson (1976) and will fit many empirically derived correlations. Here a_{sp} , b_{sp} , and c_{sp} are empirical coefficients; Anderson (1976) suggests values of 0.021 W m⁻¹ C⁻¹, 2.51 W m⁻¹ C⁻¹ and 2.0, respectively.

Radiation absorption

Because snow is translucent, solar radiation entering the surface of the snowpack is attenuated and absorbed throughout the snowpack. The net solar radiation flux at a depth z can be expressed as

$$S_z = (S_s + S_d)(1 - \alpha_{sp}) e^{-\nu z}. \quad (33)$$

where $(S_s + S_d)$ is the total solar radiation incident on the snow surface. The extinction coefficient, ν , for radiation penetration through the snow is calculated from (Anderson, 1976)

$$\nu = 100. C_v(\rho_{sp}/\rho_l)d_s^{-1/2} \quad (34)$$

where C_v is taken as 1.77 mm^{1/2} cm⁻¹ (Flerchinger et al., 1996a) and d_s is grain-size diameter of ice crystals (mm; Eqn. 11).

Latent heat of sublimation

Latent heat transfer by sublimation is a result of vapor transfer through the snowpack in response to temperature gradients. Vapor density in snow is assumed equal to the saturated vapor density over ice, and therefore is solely a function of temperature. Warmer parts of the snowpack have a higher vapor density; vapor will therefore diffuse toward cooler parts, where, due to over-saturation, sublimation will occur and latent heat is released. Vapor flux through the snowpack is calculated by

$$q_v = D_e \frac{\partial \rho_v}{\partial z} \quad (35)$$

where D_e is the effective diffusion coefficient (m s^{-2}) for water vapor in snow, and ρ_v is the temperature-dependent vapor density within the snow. The net latent heat of sublimation for a layer in the snowpack is equal to the increase in vapor density minus the net transfer of vapor to that layer.

Heat Transport Processes in the Residue

The energy balance for a layer of plant residue is expressed as

$$C_r \frac{\partial T}{\partial t} = \frac{\partial}{\partial z} \left(k_r \frac{\partial T}{\partial z} \right) - L_v \frac{\partial}{\partial z} \left(\frac{(h_r \rho_{vs}' - \rho_v)}{r_h} \right) + \frac{\partial R_n}{\partial z} \quad (36)$$

where the terms (W m^{-3}) represent, respectively: specific heat term for change in energy stored due to a temperature increase; net thermal convection/conduction into a layer; net latent heat of evaporation from residue elements; and net absorption of radiant heat. Symbols here are defined as follows: C_r and T are volumetric heat capacity ($\text{J m}^{-3} \text{C}^{-1}$) and temperature (C) of residue; k_r is heat transfer coefficient within the residue ($\text{W m}^{-1} \text{C}^{-1}$); h_r is relative humidity within the residue elements; ρ_{vs}' is the saturated vapor density (kg m^{-3}) of the residue elements; ρ_v is vapor density (kg m^{-3}) of the air within the residue layer; r_h is a boundary layer resistance (s m^{-1}) between residue elements and the air space within the residue layer; and R_n is the net downward radiation flux within the residue. This equation includes the assumption that residue elements and surrounding air voids within a layer are in thermal equilibrium.

Heat capacity

The volumetric heat capacity of residue is computed from the specific heat of residue and water, weighted according to their volume fractions and assuming the specific heat of air is negligible. It is calculated as

$$C_r = \rho_r (c_r + w_r c_l) \quad (37)$$

where ρ_r , c_r , and w_r are density, specific heat capacity (taken as $1900 \text{ J kg}^{-1} \text{C}^{-1}$), and gravimetric water content (kg kg^{-1}) of the residue, and c_l is the specific heat capacity of water ($\text{J kg}^{-1} \text{C}^{-1}$).

Thermal convection/conduction

Heat is transferred through the residue by conduction through residue elements and convection through air voids. The relative magnitude of these two processes depends on wind speed within the residue, and density and moisture content of the residue. Based on results from Kimball and Lemon (1971), Bristow et al. (1986) assumed that thermal convection through crop residue increases linearly with wind speed, but neglected the effect of residue density. The following equation was taken from Bristow et al. (1986) and modified for density of the residue:

$$k_v = k_a (1 + 0.007T) (1 + 4u_r) (1 - \rho_r / \rho_{rs}) \quad (38)$$

where u_r is wind speed within the residue, ρ_{rs} is specific density of the residue, and k_a is the thermal conductivity of still air. Wind speed at the surface of the residue is calculated assuming a logarithmic wind profile to the height of the residue or an exponentially decreasing profile within the plant canopy; wind speed within the residue is assumed to decrease linearly with depth to a value of zero at the soil surface.

Thermal conduction within the residue is dependent largely on moisture content and is calculated as a weighted average of the conductivities of residue and water:

$$k_t = k_{rs}(\rho_r/\rho_{rs}) + k_l w_r(\rho_r/\rho_l) \quad (39)$$

where k_{rs} is thermal conductivity of the residue material, k_l is thermal conductivity of water, and w_r is gravimetric water content of the residue. The total heat transfer coefficient of the residue, k_r , is the sum of the convection and conduction coefficients.

Latent heat of evaporation

Latent heat is required to evaporate liquid water from the residue elements to vapor within the air voids of the residue layer. The rate of evaporation depends on the vapor density within the void spaces and the water content of the residue. Details for evaporation from the residue is given subsequently in the section describing water flux within the residue.

Heat Transport Processes in the Soil

The state equation for temperature distribution in the soil matrix, considering convective heat transfer by liquid and latent heat transfer by vapor for a layer of freezing soil is given by:

$$C_s \frac{\partial T}{\partial t} - \rho_i L_f \frac{\partial \theta_i}{\partial t} = \frac{\partial}{\partial z} \left[k_s \frac{\partial T}{\partial z} \right] - \rho_l c_l \frac{\partial q_l T}{\partial z} - L_v \left(\frac{\partial q_v}{\partial z} + \frac{\partial \rho_v}{\partial t} \right) \quad (40)$$

where the terms (W m^{-3}) represent, respectively: specific heat term for change in energy stored due to a temperature increase; latent heat required to freeze water; net thermal conduction into a layer; net thermal advection into layer due to water flux; net latent heat evaporation within the soil layer. In the above equation, C_s and T are volumetric heat capacity ($\text{J kg}^{-1} \text{C}^{-1}$) and temperature (C) of the soil, ρ_i is density of ice (kg m^{-3}), θ_i is volumetric ice content ($\text{m}^3 \text{m}^{-3}$), k_s is soil thermal conductivity ($\text{W m}^{-1} \text{C}^{-1}$), ρ_l is density of water, c_l is specific heat capacity of water ($\text{J kg}^{-1} \text{C}^{-1}$), q_l is liquid water flux (m s^{-1}), q_v is water vapor flux ($\text{kg m}^{-2} \text{s}^{-1}$), and ρ_v is vapor density (kg m^{-3}) within the soil.

Specific Heat

Volumetric heat capacity of soil, C_s , is the sum of the volumetric heat capacities of the soil constituents:

$$C_s = \sum \rho_j c_j \theta_j \quad (41)$$

where ρ_j , c_j , and θ_j are the density, specific heat capacity and volumetric fraction of the j^{th} soil

constituent.

Latent Heat of Fusion

Due to matric and osmotic potentials, soil water exists in equilibrium with ice at temperatures below the normal freezing point of bulk water, and over the entire range of soil freezing temperatures normally encountered. A relation between ice content and temperature must therefore be defined before latent heat of fusion can be determined. The total potential of the soil water with ice present is controlled by the vapor pressure over ice, and is given by the freezing point depression equation (Fuchs et al., 1978):

$$\phi = \pi + \psi = \frac{L_f}{g} \left(\frac{T}{T_K} \right) \quad (42)$$

where π is soil water osmotic potential (m), and ψ is soil matric potential. Osmotic potential in the soil is computed from

$$\pi = -cRT_K/g \quad (43)$$

where c is solute concentration (eq kg⁻¹) in the soil solution. Given the osmotic potential, soil temperature defines the matric potential and, therefore, liquid water content. If the total water content is known, ice content and the latent heat term can be determined.

Thermal conduction

Thermal conductivity of the soil is calculated using the theory presented by De Vries (1963). A fairly moist soil is conceptualized as a continuous medium of liquid water with granules of soil, crystals of ice, and pockets of air dispersed throughout. The thermal conductivity of such an idealized model is expressed as

$$k_s = \frac{\sum m_j k_j \theta_j}{\sum m_j \theta_j} \quad (44)$$

where m_j , k_j , and θ_j , are the weighting factor, thermal conductivity, and volumetric fraction of the j^{th} soil constituent, i.e. sand, silt, clay, organic matter, water, ice and air. The method used for determining values for the weighting factor, m_j , is discussed by De Vries (1963).

Latent heat of vaporization

Net latent heat of vaporization occurring in a soil layer is computed from the rate of increase in vapor density minus the net vapor transfer into the layer. Vapor density in the soil is calculated assuming equilibrium with total water potential by:

$$\rho_v = h_r \rho'_v = \rho'_v \exp\left(\frac{M_w g}{RT_K} \phi\right) \quad (45)$$

where ρ_v is vapor density (kg m^{-3}), ρ_v' is saturated vapor density, h_r is relative humidity, M_w is molecular weight of water ($0.018 \text{ kg mole}^{-1}$), g is acceleration of gravity (9.81 m s^{-2}), R is universal gas constant ($8.3143 \text{ J mole}^{-1} \text{ K}^{-1}$), and ϕ is total water potential (m).

Water Fluxes Within the System

Water Flux Through the Canopy

Vapor flux through the canopy is written similarly to the heat flux equation:

$$\frac{\partial \rho_v}{\partial t} = \frac{\partial}{\partial z} \left(k_e \frac{\partial \rho_v}{\partial z} \right) + E_l \quad (46)$$

where the terms ($\text{kg s}^{-1} \text{ m}^{-3}$) represent: net change in vapor contained within a layer; net vapor flux into a canopy layer; and a source term for transpiration/evaporation from the canopy elements leaves within the canopy layer. Here, E_l is transpiration or evaporation from the leaves within the canopy and other term are defined previously.

Vapor transfer within the canopy

The transfer coefficient for vapor flux within the canopy, k_e , is assumed equal to that for heat transfer within the canopy and is described previously.

Vapor transfer from canopy elements

Vapor transfer for a given plant species within a canopy layer, $E_{l,i,j}$, is computed from

$$E_{l,i,j} = L_{i,j} \frac{\rho_{vs,i,j} - \rho_{v,i}}{r_{s,i,j} + r_{h,i,j}} \quad (47)$$

where $L_{i,j}$ is the leaf area index, $\rho_{vs,i,j}$ and $\rho_{v,i}$ are vapor density (kg m^{-3}) of plant canopy elements (i.e. leaves) and of air within the canopy, $r_{s,i,j}$ is stomatal resistance per unit of leaf area index (s m^{-1}), and the subscripts refer to plant species j within canopy layer i .

Evaporation from canopy elements

Evaporation within the canopy is computed for non-transpiring standing dead plant material and transpiring leaves with free water on the leaves (from dew formation or intercepted rainfall). In either case, evaporation is computed from Eqn. (47) with the stomatal resistance taken as zero.

In the case of free water on the leaves, vapor density at the surface of the leaves is taken as the saturated vapor density for the computed leaf temperature. Vapor density of the standing dead plant material is computed from:

$$\rho_{vs} = h_r \rho_{vs}' = \rho_{vs}' \exp\left(\frac{M_w g}{RT_K} \psi\right) = \rho_{vs}' \exp\left(\frac{M_w g}{RT_K} a_c w_c^{b_c}\right) \quad (48)$$

where w_c is the water content (kg kg^{-1}) of the dead plant material, and a_c and b_c are coefficients with suggested values of -53.72 m and 1.32 .

Transpiration from canopy leaves

Plant stomates are assumed to close if light or temperature conditions are not adequate for transpiration. If incoming solar radiation, S_t is less than 10 W m^{-2} , or if the air temperature T_a is colder than a specified minimum air temperature, transpiration is set to zero and there is no vapor transfer from the canopy elements for the given plant species. However, evaporation of free water from the plant leaves may be considered as described in the previous section.

Transpiration within a canopy layer, $E_{l,i,j}$ (W m^{-2}), is determined assuming a soil-plant-atmosphere continuum. Water flow is calculated assuming continuity in water potential throughout the plants as illustrated in Figure 2 and may be calculated at any point in the plant from

$$T_j = \sum_{k=1}^{NS} \frac{\psi_k - \psi_{x,j}}{r_{r,j,k}} = \sum_{i=1}^{NC} \frac{\psi_{x,j} - \psi_{l,i,j}}{r_{l,i,j}} = \sum_{i=1}^{NC} \frac{\rho_{vs,i,j} - \rho_{v,i}}{r_{s,i,j} + r_{h,i,j}} L_{i,j} \quad (49)$$

Here, T_j is total transpiration rate ($\text{kg m}^{-2} \text{ s}^{-1}$) for plant species j ; ψ_k , $\psi_{x,j}$ and $\psi_{l,i,j}$ are water potential (m) in layer k of the soil, in the plant xylem of plant species j , and in the leaves of canopy layer i ; $r_{r,j,k}$ and $r_{l,i,j}$ are the resistance to water flow ($\text{m}^3 \text{ s kg}^{-1}$) through the roots of soil layer k and the leaves of canopy layer i for plant species j ; $\rho_{vs,i,j}$ and $\rho_{v,i}$ are the vapor density (kg m^{-3}) within the stomatal cavities (assumed to be saturated vapor density) of plant species j and of the air within canopy layer

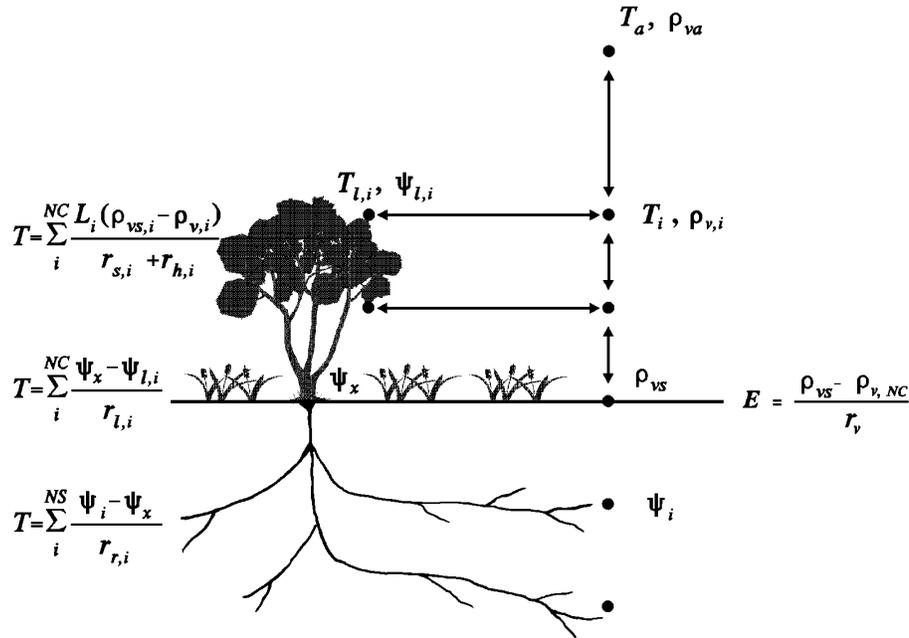


Figure 2. Physical representation of water flow through a plant in response to transpiration demands. (ρ_g is vapor density at the ground surface and r_v is resistance to vapor transfer within the canopy and equal to $\Delta z/k_e$; all other symbols are defined in the text.)

i ; NS and NC are the number of soil and canopy nodes; and other terms are as described previously. Root resistance for each plant species within each soil layer is calculated by dividing total root resistance for the plant by its fraction of roots within the soil layer. Leaf resistance for each plant species within each canopy layer is computed from total leaf resistance for the plant based on its leaf area index within each canopy layer. Transpiration from the leaves of each plant species within each canopy layer, $E_{l,i,j}$, is computed from the last term in the above equation.

Water flow within the plant is controlled mainly by changes in stomatal resistance. A simple equation relating stomatal resistance to leaf water potential is (Campbell, 1985)

$$r_s = r_{so} [1 + (\psi_l / \psi_c)^n] \quad (50)$$

where r_{so} is stomatal resistance (m s^{-1}) with no water stress (assumed constant), ψ_c is a critical leaf water potential (m) at which stomatal resistance is twice its minimum value, and n is an empirical coefficient which has typically been set to 5 (Flerchinger et al., 1996b; and Flerchinger and Pierson, 1997). Sensitivity of model simulations to stomatal resistance parameters was presented by Flerchinger and Pierson (1997). Equations relating stomatal resistance to leaf temperature, vapor pressure deficit, soil moisture deficit, and solar irradiance have been developed (e.g., Dolman, 1993, and Mihailović and Ruml, 1996), however estimation of a separate parameter is required for each of these factors. Because these factors all have an indirect effect on leaf water potential, the above relation is very effective in estimating stomatal resistance. However, this relation admittedly does not account for direct effects of extreme temperature or solar radiation on plant stress that can occur even when water is not limiting.

Mass Balance of the Snowpack

Density and ice content of each snow layer are assumed constant during each time step while the change in liquid content is computed from the energy balance. At the end of the time step, the thickness and density of each layer are adjusted for vapor transfer and change in liquid content. Excess liquid water is routed through the snowpack using attenuation and lag coefficients to determine snowcover outflow, and density of the snow is adjusted for compaction and settling.

Snowcover outflow

The amount of liquid water that can be held in the snow due to capillary tension is computed from

$$w_{sp,hold} = w_{sp,min} + \left(w_{sp,max} - w_{sp,min} \right) \frac{\rho_e - \rho_{sp}}{\rho_e} \quad \text{for } \rho_{sp} < \rho_e \quad (51)$$

where $w_{sp,min}$ is the minimum value water holding capacity ($\text{m}^3 \text{m}^{-3}$) and applies to dense, ripe snowpacks, $w_{sp,max}$ is the maximum value of w_{sp} , and ρ_e is the snow density (kg m^{-3}) corresponding to $w_{sp,min}$. The permeability of snow is quite variable and not well defined. Therefore, after the water holding capacity of the snowpack is satisfied, excess liquid water is lagged and attenuated using empirical equations. The maximum lag in hours for snowcover of depth d_{sp} (m) is

$$L_{w,max} = C_{LI} [1 - \exp(-0.025 d_{sp} / \rho_{sp})] \quad (52)$$

where C_{L1} is the maximum allowable lag (taken to be 10 hours; Anderson, 1976). The actual lag depends on the amount of excess liquid water and is determined by

$$L_w = \frac{L_{w_{\max}}}{100C_{L2}W_x + 1} \quad (53)$$

where W_x is the depth of excess liquid water (m), and C_{L2} is an empirical coefficient (assigned to 1.0 cm^{-1}). After the excess liquid water is lagged, it is attenuated and snowcover outflow is calculated from

$$W_o = \frac{S_{sp} + W_L}{1 + C_{L3} \exp[C_{L4}W_L \rho_{sp}/(\rho_l d_{sp})]} \quad (54)$$

where W_L is the depth of lagged excess water (m), S_{sp} is the excess water in storage (m), and C_{L3} (5.0 hr) and C_{L4} (450, dimensionless) are empirical coefficients.

Density change of snowcover

Snow density changes over time due to compaction, settling, and vapor transfer. Compaction and settling of the snow are discussed in the following sections, while vapor transfer was discussed previously.

Compaction of the snow

Snow deforms continuously and permanently when a sustained load is applied. A basic equation describing the rate at which snow will deform in response to a load may be written as (Anderson, 1976)

$$\frac{1}{\rho_{sp}} \frac{\partial \rho_{sp}}{\partial t} = C_1 W_{sp} \exp(0.08T - C_2 \rho_{sp}/\rho_l) \quad (55)$$

where W_{sp} is the weight of snow (expressed in terms of centimeters of water equivalent) above the layer of snow, C_1 is the hourly fractional increase in density per load of water-equivalent (taken as $0.01 \text{ cm}^{-1} \text{ hr}^{-1}$), C_2 is an empirical coefficient (approximately 21.0), and T is snow temperature (C).

Settling of the snow

After snow falls, metamorphosis of the ice crystals in the snowpack as they change shape causes the pack to settle. This process is relatively independent of snow density up to a value, ρ_d , of about 150 kg m^{-3} . Anderson (1976) suggested the following relation for fractional increase in density due to settling:

$$\frac{1}{\rho_{sp}} \frac{\partial \rho_{sp}}{\partial t} = \begin{cases} C_3 \exp(C_4 T) & \text{for } \rho_{sp} < \rho_d \\ C_3 \exp(C_4 T) \exp[-46(\rho_{sp} - \rho_d)] & \text{for } \rho_{sp} > \rho_d \end{cases} \quad (56)$$

where C_3 is the fraction rate of settling at 0°C for densities less than ρ_d , and C_4 is an empirical coefficient (taken as $0.04 \text{ }^\circ\text{C}$). The presence of liquid water will increase the rate of settling. When liquid water is present in the snow, the fractional rate of settling computed from this equation is

multiplied by a factor, C_s (assumed equal to 2.0; Anderson, 1976).

Water Flux Through the Residue

Vapor flux through the residue is described by

$$\frac{\partial \rho_v}{\partial t} = \frac{\partial}{\partial z} \left(K_v \frac{\partial \rho_v}{\partial z} \right) + \frac{\partial}{\partial z} \left(\frac{(h_r \rho_{vs}' - \rho_v)}{r_h} \right) \quad (57)$$

where the terms represent, respectively: change in vapor density within the residue layer, net vapor flux into a residue layer, and evaporation rate from the residue elements. Here, ρ_v is vapor density (kg m^{-3}) of the air space within the residue; K_v is the convective vapor transfer coefficient within the residue (m s^{-2}), taken as $k_v/\rho_a c_a$ where ρ_a and c_a the density and specific heat capacity of air; h_r is the relative humidity within the residue elements; ρ_{vs}' is saturated vapor density at the temperature of the residue elements; and r_h is the resistance (s m^{-1}) to vapor transfer between the residue elements and the air voids within the residue layer.

Evaporation within the residue

Evaporation from the residue elements depends on humidity or water potential of the water held by the residue elements. Relative humidity of the residue elements is determined from water potential of the residue by

$$h_r = \exp\left(\frac{M_w g}{RT_K} \Psi\right) = \exp\left(\frac{M_w g}{RT_K} a_r w_r^{b_r}\right) \quad (58)$$

where w_r is water content of the residue. Typical values for the empirical coefficient a_r and b_r for wheat straw are -53.72 m and 1.32 , respectively (Myrold et. al., 1981).

Suggested values for resistance to vapor transfer between residue elements and air voids, r_{vr} , is $1000\text{-}50,000$ s m^{-1} . (Potential modification to the model may include calculation of r_{vr} from windspeed within the residue.)

Water Flux Through the Soil

The soil water flux equation for with provisions for freezing and thawing soil is written as:

$$\frac{\partial \theta_l}{\partial t} + \frac{\rho_i}{\rho_l} \frac{\partial \theta_i}{\partial t} = \frac{\partial}{\partial z} \left[K \left(\frac{\partial \Psi}{\partial z} + 1 \right) \right] + \frac{1}{\rho_l} \frac{\partial q_v}{\partial z} + U \quad (59)$$

where the terms ($\text{m}^3 \text{m}^{-3} \text{s}^{-1}$) represent, respectively: change in volumetric liquid content; change in volumetric ice content; net liquid flux into a layer; net vapor flux into a layer; and a source/sink term for water extracted by roots;. In this equation, K is unsaturated hydraulic conductivity (m s^{-1}),

ψ is soil matric potential (m), and U is a source/sink term for water flux ($\text{m}^3 \text{m}^{-3} \text{s}^{-1}$).

Liquid flux

The relation assumed for the moisture characteristic equation is (Brooks and Corey, 1966; Campbell, 1974)

$$\Psi = \Psi_e \left(\frac{\theta_l}{\theta_s} \right)^{-b} \quad (60)$$

where Ψ_e is air entry potential (m), b is a pore size distribution parameter, and θ_s is saturated water content ($\text{m}^3 \text{m}^{-3}$). Unsaturated hydraulic conductivity is computed from

$$K = K_s \left(\frac{\theta_l}{\theta_s} \right)^{(2b + 3)} \quad (61)$$

where K_s is saturated hydraulic conductivity (m s^{-1}). Water flow in frozen soil is assumed analogous to that in unsaturated soil (Cary and Mayland, 1972; and Miller, 1963). Therefore, the relationships for matric potential and hydraulic conductivity of unsaturated soils are assumed valid for frozen soils. However, hydraulic conductivity is reduced linearly with ice content assuming zero conductivity at an available porosity of 0.13 (Bloomsburg and Wang, 1969).

Vapor flux

Vapor transfer in the soil is calculated as the sum of the gradient in vapor density due to a water potential gradient, q_{vp} , and that due to a temperature gradient, q_{vT} (Campbell, 1985), where

$$q_v = q_{vp} + q_{vT} = -D_v \rho_v \frac{dh_r}{dz} - \zeta D_v h_r s_v \frac{dT}{dz} \quad (62)$$

Here D_v is vapor diffusivity ($\text{m}^2 \text{s}^{-1}$) in the soil, h_r is relative humidity within the soil, s_v is the slope of the saturated vapor pressure curve ($d\rho_v'/dT$ in $\text{kg m}^{-3} \text{C}^{-1}$), and ζ is an enhancement factor. Vapor density in the soil is related to vapor diffusivity in air by

$$D_v = D_v' b_v \theta_a^{c_v} \quad (63)$$

Here, D_v' is diffusivity of water vapor in air, θ_a is air porosity, and b_v and c_v coefficients accounting for the tortuosity of the air voids with values of 0.66 and 1.0, respectively (Campbell, 1985). Observed vapor transfer in response to a temperature gradient exceeds that predicted by Eqn. (62), therefore an enhancement factor is included, which is calculated from (Cass et al., 1984)

$$\zeta = E_1 + E_2(\theta_l / \theta_s) - (E_1 - E_4)\exp\left(- (E_3 \theta_l / \theta_s)^{E_5}\right) \quad (64)$$

where E_1, E_2, E_4 and E_5 have assigned values of 9.5, 3.0, 1.0, and 4.0, respectively. E_3 is calculated from clay content by $\theta_s(1 + 26(\% \text{clay})^{-1/2})$. The slope of the saturated vapor density curve is expressed very accurately for typical temperature ranges using the empirical equation given by

$$s_v = 0.0000165 + 4944.43 \rho_v / T_K^2 . \quad (65)$$

Ice content

Unknowns in Eqns. (40), (59) and (60) are temperature, water content, ice content, and matric potential so an additional equation is needed for a solution. This is provided by the Clausius-Clapeyron equation. When ice is present, total water potential is equal to the matric potential and is related to temperature by (Fuchs et al. 1978):

$$\phi = \pi + \psi = \frac{L_f}{g} \left(\frac{T}{T_K} \right) \quad (66)$$

where g is the acceleration of gravity (m s^{-2}) and π is osmotic potential (m). Thus, as temperature drops, water potential becomes more negative, creating a gradient in water potential and causing moisture movement toward the freezing front. Osmotic potential within the soil is computed from

$$\pi = -cRT_K/g \quad (67)$$

where c is solute concentration (eq kg^{-1}) in the soil solution. From Eqns. (60), (66), and (67), liquid water content is defined by temperature during freezing conditions; soil water content greater than that computed from these relations is assumed to be ice.

Solute Fluxes

The SHAW model accounts for solute absorption by the soil matrix, and considers three processes of solute transfer: molecular diffusion, convection, and hydrodynamic dispersion. The transient, solute flux equation may be written as:

$$\rho_b \frac{\partial S}{\partial t} = \rho_l \frac{\partial}{\partial z} \left((D_H + D_m) \frac{\partial c}{\partial z} \right) - \rho_l \frac{\partial (q_l c)}{\partial z} - \rho_b V \quad (68)$$

where the terms ($\text{eq m}^{-3}\text{s}^{-1}$) represent: rate of change of total solute in a soil layer; net solute flux

due to combined effects of diffusion and dispersion; net solute flux due to convection; and a sink term for loss of solutes by degradation and root extraction. Here, ρ_b is soil bulk density (kg m^{-3}), S is total solutes present per mass of soil (eq kg^{-1}); D_H is the hydrodynamic dispersion coefficient ($\text{m}^2 \text{s}^{-1}$), D_m is the molecular diffusion coefficient ($\text{m}^2 \text{s}^{-1}$); q_l is liquid water flux (m s^{-1}); c is solute concentration in soil solution (eq kg^{-1}); and V is a sink term for solute degradation and extraction by roots ($\text{eq kg}^{-1}\text{s}^{-1}$). Several types of solutes may be modeled simultaneously with the SHAW model, however solutes are assumed to be non-interacting with other solutes.

Molecular diffusion

Diffusion of solutes through soil is affected by moisture content and tortuosity, and is related to that in free water by (Campbell, 1985; and Bolz and Tuve, 1976)

$$D_m = D_o \tau \theta_l^3 (T_K/273.16) \quad (69)$$

where D_o is the diffusion coefficient of a given solute in water at 0°C ($\text{m}^2 \text{s}^{-1}$) and τ is a soil-dependent constant for tortuosity.

Solute convection

Solute convection by moisture movement can occur only in the direction of moisture flow and is proportional to moisture flux and solute concentration. Calculation of solute transport by convection alone assumes uniform velocity in all pores and therefore does not account for dispersion of salts. Dispersion of solutes due to nonuniform velocity is accounted for in the hydrodynamic dispersion coefficient.

Solute dispersion

Solutes are transported by convection at the mean velocity of moisture flow, but are dispersed about the mean velocity due to differences in velocity between and within soil pores. The hydrodynamic dispersion coefficient depends on the average flow velocity and is calculated from (Bresler, 1973)

$$D_H = \kappa q_l / \theta_l \quad (70)$$

where κ is a soil-dependent constant (m).

Solute sink terms

Solute of a given type can be lost from the soil by degradation or extraction by roots. Solute degradation, if specified by the user, is assumed to follow an exponential decay. Solute degradation for a time step Δt (s) is computed from:

$$V = S \left[1 - \exp\left(\frac{\Delta t \ln(0.5)}{86,400 t_{1/2}}\right) \right] \quad (71)$$

where $t_{1/2}$ is the half-life (d) of the solute. Solute extraction from the soil by roots is assumed non-selective and equal to the concentration of solutes within the soil solution extracted by the roots.

Solute absorption

A linear absorption equation is assumed for the equilibrium balance between solute concentration in the soil solution and that absorbed onto the soil matrix. The relation is expressed as

$$S = \left(K_d + \frac{\rho_l \theta_l}{\rho_b} \right) c \quad (72)$$

where K_d is partitioning coefficient between the soil matrix and the soil solution (kg kg^{-1}). For a completely mobile solute (not absorbed by the soil), $K_d = 0$. A typical value of K_d for phosphorus, an ion strongly absorbed to the soil, is approximately 60 kg kg^{-1} (Campbell, 1985).

Lower Boundary Conditions

A couple of options are available for specifying the conditions for heat and water flux at the lower boundary. Soil temperature and water content at the lower boundary may be either specified by the user or model-estimated. User-specified temperature and water content at the lower boundary are input through the temperature and water input files. The model linearly interpolates between input values on different dates to obtain the temperature or water content at the lower boundary for each time step. Thus, at least two input profiles (the initial profile and another on or beyond the last day of simulation) are required for user-specified temperature or water content.

If model-estimated soil water content at the lower boundary is specified, the gradient for water flux at the lower boundary is assumed to be due to gravity alone. Under this assumption, the matric potential gradient term in Eqn. (59) becomes zero, leaving the gravity term, which is unity. Thus, this lower boundary condition is sometimes referred to as a unit gradient. Water flux is equal to the unsaturated hydraulic conductivity for the existing water content at the lower boundary.

The model will optionally estimate soil temperature at the lower boundary at the end of the time step based on a weighting the temperature at the bottom two soil layers, the depth of the profile, and the damping depth for the lower boundary. End-of-time-step temperature at the lower boundary is computed from:

$$T_{NS}^{j+1} = (1-A_T)T_{NS}^j + A_T T_{NS-1}^j \quad (73)$$

where subscripts NS and $NS-1$ denote the bottom soil layer and the layer above it, and superscripts denote beginning (j) and end ($j+1$) of time step values. The value for A_T is estimated using the annual damping depth of the depth of the bottom soil layer from:

$$A_T = \frac{\Delta t}{24} \left[-0.00082 + \frac{0.00983957 d_d}{z_{NS} - z_{NS-1}} \right] \left(\frac{z_{NS}}{d_d} \right)^{-0.381266} \quad (74)$$

where, Δt is the time step in hours. Damping depth (m) is computed from

$$d_d = \left(\frac{2k_s}{C_s \omega} \right)^{1/2} \quad (75)$$

where ω is the radial frequency ($1.99238 \times 10^{-7} \text{ s}^{-1}$) of the annual temperature oscillation.

Precipitation and Infiltration

Precipitation and snowmelt are computed at the end of each time step after the heat, water and solute fluxes are computed for the time step. Moisture and temperature conditions of the plant canopy, snow, residue and soil are adjusted for absorption, interception and infiltration of rainfall or snowmelt.

Snow Accumulation

Precipitation is assumed to be snow if one of two conditions exist: the wet-bulb air temperature is below a specified temperature; or a non-zero value for snow density is input for the time step. If temperature indicates snow but density is unknown, newly fallen snow density (kg m^{-3}) is estimated by (Anderson, 1976)

$$\rho_{sp} = 50 + 1.7(T_{wb} + 15)^{1.5} \quad (76)$$

where T_{wb} is wet-bulb temperature (C).

When snow falls on bare soil or residue, sufficient snow is melted to reduce the surface residue or soil node to 0°C . Additional snow is divided into layers of a defined thickness (approximately 2.5 cm for surface layers). New snow falling on existing snow is allowed to fill the surface snow layer to the defined thickness. Properties of the resulting layer are the weighted average of new and existing snow. Moisture and energy from rain falling on snow are included in the mass balance calculation of the surface layer.

Interception by Canopy and Residue

The maximum fraction of precipitation (or snowmelt in the case of the residue layer) is equal to the percentage of surface covered by plants or residue when viewed vertically downward. This is defined by computing τ_b (Eqns. 4 and 8) with an incident angle (β) of 90° . However, interception is limited to a maximum depth of water on the leaves of the plant canopy and the maximum water content of the residue. Transpiring plants within the plant canopy can accumulate up to 1 mm of water equivalent per unit of leaf area through interception, which is ultimately lost to evaporation. Standing or flat plant residue can intercept water up to a maximum water content (assumed equal to the water content defined by 99.9% relative humidity in the residue).

Infiltration into Soil

Rainfall, snowmelt and ponded water are infiltrated into the soil at the end of each time step. Infiltration is calculated using a Green-Ampt approach for a multi-layered soil. The infiltration rate as a wetting front passes through layer m of a multi-layered may be written as

$$f = \frac{dF'_m}{dt'} = \frac{F'_m/\Delta\theta_l + \psi_f + \sum \Delta z_k}{\frac{F'_m}{\Delta\theta_l K_{e,m}} + \sum \frac{\Delta z_k}{K_{e,k}}} \quad (77)$$

where f is infiltration rate (m s^{-1}), $K_{e,k}$ is the effective hydraulic conductivity of layer k (m s^{-1}), ψ_f is the suction head (m) at the wetting front and is assumed numerically equal to the matric potential of the layer, $\Delta\theta_l$ is the change in water content as the wetting front passes, F'_m is the accumulated infiltration (m) into layer m , t' is the time (s) since the wetting front entered layer m , and $\sum \Delta z_k$ is the depth (m) to the top of layer m . Effective hydraulic conductivity for infiltration is determined by substituting the effective porosity, computed from $(\theta_s - \theta_i)$, for θ_l in Eqn. (61). Conductivity is then reduced linearly depending on ice content and assuming zero conductivity at an available porosity of 0.13 (Bloomsburg and Wang 1969). The above equation may be integrated and written in dimensionless form as

$$t_* = (z_* - 1) \ln(1 + F_*) + F_* \quad (78)$$

where

$$f_* = \frac{f}{K_{e,m}} \quad (79)$$

$$F_* = \frac{F'_m}{\Delta\theta_l(\psi_f + \sum \Delta z_k)} \quad (80)$$

$$t_* = \frac{K_{e,m} t'}{\Delta\theta_l(\psi_f + \sum \Delta z_k)} \quad (81)$$

$$z_* = \frac{K_{e,m}}{\psi_f + \sum \Delta z_k} \sum_{k=1}^{m-1} \frac{\Delta z_k}{K_{e,k}} \quad (82)$$

Eqn. (78) is implicit with respect to F_* . By expanding the logarithmic term in to a power series, Flerchinger and Watts (1987) developed the following explicit expression for F_* :

$$F_* = \frac{1}{2} \left(t_* - 2z_* + \sqrt{(t_* - 2z_*)^2 + 8t_*} \right) \quad (83)$$

This expression is valid only if nearly-saturated flow exists behind the wetting front, which was shown to occur only if $z_* \leq 1$. When this criteria is not met, infiltration is calculated using Darcy's equation and assuming zero matric potential at the wetting front.

Rainfall or snowmelt in excess of the calculated interception and infiltration is ponded at the surface until a specified maximum depth of ponding is satisfied, after which runoff occurs. Adjustments for leaching of solutes, melting of ice and freezing any infiltrated water is addressed in the following subsections.

Solute Leaching

Molecular diffusion and hydrodynamic dispersion are neglected for solute transport and leaching upon infiltration. With these simplifications, the solute balance equation during infiltration becomes

$$\begin{aligned} \rho_b \frac{\partial S}{\partial t} &= -\rho_l \frac{\partial(q_l c)}{\partial z} \\ \rho_b(S' - S) &= \frac{\rho_l F'_{k+1}}{\Delta z_k} \left[c_{avg} - \eta c' - (1-\eta)c \right] \\ &= \frac{\rho_l F'_{k+1}}{\Delta z_k} \left[c_{avg} - \frac{\eta S' + (1-\eta)S}{K_d + \theta_l \rho_l / \rho_b} \right] \end{aligned} \quad (84)$$

where F'_{k+1} is the total water passing through layer k , c_{avg} is the average concentration of water entering the layer, S' and c' are total salts and solute concentration present in the layer after leaching, and η is a weighting factor for end-of-time-step values. Total salts in the layer after leaching can be solved directly by

$$S' = \frac{\rho_l F'_{k+1} c_{avg} / \Delta z_k + S \left(\rho_b - \frac{\rho_l (1-\eta) F'_k}{K_d + \theta_l \rho_l / \rho_b} \Delta z_k \right)}{\rho_b + \frac{\rho_l \eta F'_k}{(K_d + \theta_l \rho_l / \rho_b) \Delta z_k}} \quad (85)$$

This equation assumes moisture movement is steady state and moisture content in the layer is not changing. Therefore, S must be adjusted for the solutes entering the layer as water fills the pores prior to applying this equation.

Energy calculations

Heat carried by the infiltrating water affects the temperature and ice content of the soil. If the soil is frozen, infiltrating water may cause some ice to melt, or the infiltrating water may freeze depending on the temperature of the water and soil. The final temperature and ice content of a soil

layer is calculated using conservation of energy by

$$\rho_l c_l F_k (T_{F,k} - T') = C_s (T' - T) - \rho_l L_f (\theta_i' - \theta_i) \quad (86)$$

where $T_{F,k}$ is the temperature (C) of the water entering layer k ; T' and θ_i' are the temperature (C) and ice content ($\text{m}^3 \text{ m}^{-3}$) of the layer after infiltration; and all water draining out of layer k is at temperature T' . If θ_i' is known to be zero, T' can be solved directly. Otherwise, ice content is a function of the total water content and final temperature. In this case, T' is initially assumed equal to the freezing point of water in the soil, which is calculated from

$$T_{frz} = 273.16 \left(\frac{\phi}{L_f / g - \phi} \right) \quad (87)$$

where ϕ is total water potential if all water is liquid. Ice content θ_i' is then estimated using Eqn. (86). Liquid content θ_l , matric potential ψ , concentration of soil solution c , and total water potential ϕ are determined from the estimated ice content and total water content. With this information, the temperature T' and a second approximation of ice content is calculated. These updated values are sufficiently close to the true values required for energy balance because the specific heat term in Eqn. (87) is quite small compared to the latent heat term.

Numerical Implementation

The one-dimensional state equations previously presented describe energy, water and solute balance for infinitely small layers. The energy and water balance equations for layers within the plant canopy, snow residue and soil are written in implicit finite difference form and solved using an iterative Newton-Raphson technique. Finite difference approximation enables us to apply these equations to nodes representing layers of finite thickness. Flux between nodes is calculated assuming linear gradients. Energy storage for each node is based on layer thickness. A balance equation is written in terms of unknown end-of-time step values within the layer and its neighboring layers. Partial derivatives of the flux equations with respect to unknown end-of-time step values are computed, forming a tri-diagonal matrix from which the Newton-Raphson approximations for the unknown values are computed. Iterations are continued until successive approximations are within a prescribed tolerance defined by the user.

The solution for each time step involves alternating back and forth between a Newton-Raphson iteration for the heat flux equations and one for the water flux equations. An iteration is conducted for the heat flux equations and temperature estimates (water content in the case of melting snow) for the end of the time step are updated. This is followed by an iteration for the water flux equations, where updated vapor density within the canopy and residue, matric potential in unfrozen soil layers, and ice content in frozen soil layers are determined. Upon completion of the iteration for the water flux equations, the solution reverts back to an iteration for the heat flux equations with the updated values. Iterations continue until all subsequent iterations of both heat and water flux equations for each layer are within a prescribed tolerance. Thus, the heat and water flux equations are solved simultaneously, maintaining a correct balance between the two coupled equations.

After iterations for the heat and water flux equation have reached convergence, solute transport is computed using liquid fluxes from the water balance calculations. If more than one iteration is required for energy and water balance convergence, it is likely that there was sufficient moisture movement to affect solute concentrations, and the newly-calculated solute concentrations will be significantly different from those used in the energy and water balance calculations. In this case, the program returns to the energy and water balance calculations with the new solute concentrations and iterates until convergence is met.

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Appendix 1: Notation

a_c	coefficient for computing matric potential of dead plant canopy elements (m)
a_r	coefficient for computing matric potential of residue elements (m)
a_{sp}	coefficient for computing thermal conductivity of snow ($0.021 \text{ W m}^{-1} \text{ C}^{-1}$)
a_a	exponent for calculating albedo of soil surface
a_ε	empirical coefficient for calculating clear-sky emissivity (taken as 0.261)
A_T	weighting coefficient for estimating temperature of bottom soil layer
b	pore-size distribution parameter
b_c	exponent for computing matric potential of dead plant canopy elements
b_r	exponent for computing matric potential of residue elements
b_{sp}	coefficient for computing thermal conductivity of snow ($2.51 \text{ W m}^{-1} \text{ C}^{-1}$)
b_v	coefficient accounting for tortuosity in computing vapor diffusion through soil
b_ε	empirical exponent for calculating clear-sky emissivity (taken as $7.77 \times 10^{-4} \text{ C}^{-1}$)
B	maximum clear-sky solar transmissivity of the atmosphere
c	solute concentration in soil solution (eq kg^{-1})
c'	solute concentration in soil solution after infiltration event (eq kg^{-1})
c_a	specific heat capacity of air ($\text{J kg}^{-1} \text{ C}^{-1}$)
c_{avg}	average concentration of water entering soil layer during infiltration (eq kg^{-1})
c_i	specific heat capacity of ice ($2,100 \text{ J kg}^{-1} \text{ C}^{-1}$)
c_j	specific heat capacity of j^{th} soil constituent ($\text{J kg}^{-1} \text{ C}^{-1}$)
c_l	specific heat capacity of water ($4,200 \text{ J kg}^{-1} \text{ C}^{-1}$)
c_r	specific heat capacity of residue elements ($\text{J kg}^{-1} \text{ C}^{-1}$)
c_{sp}	exponent for computing thermal conductivity of snow (2.0)
c_v	exponent accounting for tortuosity in computing vapor diffusion through soil
C	fraction of cloud cover
C_1	fractional increase in density per cm load of water equivalent ($\text{cm}^{-1} \text{ h}^{-1}$)
C_2	compaction parameter for snow
C_3	fractional settling of snow at densities less than ρ_d (h^{-1})
C_4	settling parameter for snow (C^{-1})
C_5	ratio of fractional settling rate for wet snow compared to dry snow (h^{-1})
C_{L1}	maximum allowable lag for water being routed through the snowpack (h)
C_{L2}	lag-time parameter for for water being routed through the snowpack (cm^{-1})
C_{L3}	recession parameter for water being routed through the snowpack (h)
C_{L4}	attenuation parameter for water being routed through the snowpack (h)
C_r	volumetric heat capacity of residue layer ($\text{J m}^{-3} \text{ C}^{-1}$)
C_s	volumetric heat capacity of soil ($\text{J m}^{-3} \text{ C}^{-1}$)
C_v	coefficient for radiation extinction coefficient in snow ($\text{mm}^{1/2} \text{ cm}^{-1}$)

d	zero displacement plane for residue or canopy (m) (or derivative function)
d_d	damping depth for annual temperature oscillation (m)
d_l	characteristic dimension of canopy leaves or elements (m)
d_s	grain-size diameter of ice crystals in snow layer (mm)
d_{sp}	depth of snowpack (m)
D_e	effective diffusion coefficient for water vapor in snow ($\text{m}^2 \text{s}^{-1}$)
D_H	hydrodynamic dispersion coefficient for solute transport in soil ($\text{m}^2 \text{s}^{-1}$)
D_o	molecular diffusion of a given solute in water ($\text{m}^2 \text{s}^{-1}$)
D_m	molecular diffusion for solute transport in soil ($\text{m}^2 \text{s}^{-1}$)
D_v	effective vapor diffusion coefficient through soil ($\text{m}^2 \text{s}^{-1}$)
D_v'	vapor diffusivity in air ($\text{m}^2 \text{s}^{-1}$)
E	evaporative flux from system profile ($\text{kg s}^{-1} \text{m}^{-2}$)
E_1	parameter for vapor flux enhancement factor
E_2	parameter for vapor flux enhancement factor
E_3	parameter for vapor flux enhancement factor
E_4	parameter for vapor flux enhancement factor
E_5	parameter for vapor flux enhancement factor
E_l	evaporative flux from canopy elements, i.e. leaves ($\text{kg s}^{-1} \text{m}^{-2}$)
$E_{l,i,j}$	evaporative flux from canopy elements (leaves) of plant species j within canopy layer i ($\text{kg s}^{-1} \text{m}^{-2}$)
f	infiltration rate into soil (m s^{-1})
f^*	dimensionless infiltration rate into soil
F^*	dimensionless cumulative infiltration into soil layer containing the wetting front
F_r	fractional area surface cover by flat residue ($\text{m}^2 \text{m}^{-2}$)
F'_{k+1}	cumulative infiltration water passing through soil layer k (m)
F'_m	cumulative infiltration into soil layer m which contains the wetting front (m)
g	acceleration gravity (9.81 m s^{-2})
G	soil heat flux (W m^{-2})
G_1	empirical coefficient for grain-size diameter (mm)
G_2	empirical coefficient for grain-size diameter (mm)
G_3	empirical coefficient for grain-size diameter (mm)
h_r	relative humidity expressed as a decimal
H	sensible heat flux from the system profile (W m^{-2})
H_l	sensible heat flux from canopy elements (leaves) to air space within the canopy (W m^{-2})
$H_{l,i,j}$	sensible heat flux from canopy elements (leaves) of plant species j to air space within the canopy layer i (W m^{-2})
k	von Karman constant (taken as 0.4)
k_a	thermal conductivity of still air ($0.025 \text{ W m}^{-1} \text{ C}^{-1}$)

k_e	convective transfer coefficient within the canopy air space ($\text{m}^2 \text{s}^{-1}$)
k_j	thermal conductivity of j^{th} soil constituent ($\text{W m}^{-1} \text{C}^{-1}$)
k_l	thermal conductivity of liquid water ($0.57 \text{ W m}^{-1} \text{C}^{-1}$)
k_r	effective thermal transfer coefficient (conductive and convective) of residue layer ($\text{W m}^{-1} \text{C}^{-1}$)
k_{rs}	thermal conductivity of residue material ($\text{W m}^{-1} \text{C}^{-1}$)
k_s	thermal conductivity of soil ($\text{W m}^{-1} \text{C}^{-1}$)
k_{sp}	thermal conductivity within snowpack ($\text{W m}^{-1} \text{C}^{-1}$)
k_r	thermal conductivity of residue layer ($\text{W m}^{-1} \text{C}^{-1}$)
k_v	convective thermal transfer within the residue layer ($\text{W m}^{-1} \text{C}^{-1}$)
K	unsaturated soil hydraulic conductivity (m s^{-1})
K_d	partitioning coefficient between solute absorbed on soil matrix and that in soil solution (kg kg^{-1})
$K_{e,m}$	effective conductivity of soil layer containing wetting front (m s^{-1})
$K_{e,k}$	effective conductivity of soil layer k during infiltration (m s^{-1})
K_j	direct radiation extinction coefficient for plant species j
K_s	saturated soil hydraulic conductivity (m s^{-1})
K_v	convective vapor transfer coefficient within residue layer ($\text{m}^2 \text{s}^{-1}$)
L_f	latent heat of fusion ($335,000 \text{ J kg}^{-1}$)
L_i	atmospheric long-wave radiation incident on the system profile (W m^{-2})
$L_{i,j}$	leaf area index for plant species j in canopy layer i ($\text{m}^2 \text{m}^{-2}$)
L_s	latent heat of sublimation ($2,835,000 \text{ J kg}^{-1}$)
L_w	actual lag of excess water being routed through the snowpack (h)
$L_{w,max}$	maximum lag of excess water being routed through the snowpack for present snow conditions (h)
L_v	latent heat of vaporization ($2,500,000 \text{ J kg}^{-1}$)
m_j	weighting factor for thermal conductivity of j^{th} soil constituent
M_w	molecular weight of water ($0.018 \text{ kg mole}^{-1}$)
n	empirical exponent for computing stomatal resistance
NC	number of plant canopy layers
NP	number of plant species present in canopy
NS	number of soil layers within soil profile
q_l	liquid water flux (m s^{-1})
q_v	water vapor flux ($\text{kg m}^{-2} \text{s}^{-1}$)
q_{vp}	soil water vapor flux due to water potential gradient ($\text{kg m}^{-2} \text{s}^{-1}$)
q_{vT}	soil water vapor flux due to temperature gradient ($\text{kg m}^{-2} \text{s}^{-1}$)
r_h	resistance to vapor transfer from residue elements to air within residue layer (s m^{-1})
$r_{h,i,j}$	resistance to convective transfer from canopy elements of plant species j within

	canopy layer i ($s\ m^{-1}$)
r_H	resistance to convective heat transfer from the surface of system profile ($s\ m^{-1}$)
$r_{l,i,j}$	leaf resistance to water flow in plant species j within canopy layer i ($m^3\ s\ kg^{-1}$)
$r_{r,i,k}$	resistance to water flow through the roots of soil layer k ($m^3\ s\ kg^{-1}$)
r_s	stomatal resistance ($s\ m^{-1}$)
$r_{s,i,j}$	stomatal resistance of plant species j within canopy layer i ($s\ m^{-1}$)
r_{so}	stomatal resistance of plant with no water stress ($s\ m^{-1}$)
r_v	resistance to convective vapor transfer from surface of the system profile ($s\ m^{-1}$)
R	universal gas constant ($8.3143\ J\ K^{-1}\ mole^{-1}$)
R_n	net all-wave radiation for the profile ($W\ m^{-2}$)
R_n	net downward radiation flux through the snow or residue ($W\ m^{-2}$)
$R_{n,i,j}$	net all-wave radiation for plant species j in canopy layer i ($W\ m^{-2}$)
s	atmospheric stability (ratio of thermal to mechanical turbulence)
s_v	slope of the saturated vapor density curve (dp_v'/dT ; $kg\ m^{-3}C^{-1}$)
S	total solutes present per mass of soil (eq kg^{-1})
S'	total solutes present in soil layer after leaching due to infiltration event (eq kg^{-1})
S_b	direct (beam) solar radiation incident on a horizontal surface ($W\ m^{-2}$)
$S_{b,o}$	solar radiation incident on a horizontal surface at the outer edge of the atmosphere ($W\ m^{-2}$)
S_d	diffuse solar radiation ($W\ m^{-2}$)
S_o	solar constant ($1360\ W\ m^{-2}$)
S_s	direct solar radiation incident on the local slope ($W\ m^{-2}$)
S_{sp}	excess water of snowpack in storage (m)
S_t	total solar radiation incident on a horizontal surface ($W\ m^{-2}$)
S_z	net solar radiation flux at a depth z within the snowpack ($W\ m^{-2}$)
t	time (s)
t'	time since infiltration wetting front entered the current soil layer (s)
t_*	dimensionless time since infiltration wetting front entered the current soil layer
$t_{1/2}$	half-life of solute (d)
T	temperature (C)
T'	temperature of soil layer and water exiting soil layer during infiltration event (C)
T_i	temperature of air within canopy layer i (C)
T_j	total transpiration rate for a given plant species ($kg\ m^{-2}\ s^{-1}$)
T_a	ambient temperature at measured reference height (C)
T_{fz}	freezing point of soil water based on water potential of the soil layer (C)
$T_{E,k}$	temperature of infiltration water entering soil layer k (C)
T_K	temperature (K)
$T_{l,i,j}$	leaf temperature of plant species j within canopy layer i (C)

T_{wb}	wet-bulb temperature (C)
u	wind speed at reference height (m s^{-1})
u_i	wind speed in canopy layer i (m s^{-1})
u_r	wind speed within residue layer (m s^{-1})
u_*	friction velocity (m s^{-1})
U	source/sink term for water flux equation ($\text{m}^3 \text{m}^{-3} \text{s}^{-1}$).
V	source/sink term for solute flux (eq $\text{kg}^{-1} \text{s}^{-1}$)
w_c	gravimetric water content of dead plant canopy material (kg kg^{-1})
w_r	gravimetric water content of residue layer (kg kg^{-1})
w_{sp}	volumetric liquid water content of snow ($\text{m}^3 \text{m}^{-3}$)
$w_{sp,hold}$	volumetric water holding capacity of snow ($\text{m}^3 \text{m}^{-3}$)
$w_{sp,max}$	maximum value of $w_{sp,hold}$ ($\text{m}^3 \text{m}^{-3}$)
$w_{sp,min}$	minimum value of $w_{sp,hold}$ ($\text{m}^3 \text{m}^{-3}$)
W_o	snowcover outflow (m)
W_L	depth of lagged excess liquid water in the snowpack (m)
W_{sp}	water equivalent of overlying snow (cm)
W_x	depth of excess liquid water in the snowpack (m)
z	vertical distance within system profile (m)
z_*	dimensionless depth of soil layers above the layer containing the wetting front
z_H	aerodynamic roughness length for heat transfer (m)
z_m	aerodynamic roughness length for momentum transfer (m)
z_{NS}	depth of bottom soil layer (m)
z_{ref}	reference height above the soil surface for meteorological measurements (m)
α_d	albedo of dry soil surface
α_s	albedo of soil surface
α_{sp}	albedo of snow surface
α_i	effective albedo of canopy layer i
α_j	albedo of plant species j
β	angle which the sun's rays make with the local slope (rad)
Δt	time increment (s)
$\Delta\theta_l$	change in water content across the infiltration wetting front ($\text{m}^3 \text{m}^{-3}$)
Δz_k	thickness of soil layer k (m)
ε_a	clear-sky long-wave emissivity of the atmosphere
ε_{ac}	long-wave emissivity of the atmosphere adjusted for cloud cover
ζ	enhancement factor for vapor flux through soil due to temperature gradient
η	weighting factor for end-of-time-step values ($0.5 \leq \eta \leq 1.0$)
θ_a	volumetric air content of soil layer ($\text{m}^3 \text{m}^{-3}$)
θ_i	volumetric ice content of soil layer ($\text{m}^3 \text{m}^{-3}$)

θ_i'	volumetric ice content of soil layer after infiltration event ($\text{m}^3 \text{m}^{-3}$)
θ_j	volumetric fraction for j^{th} soil constituent ($\text{m}^3 \text{m}^{-3}$)
θ_l	volumetric liquid water content of soil layer ($\text{m}^3 \text{m}^{-3}$)
θ_s	volumetric saturated water content of soil layer ($\text{m}^3 \text{m}^{-3}$)
κ	parameter for hydrodynamic dispersion coefficient (m)
ν	solar radiation extinction coefficient for snow (m^{-1})
π	osmotic potential of soil solution (m)
ρ_a	density of air (kg m^{-3})
ρ_b	bulk density of soil (kg m^{-3})
ρ_d	density of snow below which the settling rate equals C_3 (kg m^{-3})
ρ_e	density of snow at which $w_{sp,hold} = w_{c,min}$ (kg m^{-3})
ρ_i	density of ice (920 kg m^{-3})
ρ_j	density of j^{th} soil constituent (kg m^{-3})
ρ_l	density of water ($1,000 \text{ kg m}^{-3}$)
ρ_r	density of residue (kg m^{-3})
ρ_{rs}	specific density of residue (kg m^{-3})
ρ_{sp}	density of ice portion of snowpack (kg m^{-3})
ρ_v	vapor density of air space (kg m^{-3})
ρ_v'	saturated vapor density (kg m^{-3})
$\rho_{v,i}$	vapor density of air within canopy layer i (kg m^{-3})
ρ_{vs}	vapor density at an exchange surface (kg m^{-3})
ρ_{vs}'	saturated vapor density at an exchange surface (kg m^{-3})
$\rho_{vs,i,j}$	vapor density of canopy leaves of plant species j within canopy layer i (kg m^{-3})
ρ_{va}	ambient vapor density at reference height (kg m^{-3})
σ	Stefan-Boltzman constant ($\text{J m}^{-2} \text{s}^{-1} \text{K}^{-4}$)
τ	molecular diffusion coefficient for accounting for soil tortuosity
$\tau_{b,i}$	transmissivity to direct (beam) radiation for canopy layer i
$\tau_{b,r}$	transmissivity to direct (beam) radiation within the residue layer
τ_d	atmospheric diffuse solar radiation transmission coefficient ($S_t/S_{b,o}$)
$\tau_{d,i}$	transmissivity to diffuse radiation for canopy layer i
$\tau_{d,r}$	transmissivity to diffuse radiation within the residue layer
τ_t	atmospheric total solar radiation transmission coefficient ($S_t/S_{b,o}$)
ϕ	total water potential (m)
ϕ_H	adiabatic correction factor for thermal transfer
ϕ_s	sun's altitude angle above the horizon (rad)
ψ	soil water potential (m)
ψ_e	soil air-entry potential (m)
ψ_c	critical leaf water potential at which stomatal resistance is twice its minimum

	value (m)
Ψ_f	suction head of soil below the infiltration wetting front (m)
Ψ_k	water potential of soil layer k (m)
Ψ_l	leaf water potential (m)
$\Psi_{l,i,j}$	leaf water potential of plant species j within canopy layer i (m)
Ψ_H	adiabatic temperature profile correction for heat transfer
Ψ_m	adiabatic wind speed profile correction for momentum transfer
$\Psi_{x,j}$	xylem water potential of plant species j (m)
ω	radial frequency of annual temperature oscillation ($1.99238 \times 10^{-7} \text{ s}^{-1}$)