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OVERLAND FLOW  
Kinematic wave  
Storage-depletion model  
Precipitation intensity

## INFLUENCE OF INFILTRATION ON OVERLAND FLOW

A.T. HJELMFELT, Jr.

*Department of Civil Engineering, University of Missouri, Columbia, MO 65201 (U.S.A.)*

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A mathematical model of overland flow with constant rainfall intensity but with varying infiltration rate is presented. The flow is described by the kinematic wave equations and the infiltration by the U.S. Soil Conservation Service storage-depletion process.

The outflow hydrograph is determined for various infiltration conditions and compared with measured hydrographs. The influence of infiltration on the time of concentration is also indicated.

### INTRODUCTION

Analysis of overland flow using kinematic wave theory with constant precipitation and infiltration rates results in hydrographs with an abrupt transition from the rising limb to the constant equilibrium flow. Experimental results, however, appear to indicate a gradually rising hydrograph after the time of concentration is reached.

A decreasing infiltration rate should result in increasing flow rate after the time of concentration is reached. The storage-depletion model of the U.S. Soil Conservation Service is easily combined with the kinematic wave equations to estimate the influence of a decreasing infiltration rate.

### BASIC EQUATIONS

The kinematic wave equations are used to analyze overland flow. Development of the equations and a summary of solutions for some cases is given by Eagleson (1970).

The continuity of flow equation for overland flow takes the form:

$$\partial h / \partial t + \partial q / \partial x = p - f \quad (1)$$

in which  $h$  = depth of flow;  $q$  = flow rate per unit width;  $p$  = precipitation rate;  $f$  = infiltration rate; and  $x, t$  = space and time coordinates.

The flow rate is related to the depth by:

$$q = \alpha h^{3/2} \quad (2)$$

in which  $\alpha$  is a constant involving the slope and resistance to flow across the surface.

An expression for the quantity  $(p - f)$  must be established in order to integrate eqs.1 and 2. The U.S. Soil Conservation Service (1964) describes the use of a mass balance equation:

$$F = (P - I_a) - Q, \quad \text{for } P > I_a \quad (3)$$

in which:

$$\begin{aligned} F &= \text{accumulated infiltration} = \int f \, dt \\ P &= \text{accumulated rainfall} = \int p \, dt \\ Q &= \text{accumulated runoff} = \int (p - f) \, dt \\ I_a &= \text{initial abstraction} \end{aligned}$$

and a storage-depletion relation:

$$F/S = Q/(P - I_a), \quad \text{for } P > I_a \quad (4)$$

in which  $S$  is the potential maximum retention.

Eqs.3 and 4 result in a relationship for the accumulated runoff:

$$Q = (P - I_a)^2 / [(P - I_a) + S] = \int_{I_a/p}^t (p - f) \, dt \quad (5)$$

Ogrosky and Mockus (1964) provide a discussion of the development and application of eq.5 and Musgrave and Holton (1964) give details of parameter estimation from field measurements.

For a constant rainfall intensity,  $p$ , eq.5 becomes:

$$Q = \int_{I_a/p}^t (p - f) \, dt = (pt - I_a)^2 / [(pt - I_a) + S] \quad (6)$$

It is convenient to nondimensionalize eqs.1, 2 and 6 using the relations:

$$\begin{aligned} q^* &= q/(Lp), & h^* &= h[\alpha/(Lp)]^{2/3}, & t^* &= (t - I_a/p)[(\alpha^2 p)/L^2]^{1/3} \\ x^* &= x/L & \text{and} & & S^* &= S[\alpha/(Lp)]^{2/3} \end{aligned} \quad (7)$$

The quantity  $[L^2/(\alpha^2 p)]^{1/3}$  is the time of concentration for flow across an impermeable surface. Thus the equations become:

$$\partial h^* / \partial t^* + \partial q^* / \partial x^* = (1/p)(p - f) \quad (8)$$

$$q^* = (h^*)^{3/2} \quad (9)$$

$$\int_{I_a/p}^t (p - f) \, dt = [(Lp)/\alpha]^{2/3} [(t^*)^2 / (t^* + S^*)] \quad (10)$$

## SOLUTION OF BASIC EQUATIONS

The continuity of flow equation is a wave equation. The solution is established along a characteristic curve, and the characteristic curve is related to the  $x-t$  coordinate system.

Eqs.8 and 9 result in:

$$\partial h^*/\partial t^* + 3/2(h^*)^{1/2}(\partial h^*/\partial x^*) = (1/p)(p - f) \quad (11)$$

This partial differential equation can be solved by assuming a coordinate system,  $\eta$ , such that:

$$dh^*/d\eta = (\partial h^*/\partial t^*)(dt^*/d\eta) + (\partial h^*/\partial x^*)(dx^*/d\eta) = (1/p)(p - f) \quad (12)$$

This will be true if:

$$dt^*/d\eta = 1 \quad \text{or} \quad t^* - t_0^* = \eta - \eta_0 \quad (13)$$

and if

$$dx^*/d\eta = 3/2(h^*)^{1/2}$$

or

$$x^* - x_0^* = \frac{3}{2} \int_{\eta_0}^{\eta} (h^*)^{1/2} d\eta \quad (14)$$

The depth of flow is given by:

$$h^* = 1/p \int_{\eta_0}^{\eta} (p - f) d\eta = 1/p \int_{\eta_0}^{\eta} (p - f) (dt^*/d\eta) d\eta = [\alpha/(Lp)]^{2/3} \int_{t^*=0}^{t^*} (p - f) dt^* \quad (15)$$

or

$$h^* = (t^*)^2 / (t^* + S^*)$$

in which the initial condition  $h^* = 0$  at  $t^* = 0$  has been used.

The  $x$ -coordinates of the characteristics can now be established using eqs.14 and 15:

$$x^* - x_0^* = 3/2 \int_0^{\eta} t^*/(t^* + S^*)^{1/2} \cdot d\eta = 3/2 \int_0^{\eta} t^*/(t^* + S^*)^{1/2} (dt^*/d\eta) d\eta =$$

$$3/2 \int_0^{t^*} t^*/(t^* + S^*)^{1/2} dt^*$$

or

$$x^* - x_0^* = (t^* - 2S^*)^{1/2} + 2(S^*)^{3/2} \quad (16)$$

Time of concentration is achieved when:

$$x_0^* = x_0/L = 0 \quad \text{and} \quad x^* = x/L = 1$$

and eq.16 takes the form:

$$(t_c^* + S^*)^{3/2} - 3S^*(t_c^* + S^*)^{1/2} + [2(S^*)^{3/2} - 1] = 0 \quad (17)$$

in which  $t_c^*$  is the nondimensional time of concentration. This cubic equation for  $(t_c^* + S^*)^{1/2}$  can be solved using the standard form. For those cases in which:

$$(S^*)^{3/2} > 1/4$$

the time of concentration is given by:

$$t_c^* = S^*(4 \cos^2(\phi/3) - 1)$$

in which:

$$\phi = \cos^{-1} [ \{ 0.5 - (S^*)^{3/2} \} / (S^*)^{3/2} ] \quad (18)$$

The influence of the infiltration rate on time of concentration is displayed in Fig.1.

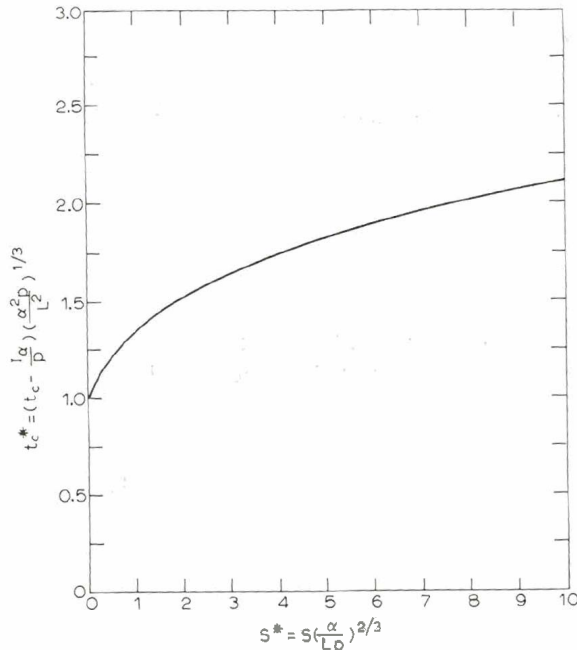


Fig.1. Influence of the maximum potential retention,  $S$ , on the time of concentration.

After time of concentration has been achieved the solutions given by eqs.10 and 14 are no longer valid because the lower limit of integration is no longer zero. Two possibilities exist for continuing the solution. The lower limit of integration may be given values greater than zero. Using this approach an analytic solution for the  $x$ -coordinate could not be obtained. An alternate possibility is to shift the time coordinate, so that the lower limit is always zero. In this case the potential maximum retention ( $S^*$ ) must be reduced to account for the quantity previously infiltrated. This is equivalent to modifying  $S$  to account for antecedent moisture conditions. An analytical solution is possible in this case. The results of the two alternatives are not identical, however, due to the nonlinear character of the infiltration relation.

The second possibility was selected. In order to establish the new  $S^*$  the dimensionless accumulated infiltration  $F^*$ , in which  $F^* = F_0 [\alpha/(Lp)]^{2/3}$  with  $F_0$  the amount accumulated to time  $t_0$ , is subtracted from the initial  $S^*$ . Thus:

$$S_{t_0}^* = S^* - F^* = S^* - t_0^* + (t_0^*)^2 / (t_0^* + S^*) \quad (19)$$

The term  $t_0$  is the initial time for the characteristic lines. Before the time of concentration was achieved  $t_0$  was zero, but the characteristic lines started at various values of  $x_0$ . After  $t_c$  is attained the  $x_0$  is zero, but the  $t_0$  varies. The depth of flow at the outfall of the strip under analysis is given by:

$$h^* = (t^* - t_0^*)^2 / [(t^* - t_0^*) + S_{t_0}^*] \quad (20)$$

The parameter  $(t^* - t_0^*)$  is the time for a parcel of fluid to traverse the surface, a time of concentration for this later portion of the storm. This parameter can be determined using eq.18 if  $S^*$  is replaced by  $S_{t_0}^*$ .

## COMPARISON WITH EXPERIMENT AND CONCLUSIONS

Field hydrographs from erosion-study plots were published by Foster et al. (1968). Points taken from the published graphs are compared with theory in Fig.2. The time scale used is the nondimensional deviation from the time of concentration. This eliminated the difficulty of estimating the initial abstraction,  $I_a$ . The time of concentration for the experimental hydrographs was estimated from the shape of the curve.

A comparison between the hydrograph resulting from constant infiltration rates and the variable infiltration rate is shown in Fig.3. An appropriate constant average infiltration rate is not readily apparent; so two values were used for the comparison.

No rigorous attempt was made to fit the kinematic equations with the plotted points. Only the general trend is to be inferred. The infiltration does allow flow to increase after reaching the time of concentration. The flow is overestimated in most cases, however. This overestimation can arise from many possibilities within the kinematic wave equations, within the conceptual form of

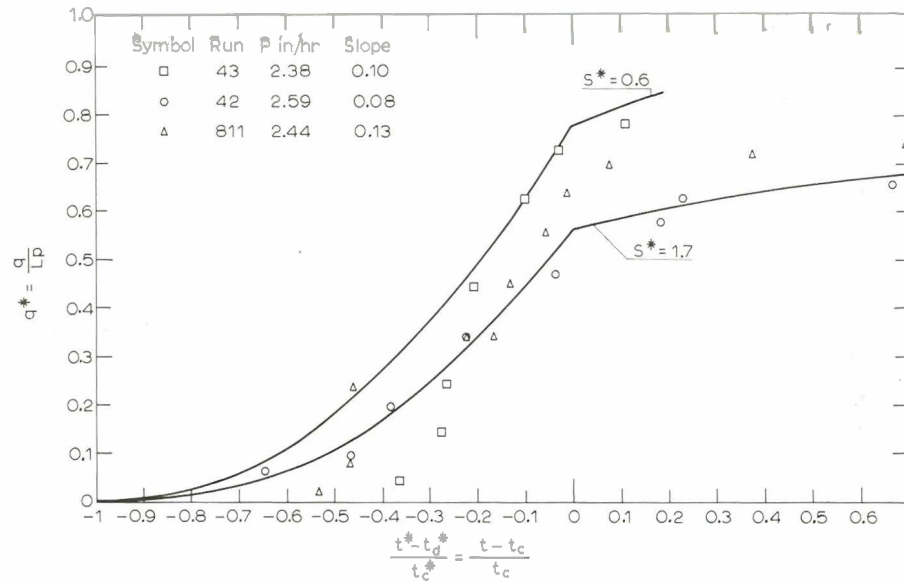


Fig. 2. Comparison of overland flow theory with experimental results reported by Foster et al. (1968).

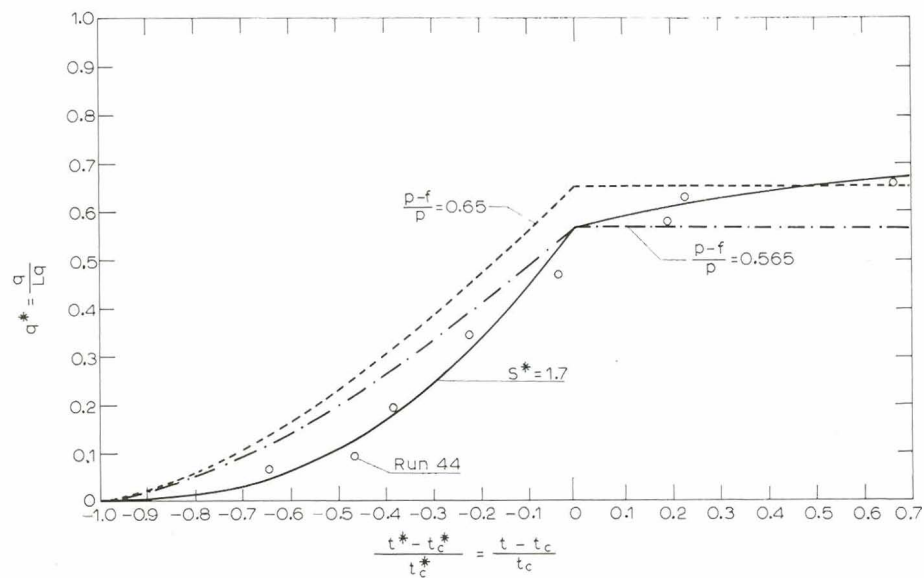


Fig. 3. Comparison of overland flow theory with and without infiltration and with experimental Run 44 of Foster et al. (1968).

the infiltration equation, and in the use of the reduced storage potential,  $S^*$ , in this formulation.

The variation of infiltration rate during a storm has a significant effect on the time of concentration and on the shape of the runoff hydrograph. These factors must be recognized in analysis of measured hydrographs to estimate parameters in kinematic wave modeling.

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