Accuracy of Soil Hydraulic Property Estimation Using Infiltrometers Having Different Disk Sizes

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Abstract. Soil hydraulic properties, such as the saturated hydraulic conductivity (K_s) and the parameter α used in exponential expressions of the hydraulic conductivity function, are imporant in modeling water flow and solute transport in unsaturated soil profiles. Tension infiltrometers have become popular instruments for the determination of soil hydraulic properties under field conditions. However, estimated K_s and α values using other independent field or laboratory measurements are often found to be different from those obtained with the infiltrometer method using approximate steady-state solutions. This is likely caused by the variable sizes of the infiltrometer disk used for the infiltration measurement and/or the limitations of steady-state solutions for small disk dimensions. To determine the effect of disk size on parameter estimation, we measured the infiltration in two soils (Arlington sandy loam and Sparta sand) with tension infiltrometers having several disk diameters (5.5-34.5 cm). For each disk size, the infiltration was repeated at multiple supply potentials, while measurements continued until steady-state, so that replicated parameter estimates were obtained. Results suggest that estimated values of K_s and α appeared to vary with the size of the infiltrometer disk used. Variations in estimated K_s and α values for different disk sizes, or for different potential increments for the same disk, were greater than the possible overestimation with the steady-state solution, as compared to an improved solution for small disk sizes. Discrepancies between tension infiltrometer and other methods in practice are caused probably more by variability within each method such as soil heterogeneity or simplification of the hydraulic conductivity function to an exponential expression, rather than by inherent limitations of the steady-state solutions.

INTRODUCTION

Parameters that are representative of in-situ soil hydraulic properties are very important for accurately describing the dynamic processes of water and solute transport in the soil. Direct measurements of the hydraulic parameters can be made with either standardized laboratory procedures using small soil cores taken from the field, or in-situ with specially-designed instruments [*Klute*, 1986]. Most of these conventional methods are difficult to use and are often very time consuming.

Tension disk infiltrometers [*Ankeny et al.*, 1988; *Perroux and White*, 1988] are designed to offer a simple and fast means of estimating soil hydraulic properties and structural soil characteristics based on infiltration measurements at the soil surface. Water flow from a tension infiltrometer disk to the underlying soil follows a three-dimensional flow process. Temporal changes in soil water content can be described with the Richards equation using initial and boundary conditions defined for geometric and hydraulic parameters specific to the infiltrometer. Because no exact analytical solution exists to the resulting transient three-dimensional flow problem, numerical inversion has been used to solve for hydraulic parameters based on known flow variables such as the transient water content or the infiltration rate [*Simunek and van Genuchten*, 1996, 1997]. From a practical standpoint, the most widely invoked method for parameter estimation using tension disk infiltrometer measurements is the use of an approximate analytical solution introduced by *Wooding* [1968] for steady-state water flow from a surface pond. Other methods of data analysis using

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tension disk infiltrometer data often require the determination of transient water flow characteristics, such as the sorptivity and macroscopic capillary length as with the method of *White and Sully* [1987].

The steady-state approach using Wooding's approximate solution remains attractive since only steady-state infiltration rates are needed for the parameter estimation. However, the accuracy of soil hydraulic parameters estimated with Wooding's approximate solution may be in question as evidenced by large variations or discrepancies reported in the literature when comparing results with other methods. For example, estimated values of the saturated hydraulic conductivity were found to be within 5~300% of the mean using numerical simulations or other laboratory procedures [*Reynolds and Elrick,* 1991; *White and Perroux,* 1989; *Ankeny et al.,* 1991]. Besides spatial variability in the soil hydraulic properties, one possible source of error with Wooding's method is its limitation for relatively small disk sizes. After carefully reexaming Wooding's solutions for water infiltration from small shallow ponds, *Weir* [1987] provided a refinement of Wooding's solution for small disk diameters. The inadequacy of Wooding's solution for small disk sizes was also recognized by *Warrick* [1992] who provided a comparison with numerical simulations.

The objective of this study was to determine the effect of disk size on the estimated soil hydraulic parameters using either the traditional Wooding's solution, or the refined solution by *Weir* [1987]. We also provide an assessment of the practicality of using tension infiltrometers having a small disk, and potential errors in the estimated hydraulic parameters using the simple form of Wooding's approximate solution.

THEORY

The rates of water flow and solute transport in an unsaturated soil are greatly affected by the hydraulic conductivity relationship, $K(h)$ (L T^{-1}) under a given soil matric potential, h (L). Many forms have been proposed for $K(h)$, including those by *Childs and Collis-George* [1950], *Mualem* [1976], and *van Genuchten* [1980]. A simple alternative approach for $K(h)$ was introduced by *Gardner* [1958] using an exponential expression:

$$K(h) = K_s \exp(\alpha h) \tag{1}$$

where K_s is the saturated hydraulic conductivity (L T^{-1}), and α (L^{-1}) is an empirical fitting parameter.

Wooding's Method

Assuming steady-state flow, *Wooding* [1968] solved for the infiltration rate from a shallow circular pond of radius r (L) using Gardner's exponential hydraulic conductivity function, and found the following approximate solution

$$Q(h) = \pi r^2 K_s \exp(\alpha h_0)\left(1 + \frac{4}{\pi r \alpha}\right) \tag{2}$$

where $Q(h_0)$ is the steady-state flow rate (L^3 T^{-1}) under a given supply potential h_0 (L). Because the only unknowns in this equation are K_s and α , these parameters can be estimated by making measurements with a fixed disk radius at multiple supply potentials, or at a fixed potential with disks having variable radii. Detailed procedures of solving the equation for K_s and α can be found in *Hussen and Warrick* [1993].

Weir's Refinement for Small Disks

For water flow from a small surface source such as a small tension infiltrometer disk, *Weir* [1987] found that Wooding's approximate solution was inaccurate. If the flow rate $Q(h_o)$ and the infiltrometer disk radius r are normalized into the following dimensionless forms

$$Q^* = \frac{\alpha}{rK_s \exp(\alpha h_0)} Q(h_0) \tag{3}$$

$$r^* = \alpha r / 2 \tag{4}$$

then Wooding's relationship can be simplified to

$$Q^* = 4 + 2\pi r^* \tag{5}$$

For $r^* < 0.4$, *Weir* [1987] found that Eq. (5) was no longer accurate and derived the following alternative approximation for Q^*

$$Q^* = \frac{4\pi \sin^2(r^*)}{r^* \pi \sin(r^*)\cos(r^*) + 2r^* \sin^2(r^*)\ln(r^*) - 1.073(r^*)^3} \tag{6}$$

The empirical fitting parameter α should be generic to the solutions of both Wooding and Weir, and may be found by measuring the steady-state flow rate at two different supply potentials (such as h_1 and h_2) for the same disk radius, and solving either Eq. (2) or (3) to obtain

$$\alpha = \frac{\ln[Q(h_2)/Q(h_1)]}{h_2 - h_1} \tag{7}$$

The saturated hydraulic conductivity, however, will become different when α is substituted in either Wooding's Eq. (2):

$$K_s = \frac{Q(h_1)}{\pi r^2 \exp(\alpha h_1)\left(1 + \frac{4}{\pi r \alpha}\right)} \tag{8}$$

or Weir's Eqs. (3) and (6):

$$K_s = \frac{\alpha}{r} \exp(-\alpha h_1) \frac{Q(h_1)}{Q^*} \tag{9}$$

FIELD MEASUREMENTS

Water infiltration in two different soils was measured using tension infiltrometers having different disk diameters. The first soil is an Arlington fine sandy loam (coarse-loamy, mixed, thermic, Haplic Durixeralf) with an Ap horizon for the surface 10 cm. The particle size distribution of this horizon was of 63% sand, 30% silt, and 7% clay. The soil bulk density was 1.53±0.03 (g cm^{-3}). Since no definable structure could be observed, the soil is considered massive. The second soil is a Sparta sand (mesic, uncoated, Typic Quartzipsamment) and consisted of approximately

95% sand, 3% silt, and 2% clay in the surface layer. Soil bulk density near the surface was 1.50±0.08 (g cm^{-3}). The structure of the soil is considered subangular blocky [*Wang et al.*, 1994].

The tension infiltration measurements for the Arlington fine sandy loam were made with four disk diameters: 5.5, 10, 15, and 20 cm. For each disk size the measurements were repeated using eight supply potentials: 0, -1, -3, -5, -7, -10, -15, and -20 cm, which provided 28 estimates of α and K_s for each disk diameter. A layer of about 1 mm of no. 60 silica sand (diameter ~ 250 mm; K_s ~ 0.33 cm s^{-1}; water entry ~ 22 cm) was used between the disk membrane and the smoothed soil surface to improve hydraulic contact. Measurements at the Sparta sand site were made with three disk diameters: 6.4, 8.7, and 34.5 cm, with the measurements repeated using five supply potentials: 0, -1, -3, -6, and -12 cm for each disk size. This yielded 10 estimates of α and K_s for each disk diameter. Because of the large sand fraction and the relatively smooth soil surface, we did not use any contact material for this soil.

To obtain for both soils additional estimates of the soil hydraulic conductivity that would be independent of the tension infiltrometer method, six bore holes each to 15 cm from the soil surface were used for replicated Guelph permeameter measurements in areas next to the infiltrometer measurements. Soil K_s values were calculated from the Guelph permeameter measurements following procedures similar to those of *Reynolds and Elrick* [1985] and *Elrick et al.* [1990], and were used as a comparison with the tension infiltrometer estimates.

RESULTS AND DISCUSSION

A comparison of the Wooding and Weir solutions for small disk sizes showed a consistent and significant difference in the predicted normalized flow rate, Q^* (Fig. 1). Because numerical simulations by *Weir* [1987] and *Warrick* [1992] showed that the refined solution of *Weir*[1987], i.e., Eq. (6), is more exact than Wooding's solution for $r^* < 0.4$, the normalized flow rate could be underestimated by about 6% for $r^* = 0.1$ to 8% for $r^* = 0.4$ if the conventional Wooding's equation were used. The definition of a small infiltrometer disk radius depends not only on the physical dimension of the disk itself (i.e., r), but also on the properties of the soil. As defined by Eq. (4), the

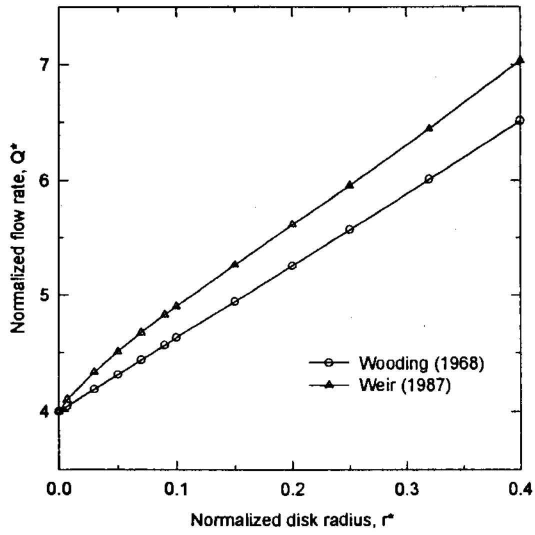

Fig. 1. Predicted water flow from small circular sources using the analytical solutions of *Wooding* (1968) and *Weir* (1987).

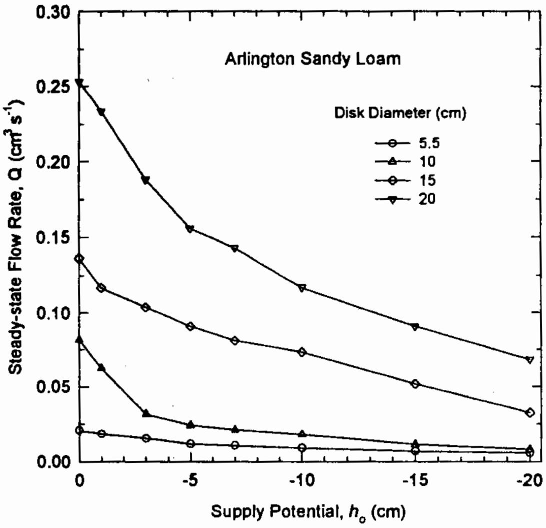

Fig. 2. Measured steady-state water flow as a function of the supply potential using different infiltrometer disks for Arlington sandy loam.

normalized disk radius could be considered small for one soil (with a small α number) but not for a different soil (having a large α value), even though the physical radius of the infiltrometer disk remains the same. The parameter α often ranges from about 0.008 cm^{-1} for a fine-textured soil such as clay, to about 0.145 cm^{-1} for a coarse soil such as sand [*Carsel and Parrish,* 1988]. To use Wooding's solution without the necessity of correcting for small disk sizes or $r^* < 0.4$, the minimum disk radius (r_{min}) should be 100 cm for clay and 5.52 cm for sand. It would be very impractical to construct and use an infiltrometer with a disk diameter of 2 m.

For each size of the infiltrometer disk used in this study, the measured steady-state infiltration rate decreased with decreasing supply potential (Figs. 2 and 3). For the Arlington sandy loam, the steady-state infiltration rate at zero potential was about 0.02 and 0.08 $cm^3\ s^{-1}$ for disk diameters of 5.5 and 10 cm, respectively. For the Sparta sand, the steady-state rate at zero potential reached about 0.21 and 0.35 $cm^3\ s^{-1}$ for the two comparable disk sizes (i.e., diameters of 6.4 and 8.7 cm). This is about an order of magnitude larger, which may indicate that for a normal infiltration event, such as rain or sprinkler irrigation where h_0 can be considered zero, the Sparta sand has a much larger infiltration capacity than the Arlington sandy loam. The steady-state infiltration rate at $h_0 = -10$ cm, however, was 0.009 and 0.018 $cm^3\ s^{-1}$ for disk diameters of 5.5 and 10 cm for the Arlington sandy loam, while the steady-state rate for the Sparta sand at $h_0 = -12$ cm was 0.011 and 0.021 $cm^3\ s^{-1}$ for disk diameters of 6.4 and 8.7 cm, respectively. Similar infiltration rates for the lower supply potentials seem to indicate that much of the infiltration in the Sparta sand was due to gravity flow.

According to Eqs. (7) to (9), any combination of two supply potentials for each disk size would produce a pair of α and K_s estimates. The multiple measurements (i.e., $n > 2$) enabled us to use nonlinear regression [*Logsdon and Jaynes,* 1993] to obtain estimates of α and K_s for each disk size. Table 1 shows that the overall average of the estimated α values was 0.086±0.051 and 0.208±0.055 cm^{-1} for the Arlington sandy loam and the Sparta sand, respectively. These values are reasonable compared to the overall mean: 0.075±0.037 cm^{-1} (n = 1183) for sandy loam and 0.145±0.029 cm^{-1} (n = 246) for sand as obtained by *Carsel and Parrish* [1988] from a much larger database. According to the estimated α values, the normalized disk radius was less than 0.4 for all disk sizes used for the Arlington sandy loam. This would imply that corrections with Weir's refinement

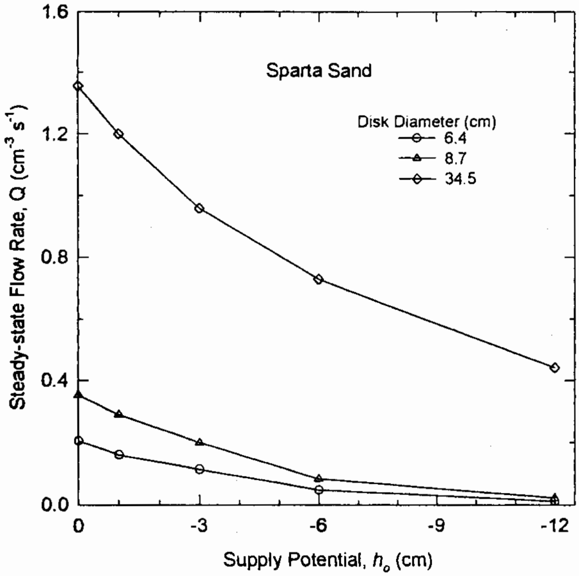

Fig. 3. Measured steady-state water flow using different infiltrometer disks for Sparta sand.

solution are required to obtain more exact estimates of K_s. For the Sparta sand, however, only the $r = 3.2$ cm disk needed the correction for small disk sizes since for the other two disk sizes $r^* > 0.4$. It is clear that soil properties (i.e., α values) contributed significantly to the determination of r^* since even a disk radius of 10 cm would be considered small and required a correction for the relatively fine-textured Arlington sandy loam.

TABLE 1. Measured Soil Hydraulic Parameters†

Disk (cm)	α (cm^{-1})	r^*	Method for Measuring K_s (x10^{-5} cm s^{-1}) *Wooding* [1968]	*Weir* [1987]	Guelph Permeameter
			Arlington Sandy Loam		
5.5	0.074±0.024	0.102±0.033	11.64±1.28	10.99±1.21	
10	0.134±0.079	0.334±0.197	31.38±9.66	29.23±9.00	
15	0.068±0.021	0.253±0.078	21.31±1.18	19.92±1.10	
20	0.071±0.017	0.353±0.084	27.94±1.98	25.99±1.84	
Average	0.086±0.051	0.260±0.099	23.07±9.04	21.53±8.37	25.49±19.63
			Sparta Sand		
6.4	0.233±0.038	0.373±0.061	232.3±17.93	215.7±16.65	
8.7	0.233±0.008	0.508±0.017	258.8±7.30	234.6±6.61‡	
34.5	0.132±0.039	1.138±0.336	116.9±22.72	2.98±0.58‡	
Average	0.208±0.055	0.673±0.333	208.1±59.46	215.7±16.65	252.7±96.30

†Estimated parameters are mean±standard deviation; $n = 28$, 10, and 6 for Arlington sandy loam, Sparta sand, and the Guelph permeameter measurements; α = a fitting parameter; r^* = scaled disk radius; K_s = saturated hydraulic conductivity.

‡ Exceeded recommended maximum for disk size correction, or $r^* > 0.4$.

With Wooding's method, estimated K_s values averaged $23.07\pm9.04\times10^{-5}$ cm s^{-1} for the Arlington sandy loam and $208.1\pm59.46\times10^{-5}$ cm s^{-1} for the Sparta sand. Measurements with the Guelph permeameter produced K_s estimates of $25.49\pm19.63\times10^{-5}$ and $252.7\pm96.30\times10^{-5}$ cm s^{-1} for the two soils, respectively. A statistical mean comparison (with a t-statistic for two population means with small sample sizes) indicated that the estimated K_s values were not significantly different (at the $P = 0.05$ level) between Wooding's method and the Guelph permeameter measurements. This seems to imply that the infiltrometer method in combination with Wooding's approximation can provide a good estimate of K_s under field conditions. Close examination also shows that the Guelph permeameter estimates varied more than values obtained with the infiltrometer-Wooding approximation method; therefore, the infiltrometer-Wooding method may produce more repeatable K_s results than other field techniques such as the Guelph permeameter.

Corrections using Weir's solution reduced the average K_s by 7.2% for the Arlington sandy loam. For the Sparta sand, a 7.7% reduction in the estimated K_s was obtained for the 3.2-cm radius disk. Large variations were found in the estimated α and K_s values between different disk sizes, and between different supply potentials for each disk size. The coefficient of variation (CV) for α was 59.3 and 26.4% for the Arlington sandy loam and Sparta sand, respectively. The calculated CV for K_s, was 39.2% between the four disk sizes for the Arlington sandy loam and 28.6% for the Sparta sand. These variations are clearly much greater than the potential inaccuracies that Wooding's approximate solution would produce. The large variation in estimated α and K_s values between different disk sizes is very likely caused by soil spatial heterogeneity since soil hydraulic properties may change over a very short distance [*Mohanty et al.*, 1994]. The variations in estimated α and K_s values between different supply potentials, however, may be attributed to soil macropores. Whereas the presence of soil macropores may not be visually apparent, functionally they will be effective only when the supply potential is greater than some threshold value above which the macropores start to conduct water [*Logsdon et al.*, 1993; *Wang et al.*, 1996].

CONCLUSIONS

This study was conducted to determine the effect of tension infiltrometer disk size on measurements of soil hydraulic parameters using the traditional approximate solutions of *Wooding* [1968] for water flow from a shallow pond. The infiltration in two soils (Arlington sandy loam and Sparta sand) was measured to steady-state with tension infiltrometers having several disk diameters, with the measurement being repeated at multiple supply potentials for each disk diameter. The saturated soil hydraulic conductivity K_s and the parameter α used in Gardner's exponential saturated-unsaturated hydraulic conductivity function were estimated from the infiltrometer measurements using both Wooding's method and a refined solution by *Weir* [1987] for small sizes. Variations in the estimated values of K_s and α between different disk sizes or different supply potentials for the same disk size were much greater than the possible overestimation using Wooding's solution as compared to Weir's refined solution for small disk sizes. Soil spatial variability and macropores most likely are more important than the physical size of the tension infiltrometer disk when measuring the soil hydraulic properties. Wooding's approximate solution should be sufficient for most disk sizes currently used for tension infiltrometer measurements.

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