# Drop Size Distributions for Irrigation Spray Nozzles

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## ABSTRACT

DROP size distributions for irrigation spray nozzles, such as may be used in low or reduced pressure sprinkler systems, were measured with a calibrated stain technique. Similar data from other sources, measured with photographic or pellet techniques, were also obtained. The distributions were fitted with the upper limit log normal (ULLN) distribution function. ULLN parameters for each distribution are tabulated. Distribution characteristics such as the volume median drop size may be calculated directly from the ULLN parameters.

A simple regression model for predicting ULLN parameters as functions of nozzle style, size and pressure is proposed and fitted to data for flooding and smooth flat plate spray nozzles. The fit of model to data was evaluated by comparing measured and predicted values for 50th (median) and 99th (volume) percentile drop sizes, and by directly comparing measured and predicted distribution functions. The distance between functions was defined analogous to the Euclidean distance between points in space, leading to definition of a pseudo $r^2$ for the (functional) regression model. The fit between data and model for the two nozzle types was quite good. The models were used to explore the influence of nozzle size and pressure on drop size distributions for the two types of nozzles.

## INTRODUCTION

Many low and reduced pressure sprinkler systems use irrigation spray nozzles to distribute water. The sizes of the water drops from irrigation spray nozzles bear on important areas of irrigation study, including the extent of wind drift and evaporation losses during irrigation, distortion of the spray patterns by the wind, and reduction of the soil's infiltration rate due to drop impact on the soil surface. Few researchers have published data sets on irrigation nozzle drop size distributions, and none have quantitatively expressed the relationship between nozzle size, operating pressure and distribution characteristics.

The objectives of this work are: to review the literature regarding drop size distributions from irrigation spray nozzles, and to summarize this information in a readily useable form; to present further measured drop size distributions; and to develop and evaluate a simple model for predicting drop size distributions, given nozzle style, size and operating pressure. Special values of interest, such as maximum or median drop sizes can be calculated from the predicted distributions.

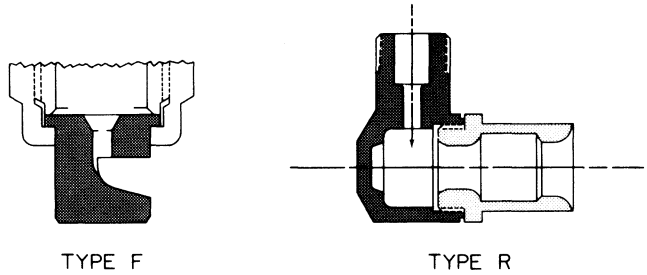


**Fig. 1—Flooding (Type F) and drift reduction (Type R) style irrigation spray nozzles (adapted from Tate, 1977).**

## IRRIGATION SPRAY NOZZLES

Agricultural spray nozzles, such as those commonly used for the application of pesticides or other chemicals, are sometimes used in irrigation systems. Two such irrigation spray nozzles are shown in Fig. 1, adapted from Tate (1977). The Type F nozzle is commonly referred to as a flooding nozzle. After leaving an orifice, water impacts a deflector surface which changes the direction of the water flow, and spreads the water into a fan. The Type R nozzle is called a drift reduction nozzle. This nozzle incorporates a vortex chamber, and is said to produce fewer fine drops than Type F nozzles for the same operating conditions. Another style of irrigation spray nozzle is shown in Fig. 2. With these nozzles, water leaves the orifice and impinges on a splash plate. Various

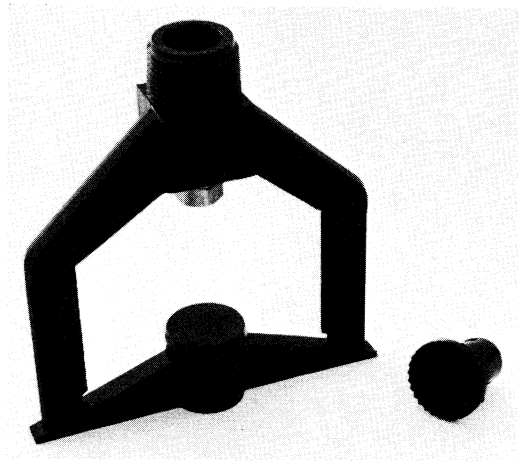

**Fig. 2—Flat plate (FP) irrigation spray nozzle and serrated plate (SP) insert.**

Article was submitted for publication in April, 1985; reviewed and approved for publication by the Soil and Water Div. of ASAE in August, 1985. Presented as ASAE Paper No. 85-2062.

Contribution from the USDA-ARS in cooperation with the University of South Carolina.

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splash plate designs are available, with convex, concave or flat plates, with smooth or serrated surfaces. Smooth flat plate (Type FP) and serrated flat plate (Type SP) configurations were examined in this study.

## SPRAY NOZZLE DROP SIZES

A variety of techniques have been used to measure the sizes of drops from irrigation nozzles. Kohl and DeBoer (1983a) used the pellet method to measure drop sizes for Type FP and Type SP spray nozzles. Drops were caught in pans of flour, and a previously developed calibration equation was used to convert the size of the resulting pellets into drop diameters. Drop size samples were taken at several distances from the nozzle, weighted in proportion to the volume of water applied by the nozzle at that distance, and combined to develop the drop size distribution for the spray as a whole.

Published drop size information on Type F and Type R irrigation spray nozzles is almost exclusively that obtained by the nozzle manufacturers. Data from the Delavan Corporation* is obtained photographically, using two different techniques. For higher discharge nozzles, measurements are taken from photographs of drops "on the fly," using the technique of Hoffman (1977). The other method, immersion sampling, involves collecting drops in a sampling cell filled with a solvent. Photographs of the samples are electronically analyzed to determine drop sizes (Tate, 1961 and Delavan, 1982b). The Spraying Systems Co.* (1966) used an inflight spray analyzer to measure drop sizes.

Further drop size measurements were made for this study under the direction of the second author using a stain technique similar to that of Inoue and Jayasinghe (1962), Inoue (1963) and Seginer (1963). Drops were allowed to impact a piece of collector paper, where they formed a stain. A calibration equation was developed to convert the stain diameters to drop diameters. Drop sizes were sampled along a radial line from the nozzle, at 1 m intervals. The distribution for each sampling location was weighted in accordance to the volume of water applied by the nozzle at that location, and summed to form the distribution for the spray as a whole.

Spraying Systems Co. (1968) presented volume median drop diameters for their flooding nozzles for various nozzle sizes and operating pressures. Table 1 is adapted from these data. Delavan (1982a, page 27) presented volume median drop diameters for both flooding and drift reduction style nozzles for a range of nozzle sizes at an operating pressure of 276 kPa (40 psi). The data in Table 2 are adapted from this information.

Complete drop size distributions for irrigation spray nozzles are relatively rare. Tate and Janssen (1965) and Tate (1968 and 1977) presented a total of three distributions for flooding style spray nozzles. The Delavan Corporation (1978) has made available five previously unpublished distributions for flooding style nozzles. Kohl and DeBoer (1983a) present fourteen distributions for Type FP nozzles and two distributions for Type SP nozzles. They have also made available two other unpublished distributions for Type FP nozzles (Kohl and DeBoer, 1983b).

*Mention of company names is for convenience in reference only, and does not imply recommendation or endorsement over other companies not mentioned.

**TABLE 1. VOLUME MEDIAN DROP DIAMETER (MM) FOR SPRAYING SYSTEMS FLOODING SPRAY NOZZLES***

| Pressure | | Nominal nozzle diameter, mm | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| kPa | psi | 0.8 | 1.9 | 2.6 | 3.8 | 4.6 | 5.3 | 6.1 | 6.5 |
| 34 | 5 | 0.71 | 0.94 | 1.11 | 1.31 | 1.46 | 1.61 | 1.75 | 1.89 |
| 68 | 10 | 0.63 | 0.86 | 1.03 | 1.22 | 1.37 | 1.52 | 1.66 | 1.79 |
| 103 | 15 | 0.57 | 0.80 | 0.96 | 1.16 | 1.40 | 1.45 | 1.59 | 1.71 |
| 138 | 20 | 0.52 | 0.75 | 0.91 | 1.10 | 1.24 | 1.39 | 1.53 | 1.63 |
| 172 | 25 | 0.48 | 0.70 | 0.86 | 1.04 | 1.19 | 1.33 | 1.47 | 1.57 |
| 207 | 30 | 0.44 | 0.65 | 0.81 | 1.00 | 1.14 | 1.27 | 1.42 | 1.52 |
| 241 | 35 | 0.40 | 0.61 | 0.77 | 0.95 | 1.09 | 1.23 | 1.37 | 1.46 |
| 276 | 40 | 0.37 | 0.57 | 0.73 | 0.91 | 1.05 | 1.18 | 1.32 | 1.42 |
| 310 | 45 | 0.33 | 0.54 | 0.69 | 0.88 | 1.02 | 1.14 | 1.27 | 1.37 |
| 345 | 50 | 0.30 | 0.50 | 0.66 | 0.84 | 0.98 | 1.10 | 1.23 | 1.32 |
| 379 | 55 | 0.27 | 0.47 | 0.62 | 0.81 | 0.94 | 1.06 | 1.18 | 1.28 |
| 414 | 60 | 0.23 | 0.43 | 0.59 | 0.78 | 0.91 | 1.02 | 1.14 | 1.24 |

*Adapted from Spraying Systems Co. (1968).

## ULLN DESCRIPTION OF DROP SIZE DATA

Mugele and Evans (1951) proposed the upper limit log normal (ULLN) distribution function to describe spray droplet data. The ULLN frequency distribution is defined on the interval [0,D] as follows:

$$f(d:\underline{P}) = Ae^{B} \quad \text{[1a]}$$

$$A = D/[d(D-d)s(2\pi)^{1/2}] \quad \text{[1b]}$$

$$B = -\left\{[\ln(d/[D-d])-m]/s\right\}^{2}/2 \quad \text{[1c]}$$

where

$f(d:\underline{P})$ = value of frequency function for drop diameter d, given $\underline{P}$

$\underline{P}$ = (D,m,s) the vector of ULLN distribution parameters

d = drop diameter, mm

D = maximum drop diameter, mm

m,s = dimensionless parameters

The value of $f(d:\underline{P})$ is zero outside the interval [0,D]. This distribution function can be derived from the assumption that d is related to a pseudo drop diameter y that is log-normally distributed. y and d are related by the tranformations:

$$y = d/(D-d) \quad \text{[2a]}$$

$$d = D[y/(y+1)] \quad \text{[2b]}$$

**TABLE 2. VOLUME MEDIAN DIAMETER (MM) FOR DELAVAN FLOODING AND DRIFT REDUCTION SPRAY NOZZLES AT 276 KPA (40 PSI)***

| Nominal nozzle diameter, mm | Nozzle style: Flooding type F | Nozzle style: Drift reduction type R |
|---|---|---|
| 2.6 | 0.40 | 1.25 |
| 3.3 | 0.47 | 1.31 |
| 3.8 | 0.55 | 1.37 |
| 4.2 | 0.62 | 1.39 |
| 4.6 | 0.70 | 1.41 |
| 5.0 | 0.78 | 1.43 |
| 5.3 | 0.85 | 1.45 |
| 5.6 | 0.92 | 1.47 |
| 6.1 | 1.00 | 1.49 |
| 6.5 | 1.12 | 1.50 |
| 7.1 | 1.23 | 1.51 |
| 7.5 | 1.32 | 1.52 |
| 8.0 | 1.40 | 1.53 |
| 8.4 | 1.48 | 1.54 |

*Adapted from Delavan (1982a).

The ULLN distribution function can refer to either number or volume frequency (Goering and Smith, 1976). It seems much more useful to work in terms of water volume, than in terms of numbers of drops of a certain size (Kohl, 1974), so all drop size distributions in this study, and their ULLN representations, are volume frequency distributions. Therefore, integrating $f(d:\underline{P})$ from zero to d yields the cumulative volume distribution function $F(d:\underline{P})$, which specifies the fraction of the total water volume occurring in drops of diameter d or smaller.

Volume percentile drop diameters can be calculated directly from the ULLN parameters. Suppose $d_x$ is such that $F(d_x:\underline{P}) = x/100$. $d_x$ is called the $x^{th}$ percentile drop diameter, and x per cent of the water volume will occur in drops of diameter $d_x$ or smaller. It is not hard to show that:

$$d_x = D[y_x/(1+y_x)] \quad [3a]$$

$$y_x = \exp(m+Z_x s) \quad [3b]$$

where $y_x$ is the $x^{th}$ percentile pseudo drop diameter, and $Z_x$ is the $x^{th}$ percentile value of a standard normal variate. Commonly encountered values of $Z_x$ are listed in Table 3. In particular, the volume median drop diameter is given by $d_{50} = D[e^m/(1+e^m)]$, and is independent of s.

Inoue and Jayasinghe (1962), Inoue (1963) and Okamura (1968) have all used the ULLN distribution very successfully to represent their sprinkler drop size data. Goering and Smith (1976) found that the ULLN distribution fit well the droplet size distributions from a wide variety of agricultural spray nozzles. Bezdek and Solomon (1983) show good ULLN fits to both sprinkler and spray nozzle drop size data.

For a distribution function as analytically complex as the ULLN distribution, choosing a parameter vector $\underline{P}$ that "best fits" a given data set is difficult. Mugele and Evans (1951) present a trial and error graphical technique, but as Goering and Smith (1976) point out, this method will yield physically impossible values (e.g. negative D) for some types of distributions. Bezdek and Solomon (1983) present three parameter estimation schemes. The first is a generalization of Mugele and Evans' graphical technique, patterned after the "percentile" approach of Johnson and Kotz (1970). The others involve different numerical schemes for determining maximum likelihood parameter estimates. They recommended an algorithm (MLSYS) which uses Newton's metod for systems of equations to solve the necessary maximum likelihood conditions. All ULLN parameters reported in the present study were determined with the MLSYS algorithm, except for those specifically indicated as being determined graphically.

TABLE 3. VALUES OF $Z_x$ FOR USE WITH EQUATION [3]

| x | $Z_x$ | x | $Z_x$ |
|---|---|---|---|
| 0.01 | -3.72 | 75 | 0.67 |
| 0.1 | -3.09 | 80 | 0.84 |
| 1 | -2.33 | 90 | 1.28 |
| 2 | -2.05 | 95 | 1.64 |
| 5 | -1.64 | 98 | 2.05 |
| 10 | -1.28 | 99 | 2.33 |
| 20 | -0.84 | 99.9 | 3.09 |
| 25 | -0.67 | 99.99 | 3.72 |
| 50 | 0.0 | | |

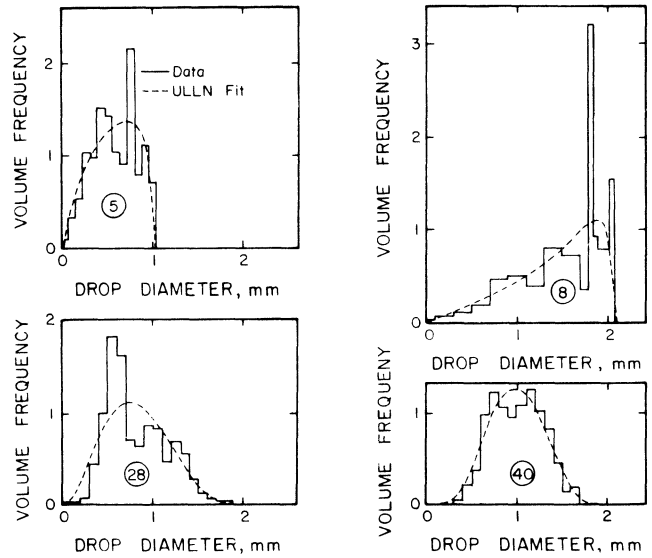


**Fig. 3—Typical ULLN fits to measured drop size distributions for irrigation spray nozzles. Circled numbers refer to distribution numbers in Table 4.**

All irrigation spray nozzle drop size distributions obtained from other sources, and those measured for this study, were fitted with the ULLN distribution function. Because of the generally excellent fits obtained (see Fig. 3), it is herein assumed that a measured distribution can be adequately represented by the fitted ULLN parameter vector $\underline{P} = (D,m,s)$. Table 4 summarizes the available drop size distributions for irrigation spray nozzles, giving the fitted ULLN parameter values for each distribution. Since it is not easy to interpret directly the physical significance of the ULLN parameters, Table 4 also lists the $10^{th}$, $50^{th}$ (median), and $90^{th}$ percentile diameters for each distribution, calculated from [3].

## A SIMPLE DROP SIZE DISTRIBUTION MODEL

Since an irrigation spray nozzle drop size distribution is characterized by the values specified for the ULLN parameter vector $\underline{P} = (D,m,s)$, it is reasonable to study the effect of nozzle size (N) and pressure head (H) on drop sizes by examining their effect on $\underline{P}$. If D, m and s could be expressed as functions of nozzle size and pressure, then these functions would implicitly express the influence of nozzle size and pressure on drop size distribution. These functions would constitute a drop size distribution model. Simple functional forms that have been used with some success (Solomon and von Bernuth, 1981) for this purpose are:

$$\underline{P} = (D,m,s) \quad [4a]$$

$$D = \alpha_1 N^{\alpha_2} H^{\alpha_3} \quad [4b]$$

$$m = \ln(\alpha_4 N^{\alpha_5} H^{\alpha_6}) \quad [4c]$$

$$s = \alpha_7 N^{\alpha_8} H^{\alpha_9} \quad [4d]$$

$$\underline{\alpha} = (\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5,\alpha_6,\alpha_7,\alpha_8,\alpha_9) \quad [4e]$$

TABLE 4. DROP SIZE DISTRIBUTIONS FOR IRRIGATION SPRAY NOZZLES

| # | * | Nozzle diameter, mm | Nozzle diameter, inch | Operating pressure, kPa | Operating pressure, psi | † | ULLN Parameter vector P, D mm | m | s | ‡ | Volume percentile drop diameters, $d_{10}$ mm | $d_{50}$ mm | $d_{90}$ mm | Reference |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | F | 1.3 | 0.05 | 276 | 40 | Ph | 0.42 | 0.322 | 0.906 | M | 0.13 | 0.24 | 0.34 | Tate & Janssen (1965) |
| 2 | F | 0.8 | 0.03 | 276 | 40 | Ph | 0.38 | 0.126 | 0.956 | M | 0.10 | 0.20 | 0.30 | Tate (1968) |
| 3 | F | 1.3 | 0.05 | 90 | 13 | Ph | 0.76 | -0.357 | 1.219 | G | 0.10 | 0.31 | 0.58 | Tate (1977) |
| 4 | F | 2.6 | 0.10 | 276 | 40 | Ph | 0.76 | 0.144 | 1.253 | M | 0.14 | 0.40 | 0.64 | Delavan (1978) |
| 5 | F | 4.2 | 0.17 | 276 | 40 | Ph | 1.05 | 0.249 | 1.171 | M | 0.23 | 0.59 | 0.89 | Delavan (1978) |
| 6 | F | 6.1 | 0.24 | 276 | 40 | Ph | 1.43 | 0.686 | 1.322 | M | 0.38 | 0.95 | 1.31 | Delavan (1978) |
| 7 | F | 8.4 | 0.33 | 276 | 40 | Ph | 2.09 | 0.828 | 1.194 | M | 0.69 | 1.45 | 1.91 | Delavan (1978) |
| 8 | F | 8.4 | 0.33 | 138 | 20 | Ph | 2.10 | 1.030 | 1.196 | G | 0.79 | 1.55 | 1.95 | Delavan (1978) |
| 9 | SP | 6.4 | 1/4 | 100 | 15 | Pe | 10.97 | -1.124 | 0.367 | G | 1.85 | 2.69 | 3.75 | Kohl & DeBoer (1983a) |
| 10 | SP | 7.9 | 5/16 | 150 | 22 | Pe | 5.95 | -0.003 | 0.596 | M | 1.89 | 2.97 | 4.05 | Kohl & DeBoer (1983a) |
| 11 | FP | 4.8 | 3/16 | 50 | 7 | Pe | 2.98 | -0.522 | 0.666 | M | 0.60 | 1.11 | 1.73 | Kohl & DeBoer (1983a) |
| 12 | FP | 6.4 | 1/4 | 50 | 7 | Pe | 3.07 | -0.311 | 0.746 | M | 0.68 | 1.30 | 2.01 | Kohl & DeBoer (1983a) |
| 13 | FP | 7.9 | 5/16 | 50 | 7 | Pe | 3.64 | -0.356 | 0.798 | M | 0.73 | 1.50 | 2.40 | Kohl & DeBoer (1983a) |
| 14 | FP | 9.5 | 3/8 | 50 | 7 | Pe | 4.63 | -0.256 | 0.871 | M | 0.94 | 2.02 | 3.25 | Kohl & DeBoer (1983a) |
| 15 | FP | 4.8 | 3/16 | 100 | 15 | Pe | 2.90 | -0.564 | 0.666 | M | 0.57 | 1.05 | 1.66 | Kohl & DeBoer (1983a) |
| 16 | FP | 6.4 | 1/4 | 100 | 15 | Pe | 3.03 | -0.447 | 0.749 | M | 0.60 | 1.18 | 1.89 | Kohl & DeBoer (1983a) |
| 17 | FP | 7.9 | 5/16 | 100 | 15 | Pe | 3.43 | -0.365 | 0.772 | M | 0.70 | 1.41 | 2.23 | Kohl & DeBoer (1983a) |
| 18 | FP | 9.5 | 3/8 | 100 | 15 | Pe | 4.35 | -0.486 | 0.829 | M | 0.76 | 1.66 | 2.78 | Kohl & DeBoer (1983a) |
| 19 | FP | 4.8 | 3/16 | 150 | 22 | Pe | 9.50 | -2.200 | 0.414 | M | 0.58 | 0.95 | 1.50 | Kohl & DeBoer (1983a) |
| 20 | FP | 9.5 | 3/8 | 150 | 22 | Pe | 4.06 | -0.549 | 0.747 | M | 0.74 | 1.49 | 2.44 | Kohl & DeBoer (1983a) |
| 21 | FP | 4.8 | 3/16 | 200 | 29 | Pe | 3.82 | -1.150 | 0.554 | M | 0.51 | 0.92 | 1.50 | Kohl & DeBoer (1983a) |
| 22 | FP | 6.4 | 1/4 | 200 | 29 | Pe | 3.28 | -0.769 | 0.691 | G | 0.53 | 1.04 | 1.73 | Kohl & DeBoer (1983a) |
| 23 | FP | 7.9 | 5/16 | 200 | 29 | Pe | 2.98 | -0.433 | 0.755 | M | 0.59 | 1.17 | 1.88 | Kohl & DeBoer (1983a) |
| 24 | FP | 9.5 | 3/8 | 200 | 29 | Pe | 3.48 | -0.553 | 0.743 | M | 0.63 | 1.27 | 2.08 | Kohl & DeBoer (1983a) |
| 25 | FP | 6.4 | 1/4 | 150 | 22 | Pe | 3.00 | -0.494 | 0.665 | G | 0.62 | 1.14 | 1.77 | Kohl & DeBoer (1983a) |
| 26 | FP | 7.9 | 5/16 | 150 | 22 | Pe | 3.55 | -0.644 | 0.691 | M | 0.63 | 1.22 | 1.99 | Kohl & DeBoer (1983a) |
| 27 | FP | 3.2 | 1/8 | 69 | 10 | S | 1.61 | -0.126 | 0.784 | M | 0.39 | 0.75 | 1.14 | |
| 28 | FP | 3.2 | 1/8 | 103 | 15 | S | 1.88 | -0.310 | 0.788 | M | 0.40 | 0.80 | 1.26 | |
| 29 | FP | 3.2 | 1/8 | 138 | 20 | S | 1.48 | 0.254 | 0.771 | M | 0.48 | 0.83 | 1.15 | |
| 30 | FP | 3.2 | 1/8 | 172 | 25 | S | 4.54 | -1.620 | 0.356 | M | 0.51 | 0.75 | 1.08 | |
| 31 | FP | 3.2 | 1/8 | 207 | 30 | S | 4.00 | -1.519 | 0.336 | G | 0.50 | 0.72 | 1.01 | |
| 32 | FP | 4.8 | 3/16 | 69 | 10 | S | 2.20 | -0.221 | 0.750 | M | 0.52 | 0.98 | 1.49 | |
| 33 | FP | 4.8 | 3/16 | 103 | 15 | S | 2.03 | -0.094 | 0.579 | M | 0.61 | 0.97 | 1.33 | |
| 34 | FP | 4.8 | 3/16 | 138 | 20 | S | 4.44 | -1.310 | 0.457 | M | 0.58 | 0.94 | 1.45 | |
| 35 | FP | 4.8 | 3/16 | 172 | 25 | S | 7.78 | -2.060 | 0.393 | M | 0.56 | 0.88 | 1.35 | |
| 36 | FP | 4.8 | 3/16 | 207 | 30 | S | 3.00 | -0.904 | 0.405 | G | 0.58 | 0.86 | 1.21 | |
| 37 | FP | 6.4 | 1/4 | 69 | 10 | S | 5.19 | -1.260 | 0.438 | M | 0.72 | 1.15 | 1.72 | |
| 38 | FP | 6.4 | 1/4 | 103 | 15 | S | 5.00 | -1.200 | 0.506 | G | 0.68 | 1.16 | 1.83 | |
| 39 | FP | 6.4 | 1/4 | 138 | 20 | S | 23.06 | -3.044 | 0.372 | M | 0.66 | 1.05 | 1.64 | |
| 40 | FP | 6.4 | 1/4 | 172 | 25 | S | 2.04 | -0.064 | 0.615 | M | 0.61 | 0.99 | 1.37 | |
| 41 | FP | 6.4 | 1/4 | 207 | 30 | S | 10.58 | -2.220 | 0.358 | M | 0.68 | 1.04 | 1.55 | |
| 42 | FP | 5.6 | 7/32 | 69 | 10 | S | 3.83 | -0.625 | 0.600 | M | 0.76 | 1.34 | 2.05 | |
| 43 | FP | 5.6 | 7/32 | 138 | 20 | S | 2.19 | 0.218 | 0.812 | M | 0.67 | 1.21 | 1.71 | |
| 44 | FP | 5.6 | 7/32 | 207 | 30 | S | 3.40 | -0.758 | 0.709 | M | 0.54 | 1.08 | 1.83 | |

*Nozzle type:
F = Flooding
FP = Flat smooth plate
SP = Flat serrated plate
†Measurement method
Ph = Photo
Pe = Pellet
S = Stain
‡Fitting method:
M = MLSYS algorithm
G = Graphical

where

$\underline{P}$ = An estimate of the ULLN parameter vector P

$D, m, s$ = Estimated values for ULLN parameters D,m,s

N = Nominal nozzle diameter, mm

H = Operating pressure, kPa

$\alpha_i$ = Constants to be determined (i = 1 to 9)

$\underline{\alpha}$ = Vector of model parameters

Equations [4] and the remainder of this paper employ the notational convention that Roman letters refer to quantities that are measured or computed directly from measured data, while letters in italics refer to estimates of their Roman counterparts. Although the model [4] will produce estimates for D, m, and s, the goal is to produce f = f(d:$\underline{P}$), an estimate of the distribution function f = f(d:$\underline{P}$) that has been derived from measurement. With suitable values for $\underline{\alpha}$, this model is capable of generating estimated distribution functions that mimic the measured ones fairly well (just how well will be the topic of a later section).

Let $N_j$ and $H_j$ be the nozzle diameter and pressure for which drop size distribution j was measured, and let $\underline{P}_j$ = ($D_j$, $m_j$, $s_j$) be the ULLN parameter vector determined for that distribution. Finally, let $\underline{P}$ = ($D_j$, $m_j$, $s_j$) computed from [4] for nozzle size $N_j$ and pressure $H_j$. The problem

is to choose $\underline{\alpha}$ such that $f_j = f(d:\underline{P}_j)$ is, in some sense, close to $f_j = f(d:\underline{P}_j)$ for each j.

It should be noted that determining $\underline{\alpha}$ by using traditional regression techniques on the sets of measured triples $\{(N_j, H_j, D_j)\}$, $\{(N_j, H_j, m_j)\}$ and $\{(N_j, H_j, s_j)\}$, will usually give very poor results. The $f_j$'s so estimated don't match the measured $f_j$'s very well. The reason for this is related to the following property of the ULLN distribution: it is possible for two ULLN distribution functions $f(d:\underline{P}_1)$ and $f(d:\underline{P}_2)$ to be similar, even when $\underline{P}_1$ and $\underline{P}_2$ are dissimilar. This can be seen in parts of Table 4. Consider, for example, distributions 38, 39, 40 and 41. D for these distributions ranges from 2 to over 20. The parameters m and s for these distributions are also dissimilar. Yet the distributions themselves, as evidenced by the tabulated percentile diameters, are quite similar. Changes in distribution curve shape that result from an increase (or decrease) in D are counterbalanced to some extent if the values of m and s are lowered (or raised). Regression on the set $\{(N_j, H_j, D_j)\}$ would penalize differences between $D_j$ and the regression estimated $D_j$ without considering the extent to which this error could be mitigated by properly over- or under-predicting $m_j$ and $s_j$. Similarly, expressions to produce the estimates $m_j$ (and $s_j$) should not attempt to match the $m_j$ (and $s_j$) as accurately as possible, but rather should match values that minimize the errors produced by any difference between $D_j$ and the estimated $D_j$.

Therefore, initial estimates for $\underline{\alpha}$ were developed using the following three stage process. First, $\alpha_1$, $\alpha_2$ and $\alpha_3$ were chosen to reflect the influence of N and H on the maximum drop diameter, though even this is not a straight forward process. While D is the theoretical maximum drop diameter, it may be a misleading indicator of the maximum diameter that can reasonably be expected in actual practice. Consider distribution number 39, for example, where Table 4 lists D as 23.06 mm. The 99th percentile diameter for this distribution, however, is calculated to be only 2.35 mm, and the 99.99th percentile diameter in only 3.56 mm. Let $d_{xj}$ be the xth percentile diameter $d_x$ for distribution j. Values of $d_{99j}$, $d_{99.9j}$, and $d_{99.99j}$ were determined for each distribution. Assuming $d_x$ is related to N and H by:

$$d_x = KN^{\alpha_2}H^{\alpha_3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [5]$$

three candidates each for $\alpha_2$ and $\alpha_3$ can be obtained by using traditional regression on the sets of triples $\{N_j, H_j, d_{99j}\}$, $\{(N_j, H_j, d_{99.9j})\}$, and $\{(N_j, H_j, d_{99.99j})\}$. After viewing the candidates, judgement was used to select initial estimates for $\alpha_2$ and $\alpha_3$. Because the $d_{xj}$ are systematically less than $D_j$, K cannot be used to estimate $\alpha_1$. Rather, an initial estimate for $\alpha_1$ was chosen after observing the values $D_j/N_j^{\alpha_2}H_j^{\alpha_3}$.

Having obtained estimates for $\alpha_1$, $\alpha_2$ and $\alpha_3$, $D_j$ for each distribution was computed from [4b]. The original data for each distribution were transformed according to $y = d/(D_j\text{-}d)$, and plotted on log-probability paper. If d is actually ULLN distributed, the transformed variables y would plot as straight lines on log-probability paper. The 50th percentile values $y_{50j}$ were read directly from the graphs, and because in general, $m = \ln(y_{50j})$, traditional regression on the set of triples $\{(N_j, H_j, y_{50j})\}$ will yield estimates for $\alpha_4$, $\alpha_5$, and $\alpha_6$. Note that the model for calculating the estimate $m$ is determined after, and fully incorporates, the model for determining the estimate $D$.

The third step also involves the log-probability plots of y. Using [4c], an $m_j$, is estimated for each distribution, and a corresponding $y_{50j}$ is calculated such that $m = \ln(y_{50j})$. The straight line passing through $y_{50j}$ that best fit the data was drawn by eye. The ULLN parameter s is related to the slope of this line, and may be calculated (Bezdek and Solomon, 1983) as $[\ln(y_{90}/y_{10})/2.563]$. Values of $y_{10}$ and $y_{90}$ may be read directly from the plot, allowing graphical determination of an estimate of s ($S_j$) for each distribution. Regression on the set of triples $\{(N_j, H_j, S_j)\}$ yields estimates for the three remaining model parameters $\alpha_7$, $\alpha_8$ and $\alpha_9$.

## EVALUATING THE DROP SIZE DISTRIBUTION MODEL

The criteria for evaluating the model [4] should not rest on comparisons between $\underline{P} = (D,m,s)$ determined from measurements, and the corresponding estimate $\underline{P} = (D,m,s)$ The goal of the model is to produce estimated functions $f = f(d:\underline{P})$ that match the characteristics of the measured functions $f = f(d:\underline{P})$. So, to evaluate the model, one might study the distance between each measured f and the corresponding predicted $f$.

If f and $f$ were two points in space, the distance between them could be computed from the usual formula for Euclidian distance. However, f and $f$ are functions, so the following functional analog to that distance formula (Olmstead, 1959) is used:

$$\langle g,h\rangle = \left\{ \int_a^b [g(x)-h(x)]^2 dx \right\}^{1/2} \quad \ldots\ldots\ldots\ldots [6]$$

where

g,h = two functions defined on the closed interval [a,b]

$\langle g,h\rangle$ = the distance between functions g and h

x = a dummy variable

When using [6] to measure the distance between ULLN distribution functions f and $f$, the interval [a,b] is taken as [0,D'], where D' is the maximum of D and $D$. The integration must be done numerically.

Using this notion of the distance between functions, it is possible to extend the concept of correlation coefficient to define a pseudo $r^2$ for the model [4]. If $\{x_j\}$ and $\{x_j\}$ are sets of values respectivley measured and predicted according to some linear regression model, then $r^2$ for that model (Walpole, 1968) can be calculated by:

$$r^2 = 1 - [\Sigma(x_j-x_j)^2 / \Sigma(x_j-\bar{x})^2] \quad \ldots\ldots\ldots\ldots [7]$$

where $\bar{x}$ is the mean of the $x_j$. The mean $\bar{g}$ of a number of functions $g_j$ (j = 1 to n) is defined by $\bar{g}(x) = (1/n)[\Sigma g_j(x)]$. A pseudo $r^2$ for the model [4] is obtained by replacing the differences $(x_j\text{-}x_j)$ and $(x_j\text{-}\bar{x})$ with the distances $\langle f_j,f_j\rangle$ and $\langle f_j,\bar{f}\rangle$:

$$r^2 = 1 - [\Sigma\langle f_j,f_j\rangle^2 / \Sigma\langle f_j,\bar{f}\rangle^2] \quad \ldots\ldots\ldots\ldots [8]$$

Based on [8], it is a straightforward process to determine $r^2$ for any given choice of $\underline{\alpha}$ in model [4]. In fact, [8] can form the basis for improving the estimates for $\underline{\alpha}$ derived with the ad hoc process described above. A pattern search technique can be used to identify perturbations in $\underline{\alpha}$ that will increase $r^2$.

TABLE 5. DROP SIZE DISTRIBUTION MODEL PARAMETERS FOR TYPE F AND FP IRRIGATION SPRAY NOZZLES

| Parameter | Model I Nozzle type FP | Model II Nozzle type F | Model III Nozzle type F |
|---|---|---|---|
| $\alpha_1$ | 1.66 | 3.40 | 2.92 |
| $\alpha_2$ | 0.66 | 0.64 | 0.76 |
| $\alpha_3$ | -0.13 | -0.35 | -0.35 |
| $\alpha_4$ | 0.96 | 0.29 | 0.32 |
| $\alpha_5$ | -0.17 | 0.35 | 0.36 |
| $\alpha_6$ | -0.03 | 0.18 | 0.18 |
| $\alpha_7$ | 0.95 | 1.87 | 2.10 |
| $\alpha_8$ | 0.22 | 0.11 | 0.11 |
| $\alpha_9$ | -0.16 | -0.14 | -0.14 |
| $r^2$ | 0.78 | 0.82 | 0.75 |

Model I: Parameters chosen to maximize $r^2$ for predicted and measured distributions 11-44.
Model II: Parameters chosen to maximize $r^2$ for predicted and measured distributions 1-8.
Model III: Parameters chosen to retain a high $r^2$ for predicted and measured distributions 1-8 and also produce a good fit to the volume median data of Delavan (1982a).

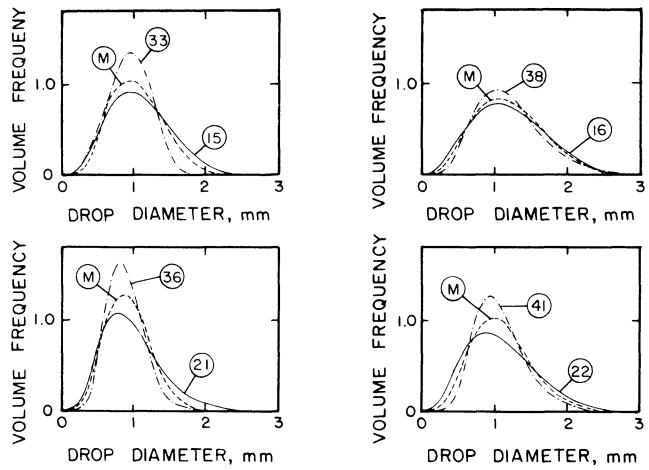


**Fig. 4—Drop size distributions measured and predicted for nearly identical conditions. Numbers refer to distribution numbers in Table 4, distributions predicted by Model I of Table 5 are marked M.**

Another approach to evaluating model [4] would be to compare drop size diameters $d_x$ predicted by [3] and [4] with those calculated from the measured distributions, or with previously published data (Delavan, 1982a and Spraying Systems, 1968).

## RESULTS

Distributions 1 through 8 from Table 4 were used to develop a drop size distribution model for Type F nozzles, while distributions 11 through 44 formed the basis of a model for Type FP nozzles. For each nozzle type, an initial estimate for $\underline{\alpha}$ was developed with the ad hoc procedure, and this estimate was adjusted to maximize $r^2$ for the models. The results of these analyses are given as Models I (for FP nozzles) and II (for F nozzles) in Table 5.

When interpreting the $r^2$ values, one should bear in mind that $r^2$ as defined here measures not only the goodness of fit of the model [4], but also the degree to which drop size distributions may be measured consistently and accurately. Regarding this latter point, it is instructive to note that four of the distributions measured for this study came very close to duplicating distributions measured previously by Kohl and DeBoer (1983a). Distributions 33, 36, 38 and 41 represent the same nozzle sizes and nearly the same pressures as, respectively, distributions 15, 21, 16 and 22. Plots of these distributions are shown in Fig. 4, and give some indication of the consistency of the drop size distribution measurement process. Also plotted (marked M) in Fig. 4 are the distributions predicted by Model I of Table 5 for these nozzle sizes and pressures. In all four cases, the distributions measured by the stain technique were more sharply peaked about their mode than were those measured with the pellet technique (Kohl and DeBoer, 1983a). This may be coincidental, or there may be some systematic bias in one or both of the measurement methods that causes this. Nevertheless, both measurement approaches gave distributions with generally similar characteristics.

Fig. 5 illustrates the fit between Type FP nozzle drop size distributions predicted from Model I and three measured distributions (Table 3) covering a range of nozzle sizes and operating pressures. Both Figs. 4 and 5

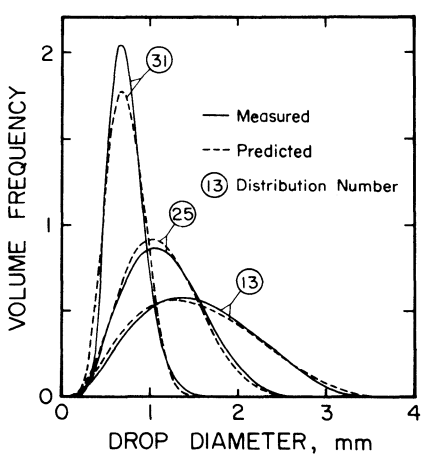


**Fig. 5—A comparison of measured distributions for Type FP nozzles and those predicted by Model I.**

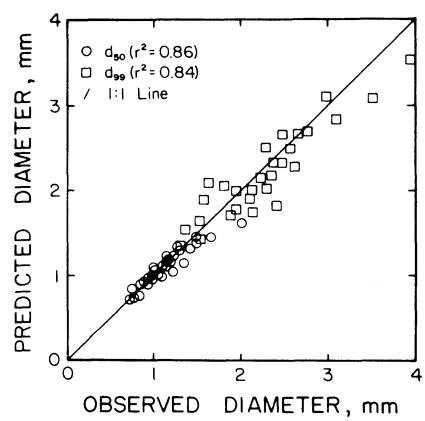


**Fig. 6—A comparison of observed values for $d_{50}$ and $d_{99}$ for Type FP nozzles and those predicted by Model I.**

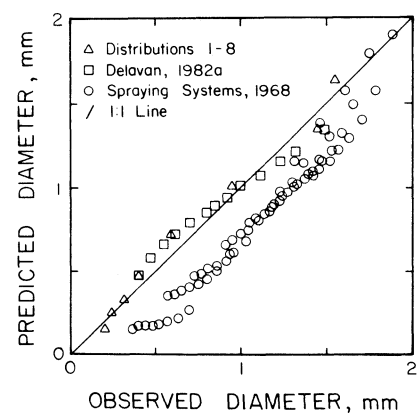


**Fig. 7—Volume median drop diameters for Type F nozzles predicted by Model III compared to measured and previously published data.**

give an indication of the degree to which distributions predicted by Model I match measured distributions. Model I attains an $r^2$ of 0.78 which, considering the test to test variation indicated by Fig. 4, is quite good. Fig. 6 indicates the agreement between observed and predicted values of $d_{50}$ and $d_{99}$ for Type FP nozzles. Again, the predictions of Model I seen reasonably accurate.

The parameters of Model II (Table 5) for Type F nozzles were based on measured distributions 1 through 8, and were selected so as to maximize $r^2$. As indicated by an $r^2$ of 0.82, the agreement between predicted and measured distributions was very good. Unfortunately, this model did not predict volume median drop sizes $d_{50}$ which correlated very well with the data published by either Delavan (1982a) or Spraying Systems (1968). Trail and error adjustments to Model II resulted in Model III, which fits reasonably well both Delavan's data (Fig. 7), and distributions 1 through 8 ($r^2 = 0.75$). As shown in Fig. 7, Model III does not fit the published data of Spraying Systems (1968). There may be differences between the flooding nozzles manufactured by the two companies that explain this discrepancy, or differences in the drop size measuring methods employed by the two companies may be involved. Literature from the manufacturers (Delavan, 1982a and Spraying Systems, 1978) include nothing to suggest any significant differences in design between their flooding nozzles. Distributions 1 through 8 were measured on Type F nozzles as manufactured by Delavan, so it is reasonable that a single model should fit both these distributions and Delavan's volume median figures.

It should be noted that the response of Model III to pressure is based on scant data: Delavan (1982a) gives data only for 276 kPa, and only distributions 3 and 8 are measured at pressures other than 276 kPa. Thus Model III should be regarded as only a tentative predictor of the performance of Type F nozzles not manufactured by Delavan, or at pressures other than 276 kPa. Fig. 8 illustrates the fit between Model III and Type F distributions 1, 5 and 7.

Figs. 9 and 10 show the effect of nozzle size and pressure on drop size distributions for Type FP and Type F nozzles as predicted by Models I and III respectively. In both cases, trends follow expected patterns: larger nozzles and lower pressures result in larger drop sizes, while higher operating pressures and smaller nozzle sizes reduce drop sizes. Fig. 11 compares the shapes of drop

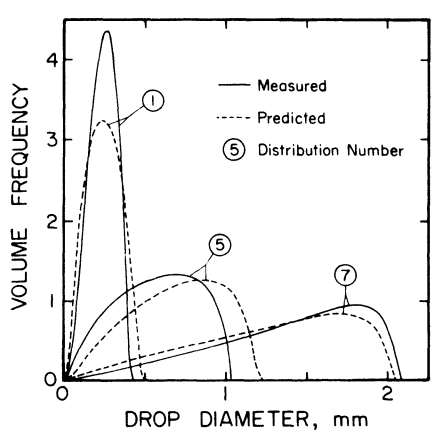


**Fig. 8—A comparison of measured distributions for Type F nozzles with the predictions of Model III.**

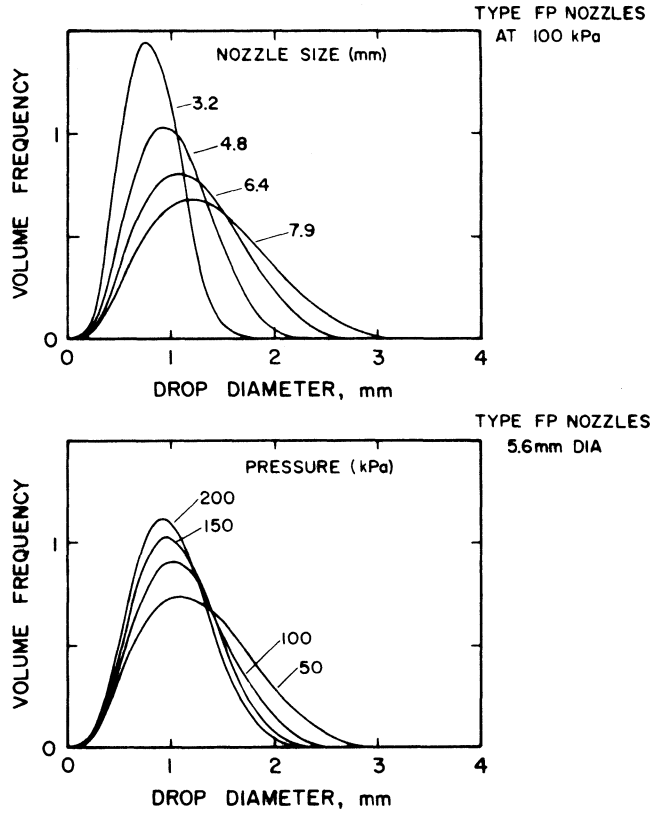


**Fig. 9—The effects of nozzle size and pressure on drop size distributions for Type FP nozzles, according to Model I.**

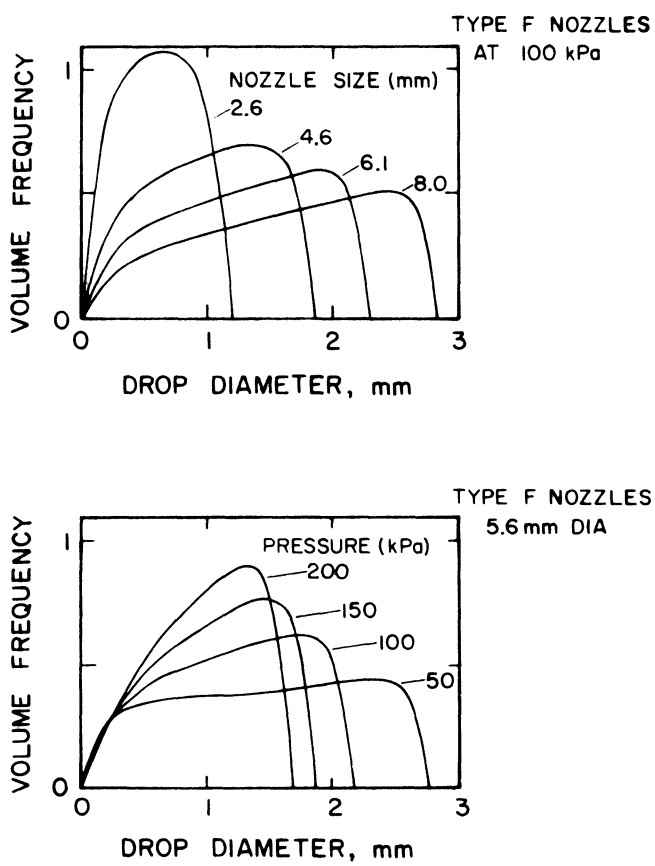


**Fig. 10—The effects of nozzle size and pressure on drop size distributions for Type F nozzles, according to Model III.**

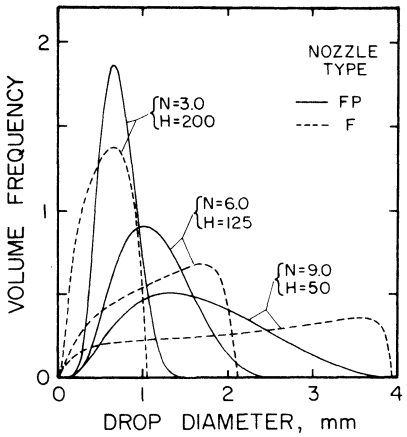


**Fig. 11—A comparison of drop size distribution shapes for Type FP and Type F nozzles.**

size distributions for the two types of nozzles. The flooding nozzle generally produces a broader spectrum of drop sizes, and, particularly for large nozzle sizes and low pressures, seems to produce more drops of the larger sizes. The Type FP nozzle produces distributions that are more concentrated about an intermediate drop size, with relatively few drops of the largest sizes.

## SUMMARY AND CONCLUSIONS

Irrigation nozzle drop sizes are important because they can influence wind drift and evaporation losses for the spray, wind distortion of the spray patterns, and the reduction of soil infiltration rates. Available data, obtained with either photographic or pellet measuring techniques, were summarized. Further drop size distributions for smooth flat plate spray nozzles were measured for this study using a calibrated stain technique.

The upper limit log normal (ULLN) distribution function was used to characterize measured drop size distributions in a concise, quantitative form, from which any desired characteristics of the distribution may be analytically determined. For example, the volume median drop size, or other volume percentile drop sizes can be calculated directly from the ULLN parameters.

A simple model was developed to predict ULLN parameters as a function of nozzle type, size and operating pressure, and was evaluated for goodness of fit by comparing measured and predicted distributions directly, and by comparing measured and predicted values for particular volume percentile drop sizes. The former approach was based on a measure of the distance between functions, and the definition of a pseudo $r^2$ measuring the degree of fit between measured and model predicted distribution functions.

A quasi analytical, ad hoc procedure was developed to obtain initial estimates for the constants of the predictive model, which were then adjusted to maximize $r^2$. In the case of flooding nozzles, further adjustments were made to develop a model which retained a reasonably high value of $r^2$, but which also produced a good fit to previously published information on flooding nozzle median drop sizes. The resulting regression models (I and III) attained $r^2$ values of 0.78 and 0.75 for smooth flat plate and flooding nozzles respectively. The model for smooth flat plate spray nozzles was based on 34 measured distributions covering a wide range of nozzle sizes and pressures, from two sources, and may be regarded as a reliable model of performance for these nozzles. However, the model for flooding nozzles was based on just 8 distributions, which did not adequately cover the range of expected operating pressures, and must be viewed as a tentative model only. Furthermore, this model rests on data for nozzles of only one manufacturer, and since volume median drop size data from another manufacturer of flooding nozzles differ, the model may be representative only of nozzles from the one manufacturer.

Drop sizes for both styles of nozzle follow similar trends. Larger drop sizes are produced by increases in nozzle diameter, and by decreases in operating pressure. The flooding style nozzle seems to produce a broader spectrum of drops, with relatively more water occurring as larger size drops, than does the smooth flat plate nozzle. This is particularly true for large nozzle sizes and low pressures.

The specific results presented here will be of particular interest to those who work with irrigation spray nozzles. The analytical techniques described, of course, can be applied to any other available drop size data. The authors are currently involved in such a project for irrigation sprinkler data from a variety of sources. The regression model approach used here may be implemented with other functional forms, perhaps with even better results. Little experimentation with other forms was done, and no guidance can be offered as to other potentially successful forms. The modeling of drop size distributions, and the influence of nozzle style, size and pressure thereon, will help in planning experimental programs to determine nozzle performance, and to extract the most information possible from given experimental results.

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