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Efficient Allocations of Irrigation Water and Nitrogen Fertilizer in Corn Production

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ABSTRACT. N-fertilizer and irrigation water are major inputs to corn production and efficient use of these inputs is essential for profit maximization and resource conservation. To use these inputs efficiently, knowledge about plant responses to N-fertilizer and irrigation water, or production functions, is essential. Corn production functions were estimated using the data from experimental plots in Florence, South Carolina, U.S.A., from 1999 through 2001. There were three irrigation treatments and four N-fertilizer regimes. Several forms of production functions were fitted to the data and the quadratic form of the production function was found to have the best fit for the data. The estimated production functions were then used to determine the optimal levels of water and N-fertilizer applications under both yield-maximizing and profit-maximizing strategies. Results indicate that the yield-maximizing strategy called for more water and N-fertilizer and yielded smaller net returns than the profit-maximizing strategy. In 1999, for example, under the current average prices of corn, water, and N-fertilizer, the yield-maximizing strategy required 667 ha-mm of water and 224 kg of N-fertilizer to produce 10.4 Mg/ha of corn

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and \$5.42 of net returns; whereas the profit-maximizing strategy required only 556 ha-mm of water and 174 kg of N-fertilizer to produce 9.87 Mg of corn and \$57.38 of net returns. The least-cost combinations of water and N-fertilizer application levels for a given output were also determined. The results provide useful information to farmers to make N-fertilizer and irrigation decisions for profit maximization and for resource conservation. *Article copies available for a fee from The Haworth Document Delivery Service: 1-800-HAWORTH. E-mail address: <docdelivery@haworthpress.com> Website: <http://www.HaworthPress.com>*

KEYWORDS. Water conservation, production functions, site-specific irrigation, profit-maximization, yield-maximization, N-fertilizer

INTRODUCTION

Water is used in many competing industries. Since its supply is limited in most irrigation projects, farmers must compete for water on the basis of crop productivity and economic efficiency. Interacting with water, N-fertilizer can increase crop yields and thus improve water use efficiency. To use water and N-fertilizer efficiently in crop production, knowledge about plant responses to N-fertilizer and irrigation water, or production functions, is essential. The production function can be used to determine the levels of irrigation water and N-fertilizer application that maximize profits. It can also be used to determine the least-cost combinations of these inputs for any given output level.

Farmers, agricultural extension agents, and some agricultural scientists attempt to maximize crop yields per ha, believing that maximizing yields will result in maximum profits. But maximum yields are seldom consistent with profit maximization and economic efficiency in resource allocations. Several studies reported that water and N-fertilizer application rates required to maximize crop yields were not the same as the application rates required to maximize profits (Kloster and Whittlesey, 1971; Hallett, 1967; Kloster and Whittlesey, 1971; Lu, et al., 2002).

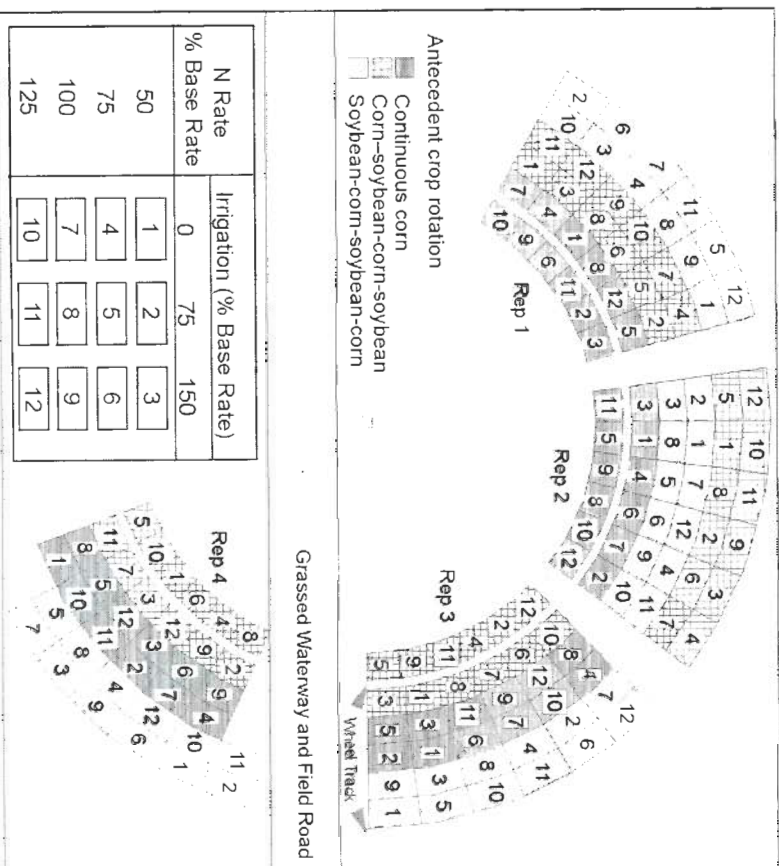
In this paper, we estimated corn production functions using data from experimental plots in Florence, South Carolina, U.S.A., from 1999 through 2001 and determined the optimal levels of water and N-fertilizer applications under yield-maximizing and profit-maximizing strategies. Information derived from this study can be used by agricultural extension agents in forming water and N-fertilizer use recommendations to help farmers improve N-fertilizer and water use efficiencies and to improve profitability.

MATERIALS AND METHODS

Source of Data

The experimental design was a split randomized complete block with irrigation water and N-fertilizer application treatment combinations in completely randomized blocks. There were three irrigation treatments (0%, 75%, and 150% of the base rate) and four N-fertilizer regimes (50%, 75%, 100%, and 125% of the base rate), three antecedent crops (corn-corn, corn-soybean, and soybean-corn) during the preceding four years, and four replicates, for a total of 144 plots as shown in Figure 1. Corn was grown under conservation tillage from 1999 through 2001 on a 6-ha site of relatively uniform Norfolk loamy sand near Florence, South Carolina.

FIGURE 1



Variable rate water applications were made using combinations of three sets of nozzles, each delivering a different rate. N-fertilizer was applied via fertigation by varying the application depth of water with a constant nutrient concentration. The center pivot irrigation system had been modified to permit variable applications to individual areas 9.1m by 9.1m in size. A more detailed description of the water delivery system may be found in Omary et al. (1997) and of the control system in Camp et al. (1998).

Soil water potential (SWP) was measured using tensiometers at two depths (30 and 60 cm) and multiple locations within the experimental site. Measurements were recorded three times each week and data were used to determine irrigation initiation and to monitor SWP. Irrigation was initiated when SWP value at 30-cm depth in the hypothetical irrigation base rate (IBR) reached -25 kPa. The IBR was intended to provide adequate water for crop growth and development. It varied during the growing season (4–13 mm/application) depending upon crop growth stage and weather conditions.

A 6.1-m length of two rows near the center of each plot was harvested during the periods 8–10 September 1999, 29 September–2 October 2000, and 20 September 2001, using an Almaco plot combine with corn header (Almaco, Nevada, Iowa). Corn grain yields were determined by weighing the harvested grain and correcting to 15.5% moisture. A detailed description of this experiment is described in Camp et al. (2002).

Returns over variable costs, defined as total returns minus variable costs (hereafter refer to as net returns), are used to measure profitability. The cost data were obtained from the enterprise budget of the Clemson Extension Service, Clemson University (2002). The variable costs include costs of seeds, fertilizers, lime, herbicides, insecticides, irrigation, drying and hauling, operation of tractors and machinery, labor, and interest on operating capital. The irrigation cost was estimated at \$4/acre-inch, or 39 cents/ha-mm. The price of corn was obtained from USDA Agricultural Statistics (2002). In the last 20 years, the price of corn has fluctuated from \$1.50/bushel (\$59/Mg) in 1986 to \$3.24/bushel (\$128/Mg) in 1995. The USDA Economic Research Service (2002) estimated the 2001–2002 season average at \$1.85 to \$1.95/bushel (\$73–77/Mg). The Farm Security and Rural Investment Act of 2002 will raise the corn support price from the current \$1.89/bushel to \$1.98/bushel (\$78/Mg) this year and next (University of Illinois, 2002).

The price of anhydrous ammonia, the most common source of nitrogen fertilizer used in corn production, is derived from natural gas. The price of natural gas increased rapidly over the last ten years. As a result, N-fertilizer prices soared. The prices of anhydrous ammonia were about 22–25 cents/lb (49–55 cents/kg) in January 2001 as compared with 12–13 cents/lb (26–29 cents/kg) in January 2000 (Thiessen, 2002). In this study, the recent average prices of \$80/Mg for corn, 40 cents/ha-mm for water, and 60 cents/kg for N-fertilizer were

used in this analysis. Since the prices of water, N-fertilizer, and corn vary considerably from year to year, they were changed in the sensitivity analysis to see how changes in relative prices will affect the optimal levels of irrigation water, N-fertilizer, and net returns.

Profit Maximizing Levels of N-Fertilizer and Irrigation Water Applications

Given a production function

$$Y = f(W, N / W) \quad (1)$$

where Y = yield

W = the amount of water,

N = the amount of N-fertilizer,

W = all other inputs.

The profit (π) is defined as total revenue minus total cost:

$$\pi = pY - C \quad (2)$$

where p is the price of corn, and C is the total cost and is given as

$$C = r_w W + r_n N + b \quad (3)$$

where r_w and r_n are prices of water and N-fertilizer, respectively, and b is the fixed cost.

Substituting equations (1) and (3) into equation (2) yields

$$\pi = pf(W, N) - (r_w W + r_n N + b) \quad (4)$$

The first-order condition for profit maximization requires that the partial derivatives of π with respect to W and N be equal to zero. That is,

$$\begin{aligned} \frac{\partial \pi}{\partial W} &= pf_w - r_w = 0 \\ \frac{\partial \pi}{\partial N} &= pf_n - r_n = 0 \end{aligned}$$

or

$$pf_w = r_w$$

$$pf_n = r_n$$

where pf_i is the value of marginal product (VMP) for the i th input. It is the rate at which the revenue will increase with further application of the i th input. As long as the VMP is greater than the price of that input, it is profitable to apply more input. The maximum profit is reached when the VMP is equal to the price of the input. The second-order condition requires that the principal minors alternate in sign, starting with negative.

The Least-Cost Combinations of Water and N-Fertilizer

To some extent, farmers can achieve a certain level of output with different combinations of water and N-fertilizer. The locus of input combinations that produce a certain level of output (Y_0) is called an isoquant. To obtain the least-cost combinations of water and N-fertilizer to attain a given level of output, we minimize the total cost function (3) subject to the constraint that output (Y) is fixed at a given level Y_0 . Thus, we formulate the Lagrangian function

$$L(W, N) = r_w W + r_n N + b + \lambda(Y_0 - f(W, N))$$

where λ is the Lagrangian multiplier. The first-order condition for minimizing cost requires the following conditions be satisfied:

$$L_\lambda = Y_0 - f(W, N) = 0$$

$$L_{r_w} = r_w - \lambda f'_w = 0$$

$$L_{r_n} = r_n - \lambda f'_n = 0$$

From the last two equations, we obtain

$$\frac{r_w}{f'_w} = \frac{r_n}{f'_n} = \lambda \quad (5)$$

Equation (5) means that the minimum cost combinations of inputs require that the ratio of input price to the marginal product (MP) for each input must be equal.

Equation (5) can be also rewritten as

$$\frac{r_n}{r_w} = \frac{f'_n}{f'_w} \quad (6)$$

where the ratio r_n/r_w is the negative of the slope of an isocost function, defined as the locus of input combinations that may be purchased for a given total cost. The

ratio f'_n/f'_w is the negative of the slope of an isoquant and is a measure of the marginal rate of substitution of N for W. Condition (6) means that, at the point of least-cost combination of W and N, the isocost function is tangent to the isoquant.

Estimation of Production Functions

Before estimating a production function, we need to specify the algebraic form of the production function and determine which variables must be aggregated and how might such aggregation be carried out (Heady and Dillon, 1961). Many algebraic functional forms have been used as production functions (Griffin, 1984). Many studies indicated that the quadratic function is most appropriate for crop production functions (Barrett and Skogerboe, 1978; Hexem and Heady, 1978; Musick et al., 1976; Stewart et al., 1973; Watkins et al., 1998). In this study, several forms of production functions, including quadratic, squared root, and double-log polynomial functions, were estimated with ordinary least squares, and the results confirmed that the quadratic equation was the most appropriate for the particular set of data used in this study.

The following form of production function was estimated using ordinary least squares:

$$Y = \alpha + \beta W + \gamma W^2 + \chi N + \delta N^2 + \epsilon W N \quad (7)$$

where α , β , χ , δ , and ϵ are coefficients to be estimated.

As indicated earlier, three antecedent crops (corn-corn, corn-soybean, and soy-bean-corn) were planted during the preceding four years of experiment. To account for variation due to the three antecedent crops, dummy variables were included for the antecedent crops in the regression. The results showed that the coefficients for the dummy variables were not statistically significant, and thus they were excluded in the final estimation.

The results for the estimated production function for the three years are

$$1999: Y = -1643 + 0.02533W^{**} + 0.02787N^{**} \\ - 0.00001978W^2 - 0.000007390N^2^{**}$$

$$+ 0.000009605WN; \quad R^2 = 0.5717, n = 14 \\ 2000: Y = -10615^{**} + 0.05471W^{**} + 0.1914N \\ - 0.000006006W^2 - 0.0001181N^2 - 0.000006006WN^{**}$$

$$+ 0.00009527WN^{***}; \quad R^2 = 0.7093, n = 13 \\ 2001: Y = 9018 - 0.01649W + 0.03495N^{**} \\ + 0.00001477W^2 - 0.000003417N^2 \\ + 0.000006157WN; \quad R^2 = 0.5701, n = 96$$

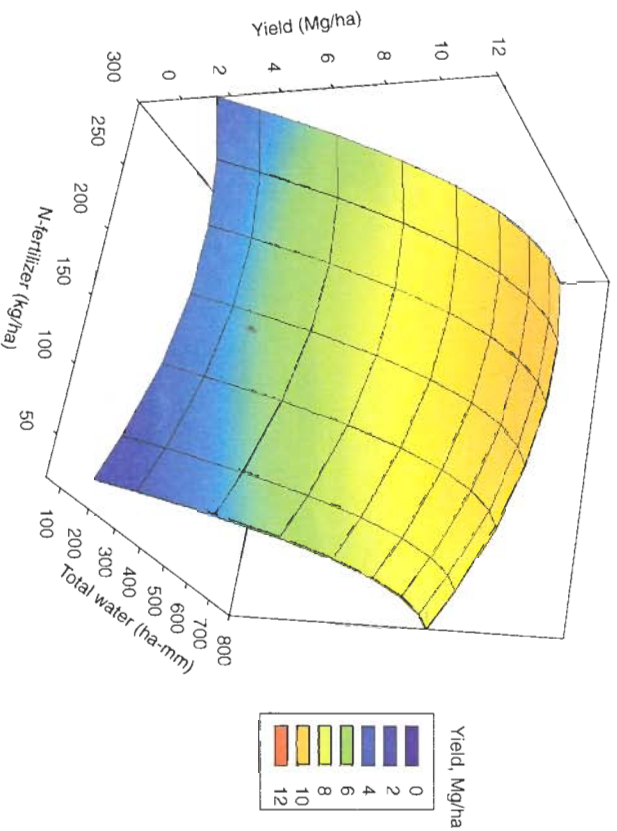


FIGURE 2

FIGURE 3. N-fertilizer Response Curve, 1999, for the optimal water application rate of 557 ha-mm

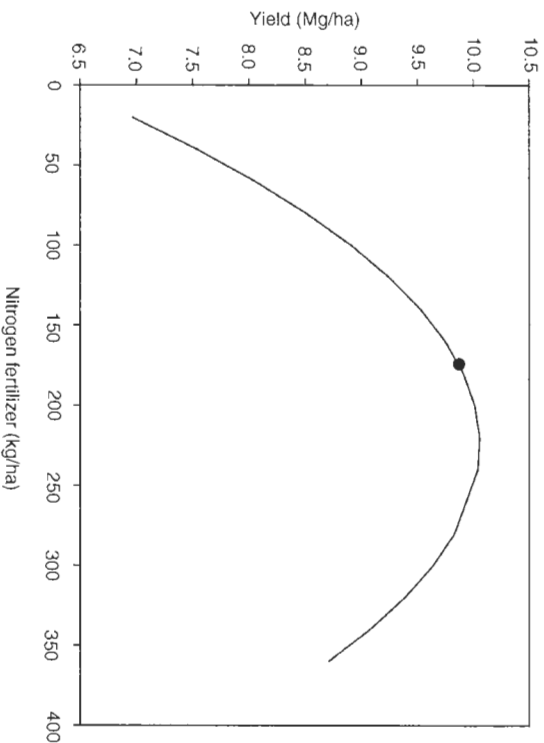
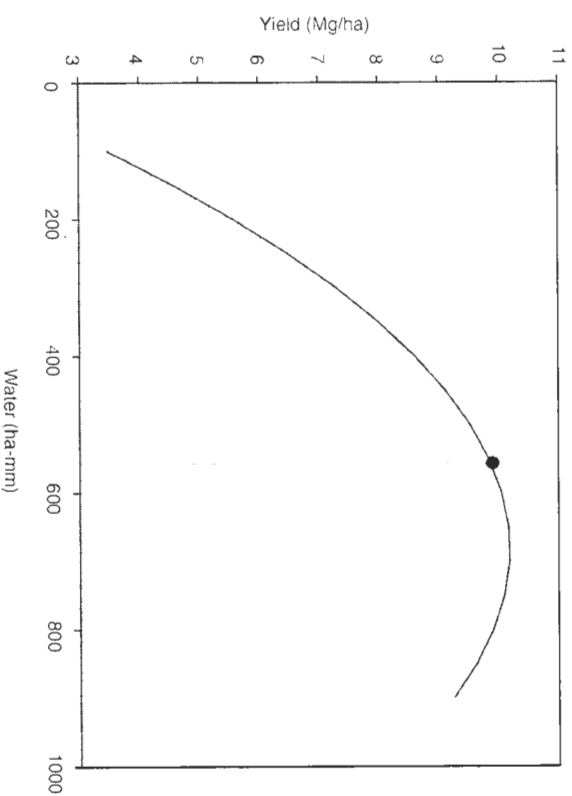


FIGURE 4. Water Response Curve, 1999, for the optimal level of N-fertilizer of 174 kg/ha



prices of water and N-fertilizer are held at 30 cents/ha-mm and 40 cents/kg, respectively, increases of corn prices from \$70 to \$80 and \$90/Mg will increase water application levels from 577 to 592 and 604 ha-mm, N-fertilizer application levels from 187 to 193 and 198 kg, and net returns from \$49 to \$149 and \$250/ha.

Increases in the price of water will result in reduced water and N-fertilizer applications and smaller net returns. By the same token, increases in the price of N-fertilizer will also result in reduced water and N-fertilizer applications and smaller net returns.

The Isoquant and Input Substitution

Equation 6 contains a family of isoquants. Each isoquant represents a given level of yield that can be achieved with a different combination of inputs. Figure 5 shows the isoquant for the yield of 9.87 Mg/ha. When the price of water is 40 cents/ha-mm and the price of N-fertilizer is 60 cents/kg, the least-cost combination of water and N-fertilizer are 557 ha-mm and 174 kg/ha, respectively, as shown at point a in Figure 5. Changes in the relative prices of these two inputs will change the least-cost combination. For example, if the price of water increases to 50 cents/ha-mm and the price of N-fertilizer decreases to 40

TABLE 2. Optimal levels of water and N-fertilizer and net returns under different combinations of prices, 1999

Price of Corn	Price of Water	Price of N-fertilizer at								
		40 cents/kg		60 cents/kg		80 cents/kg				
		Water	Nel	Water	Nel	Water	Nel			
S/Mg	cents/mm-ha	ha-mm	kg/ha	ha-mm	kg/ha	ha-mm	kg/ha			
			\$/ha		\$/ha		\$/ha			
70	30	577.47	187.43	108.81	572.70	167.79	73.94	567.93	148.15	43.23
70	40	540.78	185.05	53.70	536.01	165.41	19.34	531.24	145.76	-10.87
70	50	504.09	182.66	2.47	499.32	163.02	-31.38	494.55	143.38	-61.08
80	30	592.42	193.24	209.48	588.24	176.05	173.05	584.07	158.86	140.24
80	40	560.31	191.15	152.46	556.14	173.96	116.47	551.97	156.78	84.09
80	50	528.21	189.06	98.81	524.04	171.88	63.26	519.86	154.69	31.32
90	30	604.05	197.75	311.02	600.34	182.47	273.39	596.63	167.20	238.96
90	40	575.51	195.90	252.53	571.80	180.62	215.29	568.09	165.34	181.25
90	50	546.97	194.04	197.02	543.26	178.76	160.17	539.55	163.49	126.51

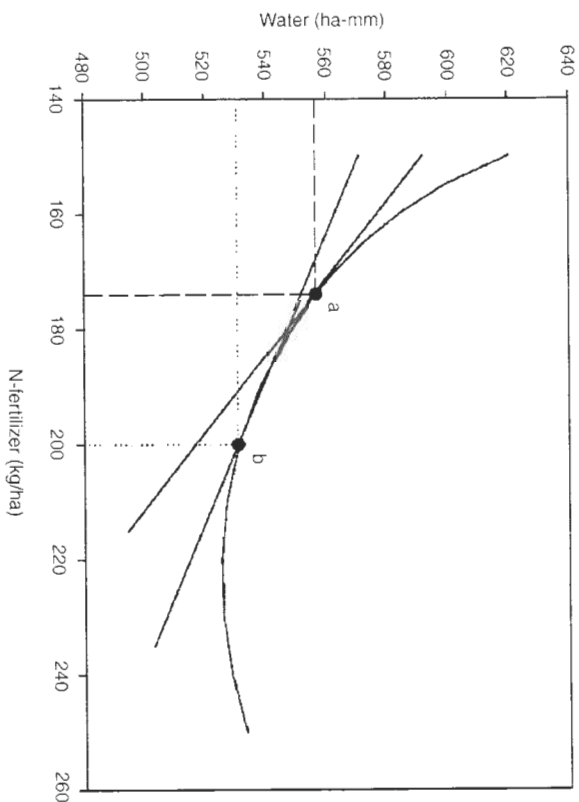
cents/kg, the cheaper N-fertilizer will be substituted for more expensive water and the least-cost combination of water and N-fertilizer will move from point a to point b. The new input combination is 531 ha-mm of water and 200 kg of N-fertilizer.

SUMMARY AND CONCLUSIONS

To use irrigation water and N-fertilizer efficiently in corn production, knowledge about plant responses to irrigation water and N-fertilizer, or production functions, is essential. In this paper, corn production functions were estimated using the data from experimental plots in Florence, South Carolina, U.S.A., for 1999 and 2001. There were three irrigation treatments and four N-fertilizer regimes in the experiment.

Most commonly used forms of production functions were evaluated and the quadratic equation was found to be most appropriate for the data used in this study. The optimal levels of water and N-fertilizer applications were determined under the yield-maximizing strategy often used by farmers and the profit-maximizing strategy used by economists. The results showed that the yield-maximizing strategy called for more water and N-fertilizer applications and produced larger yields than those used in the profit-maximization strategy, but yielded smaller net returns than the profit-maximization strategy. In 1999, for example, under the current average prices of \$80/Mg for corn, 40

FIGURE 5. Isoquant for corn, 1999



cents/ha-mm for water, and 60 cents/kg for N-fertilizer, the yield-maximizing strategy applied 667 ha-mm of water and 224 kg of N-fertilizer to produce 10.4 Mg/ha of corn and \$5.42 of net returns; whereas the profit-maximizing strategy required only 556 ha-mm of water and 174 kg of N-fertilizer to produce 9.87 Mg of corn and \$57.38 of net returns.

The results obtained using profit-maximization strategy depend on the relative prices of corn, water, and N-fertilizer. To see the effects of changing relative prices on the optimal levels of water, N-fertilizer, and net returns, different price combinations of corn, water, and N-fertilizer were used. Other things being equal, increases in corn price will cause more water and N-fertilizer to be applied, resulting in larger net income. Increases in the price of water will result in reduced water and N-fertilizer applications and smaller net returns. By the same token, increases in the price of N-fertilizer will also result in reduced water and N-fertilizer applications and smaller net returns.

Within the limited range of water and N-fertilizer used in the experiment, one input can be substituted for the other to attain a given level of output. The least-cost combinations of N-fertilizer and water applications that produce a given output also were determined. The results of this study provide useful information to farmers to make N-fertilizer and irrigation decisions for profit maximization and for resource conservation.

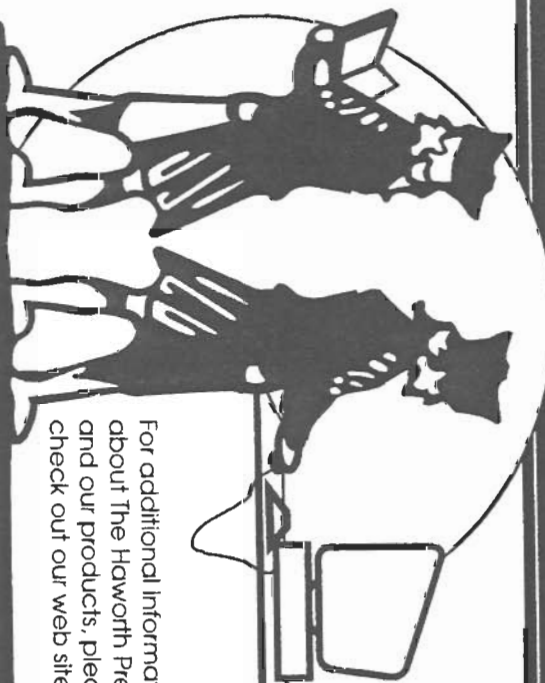
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