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ANTECEDENT RETENTION INDEXES PREDICT SOIL MOISTURE

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INTRODUCTION

Methods used for predicting runoff from rainfall are many and varied; however, most are based on empirical relations. The major variables involved in rainfall-runoff relations are those associated with rainfall, soils, vegetation, and the physical land features. The soil moisture condition just before a rainfall event is always a major consideration, because infiltration is, by far, the most important process by which rainfall is dissipated. Both the rate of infiltration and the amount of available soil moisture storage are closely related to the soil moisture content.

For most hydrologic analyses, soil moisture measurements are not available. And, if they were, there would be few cases when measurements were obtained just before a rain. Therefore, antecedent soil moisture content is usually estimated. Many methods are used—some based on interpolations between soil moisture measurements, some on budgeting procedures, and some on daily precipitation amounts.

Daily precipitation is usually the best, and often the only, available information related to antecedent moisture that is near enough to the study to be of value. An index based on previous precipitation is one of the most common approaches to define an antecedent moisture condition. By this procedure, the effect that prior precipitation has on a hydrologic event is recognized.

Precipitation occurring several days to a few weeks before an event can only affect that event by its influence on some variable that carries through the intervening time period. Soil moisture is, logically, this variable. Thus, an antecedent precipitation index (API) is really an attempt to characterize the antecedent soil moisture condition.

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Hydrologists recognize that an API cannot fully reflect all of the variables that relate to the gain, loss, and distribution of moisture within the soil. However, the antecedent soil moisture condition is so important to the dissipitation of rainfall that even an index based on the single variable of precipitation will generously improve the prediction of rainfall-runoff relations.

The actual accretion to soil moisture is not precipitation, but infiltration or retention, R, which can be represented as precipitation, P, minus runoff, Q, when minor losses such as interception are not considered. Retention is often used instead of P when values of Q are obtainable, and the antecedent precipitation index (API) then logically becomes an antecedent retention index (ARI).

The various factors affecting ARI calculations are similar to the factors affecting changes in soil moisture content. Accretion to ARI values represents infiltration into the soil, in inches. Depletion of ARI values can be compared with evapotranspiration ET, in inches—the principal cause of soil moisture depletion. Values of the ARI are based on the same physical processes that determine the amount of soil moisture; therefore, they can be compared with observed soil moisture content. Values of ARI are comparable with only available soil moisture—not total soil moisture—because evapotranspiration removes only that water held in the soil above the wilting point.

The objective of this study was to evaluate two models for computing ARI values by comparisons with observed available soil moisture. The first model was the traditional API equation based on an exponential depletion of retention, and the second model was based on a depletion more closely comparable with an ET-soil moisture relation. Some details and considerations of both models are given before relating the comparisons with soil moisture.

MODELS FOR COMPUTING ANTECEDENT RETENTION INDEXES

The Exponential Model.—Hydrologists have reasoned that the greater the time lapse between a rainfall event and a given day, the less influence the rain has on the soil moisture content of that day. For example, a 1-in. rain 2 days prior to a selected day may have a marked influence on the soil moisture; but, had this same rain occurred 30 days before, its influence would have been much less.

To obtain this reduction of influence, the precipitation effect in API equations has been considered to be inversely related to time. More commonly, however, this reduction is obtained by a decreasing exponential relation, K^t, in which K is a factor with a value less than 1.00, and t = time in days. Considering a single rain, P_t, t days before a selected day, i, the relation is

$$API_i = P_t K^t \quad (1)$$

Usually, more than one rain occurs within the effective period before a selected date. Rather than total their combined influence, daily indexes are usually calculated for the prior period by adding precipitation to that day's index before calculating the next day's index. Thus, for daily calculations, t equals 1, and Eq. 1 becomes

$$API_i = (API_{i-1} + P_{i-1}) K \quad (2)$$

which is the equation most commonly used for computing antecedent precipitation indexes.

When retention, R, is substituted for precipitation, P, the API term becomes ARI. This change in no way alters the form of the equation or its characteristics. ARI is simply more descriptive of the values calculated and sets them apart from those that would result if only precipitation were used. Thus, Eq. 2 becomes

$$ARI_i = (ARI_{i-1} + R_{i-1}) K \quad (3)$$

ARI values calculated by the exponential model for a typical period are shown in Fig. 1. Either of the two lines is representative of a series of daily ARI values. The dash line includes 0.90 in. of retention on May 1, which was not included when computing the solid line. All other retention values are the same, and a K value of 0.97 was used for both lines. The convergence of the two lines shows that the influence of retention decreases with time.

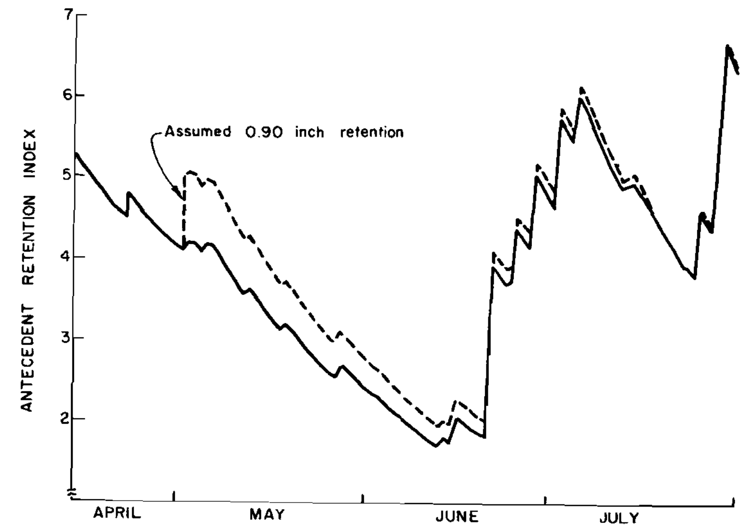

FIG. 1.—ARI VALUES BY THE EXPONENTIAL MODEL

To use Eq. 3, two values must be established in addition to the daily retention. These are: (1) initial ARI value, and (2) recession factor, K.

The following concerns the assigning of these values and the required length of calculation period before a selected date.

Initial ARI Value (ARI_0).—Most hydrologists have estimated ARI_0 as a portion of the accumulated precipitation for a period prior to the first day of calculations. For example, some have taken one-half of the precipitation for the previous month. Regardless of the method used, hydrologists recognized that the error in establishing an ARI_0 will decrease with time. For this reason, no great attention has been given to the accuracy of its determination.

However, error introduced in the ARI_0 is decayed in the manner shown in Fig. 1. If two sets of calculations began on May 1 with their ARI_0 values 0.90

apart, as the lines are shown in Fig. 1, about 40% of this difference would remain after 1 month and about 16% after 2 months when using a K value of 0.97. This demonstrates that the error in establishing an ARI_0 value can have a significant effect on the ARI values calculated for 1 to 2 months after calculations begin.

Because ARI values are comparable to soil moisture, establishing ARI_0 values is similar to estimating the soil moisture on the beginning date. A time when the soil moisture content is near field capacity may be the easiest to define—for example, a few days after a heavy rain or soon after snowmelt.

Recession Factor, K.—The recession factor, K, determines the rate of decrease with time of the ARI values when calculated by Eq. 3. The array of K^t values of Table 1 shows the variation of this factor for combinations of K and t.

TABLE 1.—VALUES OF K^t FOR SELECTED COMBINATIONS OF K AND t

t, in days (1)	Values of K							
	0.86 (2)	0.88 (3)	0.90 (4)	0.92 (5)	0.94 (6)	0.96 (7)	0.97 (8)	0.98 (9)
1	0.860	0.880	0.900	0.920	0.940	0.960	0.970	0.980
5	0.470	0.528	0.590	0.659	0.734	0.815	0.859	0.904
10	0.221	0.279	0.349	0.434	0.539	0.665	0.737	0.817
15	0.104	0.147	0.206	0.286	0.395	0.542	0.633	0.739
30	0.011	0.022	0.042	0.082	0.156	0.294	0.401	0.545
45	----	0.003	0.009	0.024	0.062	0.158	0.254	0.402
60	----	----	0.002	0.007	0.024	0.086	0.160	0.298
90	----	----	----	0.001	0.004	0.025	0.064	0.162

The K^t values can be used to predict the decay in influence of retention. The difference between the lines in Fig. 1 at any time represents the remaining influence of the 0.90 retention that occurred on May 1. At the end of each month, this difference is about 40% of that present at the end of the previous month. When t equals 30 and K equals 0.97, K^t equals 0.40, which shows that K^t defines the percentage of reduction for any time period. This would be true, regardless of other retention within the periods.

The recession of the ARI values has been compared with the loss of soil moisture, which is primarily by ET. Percolation may be significant at times, but it is usually negligible when the soil moisture content is less than field capacity, FC. Therefore, considering only ET losses, if the ARI values are to approximate soil moisture, the recession factor K must have a value that will cause a depletion rate of ARI values approximating the soil moisture depletion by ET.

Because ET varies daily and seasonally, it is reasonable to expect similar variations of K. However, adequate data are not available to define daily variations. The magnitude and trend of the seasonal changes would be much greater and probably can be defined.

Required Length of Calculations.—The period of time necessary to consider before a given date to derive reliable ARI values is governed by two factors:

(1) The magnitude and decay rate of the error in estimating an ARI_0, and (2) the decay rate of the retention. It has been shown that the decay rate of both of these depends on K^t. If a K of 0.92 were being used, only 8% of any retention or ARI_0 error would remain after 30 days. However, if a K of 0.97 were used, 40% would remain. Thus, the length of calculations required depends on the required recession factor, K.

The Evaportranspiration Model.—Soil moisture is depleted principally by ET. To be comparable, the ARI values should be depleted in a similar manner. Eq. 3, developed by rationalization, has an exponential depletion of the ARI values—that is, the amount depleted is related to the amount available. A second model was written that more closely reflects the present (1967) knowledge of soil moisture depletion by ET.

Most researchers agree that ET takes place at or near the potential evapotranspiration rate, (PET) when adequate moisture is available, and they agree that ET nearly ceases when moisture is so low that plant wilting occurs.

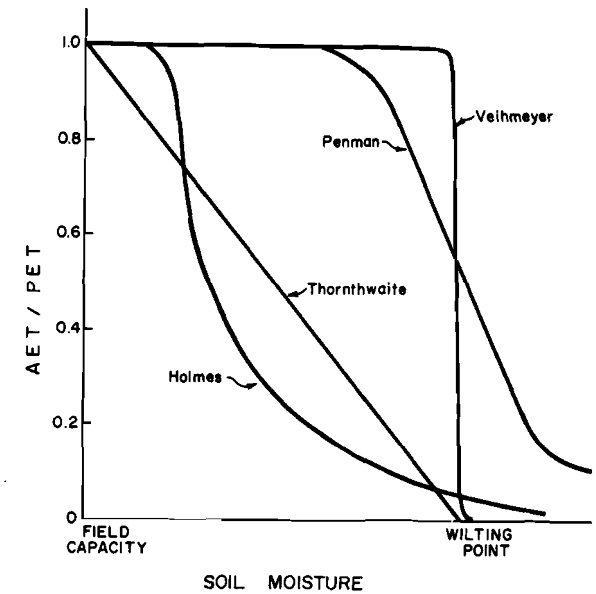

FIG. 2.—ET-SOIL MOISTURE RELATIONS

However, there are differences of opinion about the rate of actual evapotranspiration (AET) between these two points. Some of the views[3,4,5] are summarized by the curves shown in Fig. 2.

There are many factors that affect the rate of ET such as vegetation,

[3] Smith, G. W., "The Determination of Soil Moisture Under Permanent Grass Cover," Journal of Geophysical Research, Vol. 64, No. 4, 1959.

[4] Holmes, R. M., and Roberts, G. W., "Application of the Relationship Between Actual and Potential Evapotranspiration in Arid-Zone Agriculture," Transactions, American Society of Agricultural Engineers, Vol. 6, No. 1, 1963, p. 65.

[5] Veihmeyer, F. J., and Hendrickson, A. H., "Does Transpiration Decrease as the Soil Moisture Decreases?" Transactions, American Geophysical Union, Vol. 36, No. 3, 1955.

atmosphere, and the amount of soil moisture. For water to be evaporated or transpired at the soil-air or plant-air interface, both energy and water must be available. The source of energy is net radiation and advected heat, and the available water comes from storage within the soil.

For a sparsely vegetated soil, most upward movement of the soil water is by capillary action through the soil interstices. As the surface dries, suction gradients are created and water moves upward. However, the rate of this movement is slow—much slower than that required to meet the PET rate. The *ET* rate, therefore, will decline quite rapidly and may produce a relation resembling Thornthwaite's or Holmes' curves in Fig. 2.

However, a well-vegetated soil has the plant root system as an additional method of transporting water from soil storage. The root system of a developed plant extends downward and laterally, which provides an efficient system for extracting water from a large volume of soil. This supplies adequate water to the surface to maintain the *ET* rate near potential for some time. The Veihmeyer curve of Fig. 2 might represent this condition. Various combinations of soil and plant characteristics could conceivably produce relations anywhere between the Holmes and Veihmeyer curves. Most researchers currently use a relation similar to Holmes', but extend the potential rate portion until about 40% to 60% of the available soil moisture is depleted.[6]

The *ET* model written for calculating ARI values has a constant depletion rate equal to PET until 60% of the available soil moisture at field capacity remains. From this condition to the wilting point, an exponentially decreasing depletion rate was used, based on K^t—the same factor used in the exponential ARI model. Therefore, the depletion rates of this model were represented by two segments, a constant rate equal to PET and an exponentially decreasing rate. For the constant rate portion,

$$ARI_i = ARI_{i-1} + R_{i-1} - PET \quad (4)$$

and, for the exponentially decreasing rate portion, Eq. 3 applies.

Retention was used as input in this model, exactly as in the exponential model. However, the residual effect of a given amount of retention differs. The decay of a given amount of retention is shown in Fig. 3, as it was for the exponential model in Fig. 1. The dash line includes the effects of 0.90 in. of retention on August 15 which was not considered in the solid line. The differences between the lines indicate the decrease in the effect of retention. For the constant depletion part of the curve, the total retention effect is retained. Once the ARI values have been depleted sufficiently to begin using the exponential part of the model, retention is decayed according to the K^t factor, which was discussed when considering the exponential model.

It is apparent that the effect of any particular retention on subsequent ARI values depends on which method of depletion is used. There may be any combination of depletion patterns, depending on the amount and distribution of the retention. This differs from that observed when using the exponential model where it was possible to predict the retention effect, regardless of the additional retention or its distribution.

To use the *ET* model, the values needed in addition to daily retention are: (1) initial ARI value; (2) PET rates; and (3) recession factors, *K*.

[6] Shaw, R. H., "Estimation of Soil Moisture Under Corn," Research Bulletin 520, Agricultural and Home Economics Experiment Station, Iowa State University, Ames, Iowa, Dec., 1963.

These are quite similar to those required for the exponential model.

Initial ARI Value (ARI_0.)—The same consideration of ARI_0 will apply to this model as to the exponential model. However, this model will not decay an error of estimate when the depletion rate remains at the constant PET rate. If depletion is sufficient to cause the model to operate in the exponential depletion segment, the errors will decay as in the exponential model. Retention could be adequate to prevent the equation depletion from dropping to the exponential phase of the relationship for a considerable time. In this case, any error present in the ARI_0 value would be maintained in subsequent ARI values.

PET Rates.—The amount of the daily depletion for the constant portion of the model is at the PET rate. Therefore, this rate should be predictable by one of the standard equations, such as Penman's.[7]

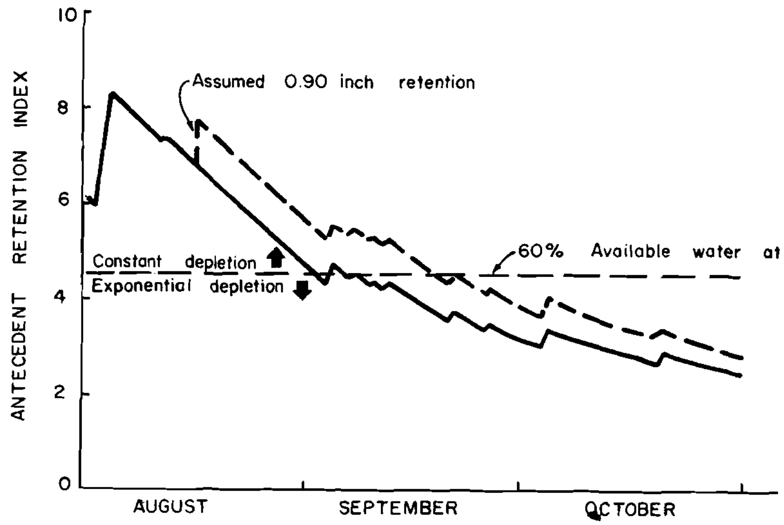

FIG. 3.—ARI VALUES BY THE *ET* MODEL

Recession Factor, *K*.—The transition from the PET depletion rate to the exponential relation should be relatively smooth; that is, the depletion by the exponential portion should be very near PET at the point of intersection. To obtain this result, a specific *K* value is needed. This *K* can be derived by knowing that the ARI value at which the transition will occur is 60% of the available water at *FC* and that *K* times this value should give a daily depletion equal to the PET rate. Each PET rate will require a different *K* value.

Required Length of Calculations.—The required period of time to carry the calculations prior to a selected date depends on the magnitude and decay of both the error of the estimated ARI_0 value and the effect of previous retention, No decay occurs when using the constant PET depletion rate, but it does occur with the exponentially decreasing depletion rate. In most cases, both types of depletion will be used, making it impossible to predict the length of calcula-

[7] Chow, V. T., "Handbook of Applied Hydrology," McGraw-Hill Book Co., Inc., New York, 1964, p. 11-26.

tions required. The calculations should begin from the point at which the best ARI_0 value can be estimated.

BASIC DATA

Soil moisture, precipitation, and runoff data were obtained from two experimental watersheds operated by the USDA Agricultural Research Service, Soil and Water Conservation Research Division. Both watersheds are in Wisconsin, one near Colby in north-central Wisconsin and the other near Fennimore in south-western Wisconsin.

The area of the Colby watershed is 345 acres; precipitation is measured by three recording rain gages, and streamflow is measured by a broad-crested V-notch weir. Land use is mixed cropping, principally hay, pasture, and small grain. The soils, Auburn and Withee, are silt loams with slow internal drainage. They have a large available moisture-holding capacity, as much as 0.30 in. of water per in. of soil at field capacity. Soil moisture measurements were taken at eight sites, under various cropping, and at 4-in. increments, to a depth of 12 in. Samples were taken with an auger and the soil moisture content determined gravimetrically. Measurements were made at 1- or 2-week intervals during the growing season for the years 1949-1955.

The Fennimore watershed area is 52 acres; it has two recording rain gages and a broad-crested V-notch weir for runoff measurements. Land use is a combination of row crops, small grain, alfalfa, and pasture. The soils, principally Tama and Dubuque, are silt loam with medium-to-good internal drainage. Soil moisture measurements were made at four to eight sites, under various crops, and at 6-in. increments to a depth of 36 in. These samples were taken with an auger and the soil moisture content determined gravimetrically. Measurements were made at weekly intervals throughout the growing season for the years 1953-1964.

Retention values were calculated by subtracting observed daily runoff from precipitation. This watershed retention represented average conditions for various soils, vegetation, and other characteristics for each watershed. To obtain a comparable soil moisture condition, the data from all of the sampling sites for each watershed were averaged. At both Colby and Fennimore, the samples were taken under various crops in approximately the same ratio as crops occurred on the watersheds.

At Colby, the measurements for the three 4-in. increments were averaged to give a single value representing the 0- to 12-in. depth over the entire watershed. At Fennimore, the samples were averaged to give both a 0- to 12-in. and a 0- to 36-in. representation.

ARI values are comparable with inches of available soil moisture; it was therefore necessary to convert the soil moisture values from percentage by weight to inches. This required establishing average bulk density, *Bd*, and wilting point, *WP*, values for the soils on each watershed. For the ARI-soil moisture comparison to remain realistic, the ARI values should not greatly exceed the field capacity (*FC*) of the soils. Therefore, an estimate of field capacity was also required. For the Colby soils, the following values were established: $Bd = 1.46$, $WP = 12\%$, and $FC = 33\%$. Those used for the Fennimore soils were: $Bd = 1.40$, $WP = 11\%$, and $FC = 26\%$. These percentages are expressed by weight.

ARI-SOIL MOISTURE COMPARISONS

To test the models, ARI values were calculated and compared with observed soil moisture. In summary, ARI values were first calculated using the exponential model, with three *K* values selected from a recommended range. The results of the ARI-soil moisture comparisons were not good. Then, *K* values were derived from the observed soil moisture data and found to differ significantly from those used in the first calculations. ARI values were again computed, using monthly averages of these derived *K* values, and were again compared with soil moisture values. The results were greatly improved.

PET values for the *ET* model were derived in much the same manner as the *K* values. ARI values, calculated by using average derived PET values,

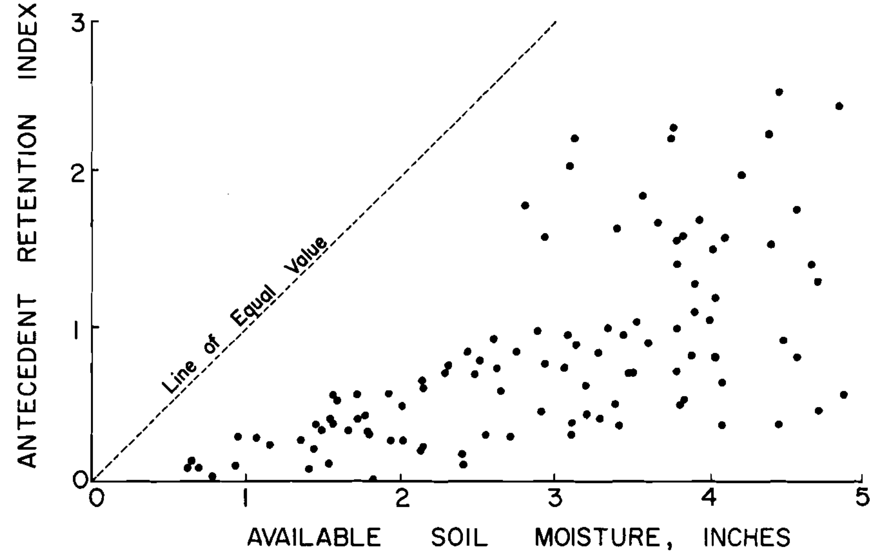

FIG. 4.—COMPARISON USING *K* OF 0.88

were compared with soil moisture—again with good results. Details of these calculations and comparisons follow:

Using Recommended *K* Factors.—Daily ARI values were calculated for the growing season for 19 yr, using the exponential model. ARI_0 values were set equal to the available water at *FC* because the calculations began each year soon after snowmelt when the soil was usually near *FC*. The recession factor *K* was taken as a constant for the entire year, and three values, 0.88, 0.90, and 0.92, were taken from the recommended range.[8,9]

Agreement of these computed ARI values with the observed available soil moisture was not very good, as shown for the Colby data in Fig. 4 where a *K* of 0.88 was used. All ARI values were far too small. It was evident that

[8] Linsley, R. K., Kohler, M. A., and Paulus, J. L. H., "Applied Hydrology," McGraw-Hill Book Co., Inc., New York, 1949.

[9] Chow, V. T., op. cit., p. 14-6.

this model did not predict actual soil moisture when constant K values of 0.88 to 0.92 were used. The most probable source of error appeared to be in the assigning of the K values.

Deriving K and PET from Soil Moisture Data.—The K and PET values required to produce ARI values comparable with soil moisture were derived between consecutive dates of soil moisture sampling. An iterative-type solution was used on an electronic computer. ARI values were assumed to be equal to the measured available soil moisture of a given date. Iterations were made to determine the K or PET value required to establish an ARI value equal to the available soil moisture on the next sampling date, considering the retention within the interval. This gave a series of K and PET values throughout the year representative of 1- or 2-week periods. These calculations were made for each of the three sets of soil moisture data for both models.

Most periods gave reliable K or PET estimates. There were some periods when obvious percolation occurred that invalidated the necessary assumption that all moisture loss was by ET. There were a few periods when the observed retention was not sufficient to account for the increase of observed soil moisture. These inadequacies were usually small and were probably the result of our inability to measure precipitation, runoff, and soil moisture to the necessary degree of accuracy. When deriving PET values, only those periods having available soil moisture near or above 60% of that of field capacity were used.

A typical set of derived recession factors, K, versus time is shown in Fig. 5. This set is for the Fennimore 0- to 12-in. soil moisture data. The large amount of scatter is probably the result of two principal effects. First, K is related to ET, which varies considerably within short time periods such as those used. Second, any inaccuracies of soil moisture measurements would be reflected in the derived K values because the ARI values were made equal to the measured available soil moisture at the beginning and end of each period. However, because the last soil moisture value of a period became the first value of the next period, any error introduced in the first period was also introduced in the second, but in an opposite sense. That is, if a soil moisture measurement error caused K to be too low in the first period, it caused K to be too high in the following period. Therefore, the K values are not entirely independent. An average of their values is probably the best estimate.

The horizontal bars in Fig. 5 indicate the monthly average values chosen to represent the derived values. The two major considerations in choosing these values were monthly arithmetic averages and the need for a smooth seasonal trend. All derived values were considerably above those usually recommended and used in the first comparisons of this study.

An example of derived PET values is shown in Fig. 6. These are from the Fennimore soil moisture data, 0- to 12-in. depth. The consideration of scatter and independence noted when describing the derived K values is also applicable to these values. The horizontal bars indicate monthly average values, again chosen by considering arithmetic averages and seasonal trend.

Summaries of the derived K and PET values for the three sets of soil moisture data considered are shown in Figs. 7 and 8. The points represent monthly arithmetic averages of the derived values. The difference between the K values of the Fennimore 0- to 12-in. soil moisture and those of Colby

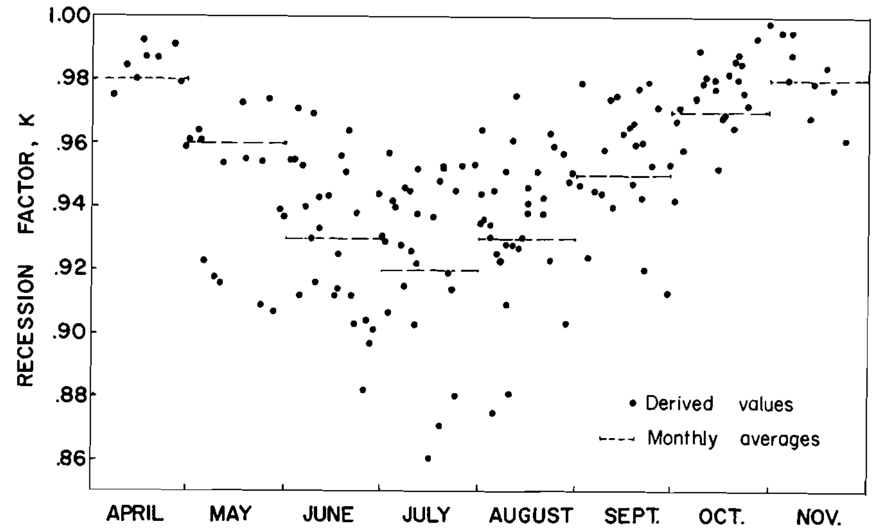

FIG. 5.—DERIVED K VALUES

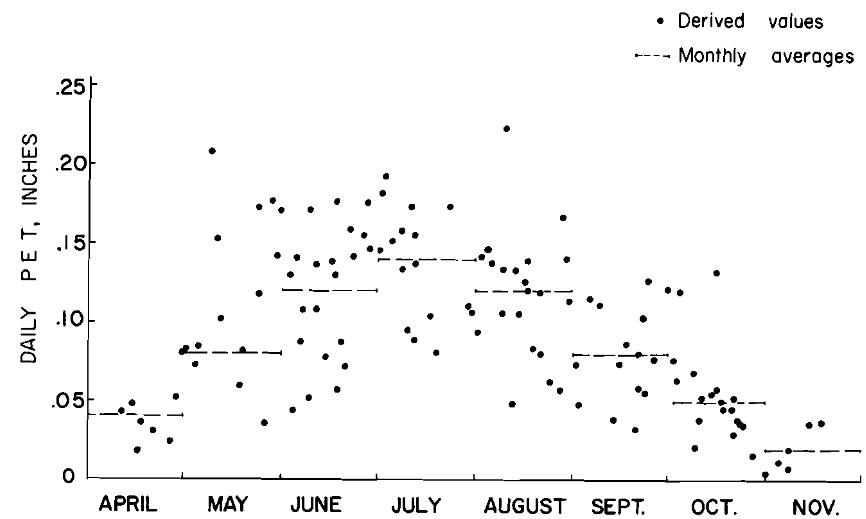

FIG. 6.—DERIVED PET VALUES

is probably a reflection of their locations (Colby is about 150 miles north of Fennimore) and the slightly higher moisture-holding capacity of the Colby soil. The PET values of Fig. 8 do not show this difference. This is probably the result of having fewer periods of the Colby data that gave reliable PET values as compared with the number usable for deriving K values.

The amount of ET coming from the 12- to 36-in. zone is indicated by the difference between the 0- to 12-in. and 0- to 36-in. Fennimore curves of Fig. 8. This increased ET results in requiring higher K values, as indicated in Fig. 7. It might be expected that K would be lower to give more depletion;

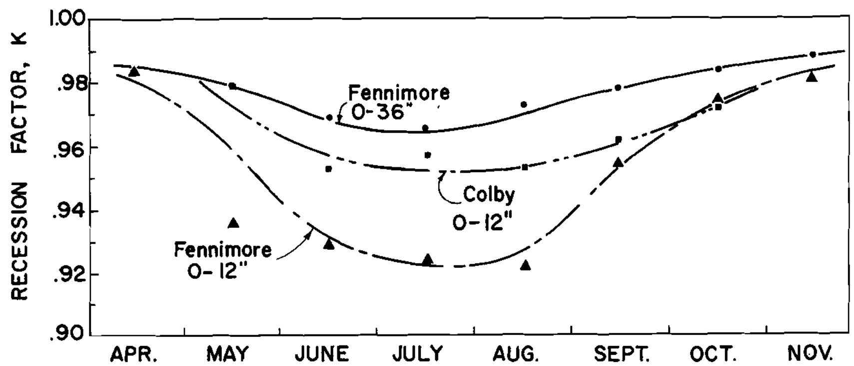

FIG. 7.—SUMMARY OF DERIVED K VALUES

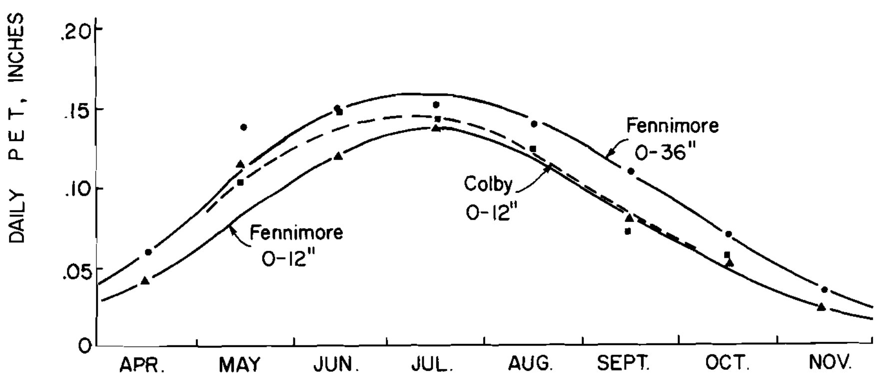

FIG. 8.—SUMMARY OF DERIVED PET VALUES

however, a higher K is necessary because it represents depletion as a percentage of available moisture. When considering greater depths of soil, the amount of available moisture increases more than the additional ET; therefore, a smaller percentage of that moisture available provides the amount to be depleted.

Use of Derived K and PET Values.—Sets of ARI values were calculated and compared with observed soil moisture, to verify that these derived average monthly K and PET values, with their respective models, produce ARI values

that relate to soil moisture. Such a comparison is not entirely independent because the K and PET values were derived from these same data; however, it does show whether the models adequately represent the depletion of soil moisture when the best average monthly parameter values are known.

Six sets of ARI values were computed, one by each model for each of the two depths considered at Fennimore and the one depth for Colby. Daily values

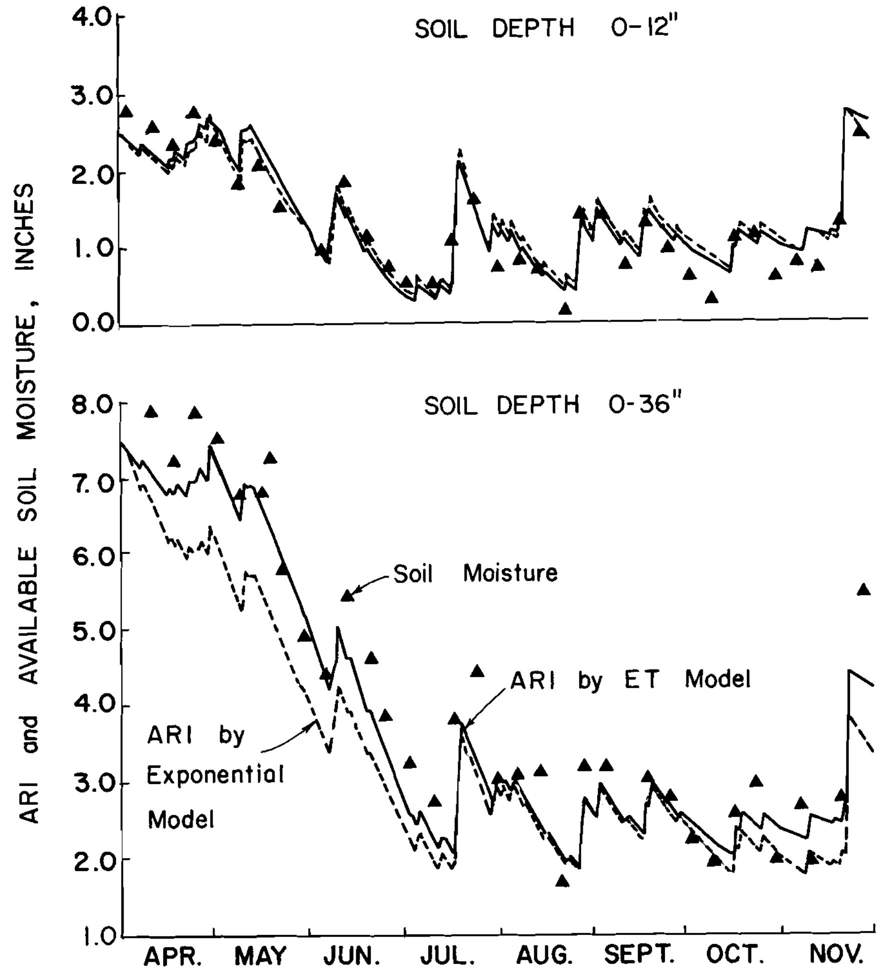

FIG. 9.—ARI AND SOIL MOISTURE VALUES

were calculated for the growing season for each year soil moisture data were available. The amount of available soil moisture at field capacity was used as the ARI_0 value.

There are times when sufficient retention occurs to raise the ARI values higher than field capacity, probably indicating periods of percolation. To

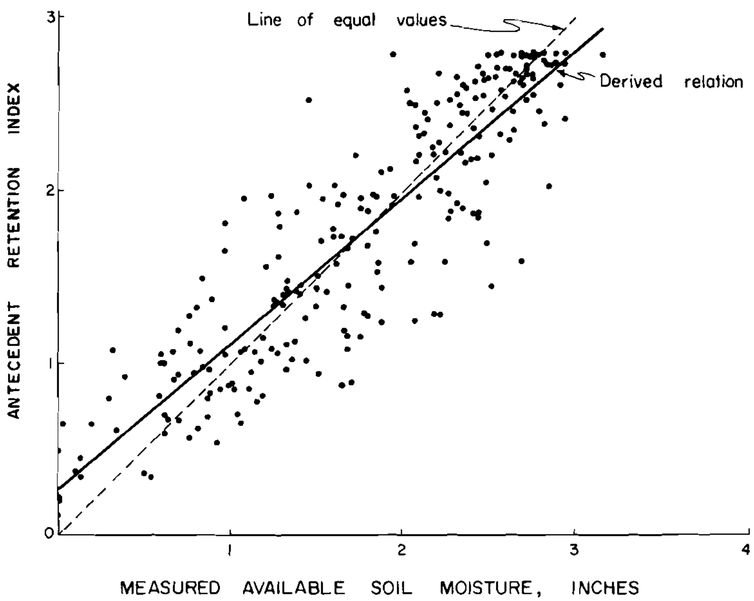

FIG. 10.—EXPONENTIAL MODEL EVALUATION

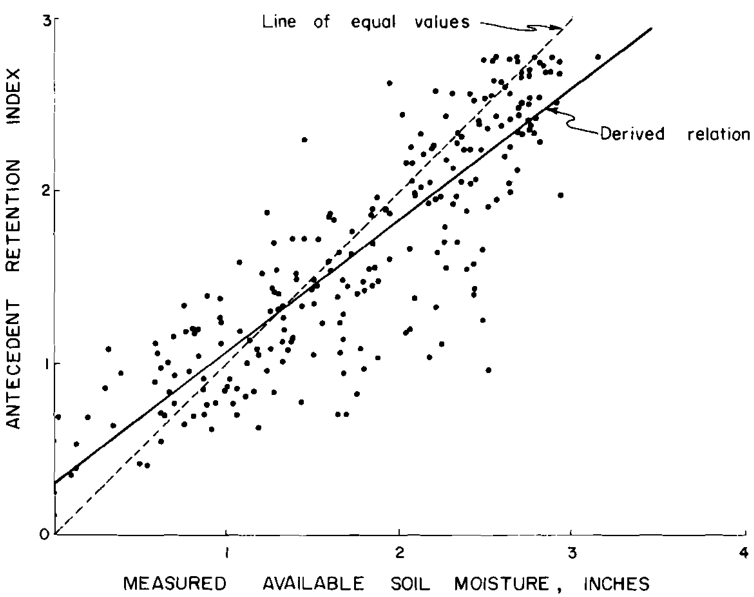

FIG. 11.—*ET* MODEL EVALUATION

maintain the ARI values in a realistic range, an upper limit was established based on the available soil moisture at field capacity. For the Fennimore soils, which have good internal drainage, this limit was placed at 1.1 times the amount of available soil moisture at field capacity; for the Colby soils, which have poorer internal drainage, a factor of 1.2 was used.

The calculated ARI values were compared with the observed available soil moisture values in two ways. The first was a visual comparison, as shown in Fig. 9. The daily ARI values calculated by both models and the measured available soil moisture values were plotted versus time. Nearly all of the plots for other years and watersheds were quite similar to those shown.

The second method of comparison was scatter plottings of ARI values versus observed soil moisture, as shown in Figs. 10 and 11 for the 0- to 12-in. Fennimore data. A line-of-best-fit, standard error of estimate and a co-

TABLE 2.—RESULTS OF COMPARING ARI WITH OBSERVED AVAILABLE SOIL MOISTURE

Data Used[a]	Coefficient of Determination, R^2 (1)	Standard Error of Estimate $S_{y.x}$ (2)	Computed Relation[b] (3)
(a) Exponential Model			
Colby 0-12 in.	0.70	0.60	y = 0.37 + 0.87 x
Fennimore 0-12 in.	0.75	0.35	y = 0.31 + 0.76 x
Fennimore 0-36 in.	0.69	0.97	y = 0.72 + 0.75 x
(b) Evapotranspiration Model			
Colby 0-12 in.	0.49	1.00	y = -0.51 + 0.85 x
Fennimore 0-12 in.	0.81	0.32	y = 0.26 + 0.85 x
Fennimore 0-36 in.	0.78	0.88	y = 1.22 + 0.85 x

[a] The Colby data had 112 dates of soil moisture for comparison; the Fennimore data had 245.
[b] x = observed soil moisture; y = calculated ARI values.

efficient of determination were calculated for each set. A summary of these values for all six comparisons is given in Table 2. Fig. 10 compared with Fig. 4 is indicative of the improvement obtained by using the seasonly varied, derived K values. The R^2 values indicate a reasonable degree of correlation between the calculated ARI and observed available soil moisture values. A high degree of correlation would not be expected because of the inherent errors in measuring the available soil moisture. The ARI values were slightly higher than the available soil moisture values in the lower range and lower in the higher range.

The Colby comparisons were probably not so good as the Fennimore comparisons because of fewer soil moisture data. The slow internal drainage and large moisture-holding capacity of the Colby soils may also have had some effect.

These comparisons show that either model will yield ARI values that are reasonably related to available soil moisture, but only if proper K or PET values are used. These results support the use of either model. Although both models gave reasonable comparisons, the ET model is on a sounder basis and will lend itself more easily to further refinement. However, the exponential model is easier to use and will probably remain popular with hydrologists.

Predicting K and PET.—To allow adoption of these models at other locations, K and PET must be predictable. Because both parameters depend on factors that determine ET, it is expected that different values will be required for other locations and conditions.

Several techniques have been developed for predicting PET values. Two of these, the PE-Index and the Blaney-Criddle, were used to calculate PET values for the Fennimore, and Colby, Wisconsin areas.[10] Table 3 provides a comparison of these calculated values with those that were derived from the

TABLE 3.—COMPARISON OF DERIVED AND CALCULATED AVERAGE DAILY PET, IN INCHES

Method	April (1)	May (2)	June (3)	July (4)	Aug. (5)	Sept. (6)	Oct. (7)	Nov. (8)
(a) Fennimore								
PE-Index	0.07	0.11	0.14	0.17	0.15	0.10	0.06	0.03
Blaney-Criddle	0.11	0.15	0.18	0.20	0.18	0.14	0.10	0.06
Soil Moisture (0-36 in.)	0.06	0.11	0.14	0.16	0.14	0.11	0.07	0.03
(b) Colby								
PE-Index	—	0.09	0.13	0.15	0.14	0.09	0.05	—
Blaney-Criddle	—	0.15	0.18	0.19	0.18	0.13	0.10	—
Soil Moisture (0-12 in.)	—	0.10	0.13	0.14	0.12	0.07	0.06	—

soil moisture data (Fig. 8). There is reasonably close agreement, particularly with the PE-Index method, indicating that PET values for the ET model can be estimated.

Prediction guidelines can also be obtained by relating the derived K and PET values to U. S. Weather Bureau pan evaporation data. The relation for the derived values of this study are shown in Figs. 12 and 13. The K and PET values used were selected from plottings of derived values such as those shown in Figs. 5 and 6. The evaporation pan data for Colby was obtained from the Marshfield Experiment Station about 25 miles away. An average of data from two pans, both 50 to 75 miles away, was used with the Fennimore data. The K values of Fig. 12 are for the exponential model. Those required in the ET model are related to the PET and soil moisture values, as previously stated.

[10] Ibid., p. 11-26.

When using these relations as guides for selecting values, it must be remembered that they were derived for areas in Wisconsin, of several acres, and with mixed land use. The Denmead[11] curve shown in Fig. 13 indicates the variation that might be expected when a single crop, such as corn, is used. These relations should be valid throughout much of the Midwest and other

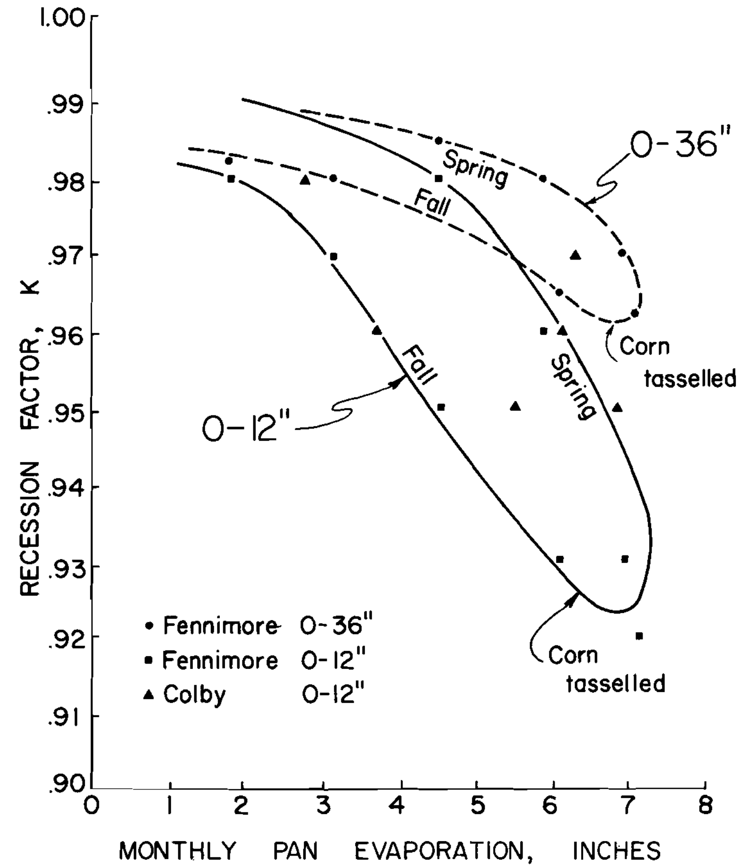

FIG. 12.—PREDICTION GUIDE FOR K VALUES

areas where the climate, soils, and vegetation are somewhat similar to the areas represented by the data of this study.

SUPPLEMENTARY ANALYSES

Antecedent conditions for rainfall-runoff relations of a particular day are estimated by some hydrologists from the amount of precipitation for the pre-

[11] Denmead, D. T., and Shaw, R. H., "Evapotranspiration in Relation to the Development of the Corn Crop," Agronomy Journal, Vol. 51, 1959, pp. 725-726.

ceding 5 days. To test the validity of such a technique, the calculated ARI values on the 214 days soil moisture was sampled at the Fennimore station were correlated with the precipitation that occurred during the preceding 5 days. The resulting R^2 was 0.12, indicating that no relation existed. This is to be expected, because the models of this study show that precipitation a month or more before a given date has a significant effect on the soil moisture of that date.

Trials were made with the exponential model using a constant K throughout the season and applying a seasonal correction, by graphical correlation, to the resulting ARI values. The results were not nearly so good as when using a seasonally varied K. The changes that occur by correcting the calculated ARI values are not the same as the changes that occur when using varying K values, because the relationship between ARI and K is not linear.

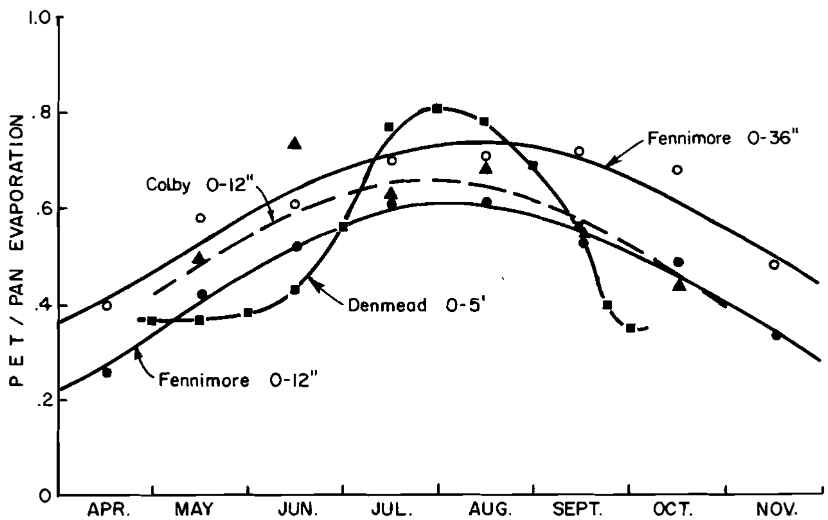

FIG. 13.—PREDICTION GUIDE FOR PET VALUES

CONCLUSIONS

The major conclusions of this study are:

1. There is a rational basis for using antecedent retention indexes ARI to describe soil moisture conditions.
2. Two models for calculating ARI values, exponential and evapotranspiration, were evaluated. The ET model has somewhat greater theoretical justification but is slightly more difficult to use.
3. Accuracy in selecting the initial antecedent retention index ARI_0 is of considerable importance to later values.
4. The factor K of the exponential model, which defines the rate at which depletion takes place, varies with season and the soil depth represented. Derived values varied seasonally from 0.92 to 0.98 when representing the 0- to 12-in soil depth and 0.96 to 0.98 when representing the 0- to 36-in. soil depth.
5. Either ARI model, exponential or ET, will produce ARI values that are reasonably related to available soil moisture, provided the proper K or PET values are used.
6. K and PET values derived from soil moisture data were related to pan evaporation measurements to facilitate their estimation at other locations.
7. There is no relation between soil moisture of a selected date and precipitation during the preceding 5 days. Precipitation during the previous 1 or 2 months largely determines the soil moisture content.

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APPENDIX.—NOTATION

The following symbols are used in this paper:

AET = actual evapotranspiration, in inches per day;
API = antecedent precipitation index;
ARI = antecedent retention index;
ARI_0 = beginning ARI value;
B_d = bulk density, in grams per cubic centimeter;
ET = evapotranspiration, in inches per day;
FC = field capacity, in percentage;
i = designates a selected day;
K = the recession factor in the ARI equation;
P = precipitation, in inches;
PET = potential evapotranspiration, in inches per day;
Q = runoff, in inches;
R = retention, in inches;
t = time, in days; and
WP = wilting point, in percentage.

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KEY WORDS: antecedent indexes, evapotranspiration, hydraulics, hydrology, infiltration, rainfall; runoff; soil moisture

ABSTRACT: Two models for calculating antecedent moisture index values are evaluated by comparisons with measured soil moisture. One model, based on the traditional antecedent precipitation index equation, depletes the index values exponentially. The second model depletes the index values by an evapotranspiration-soil moisture relation. Index values computed by the traditional exponential model with recommended K values of 0.88 and 0.92 compare poorly with measured soil moisture. Derived K values show a distinct seasonal trend and range from 0.98 in the spring and fall to 0.92 in the summer when representing the 0- to 12-in. soil depth, and 0.98 to 0.96 when representing the 0- to 36-in. soil depth. Index values by both models compare favorably with available soil moisture when derived K and PET values are used. The K and PET values are related to pan evaporation to facilitate their prediction at other locations.

REFERENCE: Saxton, Keith E., and Lenz, Arno, T., "Antecedent Retention Indexes Predict Soil Moisture," Journal of the Hydraulics Division, ASCE, Vol. 93, No. HY4, Proc. Paper 5351, July, 1967, pp. 223-241.