10987 STORAGE PROBABILITIES OF SMALL RESERVOIR

KEY WORDS: **Hydrology; Irrigation;** Irrigation design; **Probability theory;** Reservoir capacity; **Reservoirs; Reservoir storage**

ABSTRACT: Supplemental irrigation systems using a reservoir for water supply storage can be designed with a known risk of having inadequate storage to satisfy the yearly irrigation requirements. The relationships between irrigation demand, reservoir capacity, and reservoir failure probability were determined for a gaged reservoir watershed by applying a Markov Chain reservoir storage probability model. Twenty-seven years of observed and modeled reservoir budget data for a 97-acre-ft (120,000-m^3) reservoir and a 154-acre (623,000-m^2) watershed were used to develop the demand- capacity-probability relationships for demands ranging from 0 acres to 100 acres (0 m^2 to 405,000 m^2) of irrigated corn, reservoir capacities from 40 acre-ft to 200 acre-ft (49,000 m^3 to 247,000 m^3), and failure probability of 1% to 50%. The effect of varying irrigation efficiency on these relationships was also examined.

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IRRIGATION STORAGE PROBABILITIES OF SMALL RESERVOIR[a]

By Larry A. Kramer[1]

INTRODUCTION

Throughout much of the humid and semihumid areas of the United States, rainfall shortages during a crop growing season often result in critical soil moisture deficits. Irrigation water is needed to supplement rainfall during these deficit periods. The uncertainty of water supply for irrigation is, in some regions, a limiting factor to the design of a supplemental irrigation system. Underground water sources frequently are not available in the quantity or the quality needed. If there is a suitable site on the farm or ranch, a surface reservoir can be constructed to store watershed runoff throughout the year for supplemental irrigation needs.

In this paper, the writer presents an application of a reservoir storage probability model to hydrologic data from a small, gaged reservoir watershed to derive the relationships between reservoir capacity, irrigation water demand, and reservoir failure probability. Reservoir failure is defined as the annual probability of storage depletion. Failure occurs when there is insufficient water in storage to satisfy the annual irrigation demand.

The method of analysis is briefly described as follows. Observed watershed and reservoir hydrologic records were combined with modeled irrigation demand to generate accumulated monthly reservoir storage data for the period of observation. Annual maximum and minimum reservoir storage volumes and the volume increments and decrements between those maximums and minimums were determined. The probability distributions of these increments and decrements were used to develop reservoir failure probability curves for the various levels of irrigation demand and reservoir storage capacities assumed.

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Procedures

Reservoir Storage Data.—Hydrologic data used in this analysis were observed by the U.S. Department of Agriculture Agricultural Research Service on the McCredie reservoir and watershed (4). This 154-acre (623,000-m^2) watershed, which is about 25 miles (40 km) east of Columbia, Mo., is located in the central Missouri claypan soils area. Data from 27 yr of record, 1942-1968, were analyzed. At spillway elevation, the McCredie reservoir has a surface area of about 16 acres (65,000 m^2) and a capacity of 97 acre-ft (120,000 m^3). The land use on this gently rolling watershed during the record period was about one-third row crops and two-thirds grass and meadow. Monthly volumes of runoff from the watershed, Q, and rainfall on the reservoir, P, were summarized from detailed continuous measurements. Reservoir evaporation-and-seepage volumes $E + S$ were computed for each month from a reservoir mass balance.

Irrigation demand volume I_x for a corn crop of x acres was modeled for the period of record with a method described by Woodruff (6) which is based on observed daily precipitation data, a constant daily consumptive use rate, and soil water storage parameters. Irrigation demands were adjusted for a field application efficiency of 75% and were summed each month of the irrigation season for various assumed acreages.

The net change in reservoir storage, ΔV, at the end of each month was computed as a balance of monthly reservoir inputs and losses according to

$$\Delta V = Q + P - (E + S) - I_x \qquad (1)$$

Eq. 1 was applied to all months of the 27-yr record period for several assumed acreages of irrigated corn or levels of irrigation demand x.

An average monthly summary of the variables of Eq. 1 will serve to illustrate the seasonal trends in the general hydrologic characteristics of the McCredie reservoir watershed. The 27-yr averages of the variables in Eq. 1 for each month of the year are shown in Fig. 1. For example, irrigation withdrawal I_{50} is shown for 50 acres (200,000 m^2) of irrigated corn. Irrigation demands typically increased each month of the 3-month irrigation season and coincided with periods of low runoff, decreasing precipitation, and maximum evaporation and seepage.

To apply the detailed McCredie data to a storage probability model, the series of monthly net storage volumes, as computed by Eq. 1, was summed to determine accumulative monthly net storage, V, for the period of record for each irrigation demand level tested, x. For the McCredie reservoir, V typically cycled between an annual maximum and an annual minimum each year. The change from the annual minimum of V to the next annual maximum was determined and called the annual storage increment, X. The change from the annual maximum of V to the minimum of V was also determined and called the annual storage decrement, Y. An example of identifying the annual storage increments and decrements from the cumulative monthly net storage is shown in Fig. 2 for the first 4 yr recorded, with 50 irrigated acres (200,000 m^2). The annual increments and decrements for each of the 27 yr studied were used as the data input to the reservoir storage probability model described in the next section.

Reservoir Storage Probability Model.—A Markov Chain, variable-season reservoir storage probability model described by White (5) and Harris (2) was utilized

to derive reservoir failure probabilities of an irrigation supply reservoir. This model represented a storage process in which the reservoir storage moved from

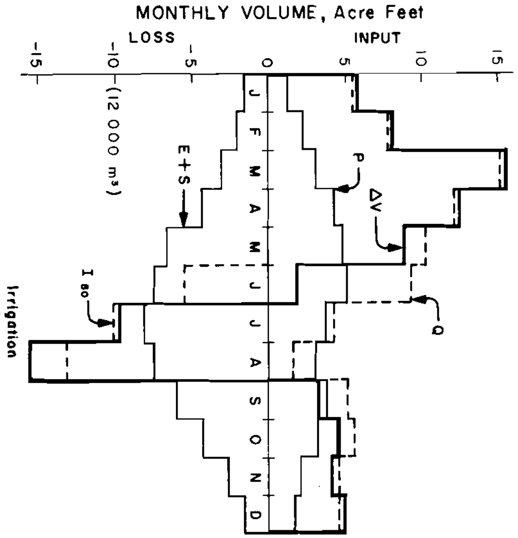

FIG. 1.—Average Monthly Inputs, Losses, and Net Change in Reservoir Storage (1942–1968)

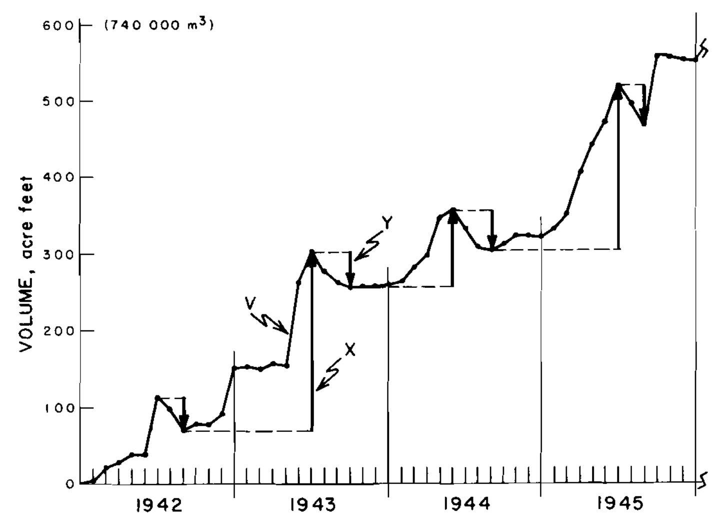

FIG. 2.—Annual Increments X and Decrements Y Derived from Cumulative Monthly Net Storage V for 4-yr Record

an annual minimum through an increment period followed by a decrement period to the next annual minimum. The variable-season characteristic of this storage model related to the variable time periods during which storage increments and decrements occurred each year.

To quantify the reservoir storage process, the reservoir was partitioned into discrete volume storage zones z which were assigned values of 0, 1, 2, ..., $k - 1$, or k. The storage zone, $z = k$, was the maximum zone for a specific reservoir capacity. Thus, the reservoir storage could take on any value of z from 0 to k at the end of storage increment and decrement periods.

TABLE 1.—General Relationship between Continuous and Discrete Random Variables

X or Y (1)	i or j (2)
0 to $1/2\,\Delta S$	0
$1/2\,\Delta S$ to $3/2\,\Delta S$	1
$3/2\,\Delta S$ to $5/2\,\Delta S$	2
—	—
—	—
$(K - 3/2\,\Delta S)$ to $(K - 1/2\,\Delta S)$	$k - 1$
$(K - 1/2\,\Delta S)$ to $K+$	k

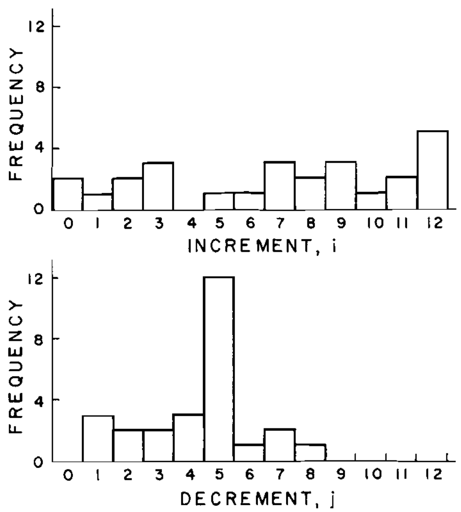

FIG. 3.—Increment and Decrement Histograms for 50-acre (200,000-m²) Irrigation Demand and 120-acre-ft (148,000-m³) Reservoir Capacity

The storage zone interval, ΔS, is chosen sufficiently small so that the resulting discrete probability distribution approaches the results of a continuous distribution with only minimal computation. As shown by White (5), the discrete probabilities decrease to the continuous solution as the number of discrete zones increases. White indicated that about 10 zones provided an adequate solution. For the solutions described in this paper, ΔS = 10 acre-ft (12,000 m³). This resulted in five to 21 discrete zones for the various reservoir capacities analyzed.

Annual increments X and decrements Y were considered a series of independent observations of continuous random variables. The variables, X and Y, were transformed into discrete random variables by classifying them into histograms with classes of i and j and a class interval volume of ΔS. The generalized relationship between the discrete and continuous random variables is indicated in Table 1, in which K is the reservoir capacity.

Frequency histograms of i and j were obtained from the series of annual increments X and decrements Y determined from V. Histogram examples are shown in Fig. 3 for 50 irrigated acres (200,000 m²) and 120-acre-ft (148,000-m³) reservoir capacity. The number of increments and decrements occurring in each class interval up to reservoir capacity is indicated. The increment distribution was relatively uniform, but the decrement distribution had a distinct peak. For a specific reservoir capacity, the increment distribution changed little with changes in the irrigation demand levels. But the decrement distribution mean increased when irrigation demand increased because of larger decrement volumes. Consequently, the peak of the distribution shifted to higher decrement classes.

The frequency histograms for annual increments and decrements were converted to sample probability distributions. The increment and decrement probability distributions were defined as

$$q_i = \text{probability (increment } X = i); \quad i = 0, ..., k \qquad (2)$$

$$p_j = \text{probability (decrement } Y = j); \quad j = 0, ..., k \qquad (3)$$

The q_i and p_j probabilities were incorporated into a set of equations that describe the probability distributions of the annual reservoir storage maximum and minimum occurring in each storage zone, z. For year t, these storage probability distributions were defined as

$$M_z = \text{probability (maximum storage}_t = z); \quad z = 0, ..., k \qquad (4)$$

$$M_z = f(N_z, q_i) \qquad (5)$$

in which $N_z = \text{probability (minimum storage}_t = z); \quad z = 0, ..., k \qquad (6)$

For the year $t + 1$, the minimum storage probability was defined as

$$N'_z = \text{probability (minimum storage}_{t+1} = z); \quad z = 0, ..., k \qquad (7)$$

$$N'_z = f(M_z, p_j) \qquad (8)$$

Combining Eqs. 5 and 8 gives

$$N'_z = f(N_z, q_i, p_j) \qquad (9)$$

Thus, the annual minimum reservoir storage probability, N'_z, was defined as a function of the previous annual minimum probability, N_z, and the increment and decrement probabilities.

To obtain numerical solutions for N'_z, a set of simultaneous probability equations, one for each storage zone, z, was defined using joint and conditional probability rules and readily solved with matrix algebra routines on a digital computer. The details of the solution are clearly presented and examined by White (5), Harris (2), and Kirby (3).

Results

Increment and decrement frequency histograms were determined from the 27 yr of accumulated monthly net storage data for irrigation demand levels of 0 acres, 25 acres, 50 acres, 75 acres, and 100 acres (0 m^2, 100,000 m^2, 200,000 m^2, 300,000 m^2, and 405,000 m^2) of corn and reservoir capacities of 40 acre-ft, 80 acre-ft, 120 acre-ft, 160 acre-ft, and 200 acre-ft (49,000 m^3, 99,000 m^3, 148,000 m^3, 197,000 m^3, and 247,000 m^3).

The assumption of independence of increments and successive decrements was tested before computing storage probabilities. The serial correlation between increments and decrements decreased from -0.279 to -0.084 as irrigation demand increased from 0 acres to 100 acres (0 m^2 to 410,000 m^2). Using Anderson's (1) test at the 95% significance level, the serial correlation would be -0.37; thus, the correlations for all levels of irrigation demand studied were considered statistically insignificant.

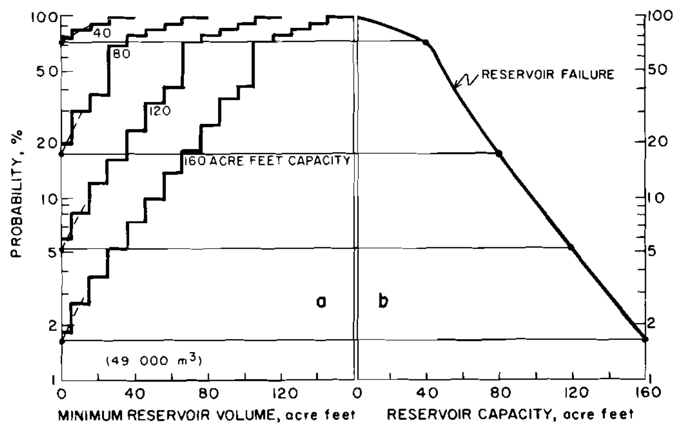

FIG. 4.—(a) Annual Minimum Reservoir Volume Probabilities; (b) Reservoir Failure Probabilities for 50-acre (200,000-m^2) Irrigation Demand

Minimum storage probability distributions N_z' were computed for each combination of irrigation demand and reservoir capacity assumed. Example results for an irrigation demand of 50 acres (200,000 m^2) and reservoir capacities of 40 acre-ft, 80 acre-ft, 120 acre-ft, and 160 acre-ft (49,000 m^3, 99,000 m^3, 148,000 m^3, and 197,000 m^3) are shown in Fig. 4(a). For the 160-acre-ft (197,000-m^3) reservoir capacity, there is a 25% probability that the annual minimum volume will be less than or equal to 80 acre-ft (99,000 m^3). similarly, for an 80-acre-ft (99,000-m^3) reservoir capacity, there is an 80% probability that the annual minimum volume will be less than or equal to 40 acre-ft (49,000 m^3). The failure probability, which is the probability of depleting the entire reservoir storage, is derived from a short extrapolation of the annual minimum volume probability distributions to the graph ordinate as indicated by the dashed lines in fig. 4(a). For the 120-acre-ft (148,000-m^3) reservoir size, there is about a 5% probability of storage depletion of reservoir failure.

The reservoir failure probability points for each capacity tested are replotted in Fig. 4(b) to derive the relationship of reservoir failure probability to reservoir capacity. A similar curve was obtained for each irrigation demand level. These curves define the relationships between reservoir storage capacity, irrigation

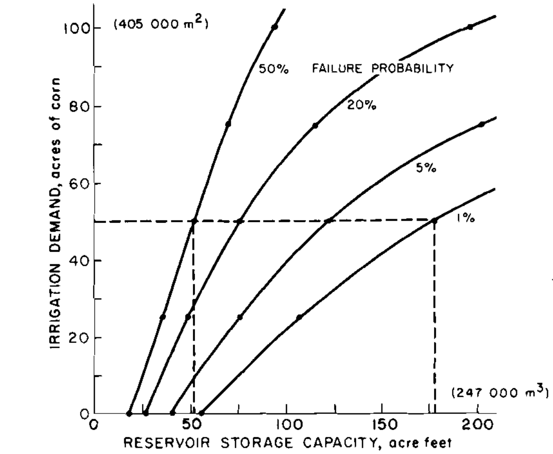

FIG. 5.—Relationship between Irrigation Demand, Reservoir Capacity, and Failure Probability

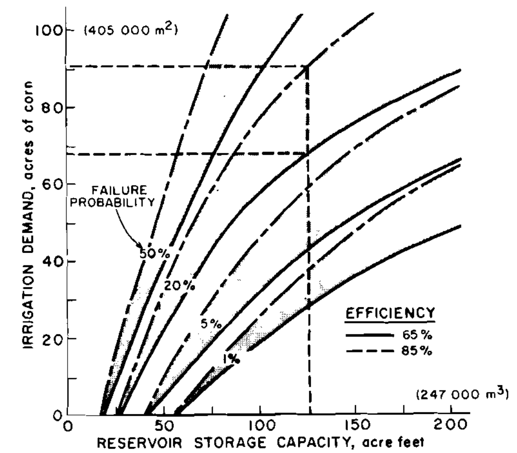

FIG. 6.—Effect of Irrigation Efficiency on Relationship between Irrigation Demand, Reservoir Capacity, and Failure Probability

water demand, and reservoir failure probability.

The results are summarized in Fig. 5 where irrigation demand, in acres of irrigated corn, is related to reservoir capacity by a family of reservoir failure probability curves of 1%, 5%, 20%, and 50%. For example, as shown by the dashed lines, when irrigating 50 acres (200,000 m^2), the failure probability would be 50% with a 50-acre-ft (62,000-m^3) reservoir and 1% with a 180-acre-ft (222,000-m^3) reservoir. As expected for a constant irrigation demand, the failure probability decreases as the reservoir capacity increases. Similarly, for a given reservoir capacity, the probability of failure increases as the demand increases.

The results in Fig. 5 can be used to evaluate supplemental irrigation system designs on the basis of the degree of failure probability, whereas supplemental systems have often been designed and operated without a known failure probability. Common design principles have been "rule of thumb" relationships, e.g., storing from 1 acre-ft to 2 acre-ft (1,000 m^3 to 2,000 m^3) of water per acre (4,000 m^2) irrigated. From Fig. 5, 1 acre-ft (1,000 m^3) of storage per acre (4,000 m^2) irrigated for the McCredie reservoir would result in a failure probability of nearly 50%, but 2 acre-ft (2,000 m^3) of storage per acre (4,000 m^2) would result in a failure probability of only 10%. Knowledge of the failure probability associated with a specific irrigation system will help the system operator decide rational operating procedures so that the system is not subjected to failure risks greater than the design criteria.

The relationships shown in Fig. 5 were derived from measured inputs of runoff and precipitation into an existing reservoir, computed losses of evaporation and seepage, and modeled irrigation demand. An irrigation efficiency of 75% was assumed. The example relationships developed from the McCredie reservoir watershed data are primarily representative of the central claypan soils area of northeast Missouri and provide general information for the design of reservoir storage facilities in this region. The hydrologic characteristics of this study watershed are similar to other gaged watersheds in the same soil resource area, as shown by Saxton and Whitaker (4). The results presented for the McCredie reservoir watershed were derived from historical records and must be properly interpreted before extrapolation to other locations, but the technique could be applied readily where similar data are available.

The independent terms of Eq. 1 can be varied to examine their effect on the reservoir failure probabilities. Parameters such as the type of crop irrigated, the method of modeling irrigation demand, and the irrigation efficiency could be changed to compute corresponding irrigation demands. Irrigation efficiency is a parameter that can be controlled by the irrigator through system management. The effect of efficiency was studied by assuming values of 65% and 85% and computing reservoir failure probabilities in the manner previously described for 75% efficiency. The shaded areas in Fig. 6 represent constant failure probability regions of 1%, 5%, 20%, and 50%. Each region is bounded on the top by the 85% efficiency condition and on the bottom by the 65% efficiency condition. These results show (dashed lines in Fig. 6) that for a 125-acre-ft (154,000-m^3) capacity reservoir at 20% failure probability, 68 acres (280,000 m^2) could be irrigated at 65% irrigation efficiency, but 90 acres (360,000 m^2) could be irrigated at 85% efficiency.

Summary and Conclusions

A Markov Chain, variable-season reservoir storage probability model was used to develop the relationships between the irrigation demand, reservoir capacity, and reservoir failure probability for a gaged reservoir watershed. The model was used to examine these relationships for irrigation demand levels of 0 acres to 100 acres (0 m^2 to 405,000 m^2) of corn, reservoir capacities of 0 acre-ft to 200 acre-ft (0 m^3 to 247,000 m^3), and failure probabilities of 1% to 50%. Data from the McCredie reservoir watershed for the period 1942–1968 were utilized in the model. The results showed this probability model could be used to define failure probabilities of the supplemental irrigation storage reservoir for small agricultural watersheds. The effect of irrigation efficiency on failure probability was investigated with the model. The effect on storage probability of other irrigation demands modeled by different techniques and for crops other than corn could be analyzed by the method presented.

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Appendix II.—Notation

The following symbols are used in this paper:

$E + S$ = monthly volume of reservoir evaporation and seepage, in acre-feet;
I_x = monthly volume of irrigation water demand, in acre-feet;
i = discrete random variable of storage increment volume;
j = discrete random variable of storage decrement volume;
K = volume of storage at reservoir capacity, in acre-feet;
k = maximum value of i, j, or z;
M_z = annual maximum reservoir storage probability distribution;
N_z = annual minimum reservoir storage probability distribution;
N'_z = N_z in next year;

P = monthly volume of precipitation on reservoir, in acre-feet;
p_j = storage decrement probability distribution;
Q = monthly watershed runoff volume, in acre-feet;
q_i = storage increment probability distribution;
t = year index;
V = accumulated ΔV, in acre-feet;
X = continuous random variable of storage increment volume, in acre-feet;
x = irrigation demand level, in acres of irrigated corn;
Y = continuous random variable of storage decrement volume, in acre-feet;
z = discrete reservoir storage zone;
ΔS = interval volume of i, j, or z, in acre-feet; and
ΔV = monthly net change in reservoir storage, in acre-feet.