CHARACTERIZATION OF NONLINEAR ELASTIC PROPERTIES OF BEEF PRODUCTS UNDER LARGE DEFORMATION

Renfu Lu, Yud Ren Chen

ABSTRACT. *Knowledge of mechanical properties of beef is important for studying its tenderness and improving tenderness measurement techniques. This research was initiated to develop a three-dimensional constitutive equation to describe the nonlinear stress-strain relationship of beef products in large deformations, to determine the constitutive parameters, and to validate the model for multidimensional stress/strain states. Based on experimental observations and the theory of finite elasticity, a three-dimensional constitutive equation was proposed, which assumes that the materials are isotropic and incompressible. Uniaxial compression tests were performed on twenty samples each of bologna, salami, and smoked sausage to determine the two parameters in the constitutive equation. Confined two-dimensional compression tests were also conducted to validate the constitutive equation. Experimental results showed that beef products exhibited a pronounced nonlinearity under uniaxial compression; the slope of the force-deformation curves increased monotonically with the increasing deformation. The constitutive equation fitted the stress-strain curves well for the three products. The constitutive equation predicted well the stress-strain responses of the beef products under confined two-dimensional loading; the differences between predicted and measured values were mostly not significant at the 0.05 level. This constitutive equation can be used to predict stress-strain responses of beef products under different loading conditions.*
Keywords. *Meat, Beef, Tenderness, Mechanical properties, Modeling.*

Texture is an important aspect of the overall quality of most foods and agricultural products. Many textural attributes, such as firmness, hardness, and tenderness, are directly related to the mechanical properties of foods. Therefore, knowledge of mechanical properties is important for food quality evaluation and control. Considerable research has been reported on characterization of the mechanical properties of foods and agricultural materials (see, for example, Mohsenin, 1986). Most studies, however, are focused on the stress-strain responses in small deformations for which the linear elastic or viscoelastic theory can be applied. There are applications in which knowledge of mechanical properties of foods in large deformation is necessary or desirable.

For example, tenderness is considered one of the most important textural attributes for judging the overall eating quality of beef and its products. The current, widely used method for measuring tenderness is the Warner-Bratzler (WB) shear testing, which measures the maximum force required to cut through a meat sample. The measurement involves a process of compression, bending, and shearing of the sample. Due to the structure of the blade, the actual loading process is difficult to interpret. Several researchers have attempted to interpret the mechanism of WB shearing based on experimental observations and intuitive reasoning. Voisey and Larmond (1974) reported that meat samples fail primarily in tension rather than in shear during WB shearing test. Recently, Zhang and Mittal (1993) studied the effect of sample size and loading rate on tenderness measurement of three beef products, including bologna, salami, and pastrami. Based on their observations, they also concluded that meat samples were ruptured under tension instead of shear. Validity of these interpretations has yet to be demonstrated and such a verification is crucial to the understanding of factors affecting tenderness measurement and to improving existing measurement techniques (Voisey, 1976).

During WB shearing, the sample is subjected to large deformations, both elastic and plastic. Like most biological materials, beef and its products exhibit nonlinear, time-dependent force-deformation behavior, particularly under large deformations. To understand the WB shearing process, one must understand the fundamental mechanical properties of meat in large deformations as well as their failure characteristics. A considerable amount of information is available on mechanical properties of muscle tissue (Lepetit and Culioli, 1994). The data available, however, cannot be used for stress-strain analyses because they are either incomplete or the purposes of those studies are not directed toward characterization of the fundamental rheological properties. Only limited studies have been reported on characterizing the mechanical properties of meat, and they are primarily limited to the linear elastic or viscoelastic theory (e.g., Sacks et al., 1988).

There is considerable interest and activity in nonlinear mechanical properties of living tissues in the biomedical engineering area and a comprehensive review of the latest

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advances in this area can be found in Fung (1991, 1993). Studies have also been reported on the nonlinear mechanical properties of several agricultural materials and foods under large deformations. Gates et al. (1986) and Gao et al. (1989) applied the finite elastic theory to study the plant vegetative cells and constitutive equations were proposed to predict the nonlinear stress-strain relationship of cell walls. Gao et al. (1993) and Tang et al. (1997) developed constitutive equations, based on the strain energy theory, to characterize the stress-strain relationship of several gels under large deformations. Purely empirical equations were also proposed to describe the nonlinear stress-strain relationship for agricultural products, such as tobacco leaves and soybean (Foutz et al., 1993; Henry and Zhang, 1996; Peleg, 1983). These empirical equations are often obtained by using a mathematical function to fit the stress-strain curves and, therefore, they cannot be applied to multidimensional stress-strain situations.

This article reports the results from our study on characterizing the nonlinear stress-strain relationship of three beef products, i.e., bologna, salami, and smoked sausage. These products are relatively uniform and homogeneous in structure and highly elastic but conspicuously nonlinear. The ultimate goal of this research is to gain a better understanding of the relationship between the mechanical properties of meat and its products and tenderness measurements. The specific objectives of this study were to:

1. Propose a constitutive equation to describe the stress-strain relationship for the beef products under large deformation;
2. Determine the parameters in the constitutive equation for bologna, salami, and smoked sausage using data obtained from uniaxial compression tests; and
3. Validate the constitutive relations using data from confined two-dimensional loading tests.

Nonlinear Constitutive Equation

In studying the nonlinear mechanical properties of biological materials, the first, also the most important step is to select or develop an appropriate constitutive equation to describe the stress-strain relationship. Once the constitutive relationship is established, experiments can then be performed to determine the parameters characterizing the mechanical properties of a particular material. For linear elastic or viscoelastic materials, there exists a unique constitutive equation that can be derived mathematically based on the principle of solid mechanics. Most biological materials, however, are essentially nonlinear, inelastic under finite deformation; their mechanical behavior is not only dependent on the current loading but also the past loading history. It is considerably more difficult and complex to deal with nonlinear inelastic materials, both theoretical and experimental. Further, there is no unique constitutive equation to describe the stress-strain relationship for these materials. In many instances, a specific constitutive equation must be developed to describe a particular material or a class of materials.

In studying the nonlinear mechanical properties of biological materials, the concept of pseudo-elasticity is often introduced to simplify the problem. The essential of this concept is that for certain applications, inelastic materials may be treated as if they were elastic. For instance, if, after repeated loading and unloading (also called preconditioning), there exists a unique stress-strain relationship for the loading and unloading path, separately, then the material can be treated as one elastic material in loading, and another elastic material in unloading (Fung, 1993). Also in situations where only monotonic loading paths are considered, an inelastic material may also be treated as elastic (Veronda and Westmann, 1970). In this section, we propose a nonlinear constitutive equation, based on the concept of pseudo-elasticity, to describe multidimensional stress/strain states in the three beef products.

In developing the constitutive equation, it is assumed that the beef products can be considered as isotropic, incompressible pseudo-elastic materials. The isotropy assumption seems justifiable since bologna is mainly composed of finely comminuted and well commingled beef; salami of ground beef; and smoked sausage of minced beef. The three products studied were 100% beef, of which about two-thirds is water with the rest being protein, fat, carbohydrate, etc. No void spaces are present in these products. Beef, as indicated in previous studies (Lepetit and Culioli, 1994), is generally considered to be incompressible. Therefore, it seems reasonable to use the incompressibility assumption for these beef products. The appropriateness of these two assumptions will be further discussed in the "Results and Discussion" section in terms of the ability of the constitutive model to predict the stress-strain responses under a multidimensional loading state.

A survey of literature shows that several constitutive equations have been proposed to characterize nonlinear isotropic, incompressible materials. With the exception of the well-known Mooney-Rivlin equation which is popular for describing rubber-like materials, no effort was made to test the validity or suitability of other reported constitutive equations (e.g., Blatz et al., 1969; Veronda and Westmann, 1970). Preliminary analysis on the experimental data (See the "Results and Discussion" section) showed that the Mooney-Rivlin equation describes the force-deformation behavior of the beef products well under compression. The main problem with this equation is that the two parameters in the Mooney-Rivlin equation determined from experimental data showed no consistent pattern and had large variations for each product, making it impossible to differentiate different materials based on parameter values. Further, the Mooney-Rivlin equation is not adequate to describe the nonlinear stress-strain relationship of many biological materials under tensile loading conditions (Fung, 1993). For these reasons, the Mooney-Rivlin equation was not chosen and a new constitutive equation was proposed to characterize the three beef products.

From the theory of finite elasticity, it is known that the stress-strain relationship for an elastic material may be expressed as a function of the strain energy. Accordingly, the constitutive equation for an isotropic and incompressible elastic solid under finite deformations may be expressed in the following form (Fung, 1993):

$$\sigma_{ij} = \frac{\partial x_i}{\partial X_R} \frac{\partial x_j}{\partial X_S} \frac{\partial W}{\partial E_{RS}} - p\delta_{ij} \qquad (1)$$

where

i, j, R, S = indexes, taking values of 1, 2, or 3 (x, y, or z)
σ_{ij} = Cauchy stress components
δ_{ij} = the Kronecker delta; equal to one when i = j, and zero when $i \neq j$
E_{RS} = the Lagrangian (or Green's) strain components
p = hydrostatic pressure
W = strain energy function
x_i = spatial coordinates in a Cartesian coordinate system with respect to the current (deformed) configuration
X_i = spatial coordinates in a Cartesian coordinate system with respect to the initial (or undeformed) configuration.

In equation 1 and the following derivations, the conventions commonly used in the tensor analysis are implied. For instance, whenever an index is repeated once, it is a dummy index indicating a summation through the integral numbers 1, 2, and 3. The Lagrangian strains in equation 1 are defined as:

$$E_{RS} = \frac{1}{2}\left(C_{RS} - \delta_{RS}\right) = \frac{1}{2}\left(\frac{\partial x_i}{\partial X_R}\frac{\partial x_i}{\partial X_S} - \delta_{RS}\right) \quad (2)$$

in which C_{RS} are the right Cauchy-Green deformation components. Apparently, the key step to developing a constitutive equation is to find a strain energy function that is capable of describing the nonlinear behavior of the beef products. It has been shown (Spencer, 1980) that for isotropic elastic materials, the strain energy function is a function of the three strain invariants, I_1, I_2, and I_3 for the right Cauchy-Green deformation tensor **C**, i.e.:

$$W = W(I_1, I_2, I_3) \quad (3)$$

These strain invariants can be expressed in terms of the three principal stretches (or compressions), designated as λ_1, λ_2, and λ_3 (Spencer, 1980).

$$\begin{aligned} I_1 &= \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \\ I_2 &= \lambda_1^2\lambda_2^2 + \lambda_2^2\lambda_3^2 + \lambda_3^2\lambda_1^2 \\ I_3 &= \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \end{aligned} \quad (4)$$

The principal stretches or compressions are defined as the ratio of the length of lines in the three principal directions in the deformed body divided by their length in the undeformed body. When a uniaxial test is performed on a cylindrical sample in the longitudinal direction, the stretch or compression is simply the ratio of the instantaneous gauge length (L) of the deformed sample to its original gauge length (L_0):

$$\lambda = \frac{L}{L_0} = 1 + \frac{\Delta L}{L_0} \quad (5)$$

where ΔL is the net change (positive in extension and negative in compression) in the sample length.

In the past, a variety of mathematical expressions have been proposed to describe the strain energy function for living biological tissues and some engineering materials. There are essentially only two schools: one uses polynomials, while the other uses exponential functions. According to Fung (1993), the exponential functions are more popularly used for describing living tissues. Gao et al. (1989, 1993) and Tang et al. (1997) used the polynomials to describe the strain energy function of plant tissues and gels. Based on the observations of experimental data for the beef products (See the "Results and Discussion" section) and the approach used by Fung (1993), this study uses an exponential function to describe the strain energy function. In proposing a constitutive equation for the beef products, the following two conditions were considered: (1) the strain energy function must satisfy the condition in equation 3; and (2) the nonlinear constitutive equation should be reduced to the exact form of the linear theory, since the later is a special case for elastic materials under small deformations. With these two considerations, the following strain energy function was proposed:

$$W = \frac{1}{2}\frac{\beta}{\alpha}\left(e^{\alpha \operatorname{tr}\mathbf{E}^2} - 1\right) \quad (6)$$

where α and β are the parameters to be determined from experiments, and tr is the trace of a square matrix, i.e.:

$$\operatorname{tr}\mathbf{E}^2 = E_{ij}\,E_{ij} \quad (7)$$

Substituting equation 7 into 6, equation 1 can be written as:

$$\sigma_{ij} = -p\delta_{ij} + \beta\frac{\partial x_i}{\partial X_R}\frac{\partial x_j}{\partial X_S}E_{RS}\,e^{\alpha \operatorname{tr}\mathbf{E}^2} \quad (8)$$

The strain energy function in equation 6 and the constitutive equation 8 satisfy the two conditions stated previously, because:

$$\operatorname{tr}\mathbf{E}^2 = \left(\operatorname{tr}\mathbf{E}\right)^2 - 2I'_2 = I'^2_1 - 2I'_2 \quad (9)$$

in which I'_1 and I'_2 are the first and second strain invariants of the Lagrangian strain tensor **E**, given by the following equations:

$$\begin{aligned} I'_1 &= \operatorname{tr}\mathbf{E} = \frac{1}{2}\operatorname{tr}\left(\mathbf{C} - \mathbf{I}\right) = \frac{1}{2}\left(I_1 - 3\right) \\ I'_2 &= \frac{1}{2}\left[\left(\operatorname{tr}\mathbf{E}\right)^2 - \operatorname{tr}\mathbf{E}^2\right] = \frac{1}{4}\left[\left(I_2 - 3\right) - 2\left(I_1 - 3\right)\right] \end{aligned} \quad (10)$$

where **I** is the unit isotropic tensor. Notice that the principal stretches (or compressions) for the Lagrangian strain tensor **E** are $\frac{1}{2}(\lambda_i^2 - 1)$. The second equation of equation 10 can be obtained by using the definitions of invariants. Hence the strain energy function W is indeed a function of I_1 and I_2 (I_3 is equal to one for incompressible materials). The second condition is also satisfied because, under small deformations, the exponent in equation 8 will be close to zero, implying that the exponential function approaches one. The derivatives $\partial x_i/\partial X_R$ are reduced to one when i = R and zero when i ≠ R and E_{ij} are approximately the same as the infinitesimal strains ε_{ij}. Therefore, equation 8 becomes:

$$\sigma_{ij} = -p\delta_{ij} + \beta\varepsilon_{ij} \tag{11}$$

Equation 11 is the exact form of the constitutive equation for linear incompressible elastic materials, in which the parameter β is equivalent to two times the shear modulus G.

The two material parameters in equation 8 can be obtained by performing uniaxial compression tests on cylindrical samples. The undetermined hydrostatic pressure p can be determined from the boundary condition, $\sigma_{rr} = 0$; that is, the lateral or radial stresses in this case are zero. After some mathematical derivations (See “Appendix”), the axial stress T_{ZZ} (the force divided by the cross-sectional area of the undeformed sample) is given by the following equation:

$$T_{zz} = \frac{P}{A_0} = \frac{\sigma_{zz}}{\lambda_z} =$$

$$\frac{1}{2}\beta\left(1-\frac{1}{\lambda_z^3}\right)\left(\lambda_z^3-\lambda_z+1\right)e^{\frac{1}{4}\alpha\left\{\left(\lambda_z^2-1\right)^2+2\left(\frac{1}{\lambda_z}-1\right)^2\right\}} \tag{12}$$

in which P is the total applied force, A_0 is the cross-sectional area of the undeformed sample, and λ_z is the ratio of the height of the deformed sample to that of the original. The parameter α is dimensionless and β has the same unit as the stress T_{ZZ}. Equation 12 is useful for determining the two material parameters from the force-deformation curve of a cylindrical sample subjected to uniaxial loading.

For the confined loading situation in figure 1 where the cubic sample is loaded in one direction (z-axis) and can move without restriction in the second direction (x-axis) but its displacement in the y-axis is restricted, the following equation can be obtained from equation 8 (See derivations in “Appendix”):

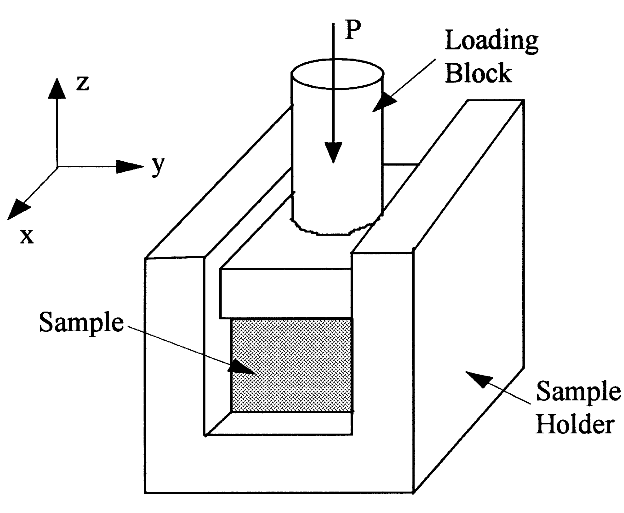


Figure 1–A schematic of the loading device for performing confined two-dimensional compression tests.

$$T_{zz} = \frac{P}{A_0} =$$

$$\frac{1}{2}\beta\lambda_z\left(\lambda_z^2-1\right)\left(1+\frac{1}{\lambda_z^6}\right)e^{\frac{1}{4}\alpha\left\{\left(\lambda_z^2-1\right)^2+2\left(\frac{1}{\lambda_z^2}-1\right)^2\right\}} \tag{13}$$

Equation 13 was used to validate the constitutive equation using the data acquired from confined compression tests for the three beef products.

For nonlinear materials under large deformations, it is often difficult, if not impossible, to interpret the physical meanings of the material parameters in the constitutive equation. Figure 2 shows the overall patterns between the stress and compression ratio, generated using equation 12 for various values of α and β. The compression ratio is defined as ΔL/L, while the compression λ_z is equal to (1 – ΔL/L), where L is the original sample length and ΔL is the sample deformation. When deformations are small, stress is linearly proportional to the strain, which is a basic feature of the constitutive equation 12. The parameter α has little effect on the slope of the curve for small deformations. However, as deformation increases, the effect of the

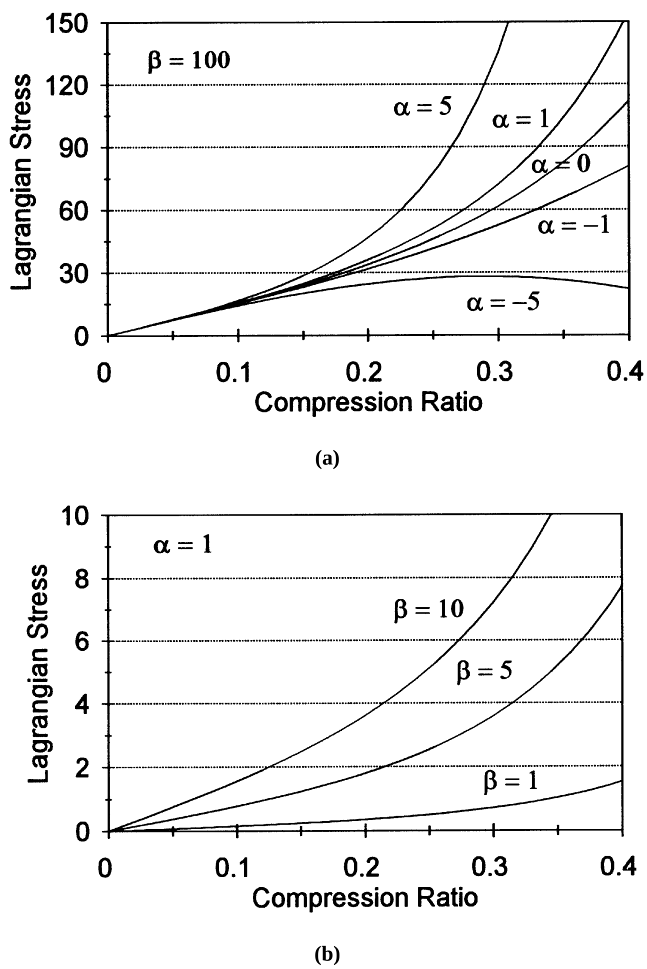


Figure 2–The stress vs compression ration (ΔL/L) curves for uniaxial compression of cylindrical samples generated using the constitutive equation 12 for different values of the parameters α and β. The curves in (a) were generated with β = 100 and those in (b) were generated with α = 1.

parameter α on the overall pattern becomes increasingly significant. The stress versus compression ratio curve changes from concave to convex as α changes from negative to positive (fig. 2a). On the other hand, the parameter β directly influences the slope of the stress versus compression ratio curve over the entire range; increasing the β value increases the curve slope (fig. 2b). Under small deformations, the parameter β is equivalent to two times the shear modulus G since equation 8 is reduced to the form of the linear theory.

PROCEDURE AND METHODS

Three beef products, i.e., bologna, salami, and smoked sausage, were used in this research. The beef products were purchased from a local grocery store and were kept refrigerated at 2°C prior to testing. Two sets of experiments were performed on the beef products. The first set of experiments was the uniaxial compression tests used to determine the material parameters in equation 12. The second set of experiments was the confined compression tests that were used to validate the constitutive equation under a multiple stress/strain state. Since it took less than two to three minutes to complete preparation and testing of beef samples, the change in sample temperatures was considered to be minimal during the entire test period.

CONSTITUTIVE PARAMETERS DETERMINATION

Uniaxial compression tests were performed on bologna, salami, and smoked sausage to determine the material parameters in the constitutive equation 12. Twenty cylinders of 15.9 mm diameter were taken for each product and they were trimmed to a length of 15.0 mm. Compression tests were performed on an Instron universal testing machine (Model TM) equipped with a 50 kg capacity load cell, with the crosshead speed set at 127 mm/min. The high loading rate was selected because it is close to the one normally used in Warner-Bratzler shear tests. All samples were compressed to a maximum deformation of 10 mm (corresponding to 66% compression ratio). Since the beef products had a considerable amount of water and fat that served as a natural lubricant, no lubricant was used in all uniaxial tests and the friction between the loading plate and the sample was considered to be insignificant. The force-deformation curve for each sample was recorded by a computer at an increment of 0.025 mm. Sample deformations were considered to be equal to the crosshead's displacements since the deflection of the load cell was negligible (< 0.1 mm at the full load capacity).

Additional tests were conducted to observe the mechanical properties of the beef products under cyclic uniaxial loading and unloading. Samples were loaded and unloaded four times (or cycles) between 0 and 33% compression ratios.

Data from monotonic uniaxial compression tests were analyzed using SAS (SAS, 1995). Nonlinear regressions were performed to fit each force-deformation curve with the constitutive equation 12 and the parameter values were obtained by the modified Gauss-Newton method. In determining the material parameters, only a portion of the force-deformation curve with the compression ratios ($\Delta L/L$) between 0 and 38% was used for bologna and salami. For smoked sausage, the compression ratio range was selected to be between 0 and 44%. These compression ranges were determined to exclude the yielding and/or failure portion of the curves and to ensure that the constitutive equation can be applied to a large range of strains within the limit of pseudo-elasticity.

CONSTITUTIVE MODEL VALIDATION

To validate the constitutive equation for multidimensional stress-strain states, confined compression tests were performed with a specially designed loading device (fig. 1). Cubes of 25.4 mm were cut from the same beef samples from which cylinders were taken for uniaxial compression tests. The cubes were placed in the slot of the sample holder shown in figure 1. When loads were applied through the loading block, the sample was only allowed to move without restriction in one lateral direction (x-axis) and was constrained in the other direction (y-axis); thus creating the two-dimensional strain status in the sample. Prior to testing, all contacting surfaces were lubricated with lubricant to reduce friction between the sample and the sample holder, and the moving block and the holder. However, according to Lepetit (1989), application of lubricant may not be necessary because the frictional forces between the meat and the sample holder and the moving block are negligible compared with the compressive forces at any compression ratio. The loading rate used in the confined compression was 127 mm/min, the same as the one for uniaxial compression. Four tests were performed for each product and the maximum compression ratio was set at 25% so that the samples would not be ruptured.

Validation of the constitutive equation was performed by comparing values of the measured stress (T_{zz}) versus compression (λ_z) with those predicted using equation 13. Predicted stresses were calculated using the constitutive parameter values obtained from uniaxial compression tests.

RESULTS AND DISCUSSION

Figure 3 shows typical stress-strain responses of the three beef products under monotonic uniaxial compression. Nonlinearity was conspicuous for all three products under uniaxial compression; the slope of the force-deformation

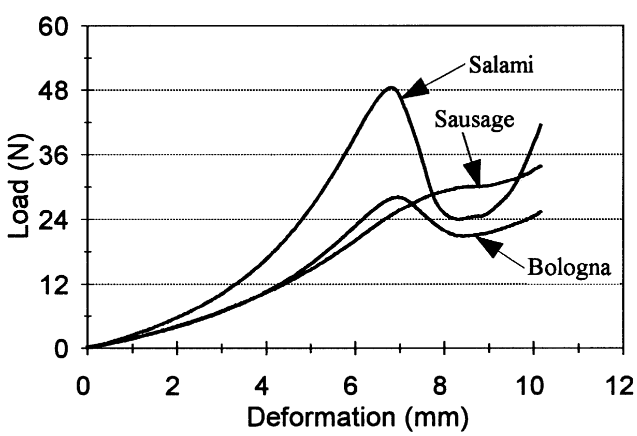
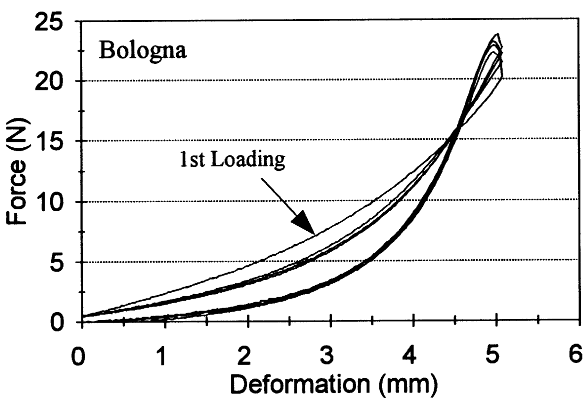


Figure 3–Typical force-deformation curves for bologna, salami, and smoked sausage under monotonic uniaxial loading at a loading rate of 127 mm/min (sample size: 15.9 mm in diameter and 15.0 mm in height).

curves increased monotonically until near the maximum force was reached which corresponded to the rupture of the samples. All three products could sustain large deformations before they were ruptured (the corresponding compression ratio, $\Delta L/L$, was greater than 30%). Salami had a much higher stress at failure than the other two products. The change in the curvature of the force-deformation curves was most pronounced for salami and least for smoked sausage. Both salami and bologna showed a dramatic drop in force with further compression after reaching the maximum, while smoked sausage did not have a distinctive peak force even when the samples were ruptured.

Typical force-deformation curves for bologna and smoked sausage under cyclic loading and unloading are shown in figure 4 and similar curves were also obtained for salami. A small loop in the force-deformation curves was observed when the cycle was changed from loading to unloading, which was caused by the accelerational or deaccelerational forces in the sample resulting from the rapid change in the crosshead speed. When the cycle was changed from unloading to loading, no hysteresis loops were observed. This is because when the crosshead was changed from unloading to loading at zero deformation, the sample was barely in touch with the loading plate. Therefore, no accelerational and deaccelerational forces induced in the sample would be transmitted to the load cell.



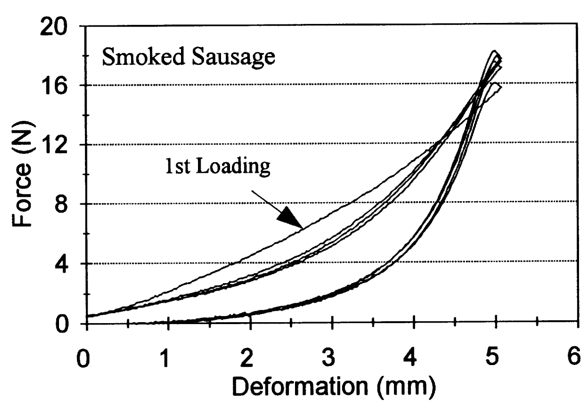


Figure 4–Typical force-deformation curves for bologna and smoked sausage samples (15.9 mm in diameter and 15.0 mm in height) subjected to four cycles of uniaxial loading and unloading at a loading rate of 127 mm/min. Small loops occurred when the cycles changed from loading to unloading, due to the accelerational (or deaccelerational) forces caused by the rapid change in the crosshead speed.

For each cycle of loading and unloading, there always existed a hysteresis loop that was smaller for bologna than for smoked sausage. As shown, the force-deformation curve for the first loading stroke was quite different from those for the subsequent loading strokes. However, the difference in the force-deformation curves for the subsequent loading strokes became almost indiscernible. There were no noticeable differences for all unloading strokes including the first one. These results indicate that the beef products exhibit a unique force-deformation behavior in loading and unloading, respectively, for compression ratios up to at least 33%.

Figure 5 shows comparisons of the measured and regression-fitted (eq. 12) stress versus compression ratio curves for bologna, salami, and smoked sausage under uniaxial loading. There are two graphs for each product; one represents the best fit of the constitutive equation to the data and the other the worst fit. As shown in figure 5, the constitutive equation 12 predicted the stress-strain responses of the three products reasonably well even under the worst cases. In the best cases, the differences between the measured and model-fitted stress versus compression ratio curves for bologna and smoked sausage were too small to distinguish visually over the entire strain range. The goodness of the constitutive equation for predicting the stress-strain responses is by no means trivial. This indicates that the basic assumptions made in this study are valid and the materials closely follow the mechanical behavior described by the constitutive equation 12.

Table 1 summarizes the parameter values and their statistics for the three products. The parameter α for bologna and salami is not significantly different at the 0.05 level but is significantly different for smoked sausage. The parameter β is statistically different at the 0.05 level for all three products. Salami showed highest values for the parameters α and β among the three products, which is consistent with the fact that nonlinearity, measured by the change of the curve slope, was greater for salami than for the other products (fig. 3). Smoked sausage had a negative α value for all twenty test samples and its nonlinearity was less severe than the other two products. The average RMSE (root mean squared error) for the three products was 1.90 kPa for bologna, 2.98 kPa for salami, and 1.88 kPa for smoked sausage. These results show that the proposed constitutive equation is capable of describing the stress-strain responses of beef products under uniaxial loading.

The capability of the constitutive equation to predict the stress-strain responses of beef products under multidimensional loading situations is demonstrated in figure 6. Predicted stresses were calculated using equation 13 with the parameters given in table 1 and compared favorably with experimental measurements. The constitutive model predicted stresses well for bologna samples. Predicted stresses were not significantly different from the measured at the 0.05 level (t-test) when the compression ratio was greater than 3.5%, even though the difference tended to increase with compression ratio. For the salami samples, model predictions also compared well with measured values; the differences were not significant at the 0.05 level for the compression ratios greater than 3%. For smoked sausage, model predictions were not significantly different from the measured except when the compression ratio was less than 2%, between 5% and 13%,

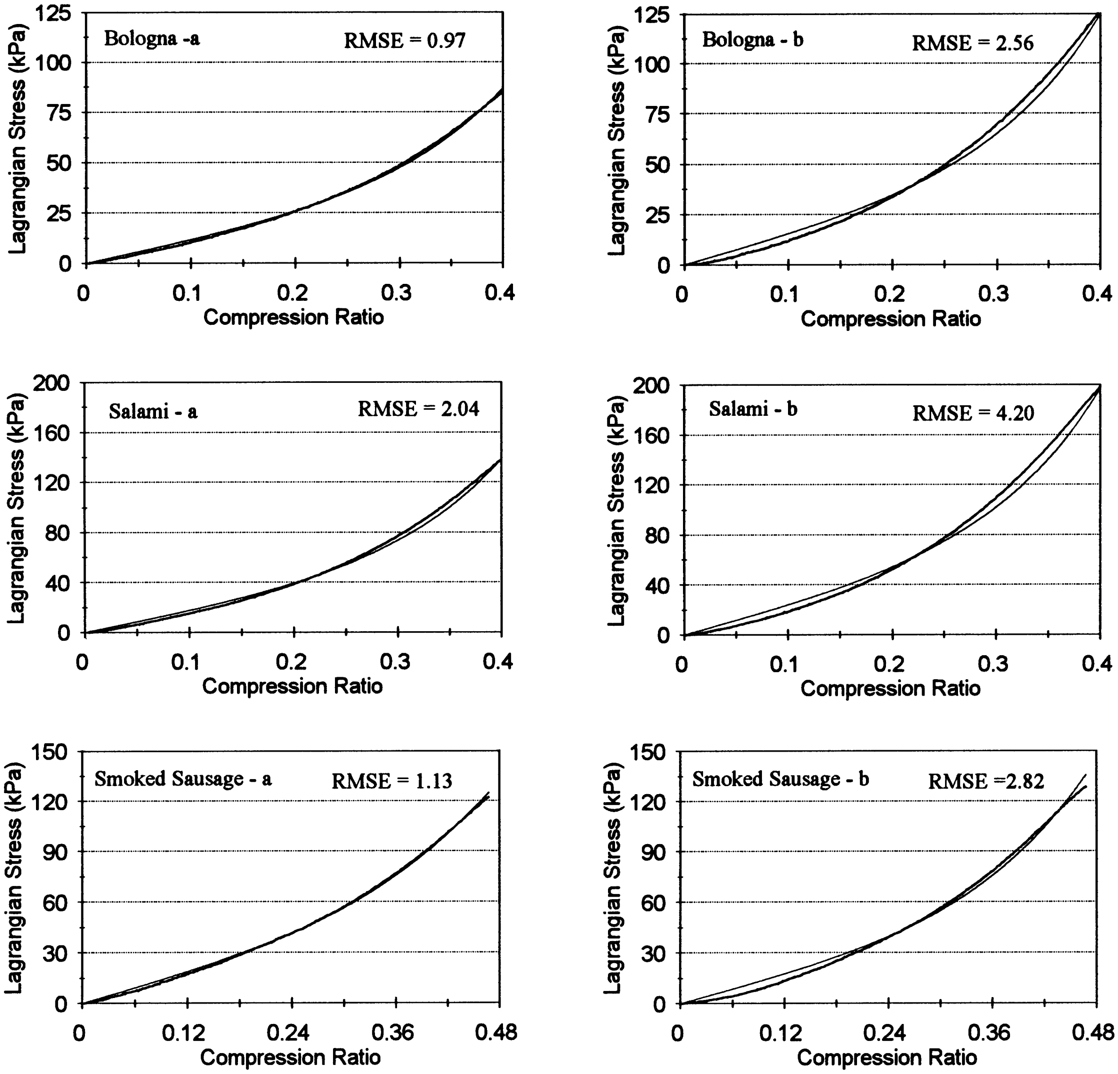


Figure 5–Comparison of measured (thick line) and model-fitted (thin line) stress vs compression ratio (ΔL/L) curves for bologna, salami, and smoked sausage samples under uniaxial compression. Two graphs are presented for each product, representing the best (a) and worst (b) fit of the constitutive equation to the experimental data, as measured by the RMSE (root mean squared error).

Table 1. Parameter values and their statistics in the constitutive equation 12 for bologna, salami, and smoked sausage obtained from uniaxial compression data*

	α†	β†	RMSE
Bologna	0.3634[b] (0.1960)	97.20[a] (10.13)	1.90 (0.46)
Salami	0.4340[b] (0.1756)	134.79[c] (13.69)	2.98 (0.79)
Smoked sausage	–0.6829[a] (0.1390)	114.33[b] (13.32)	1.88 (0.42)

* Each value in the table represents the average of 20 replications. Values in the parentheses are the standard deviations. The parameter α is dimensionless, and the parameter β and the RMSE (root mean squared error) are in kPa.

† The values for each parameter with the same superscript letter are not significantly different (LSD) at the 0.05 level.

and greater than 22%. The significant, but relatively small prediction errors were obtained for the intermediate range of compression ratios partly because the sample standard deviation was much smaller than it would have been expected. The significant difference for large deformations (> 22%) is likely attributed to the fact that stress yielding may have occurred in the test samples at large deformations, resulting in lower stresses than predicted. The increasing sensitivity of the stress to the compression ratio at higher deformation levels could also be a factor affecting prediction accuracy. Considering the inherent variability of the mechanical properties of beef products, the constitutive equation appears to predict their stress-strain responses with a reasonable accuracy.

Comparing the stress versus compression ratio curves for beef samples from the uniaxial and confined compression tests, it was found that the beef samples often yielded at a lower compression ratio in the confined compression tests than in uniaxial compression. This could be due to the fact that the stresses in the sample were multi-axial (or two-axial) under the confined compression. The yield condition for foods and biological materials normally depends on the stress/strain status or loading conditions (Holt and Schoorl, 1982).

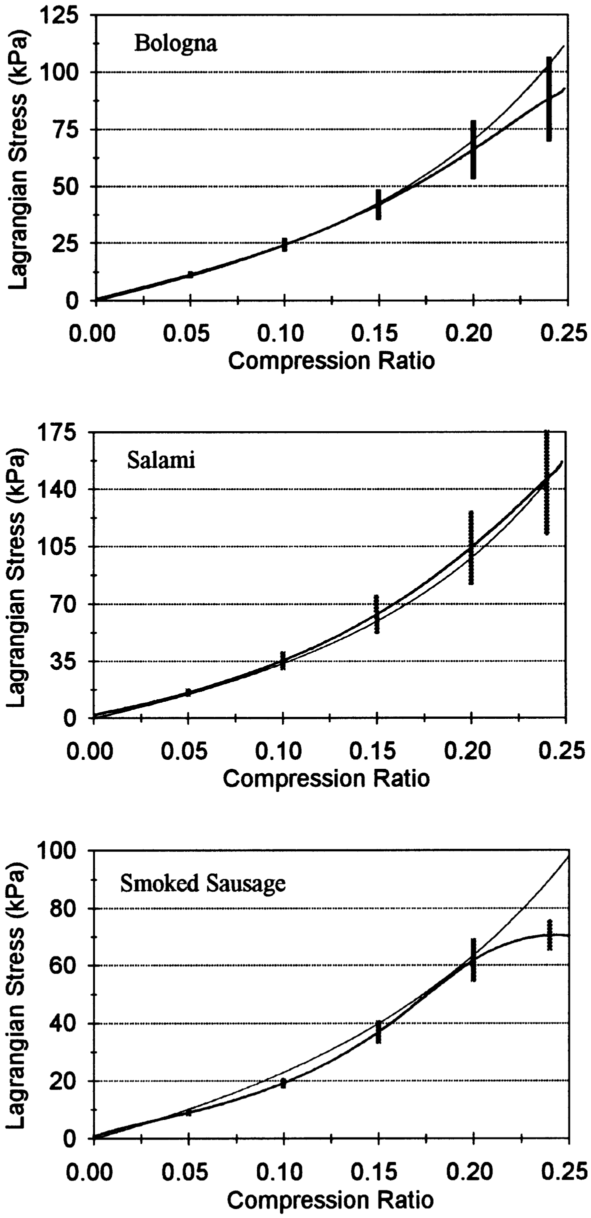


Figure 6–Comparison of measured (thick line) and predicted (thin line) stresses vs compression ratio (ΔL/L) for bologna, salami, and smoked sausage samples under confined two-dimensional compression at a loading rate of 127 mm/min. Each thick curve represents the average of four measurements. The vertical bars represent values of (2 × std).

Summary and Conclusions

A three-dimensional constitutive equation, based on the theory of finite elasticity, was developed to describe the nonlinear mechanical properties of beef products under finite deformation. Uniaxial compression and confined two-dimensional compression tests were conducted on bologna, salami, and smoked sausage to determine the material parameters and to validate the constitutive equation. Three beef products showed conspicuously nonlinear behavior under large deformations; this nonlinearity was characterized by the monotonic increase in the slope of the force-deformation curves. Nonlinearity was greatest for salami and least for smoked sausage. The proposed constitutive equation was found to fit the stress-strain curves under uniaxial compression well, with an average RMSE of 1.90 kPa for bologna, 2.98 kPa for salami, and 1.88 kPa for smoked sausage. The values of the material parameters were consistent for each beef product and clearly reflected the difference in the mechanical properties of bologna, salami, and smoked sausage. The constitutive equation predicted the stress-strain responses of the beef products well under confined two-dimensional compression; the differences were mostly not significant at the 0.05 level.

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APPENDIX DERIVATION OF EQUATIONS 12 AND 13

DERIVATION OF EQUATION 12

In deriving equation 12, the cylindrical polar coordinate system is used, in which the radial, circumferential, and axial coordinates are designated as r, ϕ, and z, respectively. When a cylindrical sample is subjected to uniaxial compression, the radial stress, σ_{rr}, is zero. Hence, the undetermined hydrostatic pressure p can be determined from equation 8, which gives:

$$p = \beta E_{rr} \lambda_r^2 e^{\alpha \mathrm{tr}\mathbf{E}^2} \quad \text{(A1)}$$

Notice that the following relationship:

$$\frac{\partial x_i}{\partial X_R} = \begin{cases} \lambda_i & \text{when } i = R \\ 0 & \text{when } i \neq R \end{cases} \quad \text{(A2)}$$

has been used in obtaining equation A1 from equation 8. From equation 2, we have:

$$E_{rr} = E_{\phi\phi} = \frac{1}{2}\left(\lambda_r^2 - 1\right) \quad \text{and} \quad E_{zz} = \frac{1}{2}\left(\lambda_z^2 - 1\right) \quad \text{(A3)}$$

and from equation 7:

$$\mathrm{tr}\mathbf{E}^2 = E_{rr}^2 + E_{\phi\phi}^2 + E_{zz}^2 = \frac{1}{2}\left(\lambda_r^2 - 1\right)^2 + \frac{1}{4}\left(\lambda_z^2 - 1\right)^2 \quad \text{(A4)}$$

where E_{rr}, $E_{\phi\phi}$, and E_{zz} are the strain components in the radial, circumferential, and axial directions, respectively.

The axial stress in the z-direction can be obtained by substituting equation A1 into equation 8, which gives:

$$\sigma_{zz} = \beta e^{\alpha \mathrm{tr}\mathbf{E}^2}\left(\lambda_z^2 E_{zz} - \lambda_r^2 E_{rr}\right) \quad \text{(A5)}$$

The incompressibility assumption implies that the volume of the deformed body is equal to that of the undeformed. Hence, we have:

$$\left(\frac{r}{R}\right)^2 = \frac{L_0}{L} \quad \text{or} \quad \lambda_r^2 = \frac{1}{\lambda_z} \quad \text{(A6)}$$

where r and R denote the radius of the deformed and undeformed cylindrical sample, respectively, and L and L_0 are the height of the deformed and undeformed sample, respectively. Substituting equations A3, A4, and A6 into equation A5 gives equation 12.

DERIVATION OF EQUATION 13

Equation 13 can be obtained by following the procedure outlined above for equation 12. For the confined loading under the rectangular coordinate system shown in figure 1, the stress in the x-direction is zero. In view of equation 8, the underdetermined hydrostatic pressure p can be expressed in the following form:

$$p = \beta \frac{\partial x}{\partial X}\frac{\partial x}{\partial X} E_{xx} e^{\alpha \mathrm{tr}\mathbf{E}^2} = \beta \lambda_x^2 E_{xx} e^{\alpha \mathrm{tr}\mathbf{E}^2} \quad \text{(A7)}$$

From equation 2, we have:

$$E_{xx} = \frac{1}{2}\left(\lambda_x^2 - 1\right), \quad E_{yy} = 0, \quad E_{zz} = \frac{1}{2}\left(\lambda_z^2 - 1\right) \quad \text{(A8)}$$

and from equation 7:

$$\mathrm{tr}\mathbf{E}^2 = E_{xx}^2 + E_{zz}^2 = \frac{1}{4}\left(\lambda_x^2 - 1\right)^2 + \frac{1}{4}\left(\lambda_z^2 - 1\right)^2 \quad \text{(A9)}$$

The incompressibility assumption requires that:

$$\lambda_x = \frac{1}{\lambda_z} \quad \text{(A10)}$$

Hence, in view of equations 8 and A7, the axial stress in the confined loading is:

$$\sigma_{zz} = \beta e^{\alpha \mathrm{tr}\mathbf{E}^2}\left(\lambda_z^2 E_{zz} - \lambda_x^2 E_{xx}\right) \quad \text{(A11)}$$

Substituting equations A8, A9, and A10 into A11 gives the relationship of equation 13.