The influence of man on the hydrological regime with special reference to representative and experimental basins — L'influence de l'homme sur le régime hydrologique avec référence particulière aux études sur les bassins représentatifs et éxperimentaux (Proceedings of the Helsinki Symposium, June 1980; Actes du Colloque d'Helsinki, juin 1980): IAHS—AISH Publ. no. 130.

The mathematical basin model of Merrill Bernard

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Abstract. A mathematical model was proposed and demonstrated by Merrill Bernard in 1937. This 40-year-old model remains relevant by today's modelling techniques. The review of Bernard's work that follows should be of interest to modellers for its historic perspective.

Le modèle mathématique de bassin de Merrill Bernard

Résumé. Un modèle mathématique a été proposé et exposé par Merrill Bernard en 1937. Ce modèle âgé de 40 ans, reste valable par rapport aux techniques actuelles de mise en valeur. La révision du travail de Bernard qui s'ensuit doit intéresser ceux qui font des modèles pour sa perspective historique.

INTRODUCTION

Basin modelling in the mid-1930s can hardly be expected to show sophistication. The digital computer was not commonly available until 20 years later. Lighthill and Whitham did not write about kinematic waves until 1955. Ten more years elapsed before Wooding (1965) published his model. In fact, Sherman had only given us unit hydrographs in 1932. Horton's (1939) equation for infiltration capacity was not to come until the end of the decade. One did, however, have the writing of Seddon (1900) and of Thomas (1934) and the early experimental work of the Soil Conservation Service. And, this was an era of explosive development in hydrological theory.

Merrill Bernard took on the task of using the results of runoff plot experiments at Bethany, Missouri, to predict the effect of land use on a 3 km^2 basin. The approach that he used, the problems that he encountered, and the results obtained are topics of discussion today; only the mechanics of computation are archaic.

Field stations for hydrological and erosion studies were established by the US Department of Agriculture during 1929—1931. The objectives were to study erosion processes and to evolve effective erosion prevention and control methods. The influence of factors such as soil, slope, cover, and rainfall intensity and distribution on rate and volume of runoff and soil loss were to be isolated and evaluated. Bernard's comments on these studies, as of 1936, were:

'Of necessity, the erosion experiment stations were limited to small farm units of about 200 acres [80 ha]. Further limitations in funds and personnel reduced the size of the individual investigations to areas ranging in size from a fraction of an acre to 4 or 5 acres [2 ha]. On these experimental areas, and within a comparatively short period, a number of rainfall and run-off observations have been made, many of which show marked contrasts in runoff and soil loss under various surface treatments and cover.

The usefulness of these data is definitely limited, for the following reasons: First, the period is not sufficiently long to have embraced the full range of all the factors influencing the result; second, there are comparatively few cases in which runoff has been observed under unusually excessive rainfall; and third, the experimental areas are so small that the results cannot be considered as having areal significance. It is the purpose of this paper to present and demonstrate a method which, in a practical degree, may be used to overcome the latter limitation.'

He went on to say, 'Obviously, a result obtained on seven-tenths of an acre [0.3 ha] or 7 acres [3 ha], cannot be extrapolated to 700 acres [300 ha] by means of simple multiplication. It is believed, however, that resorting only to well-established hydraulic assumptions, a method of combining and routing flow makes possible the projection of the results on these small areas to larger areas in the form of natural watersheds.'

THE WATERSHED MODEL

The mechanics of overland flow and of infiltration were still to be developed, so Bernard's model was formed by subdividing the basin

'. . . into elemental units which are comparable in size to the observational areas found on the erosion experimental stations. It then develops the hydrograph of flow at various points on the stream system and at the outlet of the watershed by combining and routing the flow from each of the elemental units, assigning to each the hydrograph of flow from a particular experimental area, all the result of an actual rainfall. This, in effect, assumes that the conditions on the elemental unit, and those on the experimental area whose hydrograph it has been assigned, are identical.'

The Bernard concept of subdividing a basin into elemental units has also been called the unit source area concept. In the late 1930s, the North Appalachian Experimental Watershed near Coshocton, Ohio, and the Great Plains Experimental Watershed near Hastings, Nebraska, were laid out with so-called unit-source and complex basins in a manner designed to test that concept. The test at the latter experiment station was reasonably successful (Allis, 1962), but that at Coshocton did not support the concept, probably due to the predominance of subsurface over surface flows in headwater areas (Amerman, 1965).

Bernard tested his model utilizing the synthetic basin shown in Fig. 1. It was composed of 128 elements, each element a square of 150 m each side, resulting in an

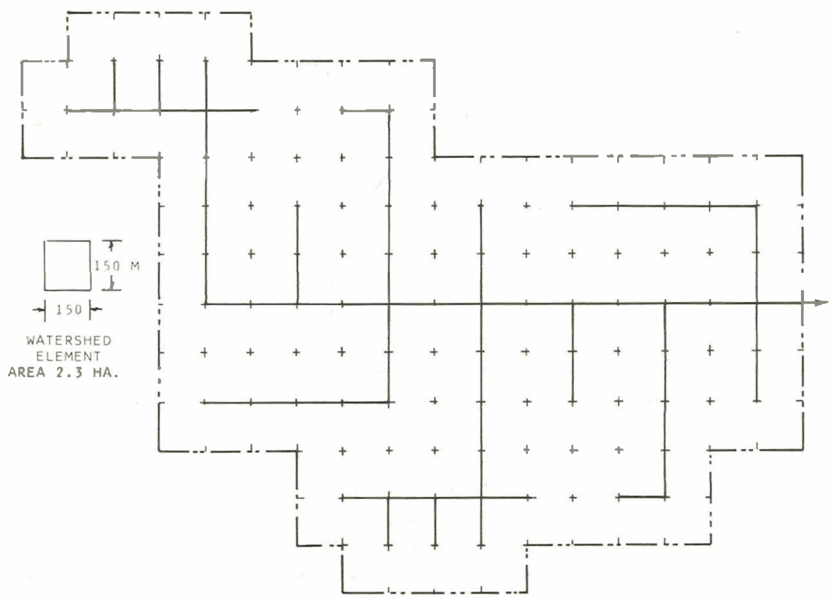

FIGURE 1. Synthetic basin used by Bernard. A total area of 290 ha is obtained by assembling 128 elements.

area of 2.3 ha. The total area was about 290 ha. A stream channel system was developed based on assumptions of slope, channel dimensions and channel conditions 'which normally would fit a watershed of equivalent area.' Six different channel cross sections were assumed.

Kinematic routing had not yet been described, so it could not be used by name, to route the hydrographs from the several elements to the outfall. Instead, Bernard used Seddon's (1900) relation for wave velocity

$$u = \frac{dQ}{dA} = \frac{Q_2 - Q_1}{A_2 - A_1}$$

which is the same as that from kinematic wave theory. L. K. Sherman had described this method in a letter to *Civil Engineering* in 1933. Bernard used Manning's equation to determine the discharge–area relation.

Application of kinematic routing, or Seddon's wave, was not without pitfalls. Kinematic shock was one such problem with which Bernard had to deal. He described the problem and his solution.

'The first water discharging into a dry channel reach enters at comparatively low velocity. It is soon overtaken and combines with the increasingly greater flows travelling at faster rate so that, for an initial period, there is no outflow from the reach, all of the water going into storage. Likewise, at the end of the period of flow, water will have ceased to enter the reach while the amount occupying storage continues to discharge. If the lower values of the hydrograph are translated through the reach with their appropriate wave velocities, the translated hydrograph is made to turn under itself, as shown in Fig. 2. The only significance given to this peculiarity is to assume that all of the water discharging into the reach throughout the time interval t_0, t_a, has accumulated into a wave-face resembling a hydraulic bore and at t_a instant

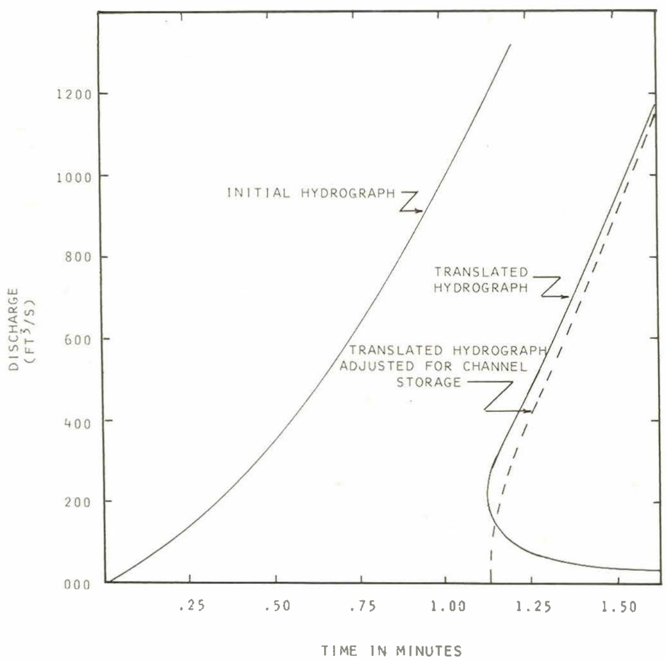

FIGURE 2. The manifestation of kinematic shock and its rectification. (100 ft^3/s = 2.83 m^3/s).

arrives at and discharges into the reach below. A correction in the shape of the outflow hydrograph for the effect of channel storage is made by bringing point *a* to the time-base as zero flow, and deducting the successive ordinates of the undertumed portion of the hydrograph from the corresponding ordinates of the normally rising hydrograph above it. The lower velocities of the lessening flows of the falling hydrograph add an equivalent area to the hydrograph at the end of the period of flow into the reach, accounting for the water originally discharged into storage. If a hydrograph is translated through successive reaches of dry channel in this manner it will be found that these deductions will soon extend to and beyond the peak, reflecting the recognized influence of storage on peak flow. In problems involving small natural watersheds and the rapid occupation of storage through simultaneous inflow at all points on the stream system, this correction is not great, but should be made principally to avoid the accumulative error in adding the ordinates of the combining hydrographs.'

APPLICATION OF THE CONCEPT

To apply the model, Bernard assumed the basin to represent conditions near Bethany, Missouri, a site of one of the USDA field experiment stations. Two of the experimental plots were selected to characterize the runoff from the various elements in the model. The two adjacent plots were on soil classified as Shelby sandy loam. One plot was in sod with a 0.5-ha drainage area and 9.6 per cent average land slope. The other was in corn with a 0.5 ha drainage area and 9.7 per cent average land slope. The elements in the model were 2.3 ha in area, so the runoff rates from the plots were linearly (by area) scaled up to the element size.

The runoff from the plots was taken as that measured during the rainfall event of 9 August 1933. The plot rainfall and runoff are shown in Fig. 3.

Bernard described his process of simulation:

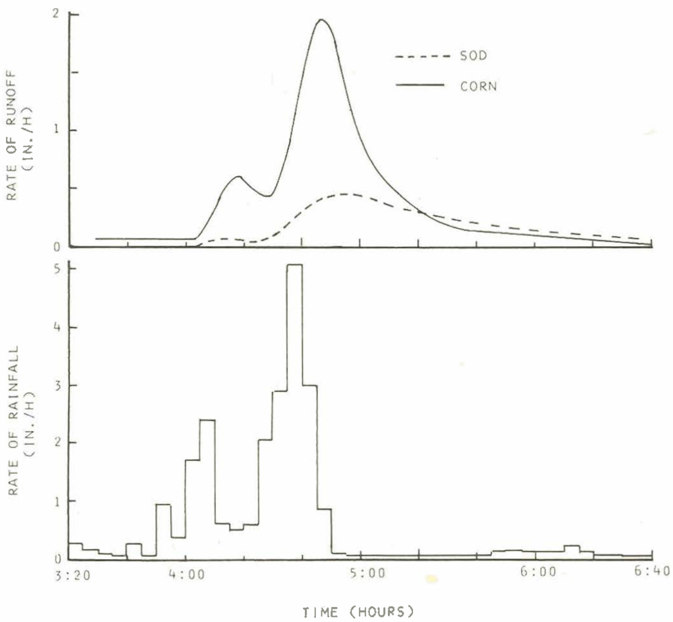

FIGURE 3. Observed rainfall and runoff hydrographs for 9 August 1933 rainfall event at Bethany, Missouri, USA. (1 in/h = 25.4 mm/h).

'This study now proposes to determine, in the form of hydrographs of flow at the outlet of the 730 acre [290 ha] watershed, the effect on the hydrograph of progressively passing from the assumption that the entire watershed is covered by sod, . . . to that under which is it entirely covered by the corn crop . . .

The procedure is to develop the hydrographs at the outlet of each of the laterals, after which the flow is assembled throughout the length of the main channel. After initial hydrographs have been assigned to the elemental units, routing schedules are prepared for each of the laterals . . .'

The lack of a digital computer is apparent in the mechanics of performing the routing. As Bernard said: 'While the combination and routing could be done analytically by an elaborate system of listing and tabulation, it is believed that the graphical method proposed is far more economical in both time and paper. It has been found best to plot the initial hydrographs, as well as their combinations, on transparent co-ordinate paper, using a sharp, hard pencil . . .'

The simulation studies indicated the influence of changing from corn to sod in the outflow hydrographs shown in Fig. 4. The influence on the peak is given in Table 1. Bernard commented as follows:

'In utilizing the method for the development of [Rational Method] run-off coefficients for this watershed, consideration should be given the fact that, while it is possible for a watershed of 730 acres [290 ha] to be planted entirely in corn or in

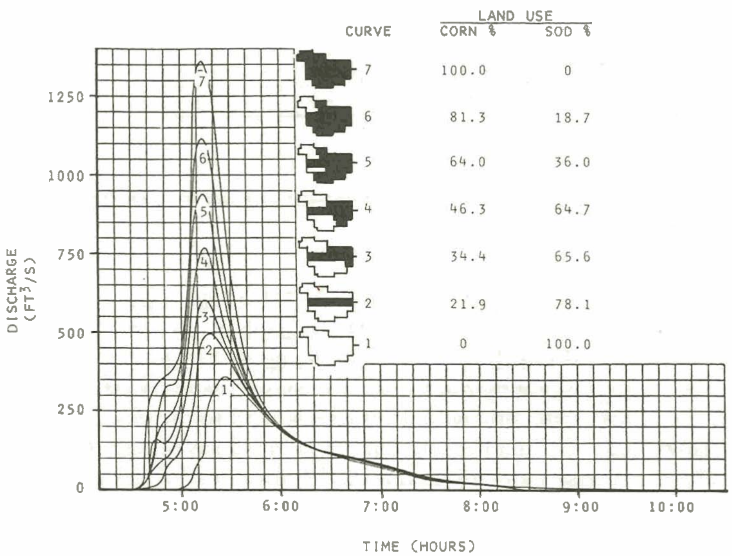

FIGURE 4. Influence of land use on outflow hydrograph. (250 ft^3/s = 7.08 m^3/s).

TABLE 1. Influence of land use on peak discharge

Hydrograph no.	% of area in sod	% reduction in peak	Hydrograph no.	% of area in sod	% reduction in peak
7 ----	0	0	3 ----	66	56
6 ----	19	17	2 ----	78	63
5 ----	36	31	1 ----	100	74
4 ----	55	44			

sod, the possibility for such to occur in a permanently subdivided farming community is remote, particularly under modern conceptions of farm management.

It is commonly believed that flood peak reduction is accomplished through retardation only. To the degree that the shape of the watershed and the pattern of its stream system affects the synchronization of the various tributary flows is the regimen of the watershed reflected in the magnitude of its flood peaks. That the regimen of the watershed can be so affected as to accelerate the run-off from certain of its subareas, thereby reducing the peak by utilizing more completely the comparatively empty channel system during the rising stage of the hydrograph, is seldom considered.

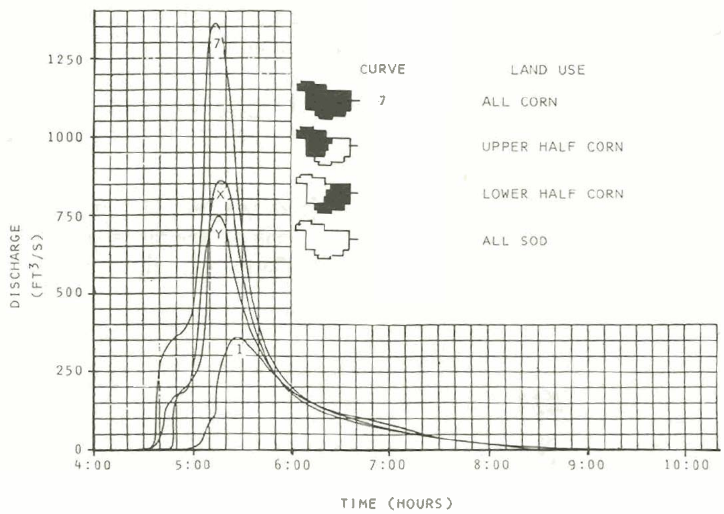

FIGURE 5. Hydrograph changes resulting from varying the location and percentage of land use. (250 $ft^3/s = 7.08\ m^3/s$).

This theory is demonstrated in Fig. 5. Here exactly one-half of the watershed is considered as being in sod and the other half in corn. Hydrograph *Y* has been developed under the assumption that the lower half is in corn and the upper half in sod. The heavier run-off from the corn is promptly discharged, filling out the rising stage of the hydrograph with flow that would otherwise have contributed to the peak flow, while the lesser and slower run-off from the sod units above are delayed until the peak has passed. Under the reverse assumption, that is, the lower half of the watershed in sod and the upper half in corn, hydrograph *X* is developed. Now the run-off from the lower half is retarded while that from the upper half has been accelerated, with the result that waters which otherwise would have been removed from the areas near the outlet before the peak arrived, are delayed until the more rapidly moving water from the corn above arrives and combined with it. The result is that, maintaining the same area in each type of cover and without changing the amount of run-off water involved, the flood peak can be affected to the extent of about 15 per cent by virtue of the relative position of the cover types alone.'

In summarizing this model, Bernard said:

'It would seem that the method of combining and routing flow presented in this paper takes fewer liberties with hydraulic laws than many model studies upon which extensive structures are designed. This, and other methods, are definitely limited in both development and application by the deplorable paucity in our knowledge of the hydrodynamics of natural stream flow. Despite this reflection, it is the hydraulic model which promises the prompt solution of such problems.'

SUMMARY

It seems that Merrill Bernard was pioneering a model that required 30 years for technology to make usable. He acknowledged '. . . his indebtedness to Ivan Bloch, Associate Engineer, of the Rural Electrification Administration, for technical assistance of the highest order. Mr Bloch assisted in the development of the method described and prepared many of the graphs and figures entailing several hundred hydrograph translations.' We are sure that Mr Bloch would have preferred the computational methods of today.

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