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Curve Numbers: A Personal Interpretation

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Abstract

Problems associated with checking validity of Curve Number Runoff Equation and of using observations of rainfall and runoff to determine Curve Numbers are discussed. Use of Curve Numbers in practice suggests checking validity of the runoff equation through its use as a transformation of a rainfall frequency distribution to a runoff frequency distribution. This concept leads to a means of determining CN values from observations that differs from that given by Soil Conservation Service.

Curve Numbers (CN) derived from rainfall-runoff observations show much scatter. Attempts to correlate this scatter with antecedent moisture have not been fruitful due to other confounding factors such as storm conditions (AMC). Treatment of CN values as random values leads to interpretation of AMC I and AMC III as measures of dispersion around the AMC II value. Acceptance of CN values as random variables permits an explanation of the reason that CN determined from rainfall-runoff data often yields values higher than expected.

Introduction

Irving Klotz (8) made the observation:

> "For the validity of their conceptual and experimental methods, most scientists depend on assurances from reputable predecessors in their field. The latter individuals in turn have usually adopted the procedures from some comparable persons who preceded them. If the forerunners in the use of a technique have not recognized its limitations or have obscured them, a tradition of analysis may develop that generates pervasive misinformation in the scientific literature."

The Curve Number procedure will be considered in the spirit of this observation. The primary source, the National Engineering Handbook of the Soil Conservation Service (SCS) (12), will be investigated as will interpretations and extrapolations that have followed. A most important element is establishment of a framework for verification of the runoff equation, the associated Curve Numbers and modifications of the Curve Numbers.

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Runoff Equation in Theory and Application

Some statements that the SCS has written about the Curve Number procedure are not correct, or are not consistent. This has been exposed many times, but some of the problems will be reviewed here with a rationale given for changes and interpretations. These difficulties seem to have little influence on procedures given by SCS, but do influence extrapolations of the procedures.

The Curve Number runoff equation is given (12) as

$$Q = \frac{(P - I_a)^2}{(P - I_a) + S} \quad ; P > I_a$$
$$Q = 0 \quad ; P < I_a \tag{1}$$

in which Q = runoff, P = rainfall, I_a = initial abstraction, and S = maximum potential retention. The initial abstraction is related to maximum potential retention by the relation

$$I_a = 0.2S \tag{2}$$

The Curve Number, CN, is related to S by

$$CN = \frac{1000}{10 + S} \tag{3}$$

if P, Q and S are in the units of inches.

Definitions and concepts that lead to difficulties include the following:

1) S includes I_a: According to SCS (12), the maximum potential retention, S, includes the initial abstraction, I_a. As many have pointed out (see, for example, Chen 1), a very large storm, $P \to \infty$, will yield a retention $(P-Q) = S + I_a$, according to the runoff equation. Thus, either the equation or the definition is incorrect. The S values, hence Curve Numbers, given by SCS were probably determined using the runoff equation, and not the definition. Therefore, it is probably most efficient to change the definition to indicate that S does not include I_a.

2) Definition of AMC II: Antecedent Moisture Condition II (AMC II) is the base from which adjustments to Curve Numbers are made. SCS (12) gives three definitions for AMC II.

a) Average conditions: It is not certain if SCS (12, Page 4.10) intended to be qualitative or quantitative, and if quantitative what should be averaged. In early editions of the National Engineering Handbook, the conditions were associated with annual floods. In other places SCS seems to imply the 5-day antecedent precipitation is the item to be averaged, though this is not included in this definition.

b) Median CN: The SCS (12, Example 5.4) illustration for determination of the CN from rainfall-runoff data defines the AMC II value as the median of many observations with AMC I and III as enveloping curves. No connection with antecedent precipitation is expressed.

c) Antecedent rainfall table: The SCS (12, Table 4.2) gives a table that shows AMC I, II and III conditions based on antecedent rainfall.

These three definitions are not, necessarily, compatible. Definition (b) seems reasonable in terms of a way to determine the CN. Definition (a) is reasonable in a qualitative sense, but is very difficult to verify in a quantitative sense. Definition (c) does not seem appropriate for Missouri and Iowa.

The preceding indicates that the primary source contains errors and ambiguities. Not everything that is done with Curve Numbers has its basis in SCS documents. There have been extrapolations that also need consideration.

3) "There is a physical basis for S." It is easy to say S is a symbol for storage, so one can measure pore space and soil moisture and determine S in the same sense that Holtan (7) determined S in his infiltration relation. This concept is not verified by SCS (12), nor any place else that the writer has seen. The concept is inherent, however, in modeling applications where S is determined on an event basis by keeping track of soil moisture.

4) "The runoff equation is an infiltration equation." SCS (12) does show how to use the runoff equation as such in developing a design flood. This is most likely due to lack of a better approach than belief in the application. A similar approach was shown by Linsley, Kohler and Paulhus (9), using a coaxial correlation diagram.

Recognizing that not everything is what it seems to be, one must try to define and verify what one can. Two elements will be considered: the runoff equation and the Curve Numbers.

Verification of The Runoff Equation

Verification of the runoff equation is not easy unless its role is carefully defined. One can wish its role to be a simple infiltration equation and try to verify that. For example, one might ask, does it agree with other infiltration theories? The answer to that question is "no" (Smith, 10, Chen, 2, Hjelmfelt, 4). One can also ask if it fits infiltration measurements? Experience indicates that most theories fit some data, but all theories do not fit the same data, nor does any theory seem to fit all data (see for example Rawls et al., 10). It is difficult to define a procedure for verifying an infiltration theory using infiltration data. When discussing this problem with theoreticians, they complain about the data, whereas experimentalists complain about the theories. Perhaps both are correct. That the runoff equation does not agree with other infiltration equations, however, suggests that trying to prove that the equation is in fact an infiltration equation would be fruitless.

An alternative way to seek a vehicle for verification is to determine its usual application. When the runoff equation was developed (and probably today), it was used to determine a design discharge (a 25 year, 100 year, probable maximum flood) based on a synthetic rainstorm. The object was to take a rainfall that was in some sense representative of the design frequency and transform that into a runoff volume for that frequency. Thus, one can test the runoff equation for its ability to convert a rainfall frequency distribution into a runoff frequency distribution.

The transformation of rainfall depth frequency to runoff depth frequency is illustrated in Fig. 1. Rainfall and runoff volumes for annual floods, based on peak discharge, on Ralston Creek, Iowa (area = 3.01 mi^2 = 7.79 km^2) were given by Dalrymple (3) for the period 1938 to 1960. Snowmelt events were not included in the series. More recent events have not been added as the watershed has been urbanizing. The rainfall and runoff values were treated separately. The Wiebull equation was used to plot the points shown in Fig. 1. A lognormal distribution was fit to the rainfall values to give the rainfall curve. Points on this curve were used to compute the runoff curve using the Curve Number runoff equation with CN = 81.

The shape of the runoff curve determined from transforming the rainfall frequency distribution appears reasonable. The curve is high compared to the observed runoff frequency distribution, so the Curve Number should be slightly less than 81. This approach was applied to several other watersheds (5). Quite good results were found in cases where runoff was a substantial fraction of the rainfall. Within limits, the runoff equation does quite well as a frequency transformer. It may also do other things, but verification is not easy.

Verification of Curve Numbers

Having a definition and method for verifying the runoff equation, it is necessary to consider the Curve Numbers, CN, or the maximum potential retention, S. A plot of rainfall and associated runoff yields a scatter diagram. This should be expected from the SCS example (12, Fig. 5.2). The question is how to interpret the scatter diagram. The CN that divides the family into two equal groups gives the value associated with AMC II, and the enveloping curves give AMC I and III, according to SCS. One should recognize, however, that not all points are

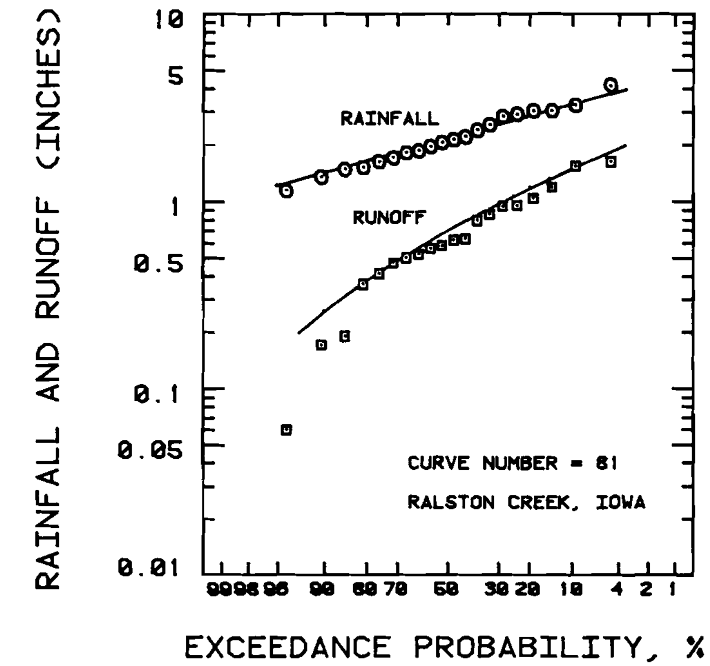

Figure 1.--Distribution of Annual Maximum Event Rainfall and Runoff for Ralston Creek, Iowa, 3.01 sq. mi. (7.79 km^2) (1 in = 25.4 mm)

plotted. Some rainfalls produce no runoff, i.e. $P < I_a$. These should be recognized in establishing the line that divides the points into two groups. If this situation is neglected, the low rainfall values that are plotted must be associated with high CN. This problem was avoided, in part, by SCS (12, Fig. 5.3) by using an annual flood series. If a partial duration series is used, the truncation must be based on rainfall. This leads to difficulties in regions where runoff is only a small fraction of the rainfall, as few events are usable.

It is tempting to explain the scatter in the Curve Numbers, or S, through the antecedent moisture condition as represented by the 5-day antecedent precipitation. This is suggested by SCS (12, Table 4.2). A graph of S versus 5-day antecedent precipitation for annual maximum runoff event at Ralston Creek is shown in Fig. 2. High antecedent precipitation does seem associated with low S (high CN), but low antecedent precipitation may be associated with the whole spectrum of S (and CN).

Another approach is to say that AMC does not imply just moisture, but includes effects of other antecedent conditions and of the storm

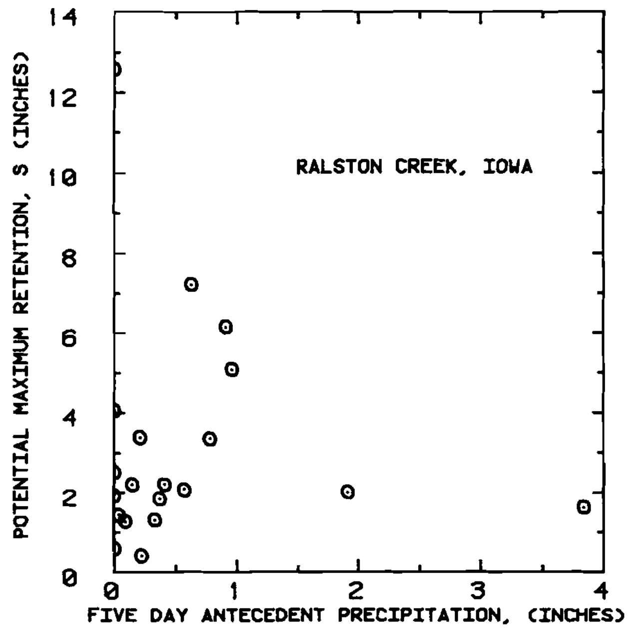

Figure 2.--Relation Between Maximum Potential Retention and Five-Day Antecedent Precipitation for Annual Maximum Events, Ralston Creek, Iowa (1 in. = 25.4 mm)

itself. In this case, one can simply state that the CN is a random variable and treat it as such (6). A lognormal distribution seems to fit the array of S values reasonably well. The Ralston Creek distribution is shown in Fig. 3. The values of S were computed for each event and the Wiebull plotting formula was used. A lognormal distribution was fit to the values to give the line shown. The mean of the logarithms corresponds to the median of the original values for a lognormal distribution, so the 50% value is representative of AMC II. The Curve Number associated with this value is 81, the value used in Fig.1.

The Curve Numbers associated with 10% and 90% probabilities are 60.5 and 92.3, respectively. Transformation of the CN=81 to the AMC I and AMC III values according to SCS (12, Table 10.1) gives CN_I=64 and CN_{III}=92. The agreement between AMC I and III values and the 90% and 10% values is reasonable. Similar results were found for other watersheds (6).

Summary

The Curve Number Runoff equation can be considered a transformation from a rainfall frequency distribution to a runoff frequency

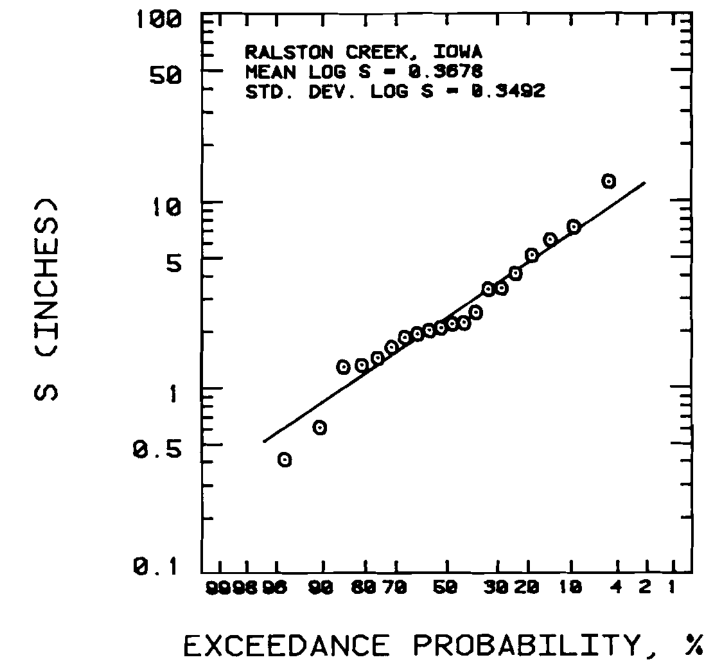

Figure 3.--Lognormal Frequency Distribution of Maximum Potential Retention for Annual Maximum Events, Ralston Creek, Iowa (1 in. = 25.4 mm)

distribution. The Curve Number is not a constant, but varies from event to event. The variability can be summarized by treating the potential maximum retention as a random variable. The AMC II value represents the central tendency and AMC I and III represent extremes.

The variability of the Curve Number leads to difficulty in application of the procedure for conditions where runoff is a small fraction of the rainfall. This difficulty is caused by the rainfall events that result in no runoff. A Curve Number should be associated with each of these events, but cannot.

Antecedent precipitation only explains a portion of the Curve Number variability. Wet antecedent conditions are associated with high Curve Numbers (S). Dry antecedent conditions are associated with a wide spectrum of Curve Numbers. Apparently, other watershed and storm characteristics become important for these latter conditions.

APPENDIX I.--References

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