Mathematical Evaluation of Factors Affecting Gully Stability[1]

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ABSTRACT

Because large amounts of sediment come from gully banks in the coarse loess soils of the Lower Missouri River Basin, a study was made of the factors influencing the stability of gully banks. The approach was to consider the action of forces within the soil mass forming the wall of the gully. A two-dimensional stability analysis was made using the Simplified Bishop Method of Slices.

If the angle of internal friction is $\leq 35°$, calculated factors of safety indicate that vertical, saturated, or near-saturated gully walls in loessial soils will fail if a ground water table exists at the base of the wall and if the cohesion for the saturated soil system is zero at a hydrostatic pressure of zero. Although gully bank stability is sensitive to the angle of internal friction, the calculated stability is relatively insensitive to slope height and soil bulk density. Tension cracks that frequently open at some distance back from the gully face do not materially influence stability. The effect of infiltration rate into the soil on the stability of the gully wall depends upon the rate of water conductivity through the soil and upon the cohesion of the soil or the level of the ground water table. Increasing the infiltration rate decreases the stability.

Additional Index Words: erosion, loessial soils, soil mechanics, sedimentation.

Suspended sediment is widely recognized as a major pollutant of the Nation's streams, lakes, and reservoirs. Although much of the total sediment that enters these waters results from material eroded from gullies, little attention has been given to the mechanics of gully erosion. Beer and Johnson (1963) and Thompson (1964) attempted to relate selected topographic and soil properties to gully growth by statistical regression; however, in this study, we approach the problem of gully development by considering the action of forces on the soil mass forming the gully wall. Our analysis is made for the coarse loess soils of the Lower Missouri River Basin where gullying is severe, but it is applicable to a wide variety of soil conditions.

Gully development may be grouped into three phases: (i) failure of gully head and gully banks, (ii) cleanout of the debris by streamflow, and (iii) degradation of the channel.

Gully erosion is impeded by limiting any one of the three phases. The process is associated with the energy generated by water falling over the headcut and flowing down the channel. If the energy of the flowing water is concentrated at the gully headcut, large masses of soil will erode. Terraces are used to decrease the water overflow energy by increasing infiltration into the soil profile. Table 1 illustrates this point. Also, if the sediment load of a stream exceeds the stream's ability to carry the load, sediment will be deposited and erosion of banks and beds will be retarded.

This study concentrated on the failure of gully heads and gully banks. In the process of gully erosion, the resisting forces of the gully walls decrease to a point at which the steep gully wall collapses. With this slumping of the near-vertical slope, a more stable slope geometry is formed. If no cleanout occurs, the reduced slope will grass over and gully development will cease.

According to Skempton and Hutchinson (1969), scientific knowledge of gully wall stability may be obtained through four areas of study:

"1) Classification and precise description of the materials involved in mass movements and the quantitative measurement of their relevant properties.

2) Methods of calculating the stability of a slope in terms of the type of failure and the material properties.

Table I—Average annual water and sediment yield (1964–1970) of Treynor, Iowa corn watersheds

Watershed number*	Conservation treatment	Streamflow		Sediment yield	
		Base	Surface	Sheet rill	Gully
		cm		t/ha	
1	Contoured	6.30	12.07	61.4	16.1
4	Terraced	15.88	1.73	1.8	0.0

* R. F. Piest and R. G. Spomer. 1968. Sheet and gully erosion in the Missouri Valley Loessial Region. Trans. Amer. Soc. Agr. Eng. 11:850–851 for description of Treynor, Iowa watersheds.

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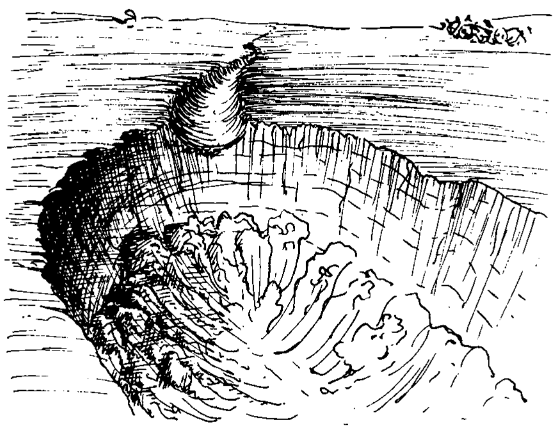

Fig. 1—Schematic of a gully showing gully walls, headcut, and streambed.

3) Classification and causes of the various types of mass movements that occur.
4) Correlation between field observations and the results of stability calculations based on measured properties of the materials."

This report is concerned with the second area only.

METHOD OF ANALYSIS

The method of analysis that was used to identify major contributors to the mass movement of soil in gullies ignores the three-dimensional features of gully geometry (Fig. 1). The errors involved in applying a two-dimensional stability analysis to what may be a three-dimensional gully wall failure in the immediate vicinity of the gully head are difficult to evaluate. For studying failure at the gully head, it seems unreasonable to assume, as in the two-dimensional stability analysis, that the length of the sliding mass is so great that the end effect is negligible. Thus, field observations should be made to establish the degree of reliability of the simpler two-dimensional analysis for predicting gully growth.

The index of stability of a soil slope with respect to general shear failure is known as the safety factor of the slope. In the broadest terms, this may be defined as the ratio of the potential resisting forces to the forces tending to cause movement. In stability analyses, the factor of safety is defined more specifically as the ratio of the moment of the available shearing forces on the trial failure surface to the net moment of the driving forces.

In reviewing the engineering literature, we note that, where the failure surface is assumed to be an arc of a circle (Bishop, 1955), the sliding tendency is created by the moment of the mass about the center of the arc. This moment is opposed by the shearing resistance developed along the sliding surface. Thus, the factor of safety for some assumed failure arc, Fs, is the ratio of the moment of the shearing force along this arc to the moment of weight of the failure mass about the center of the arc.

Referring to Fig. 2:

M_D = moment of driving forces,
= weight, W_T, times perpendicular distance, a, from axis of rotation to line of action of the force,
= $W_T \cdot a$.

M_R = moment of available shearing forces,
= total shearing strength, T, times moment arm, R, of resisting forces,
= $T \cdot R$.

Thus, the factor of safety,

$$Fs = \frac{T \cdot R}{W_T \cdot a}.$$

Instead of attempting to establish average values of weight and shearing strength for a particular gully, a method was sought that permits detailed knowledge of variations in soil conditions and soil properties to be analyzed directly. The Method of Slices appears well suited to this type of analysis (Lambe and Whitman, 1969). In this approach, the sliding mass is subdivided into vertical sections, and a step-by-step analysis is made of the forces acting on each section of the failure surface. Although there are numerous versions of the Method of Slices, the Simplified Bishop Method of Slices was used because "experience during the past 15 years has indicated that the Simplified Bishop Method is generally superior to other simplified methods" (Bailey and Christian, 1969) and because of the availability of computer programs based on this method (ICES LEASE-1). The relative accuracy of the various methods of slope stability analysis has been discussed by Whitman and Bailey (1967). Furthermore, Singh (1970) states, "Differences between the actual field strength parameters and those determined in the laboratory may overshadow all the refinements in computing the factor of safety of slopes." The variation in the factor of safety computed by different methods of analysis is usually less than the scatter in the shear strength parameters.

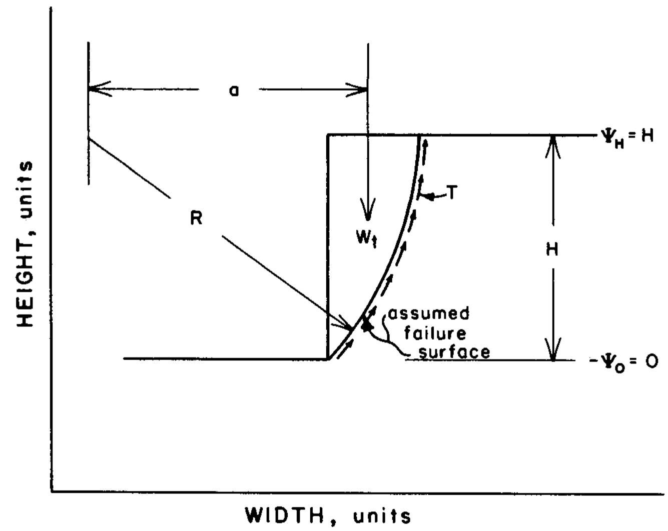

Fig. 2—Two-dimensional cross section showing the relationship of resisting and driving forces of a circular arc failure surface of a gully wall of homogeneous material.

Using the Simplified Bishop Method of Slices, programmed for computer operation in LEASE 1 (Bailey and Christian, 1969), factors of safety for various loading and boundary conditions were computed to obtain a better understanding of the range of conditions producing collapse of gully walls. Gully wall failure is due to an interaction of several soil and topographic properties; the stress state within the soil mass and the conditions that precipitate the failure may vary appreciably from one gully to the next.

In analyzing the stability of a typical gully wall formed in the loessial soils of the Midwest, a range of values was assumed for degree of slope, slope height, soil density, pore water pressures, and soil strength. Failure of a cohesive soil with internal friction is assumed to be adequately described by the Mohr-Coulomb theory. If shear strength is expressed as a function of total normal stress,

$$\tau_f = c + \sigma_n \tan \phi$$

where c denotes the apparent cohesion and ϕ denotes the friction angle or angle of shearing resistance, both with respect to changes in total stress. σ_n is the total normal stress on the failure plane at time of failure, and τ_f is the shear stress on the failure plane at failure.

If the mass in which the stresses are produced is saturated, these internal stresses may be expressed as a combination of

Table 2—Effective stress analyses of vertical slopes for various soil parameters*

Slope height, H	Wet density, ρ	Angle of friction, ϕ'	Computed factor of safety Simplified Bishop
cm	g/cm^3	degrees	
20	1.70	25	0.58
		30	0.79
		35	0.95
		40	1.14
300	1.70	25	0.62
		30	0.76
		35	0.96
		40	1.11
300	1.80	25	0.59
		30	0.74
		35	0.89
		40	1.07
300	1.90	25	0.57
		30	0.71
		35	0.86
		40	1.03

* Assumed c' = 0, c = ψi tan ϕ'.

effective or intergranular stress σ_n' and a hydrostatic excess stress u_w, i.e.,

$$\sigma_n' = \sigma_n - u_w.$$

A general expression proposed by Bishop (1959) for the effective normal stress σ_n' in partially saturated soils is:

$$\sigma_n' = \sigma_n - u_a + \chi\,(u_a - u_w)$$

where σ_n' denotes effective stress normal to the plane considered,
σ_n denotes total stress normal to the plane considered,
u_a denotes pressure in the air in the pore space,
u_w denotes pressure in the water in the pore space,
and χ denotes fractional surface area over which the hydrostatic excess pressure acts.

Expressed in terms of effective stresses, the shear strength of soil is:

$$\tau_f = c' + \sigma_n' \tan \phi'.$$

For a normally consolidated (under its own weight) soil, the apparent cohesion, c, can often be attributed solely to negative pore pressures; thus, on an effective stress basis, one may assume $c' = 0$ and therefore $\tau_f = \sigma_n' \tan \phi'$. Connection of soil pores to the atmosphere gives $u_a = 0$ and therefore $\sigma_n' = \sigma_n - \chi\, u_w$. When $\sigma_n = 0$, $\tau_f = c = -\chi\, u_w \tan \phi'$, and hence $c = -\chi\, u_w \tan \phi' = \chi\, \psi_i \tan \phi$, where ψ_i is the soil water suction.

The value of χ is unity for saturated soils and is zero for dry soils. Intermediate values are assumed to depend primarily on the degree of saturation (Lee and Donald, 1968); however, the relationship is not unique. It is also influenced by other factors such as soil type, soil structure, cycle of wetting and drying, and stress path. When the suction in the soil water exceeds the air entry value, the value needs to be determined experimentally.

Effect of Friction Angle on Gully Stability

Using the model of Fig. 2, failure conditions were initially analyzed for a soil of uniform bulk density with friction angles of 25°, 30°, 35°, and 40°. A water table is assumed to exist at the toe of vertical slopes covering a wide range of slope heights, H. Based on the Mohr-Coulomb failure law and $\chi = 1$ (saturation), the apparent cohesion of the soil at any height above the water table may, under condition of zero water flow, be taken equal to the soil water suction at that height, ψ_i, times tan ϕ'. Thus, at the toe of a 300-cm-high slope $\psi_o = 0$ and $c = 0$ and at the soil surface $\psi_H = 300$ cm water and $c = \psi_H \tan \phi'$. The true cohesion of silt loam soils is usually less than 5 g cm^{-2}. Furthermore, as pointed out by Ingles (1968), for uncemented media the granular stresses produced by capillary forces are almost three orders of magnitude greater than the van der Waal's attractive force.

Table 3—Computed factors of safety as affected by change in water table for a slope height of 300 cm

Wet density, ρ, g/cm^3 surface	toe	Angle of friction, ϕ'	Suction at soil surface, ψ_H*	Computed factor of safety Simplified Bishop
		degrees	cm in water	
1.65	1.95	25	300	0.60
1.65	1.95	25	330	0.72
1.65	1.95	25	360	0.83
1.65	1.95	25	410	1.00
1.65	1.95	25	450	1.14

* Water suction, ψ_i, is assumed to be a linear function of height above water table. Cohesion, c, is equal to ψ_i tan ϕ'. Water table at toe of slope, ψ_o = 0.

Using LEASE 1, trial failure circles were used to determine minimum factors of safety under the assumptions that (i) the critical failure surface does not cut below the toe of the gully wall and (ii) the failure surface is normal to the horizontal soil surface forming the top of the wall. Both assumptions appear reasonable. Selected failure surfaces cutting below the toe of a homogeneous soil slope were calculated to have higher factors of safety than those intersecting at the toe. The second assumption is consistent with the observation of vertical tension or shrinkage cracks in loessial areas. These cracks appear to intersect normal to the soil surface (Lohnes and Handy, 1968). Supporting evidence is the appearance of vertical failure surfaces in the upper regions of failure in gully walls immediately following massive movements of soil.

Table 2 presents minimum factors of safety for the Bishop Method. The calculated factors of safety indicate that vertical, saturated, or near-saturated gully walls will fail in most loessial soils if a ground water table exists at the base of the wall and if the cohesion for the saturated soil system is zero at a hydrostatic pressure of zero. The calculated values in Table 2 indicate that failure is, as we might expect, independent of height for vertical slopes if the effective cohesion is equal to the height at all points along any given vertical plane. Increasing the wet bulk density of the soil causes an increase in the driving forces, thereby causing a decrease in the factor of safety.

The friction angle of silt soils is not expected to be greater than 30°. Lohnes and Handy (1968), using a borehole shear test device, concluded that the internal friction angle, ϕ', of friable loess is fairly constant between 20° and 30°. The mean friction angle for Iowa bluff-line loess at 14 locations was 24.9°. G. R. Olson (1958. Direct shear and consolidation tests of undisturbed loess. M.S. Thesis. Iowa State University, Ames) conducted direct shear tests on five Wisconsin-age loessial soils from western Iowa. Consolidated, drained tests on saturated samples showed the angle of shearing resistance to vary from 23.8° to 24.7°. Consolidated, undrained triaxial shear tests on three Iowa loessial soils, wetted to near saturation, gave friction angles of 28.8° to 29.7° (F.M. Akiyama, 1964. Shear strength properties of western Iowa loess. M.S. Thesis, Iowa State University, Ames).

Effect of Water Table Depth on Gully Stability

Having shown that the Simplified Bishop equation predicts an unstable condition for a vertical gully wall with $\phi' \leq 35°$, we now examine processes thought to contribute to slope stability in loessial soils.

Water is the principal strength-modifying agent acting on loessial area slopes. Whether water is in the form of a fluctuating water table or is present because of increased infiltration from rainfall, the shearing resistance of the soil decreases. Table 3 illustrates the effect of raising or lowering the water table upon the stability of a 300-cm-high slope. In this case, we assumed that the soil bulk density increased linearly with depth and that the soil had a friction angle of 25°. If the water table reaches a level of 110 cm below the toe of the slope, failure is expected.

Observations of gullies in the loessial area of the United States indicate that the ground water level is usually above the

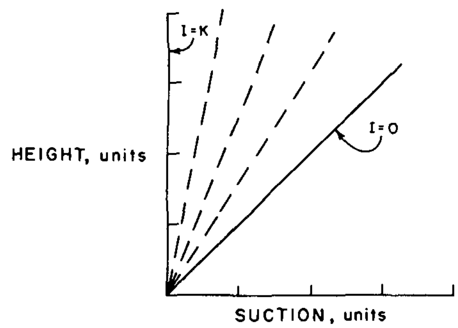

Fig. 3—Relationship of water suction and slope height for various infiltration rates into a saturated, porous media.

toe of the gully wall. Since the results in Table 3 suggest that, in such a situation, loessial-area vertical slopes 300 cm high would not exist, we conclude that the assumption of zero cohesion is not valid for all loessial soils. If the water table were at the toe of the slope analyzed in Table 3 and a stable condition existed, a cohesion greater than that developed by 110 tan ϕ', that is, 53 g cm^{-2}, would be necessary to create a stable condition.

For many engineering purposes, the cohesion of loessial soils is considered as zero; however, small values of cohesion are extremely important in slope stability analyses as seen in Table 3. From the three soils analyzed by Akiyama (1964), cohesion under saturated conditions ranged from 26 to 302 g cm^{-2}. Olson's (1958) cohesion values ranged from 14 to 126 g cm^{-2}. Thus, in analyzing the effect of a fluctuating water table on slope failure, measurements of both the fluctuating height and of the cohesion of the soil at zero suction are critical.

Effect of Tension Cracks on Gully Stability

Since the horizontal stress is tensile throughout the upper part of a vertical slope, tension cracks tend to open at the soil surface. These cracks reduce the overall stability of a slope by decreasing the cohesion which can be mobilized along the upper part of a potential failure surface. Mathematically, the maximum depth of the tension zone can be calculated; however, additional factors such as expansion caused by frost action and shrinkage of clay greatly affect the depth to which the cracks extend. If the cohesion is nonuniform in the field, the mathematical expression $Z_c = (2c'/\gamma) \tan (45 + \frac{1}{2}\phi')$ has limited use. A vertical crack 60 cm deep was introduced in the final condition analyzed in Table 3. All other conditions remained the same. The factor of safety by the Bishop Method decreased from 1.14 to 1.11. This suggests that tension cracks

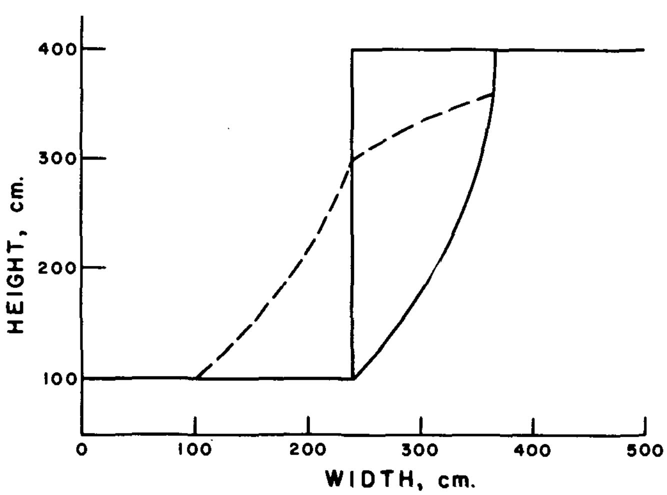

Fig. 4—Slip surface of a gully wall.

Table 4—Computed factors of safety as affected by infiltration

Wet density, ρ surface	toe	Angle of friction, ϕ'	Infiltration rate	Computed factor of safety Simplified Bishop
g/cm³		degrees	cm/hour	
1.65	1.95	25	0.0	1.14
1.65	1.95	25	0.6	1.08
1.65	1.95	25	1.2	1.02
1.65	1.95	25	1.8	0.96

may be ignored as a strength-reducing factor *per se*. However, their role in modifying the rate and degree of wetting of soils during rainfall might be a significant factor in gully bank stability.

Effect of Infiltration on Gully Stability

The movement of water through a soil can initiate gully wall failure because of the decrease in soil water suction and the resultant decrease in shear strength. When, as in our analytical treatment, the soil is assumed to be saturated, the infiltration rate at all points in the soil equals the rainfall rate and a suction profile is established immediately. The rate of infiltration, I, modifies the soil water suction-slope height curve, as shown in Fig. 3. For assumed values of the saturated hydraulic conductivity, K, of the soil and rate of water application, the suction profile may be readily calculated from Darcy's Law. From the suction profile, the relationship between apparent soil cohesion and depth may be determined.

This is, admittedly, a simplified model that enables the potential effect of infiltration on gully wall stability to be examined. In laboratory models the above assumptions can be maintained; however, under field conditions the system will be complicated by unsaturated flow or by a χ value less than unity. If the degree of saturation is $\geq$ 80%, the error involved in the assumption of a χ of unity is often insignificant (Lee and Donald, 1968).

The influence of infiltration has been evaluated for the final set of soil conditions used in Table 3, namely, a friction angle of 25°, a linear density gradient with depth, and an initial suction of 450 cm water at the soil surface. The soil is assumed to be moderately permeable with a saturated conductivity, K, equal to 3.6 cm/hour. Again we can hypothesize either a water table at the toe, $\psi_o = 0$, with a soil cohesion of 150 tan ϕ' (70 g cm^{-2}) or a water table 150 cm below the toe.

The results in Table 4 indicate the influence of infiltration rate on gully stability. Interpolation of the data in Table 4 shows that a rainfall rate of at least 1.4 cm/hour would be needed to reduce the shearing resistance of the soil to a value low enough to collapse the gully wall. Thus, if the infiltration rate is $\geq$ 38% of the saturated conductivity, the wall will collapse. Soils having higher K values or greater cohesion would require higher infiltration rates for instability.

DISCUSSION

Following failure of the gully wall, the degree of slope is reduced to a stable value. This stable gully condition usually persists until the sheared soil mass is removed by water. For example, the factor of safety, F_{SB} = 1.00, by the Bishop Method predicts failure for the gully wall analyzed in Table 3, case 4. After slumping, a more complex slope such as that shown in Fig. 4 results. For this complex slope, the minimum factor of safety is F_{SB} = 1.48, indicating stability.

Gully bank instability also depends on factors other than those included in this study, such as seepage of subsoil water, changes in electrolyte concentration of the soil solution, and the soil-modifying effects of winter freezing and spring thawing and wetting and drying cycles. Some of these factors may be incorporated into our simple analysis if their effects on the mechanical and hydraulic properties of the soil are known. Other factors such as seepage will necessitate a more complicated analysis of gully wall stability involving horizontal as well as vertical variations in the mechanical properties of the soil mass.

We have examined some of the factors influencing gully bank stability in loess soils. Particularly, we have considered the problem for loess soils found in the Lower Missouri River Basin where gullying is severe. The results of our simple analysis indicate that height of water table, cohesion of the soil, and rate of water infiltration are controlling factors affecting stability. Tension cracks have lesser effects. These conclusions agree with visual observations of gully bank failures that have occurred under field conditions.

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