Wettability effects on two- and three-fluid relative permeabilities

Scott A. Bradford a,*, Linda M. Abriola a, Feike J. Leij b

a University of Michigan, Department of Civil and Environmental Engineering, 181 EWRE, 1351 Beal Avenue, Ann Arbor, MI 48109-2125, USA
b US Salinity Laboratory, US Department of Agriculture, Agricultural Research Service, 450 W. Big Springs Road, Riverside, CA 92507-4617, USA

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Abstract

Specification of relative permeability ($k_r$)--saturation ($S$) relations for all fluid phases is required for the simulation of multiphase flow and transport in porous media. Indirect methods are frequently employed to estimate these $k_r-S$ relations owing to the time, expense, and difficulty associated with direct measurements. A common indirect approach uses capillary pressure data in conjunction with a selected pore-size distribution model to estimate $k_r-S$ relations. Such methods typically assume perfect wettability of the solid. Natural porous media, however, are composed of a variety of mineral constituents with different adsorptive properties, which can exhibit non-zero contact angles and/or fractional wettability. Consequently, fluid distributions in natural media may be more complex than those predicted by simple pore-size distribution models and, under such conditions, current estimation approaches for $k_r$ may be inadequate. In this work, the pore-size distribution model of N.T. Burdine (1953, Relative permeability calculations from pore-size distribution data. Transactions of the American Institute of Mining, Metallurgical and Petroleum Engineers 198, 71–77) is extended to incorporate wettability variations. In this model, wetting and less wetting (non-wetting or intermediate) fluid pore classes are used to calculate $k_r$ for water or organic. The wettability of the porous medium is used to determine the contributions of the pore classes to $k_r$. For both two- and three-fluid systems, the model predicts that an increase in the contact angle (measured through water) or organic-wet fraction of a medium will be accompanied by an increase in the water $k_r$ and a decrease in the organic $k_r$. In three-fluid media, $k_r$ values for water and organic depend on both liquid saturations when the solid is imperfectly wetted. The model assumes that wettability variation has no influence on the air $k_r$.

* Corresponding author. Tel.: +1 313 936 3175; fax: +1 313 763 2275; e-mail: sbrad@engin.umich.edu

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Model predictions are shown to be consistent with available experimental data. © 1997 Elsevier Science B.V.

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1. Introduction

The release of slightly miscible organic liquids to the subsurface environment poses a significant threat to soils and groundwater. To reliably predict the temporal and spatial distribution of such contaminants with multiphase flow and transport models, accurate estimates of capillary pressure ($P_c$)–saturation ($S$)–relative permeability ($k_r$) relations are required (Demond et al., 1996). The measurement of two- and, especially, three-fluid relative permeabilities, however, is difficult, time consuming, and expensive (Honarpour et al., 1986; Klute and Dirksen, 1986). Consequently, indirect approaches are frequently used to estimate $k_r$–$S$ relations from more easily measured $P_c$–$S$ data.

One indirect technique for obtaining $k_r$–$S$ relations is based on an extension of the hydraulic radius concept (Carman, 1937) to multiphase systems (Rapoport and Leas, 1951; Morrow, 1970). A second approach uses statistical pore-size distribution models to predict the permeability relations (e.g. Burdine, 1953; Mualem, 1976; and Lenhard and Parker, 1987). These indirect approaches, however, have been limited in their development and application to perfectly wetted porous media. The hydraulic radius approach has not been used to predict $k_r$–$S$ relations for systems of other wettabilities owing to the lack of reliable experimental methods or estimation procedures to obtain values of the interfacial areas that are required by this model. Statistical pore-size distribution models employed to date implicitly assume a distinct order of wettability.

The contact angle ($\theta$) is often used for quantifying the wettability (the distribution of immiscible fluids near a solid) of porous media. For organic ($o$)–water ($w$)–solid ($s$) systems, the angle is typically measured after a drop of the organic liquid is placed on a flat solid that is immersed in water. Flat polished plates of silica or quartz are used to represent sandstone, while calcite is used if carbonates are prevalent (Anderson, 1986). The horizontal force balance equation for this system, Young’s equation, defines the contact angle measured through water as:

$$\cos(\phi_{sw}) = \frac{\sigma_{so} - \sigma_{sw}}{\sigma_{ow}}$$

where $\sigma$ is the interfacial tension (N m$^{-1}$) and the subscripts indicate the phases. When $\phi_{sw} < 90^\circ$, water is the wetting fluid, whereas, for $\phi_{sw} > 90^\circ$, the water is the non-wetting fluid; the converse occurs for the organic.

The microscopic contact angle defined by Eq. (1) will vary spatially in natural porous media owing to contaminant aging (Powers and Tamblin, 1995), and/or variations in aqueous chemistry (Demond et al., 1994), mineralogy (Anderson, 1987a), organic matter distributions (Dekker and Ritsema, 1994) and surface roughness (Morrow, 1975). When both water- and organic-wet sites are present, this condition is referred to as fractional
wettability. In the petroleum literature, fractional wettability has been recognized as a ubiquitous condition (Brown and Fatt, 1956; Donaldson et al., 1969; Salathiel, 1973). Hence, a single value for the contact angle measured on a flat polished solid is only a crude approximation for natural systems. Alternatively, capillary pressure data may be used to infer the average influence of wettability (Donaldson et al., 1969; Bradford and Leij, 1995a, Bradford and Leij, 1995b). Wettability effects on capillary pressure data have been observed to be either saturation dependent or independent for various fluid and solid properties (Donaldson et al., 1969; Morrow, 1976; Bradford and Leij, 1995a,b).

Two-fluid relative permeability relations are significantly affected by the solid’s wettability (Anderson, 1987b; Honarpour et al., 1986). Research concerning the effects of wettability on \( k_r - S \) relations has been conducted by Fatt and Klikoff (1959), Donaldson and Thomas (1971), Owens and Archer (1971), Morrow et al. (1973), McCaffery and Bennion (1974), Heaviside et al. (1987) and Demond and Roberts (1993). These studies indicate that, for media containing organics and water, an increasing contact angle or organic-wet fraction is accompanied by an increase in the \( k_{rw} \) and a decrease in \( k_{ro} \) (i.e. Owens and Archer, 1971; Donaldson and Thomas, 1971; Heaviside et al., 1987). This observed change in relative permeability may be explained by considering the distributions of the fluids: water tends to reside in the smaller, less conductive, pores of water-wet media and in the larger pores of organic-wet media, with the converse true for the organic.

A number of previous studies have presented measured three-fluid \( k_r - S \) relations (Leverett and Lewis, 1941; Caudle et al., 1951; Corey et al., 1956; Reid, 1958; Snell, 1962; Donaldson and Dean, 1966; Sarem, 1966; Saraf and Fatt, 1967; Oak et al., 1990; and Dria et al., 1993). Donaldson and Kayser (1981) reviewed the early studies and found marked differences owing to experimental errors, simplifying assumptions, and wettability differences. Published data qualitatively demonstrate the significance of wettability and hysteresis on \( k_r - S \) relations (Honarpour et al., 1986). For example, water and oil relative permeabilities have been reported to be functions of their own saturation or both liquid saturations (e.g. Oak et al., 1990).

The simulation of multiphase flow and transport processes is likely limited by our ability to accurately characterize the hydraulic properties of the system. Recently, Demond et al. (1996) demonstrated that the use of inaccurate estimates of \( k_r - S \) relations results in dramatically different simulation predictions from those based on measured parameters. The primary objective of this study is to develop an approach to estimate the influence of wettability variations on two- and three-fluid relative permeability relations.

2. Theory

Throughout this paper effective saturations are employed as a measure of the pore space accessible for flow. Because wettability affects the "immobile" residual wetting fluid, the effective saturation will depend upon the medium wettability, in addition to the pore structure. Terminology employed in this work pertaining to effective saturations
under various medium wettability conditions is defined in the Appendix A. For the reader’s convenience, a notation section is also included at the end of the paper.

2.1. Hydraulic radius

The permeability of sands has been found to be correlated with the hydraulic radius, \( r_h \), of a medium (Carman, 1937). For single phase flow, \( r_h \) is defined as the ratio of the porosity (\( \epsilon \)) to the solid surface area per unit bulk volume (\( A_s \)):

\[
r_h = \frac{\epsilon}{A_s} - \frac{1}{A_s^*}
\]

(2)

where \( A_s^* \) is the solid surface area per unit pore volume. The Carman–Kozeny equation relates permeability and hydraulic radius as:

\[
k = \frac{\epsilon r_h^2}{C_k} = \frac{\epsilon}{C_k} \left( \frac{1}{A_s^*} \right)^2
\]

(3)

where \( C_k \) is the Kozeny constant which has been empirically established to have a value of approximately 5 for unconsolidated sand (Carman, 1937). Rapoport and Leas (1951) extended the hydraulic radius concept to two-phase flow by replacing: (i) \( \epsilon \) with the "effective porosity" with respect to the fluid (\( \epsilon_S \)); and (ii) \( A_s \) with the sum of the interfacial areas bounding a flowing fluid. According to their procedure (Rapoport and Leas, 1951) the two-fluid relative permeability to water is written as:

\[
k_{rw}(\bar{S}_w^{ow}) = (\bar{S}_w^{ow})^3 \left[ \frac{A_{se}^*}{A_{sw}^* + A_{ow}^*} \right]^2
\]

(4)

and the relative permeability to organic is given as (cf., Morrow, 1970):

\[
k_{ro}(\bar{S}_w^{ow}) = (1 - \bar{S}_w^{ow})^3 \left[ \frac{A_{se}^*}{A_{se}^* + A_{ow}^*} \right]^2
\]

(5)

where \( \bar{S} \) is the effective fluid saturation; the subscript on \( S \) (or \( \bar{S} \)) indicates the fluid to which the saturation pertains and the superscripts denote all fluids present in the medium. The solid–water, solid–organic, and organic–water interfacial areas per unit pore volume are given as \( A_{sw}^*, A_{ow}^*, \) and \( A_{ow}^* \), respectively, while \( A_{se}^* \) is the effective solid surface area per unit pore volume. Rapoport and Leas (1951) employed an effective solid surface area in place of \( A_{se}^* \) in Eq. (4) to account for the reduction in area owing to the presence of immobile residual wetting saturation (discontinuous wetting fluid films coating the solids) which occurs in multi-fluid systems. Hence, these researchers defined \( A_{se}^* \) as the contact area (per unit pore volume) of the mobile wetting phase and the solid + immobile residual wetting phase matrix.
Similar to Eqs. (4) and (5), the hydraulic radius approach may be extended to three-fluid relative permeabilities. For the most general case of a three-fluid fractional wettability medium, these are given as:

\[ k_{rw}(\tilde{S}_w, \tilde{S}_w) = \left( \frac{A^*_{se}}{A^*_{sw} + A^*_{ow} + A^*_{aw}} \right)^2 \]

\[ k_{ro}(\tilde{S}_w, \tilde{S}_o) = \left( \frac{A^*_{ao}}{A^*_{ao} + A^*_{ow} + A^*_{ao}} \right)^2 \]

\[ k_{ro}(\tilde{S}_w, \tilde{S}_o) = \left( \frac{A^*_{ao}}{A^*_{ao} + A^*_{aw} + A^*_{ao}} \right)^2 \]

where \( A^*_{se}, A^*_{sw}, \) and \( A^*_{ao} \) are the solid-air, air-water, and air-organic interfacial areas per unit pore volume, respectively.

The interfacial areas in Eqs. (4)–(8) depend on saturation, pore-size distribution, and medium wettability (Bradford and Leij, 1997). If the hydraulic radius approach is valid, and accurate values of the interfacial areas are available, Eqs. (4)–(8) should provide a means to predict wettability effects on the relative permeability relations. Recently, Bradford and Leij (1997) presented an approach to estimate two- and three-fluid interfacial areas for various medium wettabilities. Their approach is based upon the thermodynamic description of two-fluid capillarity presented by Morrow (1970), which relates the work required to change the fluid saturation (the area under the \( P_c-S \) curve) to changes in the interfacial areas. According to their procedure, when the contact angle is less than 90°, the values of \( A^*_{ow} + A^*_{sw} \) (cf. Eq. (4)) and \( A^*_{ow} + A^*_{ao} \) (cf. Eq. (5)) at a given saturation decrease with increasing contact angle. Hence, according to Eqs. (4) and (5), both \( k_{rw} \) and \( k_{ro} \) will increase with increasing contact angle. This prediction, however, contradicts experimental observations of \( k_{ro} \) (Owens and Archer, 1971; McCaffery and Bennion, 1974). It is likely that \( A^*_{ae} \) also depends on the medium wettability since the residual immobile wetting saturation also is a function of medium wettability (Morrow, 1990). Since there are no available techniques for the estimation of \( A^*_{ae} \), the hydraulic radius approach for the prediction of \( k_r-S \) relations was not considered further in this work.

2.2. Pore-size distribution model adaptation

Owing to the difficulties in estimating parameters for the hydraulic radius model, an alternative indirect technique was sought for the predictions of \( k_r-S \) relations. The Burdine (1953) pore-size distribution model was selected as a basis for this approach owing to the ability of this model to predict the relative permeability relations for a perfectly wetted porous medium (tested on data reported by Morrow and McCaffery (1978)); other commonly used pore-size distribution models (e.g. Mualem, 1976) predict higher wetting phase relative permeability values at a given saturation than the Burdine model. Modification of the Burdine model (Burdine, 1953) is presented below for the nonhysteretic case.
2.2.1. Two-fluid systems

The Burdine model views the porous medium as a tortuous bundle of parallel capillary tubes. The size distribution of these tubes is estimated from primary drainage $P_e-S$ data according to (Laplace, 1825):

$$ R(S_w) = \frac{2\sigma ow \cos(\phi_{ow})}{P_{ow}(S_w)} $$

(9)

where $R$ is the radius of a capillary tube, and $P_{ow} = P_o - P_w$ is the capillary pressure. Flow in the $i$th capillary tube is described by the Hagen–Poiseuille equation as:

$$ q_i = -\frac{R_i^2 \rho g H}{8\mu} \frac{dH}{dz} $$

(10)

where $q_i$ is the average volumetric flux per unit cross-sectional area, $R_i$ is the radius of the $i$th capillary tube, $\rho$ is the fluid density, $g$ is the acceleration owing to gravity, $\mu$ is the fluid viscosity, and $dH/dz$ is the hydraulic head gradient in the $z$ direction. Summation of the contributions from all fluid-filled pores leads to an expression of the average flux in a porous medium. Equating this flux to Darcy’s law yields the following expressions for the relative permeability (Burdine, 1953):

$$ k_{rw}(S_w) = \left(\frac{S_w}{S_{ow}}\right)^2 \frac{\int_0^{S_{ow}} R(x)^2 \, dx}{\int_0^1 R(x)^2 \, dx} $$

(11)

$$ k_{ro}(S_w) = \left(1 - S_w\right)^2 \frac{\int_0^{S_{ow}} R(x)^2 \, dx}{\int_0^1 R(x)^2 \, dx} $$

(12)

where the tortuosity of the flow path is empirically accounted for in Eqs. (11) and (12) by using the square of $S_w$ and $S_{ow} = 1 - S_w$, respectively. If $R(x)$ is replaced by Eq. (9), one can see that the estimates for $k_e$ given by Eqs. (11) and (12) are independent of the magnitude of $\sigma_{ow}$ and $\phi_{ow}$, as long as $\sigma_{ow}$ and $\phi_{ow}$ are independent of saturation. Note that Burdine’s model assigns the water to the “smaller” pores (see Eq. (11)) and the organic liquid to the “larger” pores (see Eq. (12)) by varying the integration limits. This implicitly assumes that water is the wetting fluid and that the organic is the nonwetting fluid.

In many cases, the fluid does not reside exclusively in a single pore class, as assumed by existing pore size distribution models. The literature suggests that fractional wettability porous media and, to a lesser extent, mineralogically homogeneous porous media exhibit spatially dependent wettability (i.e. Brown and Fatt, 1956; Morrow, 1975). For a given fluid pair, the filling sequence of pores is determined by the pore radius and pore wettability. Consequently, pore scale variability in wettability will influence the pores that are accessible for fluid flow. As the magnitude of the capillary pressure at a given saturation decreases, microscopic variations in wettability likely have a more pronounced effect on the pore filling sequence (McCaffery and Bennion, 1974).
Consider a two fluid, organic–water, porous medium system. Suppose that pores are divided into two classes (domains) by size, termed herein as “small” and “large”. In this system, the effective macroscopic wettability, as determined from $P_c-S$ data, will determine the contribution of a given pore domain to the magnitude of $k$, for each fluid. Based upon Eqs. (11) and (12), $k_{rw}$ may be written as:

$$k_{rw}(S_{w}^{ow}) = \left( S_{w}^{ow} \right)^2 \frac{\left[ 1 - f(\phi_{sow}) \right] \int_0^{S_{w}^{ow}} R(x)^2 \, dx + f(\phi_{sow}) \int_{S_{w}^{ow}}^{1} R(x)^2 \, dx}{\int_0^{1} R(x)^2 \, dx}$$

(13)

where the first and second integrals in the numerator account for the contributions to the permeability of the smaller (water as wetting fluid) and larger (water as nonwetting fluid) pore domains, respectively. Similarly, $k_{ro}$ is written as:

$$k_{ro}(S_{w}^{ow}) = \left( 1 - S_{w}^{ow} \right)^2 \frac{f(\phi_{sow}) \int_0^{1} R(x)^2 \, dx + \left[ 1 - f(\phi_{sow}) \right] \int_{S_{w}^{ow}}^{1} R(x)^2 \, dx}{\int_0^{1} R(x)^2 \, dx}$$

(14)

In Eqs. (13) and (14), $f(\phi_{sow})$ is an empirical weighting function which depends upon the macroscopic contact angle of the medium. Based upon available experimental evidence (McCaffery and Bennion, 1974), the following functional form is suggested:

$$f(\phi_{sow}) = \frac{1}{2} \left[ 1 - \cos(\phi_{sow}) \right]$$

(15)

Here $\phi_{sow}$ is the effective macroscopic contact angle defined with respect to water as the “reference wetting fluid”. This contact angle may be either saturation independent or saturation dependent. When using $\phi_{sow}(S_{w}^{ow})$, the values of $k_{rw}$ and $k_{ro}$ are determined at each saturation using a different weight.

Fig. 1 shows a plot of $f(\phi_{sow})$ when $\phi_{sow}$ is independent of saturation. The shape of $f(\phi_{sow})$ is such that increasing $\phi_{sow}$ leads to an increasing contribution of larger pores to

![Fig. 1. Plot of weighting function (Eq. (15)), $f(\phi_{sow})$ verses $\phi_{sow}$.](image-url)
the determination of $k_{rw}$ (see Eq. (13)); the converse occurs for $k_{ro}$ (see Eq. (14)). Furthermore, observe that $k_{rw}$ is uniquely determined by the smaller pores when $\phi_{ow} = 0^\circ$, equally determined by smaller and larger pores when $\phi_{ow} = 90^\circ$, and only determined by larger pores when $\phi_{ow} = 180^\circ$. According to Eq. (15), $k_{rw}$ and $k_{ro}$ are most sensitive to changes in $\phi_{ow}$ for $40^\circ < \phi_{ow} < 140^\circ$. McCaffery and Bennion (1974) found that $k_{rw}$ and $k_{ro}$ were relatively insensitive to changes in $\phi_{ow}$ when $\phi_{ow} < 49^\circ$ or $\phi_{ow} > 138^\circ$.

Scaling suggests that values of $\phi_{ow}$, for a given porous medium, can be obtained from measured organic-water and air-organic $P_c-S$ data and interfacial tensions, provided that $\phi_{ao} = 0^\circ$, using the following relation (Bradford and Leij, 1995a):

$$\cos(\phi_{ow}) = \frac{\sigma_{ao} P_{ow}(S_{ow})}{\sigma_{ow} P_{ao}(S_{ao})}$$

(16)

at points where $S_{ow} = S_{ao}$. A best fit value of $\phi_{ow}$ can be developed by considering a range of liquid saturations. Receding ($\phi_{ow}^R$) and advancing ($\phi_{ow}^A$) contact angles may be obtained by fitting $P_c-S$ data for drainage and imbibition, respectively. A nonlinear least squares fitting procedure such as that of Marquardt (1963) can be used for this purpose. For systems having a saturation dependent wettability, values for $\phi_{ow}(S_{ow})$ can be calculated at each saturation according to Eq. (16).

Many numerical multiphase flow simulators require simple closed-form analytical expressions for the relative permeability relations. Closed form expressions for the Burdine model (see Eqs. (11) and (12)) can be obtained when the model of van Genuchten (1980), with $m = 1 - 2/n$, is used to describe the $P_c-S$ data. Analytical expressions for the modified Burdine’s model can similarly be obtained.

2.2.2. Three-fluid systems

Pore-size distribution models similar to Eqs. (11) and (12) have also been developed to predict three-fluid relative permeability relations (Corey et al., 1956; Lenhard and Parker, 1987):

$$k_{rw}(S_{ow}^w) = (S_{ow}^w)^2 \frac{\int_{S_{ow}^w}^{S_{ow}^w} R(x) x^2 dx}{\int_{0}^{1} R(x) x^2 dx}$$

(17)

$$k_{ro}(S_{ow}^w, S_{ao}^w) = (S_{ao}^w)^2 \frac{\int_{S_{ao}^w}^{S_{ao}^w} R(x) x^2 dx}{\int_{0}^{1} R(x) x^2 dx}$$

(18)

$$k_{ra}(S_{al}^w) = (1 - S_{al}^w)^2 \frac{\int_{S_{al}^w}^{1} R(x) x^2 dx}{\int_{0}^{1} R(x) x^2 dx}$$

(19)

where $S_{al}^w$ is the effective total liquid saturation ($S_{al}^w = S_{ow}^w + S_{ao}^w$). Note that even though multiple capillary pressure drops ($P_{ow}$ and $P_{ao}$) occur in three-fluid systems the
same pore-size distribution is used in these models to characterize the permeabilities of the medium. The only difference between two- and three-fluid pore-size distribution models is the “assignment” of fluids to a pore domain, i.e. small and large for two-fluid systems compared with small, medium, and large for three-fluid systems. As for two-fluid systems, this is achieved by changing the integration limits.

To extend Eqs. (17)-(19) to account for various wettabilities, it is assumed herein that organic and water will be found in small and medium pore domains, and that air is restricted to the largest pore domain (air is always the non-wetting fluid). Based upon this assumption $k_{rw}$ is determined by modifying Eq. (17) similar to Eq. (13):

$$k_{rw} \left( \bar{S}_{saw}, \bar{S}_{oaw} \right) = \left( \bar{S}_{w} \right)^{2} \frac{\left[ 1 - f(\phi_{saw}) \right] \int_{0}^{\bar{S}_{saw}} R(x)^{2} \, dx + f(\phi_{saw}) \int_{\bar{S}_{saw}}^{\bar{S}_{oaw}} R(x)^{2} \, dx}{\int_{0}^{1} R(x)^{2} \, dx}$$

(20)

where the limits of integration of the first and second integrals in the numerator assign water to the small and medium pore domains, respectively. Analogously, $k_{ro}$ is determined by modifying Eq. (18) similar to Eq. (14):

$$k_{ro} \left( \bar{S}_{saw}, \bar{S}_{oaw} \right) = \left( \bar{S}_{o} \right)^{2} \frac{f(\phi_{saw}) \int_{0}^{\bar{S}_{saw}} R(x)^{2} \, dx + \left[ 1 - f(\phi_{saw}) \right] \int_{\bar{S}_{saw}}^{\bar{S}_{oaw}} R(x)^{2} \, dx}{\int_{0}^{1} R(x)^{2} \, dx}$$

(21)

The value of $k_{rn}$ is obtained according to Eq. (19). The weighting function employed in Eqs. (20) and (21) is again given by Eq. (15). In this case, Eq. (15) is used to determine the contributions of the small and medium pore domains. Note again that, depending on fluid and solid properties, $\phi_{saw}$ may or may not be a function of saturation in Eq. (15).

The value of $\phi_{saw}$ for air-organic-water systems can also be determined according to Eq. (16) since air is always (likely to be) the non-wetting fluid. This implies that $\phi_{saw}$ may be determined from the $P_{ow}-\bar{S}_{w}$ or $P_{ow}-\bar{S}_{oaw}$ relations because $P_{ow}-\bar{S}_{w}$ data can be used to predict $P_{ow}-\bar{S}_{saw}$ relationships (Bradford and Leij, 1996).

### 3. Application

The equations described above for estimating wettability effects on two- and three-fluid $k_{s}-S$ relations require knowledge of the pore-size distribution and macroscopic contact angle of the porous medium. In the examples presented below, the pore-size distribution was estimated (Eq. (9)) from experimental primary drainage air-organic (Soltrol 220) $P_{c}-S$ data which were reported by Bradford and Leij (1995a). All porous media were composed of 25% very coarse sand, 50% coarse/medium sand, and 25% fine sand according to the USDA textural classification (Soil Survey Staff, 1975).
Table 1 lists the relevant fluid and solid parameters for this air–organic porous medium system (Bradford and Leij, 1995a). Hypothetical macroscopic contact angles were chosen to vary over a wide range of wetting conditions. Actual macroscopic contact angles can be fitted or calculated from $P_c$–$S$ data and interfacial tensions according to Eq. (16). For simplicity, uniform and fractional wettability systems will be referred to herein as having saturation independent and saturation dependent contact angles, respectively.

![Diagram](image-url)

Fig. 2. (a) and (b). Estimated values of $k_{rw}$ and $k_{ro}$ according to Eqs. (13) and (14), respectively, for media having hypothetical $\phi_{uw}$ equal to 0°, 45°, 90°, 135°, and 180°.
3.1. Two-fluid systems

For the selected medium, Fig. 2(a) and Fig. 2(b) illustrate the predicted dependence of $k_{rw}$ and $k_{ro}$ (according to Eqs. (13) and (14), respectively) on a saturation independent contact angle. Note that an increasing $\phi_{ow}$ leads to an increase in $k_{rw}$ and a decrease in $k_{ro}$. This result occurs owing to the change in roles (wetting versus non-wetting) of water and organic as $\phi_{ow}$ increases. These trends are consistent with the experimental results of Owens and Archer (1971) and McCaffery and Bennion (1974).

Fig. 3(a) shows the main imbibition fractional wettability $P_{ow}-S_{ow}^w$ relations (Bradford and Leij, 1995b) for media composed of 0, 25, 50, 75, and 100% organic-wet fractions: the octadecyltrichlorosilane (OTS) treated sand fraction. Fig. 3(b) shows values of $\phi_{ow}(S_{ow}^w)$ that were calculated according to Eq. (16) from these data, the air–organic $P_{w}-S$ data given in Table 1, and the measured value of $\sigma_{ow} = 0.026$ N m$^{-1}$ (Bradford and Leij, 1995a). Fig. 4(a) and Fig. 4(b) present the corresponding predictions.
Fig. 4. (a) and (b). Calculated values of \( k_{rw} \) and \( k_{ro} \) according to Eqs. (13) and (14), using \( \phi_{row}(\tilde{S}_{row}) \) in Eq. (15), for media composed of 0, 25, 50, 75, and 100% organic-wet fractions (OTS).

for \( k_{rw} \) (Eq. (13)) and \( k_{ro} \) (Eq. (14)), respectively. Note that increasing the organic-wet fraction leads to an increase in \( k_{rw} \) and a decrease in \( k_{ro} \), similar to reported experimental data (Donaldson and Thomas, 1971; Heaviside et al., 1987) and Fig. 2. In contrast to Fig. 2, calculated values of \( k_{rw} \) and \( k_{ro} \) here also depend on the \( \phi_{row}(\tilde{S}_{row}) \) relation. In general, the saturation dependency of \( \phi_{row} \) was more pronounced for media with greater organic-wet fractions (see Fig. 3(b)).

3.2. Three-fluid systems

Predicted three-phase relative permeability dependence on contact angle is illustrated in Figs. 5 and 6. Fig. 5(a) and Fig. 5(b) present the functional dependence of the \( k_{rw} \) on \( \tilde{S}_{row} \) at fixed \( \tilde{S}_{row} = 0.2 \), and the functional dependence of \( k_{ro} \) on \( \tilde{S}_{row} \) at fixed \( \tilde{S}_{row} = 0.4 \) and 0.8, respectively, according to Eq. (20). Comparison of Fig. 5(a) and Fig. 5(b) reveals that \( k_{rw} \) is primarily a function of \( \tilde{S}_{row} \) since \( \tilde{S}_{row} \) determines the number of pores accessible for water flow. Fig. 5(b) shows that \( k_{ro} \) also depends on \( \tilde{S}_{row} \) as \( \phi_{row} \) increases, primarily at larger \( \tilde{S}_{row} \). This is due to the influence of \( \tilde{S}_{row} \) on the
configuration of water at larger contact angles. Note that a greater $\phi_{so}$ increases $k_{rw} (S_{o}^{so})$ (see Fig. 5(a)), in a similar manner as for two-fluid systems (see Fig. 2(a)), and also increases $k_{ro} (S_{o}^{so})$ (see Fig. 5(b)). This result is consistent with the observations of Caudle et al. (1951), Reid (1958), Snell (1962), and Donaldson and Dean (1966) who found that $k_{ro}$ depends on both $S_{o}^{so}$ and $S_{w}^{so}$. Note that previous three-fluid pore-size distribution models are not able to produce such trends since they assume that $k_{ro}$ is only a function of $S_{o}^{so}$.

Fig. 6(a) and Fig. 6(b) present the functional dependence of the $k_{ro}$ on $S_{o}^{so}$ at fixed $S_{w}^{so} = 0.2$, and the functional dependence of $k_{ro}$ on $S_{o}^{so}$ at fixed $S_{w}^{so} = 0.4$ and 0.8, respectively, according to Eq. (21). Similar to the behavior of $k_{ro}$, observe that $k_{ro}$ is mainly a function of $S_{o}^{so}$ (see Fig. 6(a)); $S_{o}^{so}$ determines the number of pores accessible for organic flow. The value of $k_{ro}$ also depends on $S_{w}^{so}$ when $\phi_{so} < 180^\circ$ (see Fig. 6(b)), primarily for large $S_{w}^{so}$, since water affects the configuration of organic. Note that a greater $\phi_{so}$ decreases $k_{ro} (S_{o}^{so})$ (see Fig. 6(a)), in a similar manner as for
two-fluid systems (see Fig. 2(b)), and also decreases $k_{ro}(S_{ow}^{\text{aq}})$ (see Fig. 6(b)). Leverett and Lewis (1941), Caudle et al. (1951), Corey et al. (1956) were some of the first investigators to show that $k_{ro}$ depended upon both $S_{ow}^{\text{aq}}$ and $S_{o}^{\text{aq}}$.

The three-phase relative permeability model (see Eq. (19)) assumes that $k_{ro}$ is a unique function of $S_{ow}^{\text{aq}}$ ($S_{ow}^{\text{aq}} = 1 - S_{o}^{\text{aq}}$) and independent of $\phi_{ow}$. Fig. 7 shows the calculated values of $k_{ro}$ as a function of $S_{ow}^{\text{aq}}$. Observe that increasing $S_{ow}^{\text{aq}}$ leads to a decrease in $k_{ro}$ owing to the reduction of pores accessible for air flow. The reported dependence of $k_{ro}$ on the liquid saturations (Honarpour et al., 1986) has been attributed to air entrapment (Donaldson and Kayser, 1981), which is not incorporated in the presented model. Land (1968) and Lenhard and Parker (1987) present pore-size distribution permeability models which account for non-wetting fluid entrapment. These hysteretic models also may be revised to account for wettability. The details of this modification are not discussed herein owing to the more speculative nature of such a model.
The behavior of three-phase systems having saturation-dependent wettability is similar to that presented above. Increasing the organic-wet fraction leads to a greater $k_{rw}$ and a smaller $k_{ro}$ for given saturations, and $k_{rw}$ and $k_{ro}$ are functions of both water and oil saturations. In contrast, the magnitude of $k_{rw}$ and $k_{ro}$ also depends on the saturation dependency of $\phi_{row}$. As for the two-fluid system (see Fig. 4) the saturation dependency of $\phi_{row}$ is more pronounced for media having larger organic-wet fractions.

4. Evaluation

Although experimental two- and three-fluid $k_r$-$S$ relations have previously been measured, these data are generally inadequate to compare quantitatively with the estimation procedure presented herein, since the methodology requires $P_c$-$S$ data as well as contact angles. Donaldson and Thomas (1971) measured organic-water $k_r$-$S$ relations for Torpedo sandstone having various degrees of fractional wettability; wettability alterations were achieved by treating the cores with different concentrations of organochlorosilane. Fig. 8(a) presents their measured $k_{ro}$-$\bar{S}^ow$ relations. Here $\bar{S}^ow = (S^ow - S^ow_w)/(1 - S^ow_w)$ with $S^ow_w$ set equal to the water saturation at the start of the experiment. These researchers determined the wettability of the cores according to the United States Bureau of Mines wettability index ($I_{usbm}$) (Donaldson et al., 1969). Higher values of $I_{usbm}$ indicate a larger water-wet fraction. Note that, in general, $k_{ro}$ decreases at a given $\bar{S}^ow$ with decreasing $I_{usbm}$. The exception to this trend is that the value of $k_{ro}$ for $I_{usbm} = 0.649$ is slightly lower than that for $I_{usbm} = 0.179$. It is hypothesized herein that this may be due to slight differences in the pore-size distribution between the cores, or errors in the measurement of $\bar{S}^ow$. Since $P_c$-$S$ data for these Torpedo sandstone cores were not presented in the literature, the direct estimation of these $k_{ro}$-$\bar{S}^ow$ relations is not possible.

To qualitatively assess the utility of the predictive procedure presented in this work the value of $m$ from the analytical solution of the Burdine model (cf. van Genuchten et al., 1991) was fitted to the measured $k_{ro}$-$\bar{S}^ow$ data for $I_{usbm} = -1.333$ (the system which was the most strongly wetted, in this case by organic) shown in Fig. 8(a). This fitted value of $m = 1 - 2/n = 0.4629$ was used to predict the corresponding $P_c$-$S$ curve.
Fig. 8. (a) and (b). Measured and predicted $k_{rw}-S_{w}$ relations for variously wetted Torpedo sandstone containing brine and Squirrel crude oil. Measured data are from Donaldson and Thomas (1971).

(van Genuchten, 1980): Here an $\alpha_d = 0.02536$ ($\alpha_i = 2\alpha_d$) parameter was estimated from the reported mean pore size of Torpedo sandstone (11.5 $\mu$m) (Singh, 1990) and the reported interfacial tension of Squirrel crude oil ($\sigma_{ow} = 0.0223$ Nm$^{-1}$) (Donaldson et al., 1969). Hypothetical $P_c-S$ curves for the different wettibilities were obtained by shifting the previously estimated $P_c-S$ curve downward until the appropriate $I_{bhm}$ wettability index was achieved (see Bradford and Leij, 1996). Values for $\phi_{ow}(S_{w})$ were then calculated from these estimated $P_c-S$ curves according to Eq. (16); $P_{ao}-S_{ao}$ data were replaced by the estimated $P_c-S$ curve for the perfectly wetted system. Fig. 8(b) shows the resultant estimated $k_{ro}-S_{w}$ relations which correspond to the measured data shown in Fig. 8(a). Note that Fig. 8(b) captures the general trends shown in Fig. 8(a); increasing the organic-wet fraction leads to a decrease in $k_{ro}$ at a given saturation. The influence of wettability on the measured $k_{ro}-S_{w}$ relations is less marked than that predicted in Fig. 8(b). This discrepancy may be due to inaccuracies in the estimation of $S_{rw}$ or the $\phi_{ow}(S_{w})$ relations.
5. Summary and conclusions

Two- and three-fluid $k_r$-$S$ relations are required to simulate multiphase flow and transport processes. Commonly, these permeability relations are estimated from more widely available $P_c$-$S$ data according to statistical pore-size distribution models. Previous pore-size distribution models, however, are based on restrictive wettability assumptions. A modification of the Burdine pore-size distribution model is presented herein to estimate two- and three-fluid $k_r$-$S$ relations for media having non-zero contact angles and/or fractional wettability.

For two-fluid systems, descriptions of the wetting and non-wetting fluid permeability were combined for both water and organic phases. The contributions of wetting and non-wetting pore classes to $k_w$ and $k_o$ were determined based on wettability. A constant weighting function given by Eq. (15) was proposed for non-zero contact angles. The weighting function will be a function of saturation for fractional wettability media. The resultant permeability model predicts that a greater contact angle or organic-wet fraction will lead to an increase in $k_w$ and a decrease in $k_o$ at a given saturation.

For three-fluid systems organic and water were assumed to be distributed in small and medium pore classes, while air was assumed to reside entirely in pores belonging to the "largest" pore class. The permeability of water and organic were represented, thus, as composites of wetting and intermediate fluid pore classes. The contribution of a particular pore class was again determined by wettability. Behavior at the wettability extremes is preserved by the model, while $k_w$ and $k_o$ are functions of both $S_{ow}$ and $S_{aw}$ for $0^\circ < \phi_{ow} < 180^\circ$. The permeability to a particular fluid (i.e. $k_w$, $k_o$, and $k_r$) mainly depends upon that fluid's saturation ($S_{ow}$, $S_{aw}$, and $S_{aw}$, respectively) since this saturation determines the number of pores accessible for flow. The model predicts $k_w$ will increase and $k_o$ will decrease at a given saturation for a greater $\phi_{ow}$ and/or organic-wet fraction. The value of $k_r$ was assumed to be independent of wettability and uniquely a function of the total liquid saturation.

This paper attempts to predict the influence of wettability on $k_r$-$S$ relations. Comparison with available data suggests that the presented model captures the general trends reported in the literature for wettability effects on two-fluid $k_r$-$S$ relations (Owens and Archer, 1971; McCaffery and Bennion, 1974). Furthermore, the conceptual model for three-fluid $k_r$-$S$ relations provides predictions consistent with observed differences in the saturation dependency of relative permeabilities (Honarpour et al., 1986). Additional experimental studies are required to further evaluate and refine the modelling approach.

6. Notation

- $a$: air
- $A_{kl}$: interfacial area per unit pore volume between phases $k$ and $l$ (cm$^2$ cm$^{-3}$)
- $A_s$: solid surface area per unit bulk volume (cm$^2$ cm$^{-3}$)
- $A_{sw}$: solid surface area per unit pore volume (cm$^2$ cm$^{-3}$)
- $A_{sc}$: effective solid surface area per unit pore volume (cm$^2$ cm$^{-3}$)
C_k  Kozeny constant
f  weighting function
F_o  organic wet fraction
g  acceleration owing to gravity (m s^{-2})
H  hydraulic head (cm)
I_{usbm}  United States Bureau of Mines (usbm) wettability index
k_r  relative permeability
k_{rp}  relative permeability of phase p
o  organic
P_c  capillary pressure (N m^{-2}, cm water)
P_i  pressure of phase p (N m^{-2}, cm water)
P_{kl}  capillary pressure drop over interface between fluids k and l, i.e. \( P_k - P_l \) (N m^{-2}, cm water)
q_i  flux of through ith capillary tube (cm s^{-1})
r_h  hydraulic radius (cm)
R  radius of a capillary tube (cm)
s  solid
S  saturation (cm^3 cm^{-3})
S_{ik}  "immobile" residual wetting fluid saturation of fluid k (cm^3 cm^{-3})
S_{kl}  saturation of fluid k in a medium containing the two fluids k and l (cm^3 cm^{-3})
S_{klm}  saturation of fluid k in a medium containing the three fluids k, l, and m (cm^3 cm^{-3})
S_{mklt}  saturation of maximum entrapped fluid k (cm^3 cm
S_r  residual saturation (cm^3 cm^{-3})
S_l  total liquid saturation (cm^3 cm^{-3})
S  effective saturation (cm^3 cm^{-3})
w  water
\varepsilon  porosity (cm^3 cm^{-3})
\mu  viscosity of the fluid (g cm^{-1})
\rho  density of the fluid (g cm^{-3})
\sigma_{kl}  interfacial tension at interface between fluids k and l (N m^{-1})
\phi_{	ext{eq}}  equilibrium contact angle at contact line between solid and fluids i and j (degree)
\phi_{\text{adv}}  advancing contact angle (degree)
\phi_{\text{rec}}  receding contact angle (degree)

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Appendix A

Throughout this paper effective saturations have been used to quantify the pore space accessible for flow. For organic–water systems, the effective water saturation is defined as:

\[ S_{\text{sw}} = \frac{S_{\text{ow}} - S_{\text{iw}}}{1 - S_{\text{iw}} - S_{\text{to}}} \]  

(A1)

Similarly the effective organic saturation is given as:

\[ S_{\text{o}} = \frac{S_{\text{ow}} - S_{\text{to}}}{1 - S_{\text{iw}} - S_{\text{to}}} \]  

(A2)

where \( S_{\text{iw}} \) and \( S_{\text{to}} \) are the “immobile” residual water and organic saturations, respectively, expressed as volume of fluid per volume of pore space. In a three-fluid system the effective water and organic saturations are defined analogously to Eqs. (A1) and (A2), respectively; where the superscript ow is now replaced by aow. The effective air saturation is given as:

\[ S_{\text{a}} = \frac{S_{\text{a}}^{\text{ow}}}{1 - S_{\text{iw}}^{\text{ow}} - S_{\text{to}}^{\text{ow}}} \]  

(A3)

The effective total liquid saturation \( (S_{\text{bl}}^{\text{ow}}) \) is equal to \( 1 - S_{\text{a}}^{\text{ow}} \).

The lowest water \( (S_{\text{lw}}^{\text{ow}}) \) and organic \( (S_{\text{to}}^{\text{ow}}) \) saturations may be determined from \( P_{\text{ow}} - S_{\text{sw}}^{\text{ow}} \) data. For uniformly wetted medium with \( 0^\circ \leq \phi_{\text{low}} < 90^\circ \), \( S_{\text{lw}}^{\text{ow}} \) is zero and \( S_{\text{to}}^{\text{ow}} = S_{\text{low}}^{\text{ow}} \). Analogously, \( S_{\text{low}}^{\text{ow}} \) is zero for \( 90^\circ \leq \phi_{\text{low}} \leq 180^\circ \) and \( S_{\text{to}}^{\text{ow}} = S_{\text{ro}}^{\text{ow}} \). For fractional wettability media both \( S_{\text{lw}}^{\text{ow}} \) and \( S_{\text{to}}^{\text{ow}} \) may be non-zero owing to the presence of organic- and water-wet solids. In this case, it is assumed that \( S_{\text{lw}}^{\text{ow}} - S_{\text{to}}^{\text{ow}} - S_{\text{low}}^{\text{ow}} \) and \( S_{\text{lw}}^{\text{ow}} = S_{\text{lw}}^{\text{ow}} - S_{\text{mwt}}^{\text{ow}} \), where \( S_{\text{mwt}}^{\text{ow}} \) and \( S_{\text{mwt}}^{\text{ow}} \) are the maximum entrapped organic and water, respectively. The maximum amount of entrapped fluid is not known a priori for fractional wettability systems. It is hypothesized herein that an estimate of \( S_{\text{mwt}}^{\text{ow}} \) may be obtained from scaling \( S_{\text{mwt}}^{\text{ow}} \) of an air–organic system by \( \sigma_{\text{ow}}(1 - F_{\text{o}})/\sigma_{\text{ao}} \) (see Bradford and Leij, 1997); where \( F_{\text{o}} \) is the mass fraction of organic-wet solids. Similarly, an estimate of \( S_{\text{mwt}}^{\text{ow}} \) may be obtained by multiplying \( S_{\text{mwt}}^{\text{ow}} \) by \( \sigma_{\text{ow}} F_{\text{o}}/\sigma_{\text{ao}} \). Note that \( S_{\text{mwt}}^{\text{ow}} \) and \( S_{\text{mwt}}^{\text{ow}} \) are associated with the water-wet \( (1 - F_{\text{o}}) \) and organic-wet \( F_{\text{o}} \) fractions, respectively.

Bradford and Leij (1997) discuss a method for determining \( F_{\text{o}} \) for a fractional wettability medium.

References


