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An Improved Analysis of Gravity Drainage Experiments for Estimating the Unsaturated Soil Hydraulic Functions

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The unsaturated hydraulic properties are important parameters in any quantitative description of water and solute transport in partially saturated soils. Currently, most in situ methods for estimating the unsaturated hydraulic conductivity (K) are based on analyses that require estimates of the soil water flux and the pressure head gradient. These analyses typically involve differencing of field-measured pressure head (h) and volumetric water content (θ) data, a process that can significantly amplify instrumental and measurement errors. More reliable methods result when differencing of field data can be avoided. One such method is based on estimates of the gravity drainage curve $K'(\theta) = dK/d\theta$ which may be computed from observations of θ and/or h during the drainage phase of infiltration drainage experiments assuming unit gradient hydraulic conditions. The purpose of this study was to compare estimates of the unsaturated soil hydraulic functions on the basis of different combinations of field data θ, h, K, and K'. Five different data sets were used for the analysis: (1) θ-h, (2) K-θ, (3) K'-θ; (4) K-θ-h, and (5) K'-θ-h. The analysis was applied to previously published data for the Norfolk, Troup, and Bethany soils. The K-θ-h and K'-θ-h data sets consistently produced nearly identical estimates of the hydraulic functions. The K-θ and K'-θ data also resulted in similar curves, although results in this case were less consistent than those produced by the K-θ-h and K'-θ-h data sets. We conclude from this study that differencing of field data can be avoided and hence that there is no need to calculate soil water fluxes and pressure head gradients from inherently noisy field-measured θ and h data. The gravity drainage analysis also provides results over a much broader range of hydraulic conductivity values than is possible with the more standard instantaneous profile analysis, especially when augmented with independently measured soil water retention data.

INTRODUCTION

Accurate estimates of the unsaturated soil hydraulic properties are essential for most applications involving water and solute transport in partially saturated soils. The required hydraulic properties are the water retention curve $\theta(h)$ and the hydraulic conductivity function, either $K(\theta)$, or $K(h)$, where θ is the volumetric water content, h is the soil water pressure head, and K is the unsaturated hydraulic conductivity. The soil hydraulic functions are often described with relatively simple analytical expressions that facilitate easy site characterization, allow for a rapid comparison of the hydraulic properties of different soils, and provide a means for interpolating or extrapolating hydraulic property data points of individual soils [*Brooks and Corey*, 1964; *van Genuchten*, 1980; *Russo*, 1988].

One popular method for estimating the unsaturated hydraulic properties from field data is the instantaneous profile method of *Rose et al.* [1965], also referred to as the unsteady drainage flux method [*Green et al.*, 1986]. This method requires considerable experimental effort in that water contents and pressure heads must be obtained at regular intervals in space and time. While the accuracy of the θ and h data is limited by the precision of available field instruments, the K values have additional errors arising from differencing inherently noisy in situ field-measured water content and pressure head data. The resulting field estimates for K can fluctuate widely about the fitted analytical functions [*Fluhler et al.*, 1976]. The magnitude of fluctuations in analyses using the instantaneous profile method may be reduced by smoothing the field data prior to differencing [*Ahuja et al.*, 1980; *Libardi et al.*, 1980; *Luxmoore et al.*, 1981]. However, smoothing of data requires subjective decisions about the smoothing algorithm to be implemented and the extent of smoothing to be done. While smoothing appears to have little effect on the estimated $\theta(h)$ curve, its effect on the estimated K is less understood.

An alternative to working with differenced data, with or without smoothing, is to use unit gradient water flow models in which analytical functions for $dK/d\theta$ are directly fitted to estimates for $dK/d\theta$ derived from field-measured water contents [*Sisson*, 1987]. The "observed" $dK/d\theta$ values required in the fitting process follow directly from implicit solutions of the unit gradient equation

$$K'(\theta) = dK/d\theta = \frac{z}{t} \tag{1}$$

in which $dK/d\theta$ (further referred to as the gravity drainage curve) is the slope of the $K(\theta)$ function, and z and t are the depths and times at which associated water contents or pressure heads are measured. Once the unknown parameters in the $K'(\theta)$ function are available, the $K(\theta)$ function follows immediately. Equation (1) follows from the Richards equation by using the assumption that gravity is the domi-

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nant driving force for water flow during the drainage phase of an instantaneous profile type field experiment [*Sisson et al.*, 1980; *Sisson*, 1987]. The gravity-dominant (or unit gradient) flow model was first used by *Black et al.* [1969] and has since formed the theoretical basis of a number of field methods for estimating the unsaturated hydraulic conductivity [e.g., *Davidson et al.*, 1969; *Libardi et al.*, 1980; *Chong et al.*, 1981; *Jones and Wagenet*, 1984].

Previous shortcomings to using fitted $K'(\theta)$ functions for estimating hydraulic properties were that the adopted $K(\theta)$ functions needed to be of relatively simple form (i.e., exponential or power functions) and that field-measured pressure head data or independently measured retention data points could not be used in the fitting process [*Sisson*, 1987]. Thus the main purpose of this paper is to explore the feasibility of using $K'(\theta)$ data, as estimated by (1) and in lieu of the more difficult to obtain $K(\theta)$ data, in a general hydraulic parameter optimization procedure.

HYDRAULIC PARAMETER ANALYSIS

The soil water retention curve is assumed to be of the form [*van Genuchten*, 1980]

$$\theta(h) = \theta_r + \frac{\theta_s - \theta_r}{(1 + |\alpha h|^n)^m} \qquad (\theta_r \le \theta \le \theta_s) \tag{2}$$

where θ_r and θ_s are the residual and saturated water contents, respectively, and where α, m, and n are empirical shape parameters to be estimated by fitting (2) to experimental data. This paper will consider only the restricted van Genuchten functions where m is given by $m = 1 - 1/n$. The unsaturated hydraulic conductivity function may be obtained by combining (2) with the pore-size distribution model of *Mualem* [1976] to yield the following predictive expression for the hydraulic conductivity:

$$K(\theta) = K_s \sqrt{S}[1 - (1 - S^{1/m})^m]^2 \tag{3}$$

where K_s is the saturated hydraulic conductivity, and S is effective saturation ($0 \le S \le 1$):

$$S = \frac{\theta - \theta_r}{\theta_s - \theta_r} \tag{4}$$

The $K'(\theta)$ function required for the gravity drainage data analysis is obtained by differentiating (3) with respect to θ, that is,

$$K'(\theta) = K_s \frac{(1 - A^m)(1 - A^m + 4S^{1/m}A^{m-1})}{2(\theta_s - \theta_r)\sqrt{S}} \tag{5}$$

where

$$A = 1 - S^{1/m} \tag{6}$$

Substituting (2) and (4) into (3) and (5) shows that the $K(\theta)$ and $K'(\theta)$ functions can be expressed also in terms of the pressure head h. Inspection of the above hydraulic functions reveals that they contain five unknown parameters (θ_r, θ_s, α, n, K_s) which must be estimated from observed data.

A nonlinear least squares parameter optimization program, SOHYP, was previously developed [*van Genuchten*, 1978] to estimate the unknown coefficients θ_r, α, and n from observed soil water retention data. The SOHYP code was later modified to yield the RETC code (M. Th. van Genuchten, unpublished paper, 1986) which allows some or all of the unknowns, θ_r, θ_s, α, n, and K_s, to be estimated simultaneously from measured soil water retention and/or hydraulic conductivity data. Least squares parameter optimization in both SOHYP and RETC was based on Marquardt's maximum neighborhood method [*Marquardt*, 1963]. The objective function $O(\{b\})$ minimized in RETC is of the general form

$$O(\{b\}) = \sum_{i=1}^{M} \{w_i[\theta_i^* - \theta_i(\{b\})]\}^2 + \sum_{i=M+1}^{N} \{W_1 W_2 w_i[\ln(K_i^*) - \ln(K_i(\{b\}))]\}^2 \tag{7}$$

where $\{b\}$ is a vector containing one or all of the unknown coefficients $\{\theta_r, \theta_s, \alpha, n, K_s\}$, θ_i^* and K_i^* are the measured water contents and hydraulic conductivities, respectively, $\theta_i(\{b\})$ and $K_i(\{b\})$ are the computed values of θ and K for each successive estimate of $\{b\}$ using (2) and (3), respectively, M is the number of observed retention data, and N is the total number of observed retention and hydraulic conductivity data. Three weighting coefficients are used in (7): w_i, W_1 and W_2. Of these, w_i may be used to weigh each measured data point individually (w_i was set to unity in this study). Coefficient W_2 is calculated internally in RETC using

$$W_2 = \left\{\frac{1}{M}\sum_{i=1}^{M} w_i\theta_i^*\right\} \bigg/ \left\{\frac{1}{N-M}\sum_{i=M+1}^{N} w_i|\ln(K_i^*)|\right\} \tag{8}$$

which gives the water content data approximately the same weight as the ln (K) data in the parameter optimization process [*Kool et al.*, 1985]. Parameter W_1 is an independent input parameter to allow more or less weighting of the retention data in their entirety, relative to the hydraulic conductivity data. Because conductivity data usually show considerably more scatter than retention data, and are also less precise than retention data, it is sometimes beneficial to assign relatively less weight to the conductivity data in (8). This may be accomplished by using a value of less than 1 for W_1. In our study we nearly always used a value of 0.1 for W_1.

The procedure for estimating the unknown parameter vector $\{b\}$ directly from unit gradient type data is the same as above, except that K in (7) is replaced by K', that is,

$$O(\{b\}) = \sum_{i=1}^{M} \{w_i[\theta_i^* - \theta_i(\{b\})]\}^2 + \sum_{i=M+1}^{N} \{W_1 W_2 w_i[\ln(K_i'^*) - \ln(K_i'(\{b\}))]\}^2 \tag{9}$$

where $K_i'^*$ and $K_i'(\{b\})$ are now the "measured" (equation (1)) and calculated (equation (5)) values of the gravity drainage curve $dK/d\theta$. As indicated by (1), the "measured" $K_i'^*(\theta)$ values are simply the ratios z/t at which the water

TABLE 1. Particle Size Fractions and Bulk Densities for the Norfolk Sand, Troup Loamy Sand, and Bethany Loam Soils

	Sand, g/g	Silt, g/g	Clay, g/g	Bulk Density, g/cm^3
Norfolk sand	0.78	0.18	0.04	1.79
Troup loamy sand	0.84	0.13	0.03	1.64
Bethany loam	0.32	0.46	0.22	1.54

content measurements were taken during an instantaneous profile type field drainage experiment. We note that (9) may also include $K_i'^*(h)$ data rather than, or in addition to, $K_i'^*(\theta)$ if measurements of the pressure head were taken during the drainage phase. Copies of the computer code UNGRA which implements the above gravity drainage data analysis are available upon request.

FIELD DATA SETS

The unit gradient based optimization procedure was applied to three soil data sets taken from the literature. The data sets were for a relatively coarse-textured Norfolk sand (thermic Typic Paleudult) from South Carolina [*Quisenberry et al.*, 1987], a coarse-textured variant of the Troup loamy sand (Grossarenic Paleudult) series from Alabama [*Dane et al.*, 1983], and a more fine-textured Bethany loam (thermic Pachic Argiustoll) from Oklahoma [*Nofziger et al.*, 1983]. Results of particle size analyses and bulk density determinations are given in Table 1.

Field water contents for the Norfolk soil were estimated from in situ tensiometer readings and soil water retention (drying) curves measured in the laboratory on undisturbed soil cores. After flooding the 3 by 3 m square field plot until no significant changes in tensiometer readings could be observed, the plot was covered with plastic and allowed to drain freely. Field data used in our analysis were obtained during the ensuing drainage phase from the 15.2 cm depth for run "1" at site "1". The unsaturated hydraulic functions were fitted to data listed in Tables N1.2, N1.4, and N1.5 of *Quisenberry et al.* [1987]. These tables contain, respectively, the laboratory-measured soil water retention data, estimated soil water contents during the drainage phase, and unsaturated hydraulic conductivity data computed with the instantaneous profile method. Complete experimental details are given by *Quisenberry et al.* [1987].

In contrast to the Norfolk soil, the Troup and Bethany soils were instrumented to simultaneously obtain water content and pressure head data. Thus no laboratory data were used in the parameter optimization process. Field water contents were estimated using neutron probes and pressure head values from tensiometer readings. Data analyzed in this paper were taken from Tables 1.3.3 and 1.5.1 of *Dane et al.* [1983] for the Troup Ap horizon and Tables 7.1 and 9.1 of *Nofziger et al.* [1983] for the Bethany 0–15-cm-depth interval. The Troup and Bethany soil data were again for the first depth at the first site of the field drainage experiments.

The following five combinations of data were used in the hydraulic parameter optimization process for each of the three soils:

1. Data set 1 (θ-h data) is measured $\theta(h)$ data together with one hydraulic conductivity value needed to match the predicted K function to a measured K data point, as suggested by *Jackson et al.* [1965]. The value of K selected for this matching was taken at a conductivity value somewhat

TABLE 2. Fitted Hydraulic Parameters and Associated Standard Errors (±) for the Norfolk Sand, Troup Loamy Sand and Bethany Loam Soils Used in This Study

Parameter	Data Set 1 θ-h	Data Set 2 K-θ-h	Data Set 3 K-θ	Data Set 4 K'-θ-h	Data Set 5 K'-θ
			Norfolk Sand		
θ_r, m^3/m^3	0.0*	0.064 ± 0.038	0.074 ± 0.082	0.065 ± 0.023	0.0*
θ_s, m^3/m^3	0.293 ± 0.002	0.287 ± 0.008	0.260 ± 0.007	0.290 ± 0.004	0.263 ± 0.002
α, 1/cm	0.0307 ± 0.0013	0.0302 ± 0.0040	0.0193†	0.0312 ± 0.0019	0.0179†
n	1.54 ± 0.02	1.78 ± 0.24	2.03 ± 1.12	1.78 ± 0.14	1.52 ± 0.08
K_s, cm/h	0.072‡	0.100 ± 0.041	0.029 ± 0.027	0.190 ± 0.044	0.101 ± 0.042
r^2	0.998	0.82	0.83	0.97	0.95
			Troup Loamy Sand		
θ_r, m^3/m^3	0.0*	0.094 ± 0.005	0.092 ± 0.008	0.083 ± 0.007	0.056 ± 0.021
θ_s, m^3/m^3	0.304 ± 0.010	0.323 ± 0.059	0.304§	0.284 ± 0.013	0.312 ± 0.038
α, 1/cm	0.028 ± 0.0020	0.0320 ± 0.0146	0.0250†	0.0263 ± 0.0051	0.0321†
n	2.15 ± 0.11	2.33 ± 0.44	2.18 ± 0.43	2.14 ± 0.28	1.59 ± 0.21
K_s, cm/h	6.32‡	10.6 ± 17.0	8.57 ± 4.98	4.06 ± 2.58	24.2 ± 41.6
r^2	0.990	0.986	0.975	0.995	0.993
			Bethany Loam		
θ_r, m^3/m^3	0.192 ± 0.012	0.202 ± 0.005	0.0*	0.205 ± 0.027	0.0*
θ_s, m^3/m^3	0.430 ± 0.003	0.427 ± 0.002	0.433 ± 0.001	0.379 ± 0.007	0.404 ± 0.031
α, 1/cm	0.0206 ± 0.0010	0.0199 ± 0.0005	0.530†	0.0110 ± 0.0015	0.0224†
n	1.73 ± 0.08	1.80 ± 0.04	1.167 ± 0.001	2.15 ± 0.42	1.21 ± 0.06
K_s, cm/d	0.0790‡	0.0729 ± 0.0056	2.15 ± 0.17	0.0192 ± 0.0093	0.504 ± 1.071
r^2	0.998	>0.999	>0.999	0.93	0.90

*Converged to zero; fixed at zero during parameter estimation process.
†Estimated from matching with observed $\theta(h)$ data point (all data sets 3 and 5).
‡Estimated from matching with observed $K(\theta)$ data point (all data sets 1).
§Fixed at 0.304 (data set 1) because of perfect correlation between θ_s and K_s.

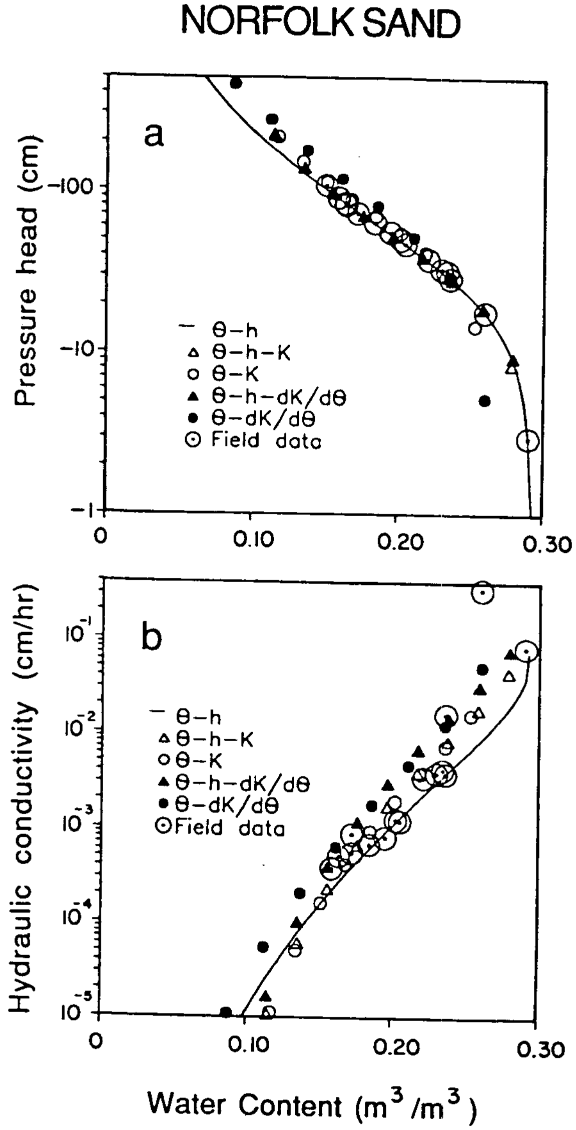

Fig. 1. Soil water retention and hydraulic conductivity curves for Norfolk sand. Open symbols denote curves fitted to K data, closed symbols denote fitted curves using K' data.

less than saturation, as recommended by *van Genuchten and Nielsen* [1985] and *Luckner et al.* [1989].

2. Data set 2 (K-θ-h data) is measured $\theta(h)$ data used simultaneously with measured $K(\theta)$ data.
3. Data set 3 (K-θ data) is measured $K(\theta)$ data, together with one measured $\theta(h)$ data point required to estimate α in (2) (note that (3) and (4) do not contain the α parameter).
4. Data set 4 (K'-θ-h data) is measured $\theta(h)$ data used simultaneously with "measured" $K'(\theta)$ and/or $K'(h)$ data, as estimated with (1).
5. Data set 5 (K'-θ data) is "measured" $K'(\theta)$ and/or $K'(h)$ data, as estimated with (1), together with one measured $\theta(h)$ point required to estimate α in (2).

RESULTS

Table 2 lists, for each soil, the fitted hydraulic parameters obtained with the five data sets described above. Calculated retention (equation (2)) and conductivity (equation (3)) curves using these parameter values are shown in Figures 1, 2, and 3 for the Norfolk, Troup, and Bethany soils, respectively. Also included in these figures are the observed soil water retention and hydraulic conductivity data points reported in the original studies by *Quisenberry et al.* [1987], *Dane et al.* [1983], and *Nofziger et al.* [1983]. As noted earlier, data set 1 requires for each soil a matching hydraulic conductivity data point to quantify K_s in the $K(\theta)$ function, while data sets 3 and 5 require a measured retention data point for determining α in the $\theta(h)$ function. In our study we always used for a given soil the same matching combination $\{\theta, h, K\}$ to estimate α and K_s. These combinations were {0.236, −29, 0.00366} for the Norfolk sand, {0.240, −39, 1.01} for the Troup loamy sand, and {0.326, −91, 0.000819}, for the Bethany loam, where h is measured in centimeters and K in centimeters per hour.

Figure 1 shows that the observed water retention and hydraulic conductivity data of the Norfolk soil are described well by curves obtained with data sets 2 and 4, both of which included observed retention data in the parameter estimation

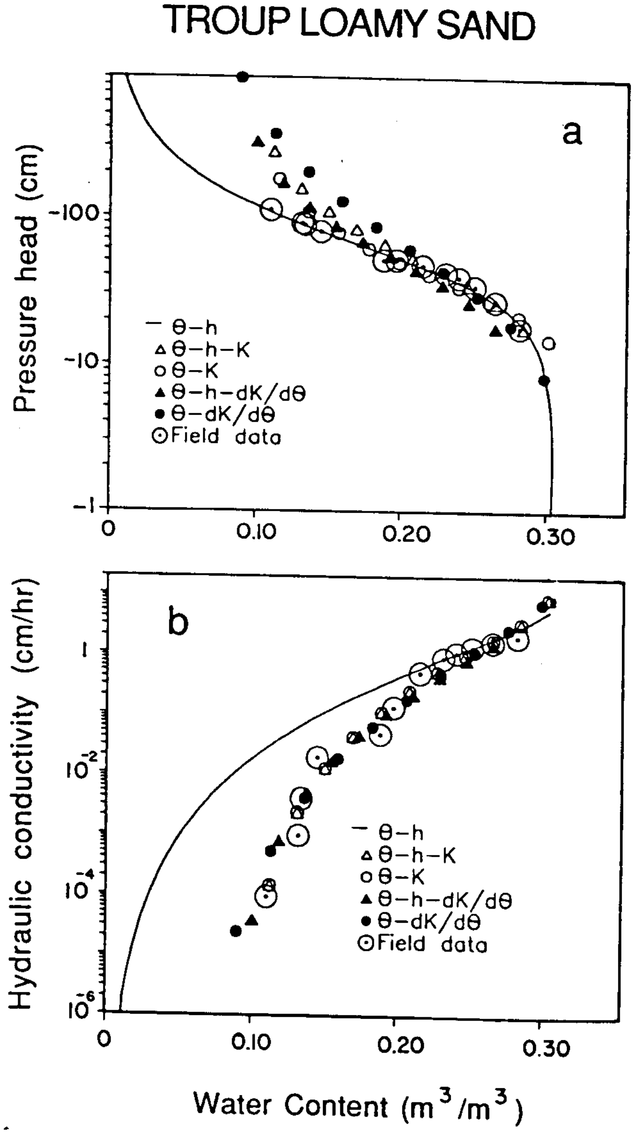

Fig. 2. Soil water retention and hydraulic conductivity curves for Troup loamy sand. Open symbols denote curves fitted to K data, closed symbols denote fitted curves using K' data.

process (the curves are indicated by the open and solid triangles, respectively). This result indicates that K' values can be reliably substituted for K values when estimating the unsaturated hydraulic properties. Results for data sets 3 and 5, which included only a single matching value for $\theta(h)$, are shown in Figure 1 by small open and solid circles, respectively. These two data sets produced relatively low estimates of θ_s, in part because of a lack of measured field data close to saturation. This suggests that retention data near saturation are important for producing reliable estimates of the hydraulic properties. The almost congruent curves produced by data sets 3 and 5, and the near identical curves resulting from data sets 2 and 4, show that $K'(\theta)$ and $K(\theta)$ data are equally useful in the parameter estimation process. Figure 1*b* also gives the predicted hydraulic conductivity curve using hydraulic parameters calculated from the observed soil water retention data (data set 1). Relatively good agreement was obtained with the observed conductivity data, except for one relatively high value near saturation (Figure 1*b*).

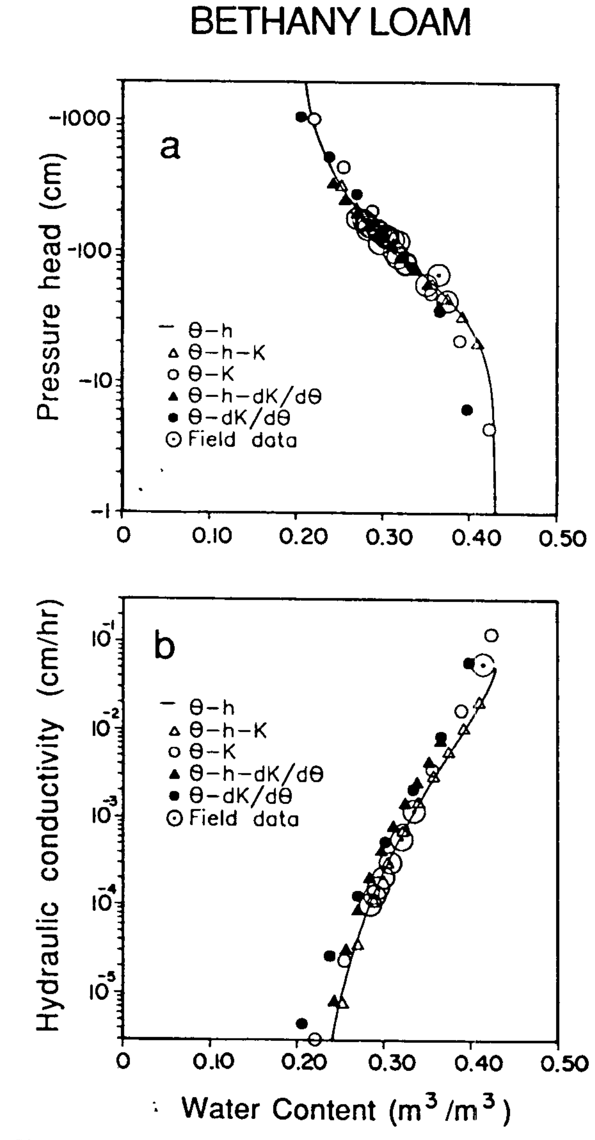

Fig. 3. Soil water retention and hydraulic conductivity curves for Bethany loam. Open symbols denote curves fitted to K data, closed symbols denote fitted curves using K' data.

As shown in Table 2, the fitted value of θ_r for the Norfolk soil ranged from 0.0 to 0.074. Since the field-measured water contents remained well above these values, θ_r must be considered an extrapolated (and mostly empirical) parameter whose value may contain considerable error. The saturated hydraulic conductivity K_s also showed some variation in the estimated values, ranging from 0.029 to 0.19 cm/h. Still, we judge the overall description of the hydraulic conductivity function to be quite acceptable in view of expected measurement errors.

Results for the Troup loamy sand (Figure 2) are similar to those of the Norfolk sand in that the fitted retention curves coalesce nicely over the range of water contents where data are available. The parameter θ_s was found to be perfectly correlated to K_s for this combination of data values. For this reason, θ_s was fixed at 0.304, being the value estimated from data set 1. The fitted soil water retention curves deviate from each other primarily at the lower water contents, largely because of differences in the estimated θ_r values (Figure 2*a*). Since the Troup field data used in our analysis were relatively far removed from the residual water content, the estimated θ_r values are again merely extrapolated values subject to considerable error and without much physical meaning. If precise values of field water contents and pressure heads had been available in the dry region of the curves, they could have been immediately incorporated into the parameter optimization process.

The hydraulic conductivity curves obtained with data sets 2 through 5 (Figure 2*b*) for the Troup soil are essentially identical and agree closely with the field-measured hydraulic conductivities. However, the solid $K(\theta)$ curve calculated from retention data only (plus a single matching point to determine K_s) deviates significantly from the observed conductivity data in the dry range. Hence we conclude that, at least for the Troup soil, observed hydraulic conductivity data $K(\theta)$, or gravity drainage data $K'(\theta)$, must be used in the fitting process to produce reliable estimates of the hydraulic conductivity function.

Figure 3 shows the experimental and fitted curves for the Bethany soil. Similar trends occur as for the Norfolk and Troup soils in that the curves tend to slightly deviate from each other at relatively low and high water contents, primarily because of a lack of measured retention data in those regions. However, all 5 data sets produced very similar curves; the overall agreement with the measured data is also good to excellent. In addition, notice that the predicted hydraulic conductivity curve derived from the retention data (data set 1) also produced a good fit with the observed conductivity data for this soil.

Use of gravity drainage (K') data for estimating the hydraulic properties is based on the assumption that unit gradient conditions exist during the drainage phase of an instantaneous profile experiment. Total hydraulic gradients between the 15 and 30 cm depths of the Norfolk soil varied from 0.5 to nearly 5 and were downward throughout the experiment [see *Quisenberry et al.*, 1987, Table N1.3]. Hydraulic gradients in the Troup fluctuated from 0.45 downward to nearly 1.0 upward, with a trend over time toward upward water movement [*Dane et al.*, 1983, Table 1.2.3]. The hydraulic gradient in the Bethany varied from 0.5 downward to 0.3 upward, with a slight trend over time to upward flow of water [*Nofziger et al.*, 1983, Table 6.1]. A review of the experimental procedures indicates that special effort was made to protect the sites from evaporation. Thus it is difficult to envision that significant water flow occurred in the soil toward the surface. Also, the gradient reversal in the Troup soil took place within 3 hours of the start of drainage, which allows little time for water to flow upward from the 15 cm depth. The gradient reversal in the Bethany took place at around 1 to 2 days, still a relatively short period of time for a significant amount of water to flow upward from the 15 cm depth to influence tensiometer readings. We speculate that tensiometer and other instrumental errors may have contributed to the apparent gradient reversals near the soil surface. For example, all three soils were instrumented with mercury manometers which are known to be temperature sensitive. Mercury manometers also require relatively large volumes of water to move from the tensiometer into the soil in response to changes in the soil water pressure head. Ironically, such measurement problems provide additional reasons for using the gravity drainage analysis, since this method of analysis avoids the need for detailed measurements of the hydraulic gradients.

CONCLUSIONS

Results for the three soils considered here show that $K'(\theta)$ data can be reliably substituted for $K(\theta)$ data in fitting retention and hydraulic conductivity curves. This finding is significant for field experimentalists, since K' is easily obtained during drainage of an initially saturated field soil. As shown by (1), $K'(\theta)$ is then simply equal to the ratio z/t at the point where θ was measured. By comparison, $K(\theta)$ data require a much more elaborate and time-consuming sampling program to insure meaningful data after interpolating and differencing field data.

Our study indicates that there are two additional extensions to the analysis of gravity drainage experiments. One important extension results from the fact that the instantaneous profile data analysis is now formulated in the form of a parameter optimization process. This approach allows one to extend the range of experimental data by augmenting the data base with measurements that are obtained independently from the gravity drainage experiment. Thus when estimates of the pressure head are available at relatively low water contents from laboratory- or field-measured soil water depletion experiments, or perhaps from psychrometer studies, then those estimates can be immediately incorporated into the optimization analysis. Another extension is obtained when K' data are substituted for K data. Since K' can be estimated with great precision, the accuracy of the estimation process should also improve.

The differences in fitted parameters from the five data sets used in our study could be reduced if special efforts had been expended to obtaining more precise estimates of θ_r, θ_s, and K_s. For example, determining water contents of soil samples obtained while instrumenting the site under dry conditions would further improve the accuracy of the θ_r estimates. Similarly, the precision of θ_s could be improved by averaging several water content values obtained immediately before initiation of the drainage phase. Also, the shape of the $K'(\theta)$ function near saturation would be more precisely known if more water content data from shallow depths were available during the first few minutes of the gravity drainage phase. With the shape of the $K'(\theta)$ more precisely known close to saturation, the precision of the fitted K_s would then also improve. Finally, we emphasize that much of the analysis presented depends on the validity of the unit gradient assumption during drainage of an initially saturated profile. Previous studies involving gravity drainage experiments [*Sisson et al.*, 1981; *Libardi et al.*, 1980; *Jones and Wagenet*, 1984] indicate that the unit gradient assumption is a good approximation during drainage, except perhaps for strongly layered soils where hydraulic conductivities change rapidly from layer to layer.

REFERENCES

Ahuja, L. R., R. E. Green, S.-K. Chong, and D. R. Nielsen, A simplified functions approach for determining soil hydraulic conductivities and water characteristics in situ, *Water Resour. Res.*, *16*(5), 947–953, 1980.

Black, T. A., W. R. Gardner, and G. W. Turtell, The prediction of evaporation, drainage and soil water storage for a bare soil, *Soil Sci. Soc. Am. Proc.*, *33*, 655–660, 1969.

Brooks, R. H., and A. T. Corey, Hydraulic properties of porous media, *Hydrol. Pap. 3*, 27 pp., Colo. State Univ., Fort Collins, 1964.

Chong, S. K., R. E. Green, and L. R. Ahuja, Simple in situ determination of hydraulic conductivity by power function descriptions of drainage, *Water Resour. Res.*, *17*(4), 1109–1114, 1981.

Dane, J. H., D. K. Cassel, J. M. Davidson, W. L. Pollans, and V. L. Quisenberry, Physical characteristics of soils of the Southern Region—Troup and Lakeland Series, *South. Coop. Ser. Bull. 262*, Ala. Agric. Exp. Stn., Auburn Univ., Auburn, 1983.

Davidson, J. M., L. R. Stone, D. R. Nielsen, and M. E. Larue, Field measurement and use of soil-water properties, *Water Resour. Res.*, *5*, 1312–1321, 1969.

Fluhler, H., M. S. Ardakani, and L. H. Stolzy, Error propagation in determining hydraulic conductivities from successive water content and pressure head profiles, *Soil Sci. Soc. Am. J.*, *40*(6), 830–836, 1976.

Green, R. E., L. R. Ahuja, and S. K. Chong, Hydraulic conductivity, diffusivity, and sorptivity of unsaturated soils: Field methods, in *Methods of Soil Analysis*, vol. 1, *Physical and Mineralogical Methods*, 2nd ed., edited by A. Klute, pp. 771–798, American Society of Agronomy, Madison, Wis., 1986.

Jackson, R. D., R. J. Reginato, and C. H. M. Van Bavel, Comparison of measured and calculated hydraulic conductivities of unsaturated soils, *Water Resour. Res.*, *1*(3), 375–380, 1965.

Jones, A. J., and R. J. Wagenet, In situ estimation of hydraulic conductivity using simplified methods, *Water Resour. Res.*, *20*(11), 1620–1626, 1984.

Kool, J. B., J. C. Parker, and M. Th. van Genuchten, Determining soil hydraulic properties from one-step outflow experiments by parameter estimation, I, Theory and numerical studies, *Soil Sci. Soc. Am. J.*, *49*(6), 1348–1354, 1985.

Libardi, P. L., K. Reichardt, D. R. Nielsen, and J. W. Biggar, Simple field methods for estimating soil hydraulic conductivity, *Soil Sci. Soc. Am. J. 44*(1), 3–7, 1980.

Luckner, L., M. Th. Genuchten, and D. R. Nielsen, A consistent set of parametric models for the two-phase flow of immiscible fluids in the subsurface, *Water Resour. Res.*, *25*(10), 2187–2193, 1989.

Luxmoore, R. J., T. Grizzard, and M. R. Patterson, Hydraulic properties of Fullerton cherty silt loam, *Soil Sci. Soc. Am. J.*, *45*(4), 692–698, 1981.

Marquardt, D. W., An algorithm for least-squares estimation of nonlinear parameters, *J. Soc. Ind. Appl. Math.*, *11*, 431–441, 1963.

Mualem, Y., A new model for predicting the hydraulic conductivity of unsaturated porous media, *Water Resour. Res.*, *12*(3), 513–522, 1976.

Nofziger, D. L., J. R. Williams, A. G. Hornsby, and A. L. Wood, Physical characteristics of soils of the Southern Region—Bethany, Konawa, and Tipton Series, *South. Coop. Ser. Bull. 265*, Agric. Exp. Stn., Okla. State Univ., Stillwater, 1983.

Quisenberry, V. L., D. K. Cassel, J. H. Dane, and J. C. Parker, Physical characteristics of soils of the Southern Region—Norfolk, Dothan, Wagram, and Goldsboro Series, *South. Coop. Ser. Bull. 263*, S. C. Agric. Exp. Stn., Clemson Univ., Clemson, 1987.

Rose, C. W., W. R. Stern, and J. E. Drummond, Determination of hydraulic conductivity as a function of depth and water content for soil in situ, *Water Resour. Res.*, *3*(1), 1–9, 1965.

Russo, D., Determining soil hydraulic properties by parameter estimation: On the selection of a model for hydraulic properties, *Water Resour. Res.*, *24*(3), 453–459, 1988.

Sisson, J. B., Drainage from layered field soils: Fixed gradient models, *Water Resour. Res.*, *23*(11), 2071–2075, 1987.

Sisson, J. B., A. H. Ferguson, and M. Th. van Genuchten, Simple method for predicting drainage from field plots, *Soil Sci. Soc. Am. J.*, *44*(6), 1147–1152, 1980.

van Genuchten, M. Th., Calculating the unsaturated hydraulic conductivity with a new closed-form analytical model, *Res. Rep. 78-WR-08*, Water Resour. Program, Dep. of Civ. Eng., Princeton Univ., Princeton, N. J., 1978.

van Genuchten, M. Th., A closed-form equation for predicting the hydraulic conductivity of unsaturated soils, *Soil Sci. Soc. Am. J.*, *44*(5), 892–898, 1980.

van Genuchten, M. Th., and D.. R. Nielsen, On describing and predicting the hydraulic properties of unsaturated soils, *Ann. Geophys.*, *3*(5), 615–628, 1985.

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