

JOURNAL OF THE HYDRAULICS DIVISION

OVERLAND FLOW FROM TIME-DISTRIBUTED RAINFALL

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INTRODUCTION

The study of hydrology of small agricultural and urban watersheds has progressed along different lines than that for medium to large size watersheds. The approach has been a more physically based analysis of infiltration, overland flow, and channel routing. Current techniques for flow analysis are often based on kinematic wave theory, whereas formerly, they were based on the overland flow equation derived by Horton (6). Horton's equation might be considered a quasiunsteady analysis because, although he used the same basic equations as those for kinematic wave theory, he assumed that the water surface profile was always geometrically similar to the equilibrium profile.

Detailed design procedures for small basins have been developed using both analytical methods. Hicks (5) and Hathaway (4) based their work on the Horton equation. Kinematic wave procedures have been described by Overton and Tsay (12) and by Chen and Evans (1). These approaches use rainfall-intensity-frequency distributions to establish the rainfall input. The average intensity for the time-of-concentration is used as rainfall of constant intensity with the Horton or the kinematic wave flow equations. Using this approach, the rainfall intensity decreases as time-of-concentration increases; thus, a correct time is necessary. Hathaway (4) showed how rainfall duration influences peak discharge. Time-of-concentration depends on watershed characteristics, but also varies with rainfall intensity which in turn varies with time, so time-of-concentration is determined by trial and error. Ragan and Duru (13) presented a nomograph to aid in solving the kinematic wave relation. Kerby (8) empirically obtained an equation based on results computed by Hathaway that is apparently appropriate for a particular rainfall frequency.

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Note.—Discussion open until July 1, 1981. To extend the closing date one month, a written request must be filed with the Manager of Technical and Professional Publications, ASCE. Manuscript was submitted for review for possible publication on November 14, 1979. This paper is part of the Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, ©ASCE, Vol. 107, No. HY2, February, 1981.

These previous efforts have used rainfall of constant intensity to estimate the overland flow hydrograph. Because constant intensity rainfall is unlikely, it is of interest to determine the influence of time distribution of rainfall on the hydrograph of overland flow. The kinematic wave equations for turbulent flow across a plane, impermeable surface are solved using a time-varying rainfall appropriate for thunderstorms. Peak discharge is shown to be a function of surface length, total precipitation, storm duration, and the time to equilibrium for an equivalent rainfall of constant intensity.

FLOW EQUATIONS

The flow per unit width across an impermeable plane surface is described by the continuity equation

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = i \dots \dots \dots (1)$$

in which h = flow depth (L); q = discharge per unit width (L^2/T); i = rainfall intensity (L/T); t = time since start of rainfall (T); and x = distance from top of plane (L). The total momentum equation is difficult to apply, so approximations are usually used. The kinematic wave approach employs the relation

$$q = \alpha h^m \dots \dots \dots (2)$$

in which α , m = constants for wide shallow flows. A detailed analytical study by Woolhiser and Liggett (15) led to the conclusion that the kinematic wave approach is appropriate for most formulations. In particular, they showed that if the kinematic wave number is greater than 10, the approximation is very good. The kinematic wave number, k , is given by

$$k = \frac{SL}{H_o F_o^2} \dots \dots \dots (3)$$

in which L = length of plane; H_o = normal depth at outfall with maximum discharge; S = slope of plane; and F_o = Froude number at outfall with maximum discharge. This was extended to $SL/H_o \geq 5$ by Morris and Woolhiser (10).

The flow on the plane may be laminar, turbulent, or both. A reanalysis of the experimental data obtained by Izzard (7) was carried out by Morgalli (9) confirming this situation. Work by Foster, et al. (3), indicated, however, that use of a constant Manning's n or Chezy's C gave good results when compared with their experimental results. In the material that follows the flow will be assumed to be fully turbulent and that Chezy's C is constant. Thus, the constants in Eq. 2 can be expressed as

$$\alpha = C \sqrt{S}; \quad m = \frac{3}{2} \dots \dots \dots (4)$$

Combining Eqs. 1 and 4 results in

$$\frac{\partial h}{\partial t} + \frac{3\alpha}{2} h^{1/2} \frac{\partial h}{\partial x} = i \dots \dots \dots (5)$$

the solution to which may be determined using the variable τ such that, by the chain rule of differentiation,

$$\frac{dh}{d\tau} = \frac{\partial h}{\partial t} \frac{dt}{d\tau} + \frac{\partial h}{\partial x} \frac{\partial x}{\partial \tau} = i \dots \dots \dots (6)$$

Eqs. 5 and 6 are identical if

$$\frac{dt}{d\tau} = 1; \quad \frac{dx}{d\tau} = \frac{3}{2} \alpha h^{1/2} \dots \dots \dots (7)$$

$$\text{or } t - t_o = \tau - \tau_o \dots \dots \dots (8)$$

$$\text{and } x - x_o = \frac{3}{2} \alpha \int_{\tau_o}^{\tau} h^{1/2} d\tau \dots \dots \dots (9)$$

Eq. 6 yields the relation of the depth of flow, h , to rainfall intensity, i , along a coordinate τ . The coordinate τ is related to the physical time and space coordinates through Eqs. 8 and 9. To solve Eqs. 6, 8, and 9 an expression must be established for rainfall intensity.

THUNDERSTORM RAINFALL

As a result of extensive studies of thunderstorms, the United States Weather Bureau (14) published average time distributions for rainfall within 1-h, high-in-

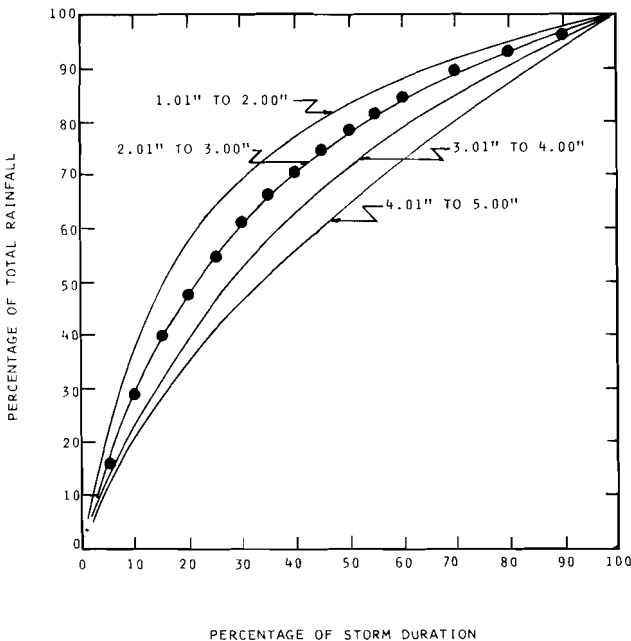


FIG. 1.—Time Distribution of Rainfall in 1-h Storms [Points Determined Using Eq. 10 with $a = 1.37$ and $b = 0.37$; Curves from United States Weather Bureau (14)]

tensity storms (Fig. 1). The relation

$$\frac{P_t}{P} = \frac{\frac{at}{D}}{b + \frac{t}{D}} \dots \dots \dots (10)$$

in which P_t = rainfall accumulation at time $t(L)$; P = total rainfall (L); D = storm duration (T); and a, b = constants can be used to describe the curves. The value $a = 1.37$ and $b = 0.37$ fit the 2.01 in./h-3.00 in./h curve quite well, as indicated by the points on Fig. 1. This relationship will be used to describe storms of all short duration.

SOLUTION OF FLOW AND RAINFALL EQUATIONS

The depth of flow is determined by integration of Eq. 6:

$$h - h_o = \int_{\tau_o}^{\tau} id\tau \dots \dots \dots (11)$$

If t_o and τ_o are made to coincide, the integral is the accumulated precipitation at time, τ , less the accumulated precipitation at time, τ_o , or

$$h - h_o = P_{\tau} - P_{\tau_o} \dots \dots \dots (12)$$

During the storm precipitation is given by Eq. 10 so

$$h - h_o = P \left(\frac{\frac{a\tau}{D}}{b + \frac{\tau}{D}} - \frac{\frac{a\tau_o}{D}}{b + \frac{\tau_o}{D}} \right); \tau < D \dots \dots \dots (13)$$

The location coordinate given by Eq. 9 can now be determined. The depth of flow is zero at $\tau = \tau_o$ so that

$$x - x_o = \frac{3\alpha}{2} \sqrt{P} \int_{\tau_o}^{\tau} \left(\frac{\frac{a\tau}{D}}{b + \frac{\tau}{D}} - \frac{\frac{a\tau_o}{D}}{b + \frac{\tau_o}{D}} \right)^{1/2} d\tau; \tau < D \dots \dots \dots (14)$$

which can be expressed

$$x - x_o = \frac{3\alpha}{2} D \sqrt{P} \sqrt{\frac{ab}{b + \frac{\tau_o}{D}}} \int_{\tau_o/D}^{\tau/D} \left(\frac{\frac{\tau}{D} - \frac{\tau_o}{D}}{\frac{\tau}{D} + b} \right)^{1/2} d\left(\frac{\tau}{D}\right); \tau < D \quad (15)$$

Integration of Eq. 15 results in

$$x - x_o = \frac{3\alpha}{2} D \sqrt{P} \sqrt{\frac{ab}{b + \frac{\tau_o}{D}}} \left[\sqrt{\left(\frac{\tau}{D} - \frac{\tau_o}{D}\right)\left(\frac{\tau}{D} + b\right)} - \left(b + \frac{\tau_o}{D}\right) \log \left(\sqrt{\frac{\tau}{D} + b} + \sqrt{\frac{\tau}{D} - \frac{\tau_o}{D}} \right) \right]_{\tau_o/D}^{\tau/D}; \tau < D \dots \dots \dots (16)$$

Evaluating at the limits and rearranging terms results in

$$x - x_o = \frac{3\alpha}{2} D \sqrt{P} \sqrt{\frac{ab}{b + \frac{\tau_o}{D}}} \left[\sqrt{\left(\frac{\tau}{D} - \frac{\tau_o}{D}\right)\left(\frac{\tau}{D} + b\right)} - \left(b + \frac{\tau_o}{D}\right) \log \left(\frac{\sqrt{\frac{\tau}{D} + b} + \sqrt{\frac{\tau}{D} - \frac{\tau_o}{D}}}{\sqrt{\frac{\tau_o}{D} + b}} \right) \right]; \tau < D \dots \dots \dots (17)$$

Eqs. 13 and 2 describe the water surface profile and hydrograph for the space-time coordinates given by Eqs. 8 and 17 during the rainstorm.

When $\tau = D$, the rainfall ceases and i in Eqs. 6 and 11 is zero. This indicates that the depth, h , and discharge, q , no longer vary with τ . The space-time coordinates of this constant flow depth are given by

$$t - t_o = \tau - \tau_o; \tau < D \dots \dots \dots (18)$$

and $x - x_o = \frac{3\alpha}{2} h^{1/2} \int_{\tau_o}^{\tau} d\tau; \tau > D$

or $x - x_o = \frac{3\alpha}{2} h^{1/2} D \left(\frac{\tau}{D} - \frac{\tau_o}{D}\right); \tau > D \dots \dots \dots (19)$

RESULTS

Examination of the results is more convenient if the equations are nondimensionalized in terms of the time to equilibrium for an equivalent steady rainfall. In this form, the influence of time distribution can be compared with a constant intensity approximation.

A total amount of rain, P , falls in time, D , so that the equivalent intensity is

$$i_{equiv} = \frac{P}{D} \dots \dots \dots (20)$$

The time to equilibrium for this steady rainfall is [see, e.g., Eagleson (2) or Overton and Meadows (11)]

$$t_e = \frac{L^{2/3}}{\left(\frac{P}{D}\right)^{1/3} \alpha^{2/3}} \dots \dots \dots (21)$$

in which L = the length of the plane. The kinematic approximation for the momentum equation (Eq. 2) can be divided by LP/D to obtain

$$\frac{q}{LP} = \alpha \frac{\sqrt{PD}}{L} \left(\frac{h}{P}\right)^{3/2} = \left(\frac{\alpha^{2/3} P^{1/3}}{D^{1/3} L^{2/3}}\right)^{3/2} D^{3/2} \left(\frac{h}{P}\right)^{3/2} \dots \dots \dots (22)$$

or using the relation for t_e , Eq. 21, this becomes

$$\frac{q}{LP} = \left(\frac{D h}{t_e P}\right)^{3/2} \dots \dots \dots (23)$$

The quantity q/L , sometimes called the runoff intensity, is the flow rate divided by the watershed area. The quantity, P/D , is the equilibrium runoff intensity for rainfall of constant intensity.

The time coordinate is given by

$$\frac{t}{D} - \frac{t_o}{D} = \frac{\tau}{D} - \frac{\tau_o}{D} \dots \dots \dots (24)$$

and the space coordinate by

$$\frac{x - x_o}{L} = \frac{3}{2} \left(\frac{\alpha^{2/3} DP^{1/3}}{D^{1/3} L^{2/3}}\right)^{3/2} \sqrt{\frac{ab}{b + \frac{\tau_o}{D}}} \left[\sqrt{\left(\frac{\tau}{D} - \frac{\tau_o}{D}\right)\left(\frac{\tau}{D} + b\right)} - \left(b + \frac{\tau_o}{D}\right) \log \left(\frac{\sqrt{\frac{\tau}{D} + b} + \sqrt{\frac{\tau}{D} - \frac{\tau_o}{D}}}{\sqrt{\frac{\tau_o}{D} + b}} \right) \right]; \quad \frac{\tau}{D} < 1$$

$$\text{or } \frac{x - x_o}{L} = \frac{3}{2} \left(\frac{D}{t_e}\right)^{3/2} \sqrt{\frac{ab}{b + \frac{\tau_o}{D}}} \left[\sqrt{\left(\frac{\tau}{D} + b\right)\left(\frac{\tau}{D} - \frac{\tau_o}{D}\right)} \right]$$

$$-\left(b + \frac{\tau_o}{D}\right) \log \left(\frac{\sqrt{\frac{\tau}{D} + b} + \sqrt{\frac{\tau}{D} - \frac{\tau_o}{D}}}{\sqrt{\frac{\tau_o}{D} + b}} \right) ; \frac{\tau}{D} < 1 \dots \dots \dots (25)$$

during the rainstorm. After rainfall stops, the space coordinate is given by

$$\frac{x - x_o}{L} = \frac{3}{2} \left(\frac{D}{t_e}\right)^{3/2} \sqrt{\frac{h}{P}} \left(\frac{\tau}{D} - \frac{\tau_o}{D}\right); \frac{\tau}{D} > 1 \dots \dots \dots (26)$$

This form uses the storm duration for the time scale factor, the watershed length, L , for the length scale factor, total precipitation as a fluid length scale factor, and the time to equilibrium for the equivalent uniform rainfall to represent the physical characteristics of the watershed.

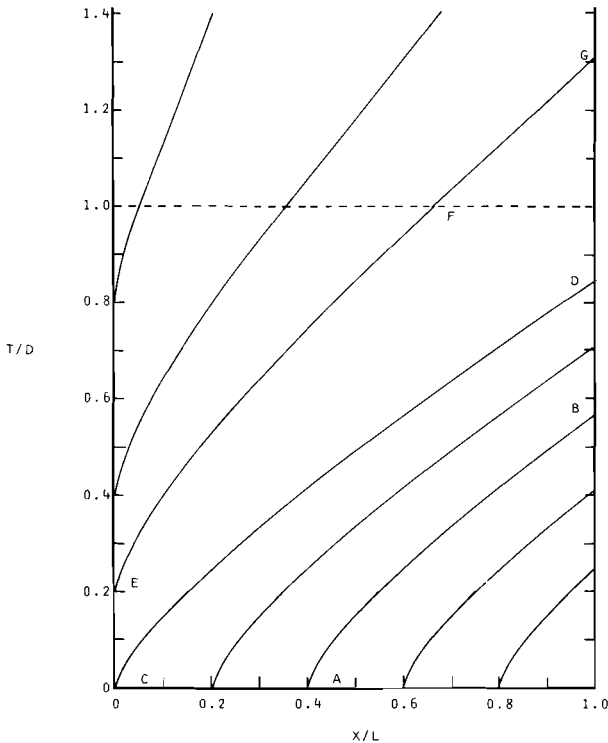


FIG. 2.—Relation between X/L and t/D for $D/t_e = 1$

The outfall hydrograph is determined using the preceding equations in the following manner. A location near the outfall with location x_o is selected. At the start of rainfall, the time t_o , and thus τ_o , is zero. The point indicated by A in Fig. 2 is an example. Values of time, τ , are selected and X/L is

determined, using Eq. 25, until the end of the plane, $X/L = 1$, is reached (unless rainfall stops before this is achieved). This is indicated as point B on Fig. 2. The time associated with point B is used to determine the stage using the nondimensional form of Eq. 13, which is

$$\frac{h - h_o}{P} = \frac{\frac{a\tau}{D}}{b + \frac{\tau}{D}} - \frac{\frac{a\tau_o}{D}}{b + \frac{\tau_o}{D}}; \tau < D \dots \dots \dots (27)$$

and the nondimensional discharge, $q/(LP/D)$, is determined using Eq. 23. Of particular interest is the curve on Fig. 2 that starts at $X/L = \tau/D = 0$, i.e., curve C-D. The time at which this curve reaches $X/L = 1$ is the time at

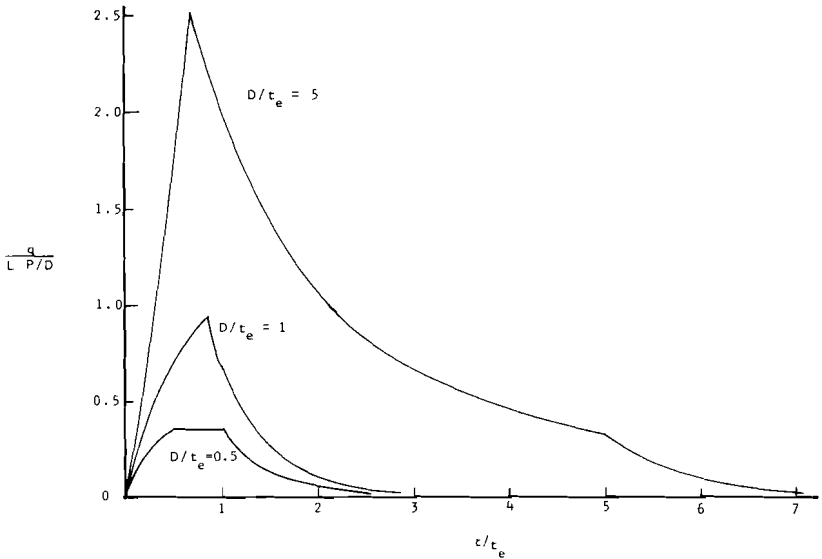


FIG. 3.—Discharge Hydrographs for Storms of Various Durations

which the total plane is contributing to the outflow, so is termed, herein, the time-of-concentration. In the example depicted in Fig. 2, this time occurs before the end of rainfall. Another situation is depicted by the curve E-F-G which starts with $X/L = 0$, and $\tau/D = 0.2$. That is, the curve starts after the storm has been in progress for 20% of the duration. The curve to point F is determined using Eq. 25. Projecting to point G is accomplished using Eq. 26, however, as rainfall has stopped for this portion. The stage is determined at point G using Eq. 27 and the discharge using Eq. 23.

Rainstorms that stop before the time-of-concentration is achieved, have flat peaks. This results because the curve starting $X/L = \tau/D = 0$ will intersect the $\tau/D = 1$ line before reaching $X/L = 1$. Considering the water surface profile at $\tau/D = 1$, indicates that the depth of flow to the right of the C-D equivalent is constant, thus, the flat peak.

Discharge hydrographs at the outfall of the plane are shown in Fig. 3. Storms of long duration are depicted by the hydrograph for $D/t_e = 5$. The total watershed is contributing before the end of the storm. The peak occurs at the time-of-concentration. The hydrograph then recedes, due to the diminution of rainfall intensity. At the end of rainfall, $D/t_e = 5$, a second recession limb appears as detention storage forms the sole source of runoff.

Storms of short duration are indicated, in Fig. 3, by $D/t_e = 0.5$. In this case, the storm ends before the total watershed is contributing to the discharge. The peak is the discharge at the end of the storm. The hydrograph remains at that discharge until the water that started at $x = 0$, $t = 0$ leaves the plane, after which the falling limb appears.

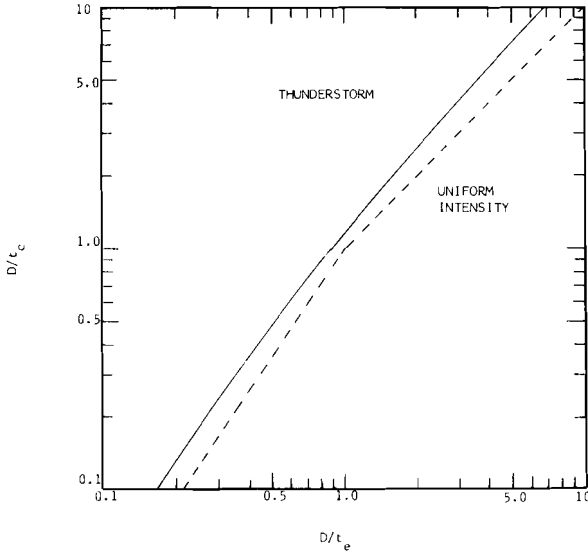


FIG. 4.—Time for Total Watershed Contribution as Function of Time to Equilibrium for Uniform Rainfall

The storm duration and the time-of-concentration are equal when D/t_e is about 0.87. That is, the time distribution of rainfall causes the watershed to respond more rapidly than does the same amount of rainfall distributed uniformly over the same time period. The time-of-concentration depends upon the storm duration and the physical characteristics of the watershed. This time is given by Eq. 25 with $(x - x_0) = L$ and $\tau_0 = 0$, the results of which are shown in Fig. 4. An explicit solution cannot be determined. The time-of-concentration for thunderstorm rainfall is always less than for rainfall of constant intensity. This results from the high rainfall intensity early in the thunderstorm coupled with the nonlinear flow relation.

The peak discharge varies with D/t_e , as shown in Fig. 5. The time distribution of rainfall has no influence on the peak discharge if the rainfall duration is less than the time-of-concentration. For the range of time, $0.87 \leq D/t_e \leq 1.08$, the uniform rainfall leads to a peak greater than the thunderstorm distribution.

This results because the slower time for total watershed contribution with uniform rainfall allows more water to accumulate. Storms of longer duration, however, yield higher peaks when distributed according to the thunderstorm pattern. This

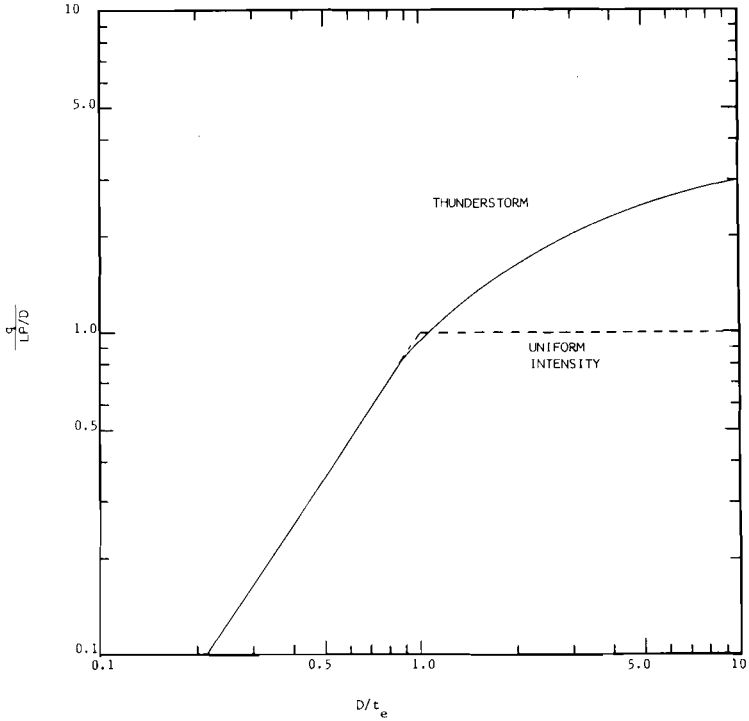


FIG. 5.—Peak Discharge for Storms of Various Durations

is because the intensity of the early portion of the thunderstorm distribution is greater than that used for the equivalent uniform rainfall. The peak discharge is determined only by this early portion.

APPLICATION

The object of this effort was to evaluate the influence of time distribution of rainfall on estimates of peak discharge. Although this can be inferred from Fig. 5, an example is helpful.

Consider a watershed with a length of 500 ft (152.4 m). The parameter $\alpha = C \sqrt{S}$ is estimated to be 1.78 ft^{1/2}/sec (0.983 m^{1/2}/s). This is subjected to 2 in. (5.1 cm) of thunderstorm rainfall in 1 h. The time to equilibrium for this intensity is determined using Eq. 21:

$$t_e = \frac{L^{2/3}}{\left(\frac{P}{D}\right)^{1/3} \alpha^{2/3}} = \frac{(500)^{2/3}}{\left[\frac{2}{12}\right]^{1/3} (1.78)^{2/3}} = 1,200 \text{ sec.} \dots \dots \dots (28)$$

Thus, $t_e = 20$ min so that $D/t_e = 3$. Using Fig. 5

$$\frac{q_{max}}{LP} = 2.05 \dots \dots \dots (29)$$

$$\frac{D}{L}$$

so $\frac{q_{max}}{L} = 2.05 \frac{P}{D} = 2.05 (2) = 4.10$ in./h (10.4 cm/h) (30)

CONCLUSION

The object of this study was to determine the influence of time distribution of rainfall on peak discharge. Usual design procedures use rainfall of constant intensity for a duration equal to the time to equilibrium. This analysis indicates that the peak discharge at the design condition will be slightly greater for rainfall of constant intensity than for rainfall with a thunderstorm time distribution. A correct value of the time to equilibrium is essential, however, for estimation of peak discharge. Difficulty in estimating this time parameter limits any value in considering the time distribution of rainfall in design applications.

The constant intensity approximation is valid for rainfall durations approximately equal to the time to equilibrium or less. As the relative duration increases, the approximation becomes less valid. Thus, an erroneous value for time to equilibrium can lead to significant under design.

ACKNOWLEDGMENT

This work is a contribution of the United States Department of Agriculture, Science, and Education Administration.

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APPENDIX II.—NOTATION

The following symbols are used in this paper:

- a = rainfall intensity parameter;
- b = rainfall intensity parameter;
- C = Chezy constant;
- D = thunderstorm duration;
- F_o = Froude number at outfall with maximum discharge;
- H_o = normal depth at outfall with maximum discharge;
- h = depth of flow;
- h_o = depth at τ_o ;
- i = rainfall intensity;
- i_{equiv} = uniform rainfall intensity needed to yield P units in duration D ;
- k = kinematic wave number = $SL/(H_o F_o^2)$;
- L = length of plane;
- m = parameter in kinematic wave equation;
- P = total storm rainfall;
- P_t = rainfall accumulated to time t ;
- q = discharge per unit width;
- q_{max} = peak discharge per unit width;
- R = hydraulic radius;
- S = land slope;
- t = time;
- t_e = time to equilibrium;
- t_o = origin of characteristic curve;
- V = flow velocity;
- x = spatial location;
- x_o = origin of characteristic curve;
- α = parameter in kinematic wave equation = $C\sqrt{S}$;
- τ = variable used in integration of kinematic wave equation; and
- τ_o = origin of characteristic curve.

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16060 OVERLAND FLOW FROM TIME-DISTRIBUTED RAINFALL

KEY WORDS: Kinematics; **Overland flow**; Peak discharge; **Rainfall**; Rainfall-runoff relationships; Thunderstorms; **Time factors**; **Time of concentration**

ABSTRACT: The influence of time-varying rainfall on overland flow is investigated. The kinematic wave equations for turbulent flow (across a plane, impermeable surface) are solved using a time-varying rainfall appropriate for thunderstorms. The peak discharge is shown to be a function of surface length, total precipitation, storm duration, and time to equilibrium for rainfall of constant intensity. For rainfall durations equal or less than the time-of-concentration, there is little difference between peak discharges estimated using time-varying rainfall and rainfall of constant intensity. For rainfall of long relative duration, the thunderstorm distribution gives much higher peak discharges.

REFERENCE: Hjelmfelt, Allen T., Jr., "Overland Flow from Time-Distributed Rainfall," *Journal of the Hydraulics Division*, ASCE, Vol. 107, No. HY2, **Proc. Paper 16060**, February, 1981, pp. 227-238