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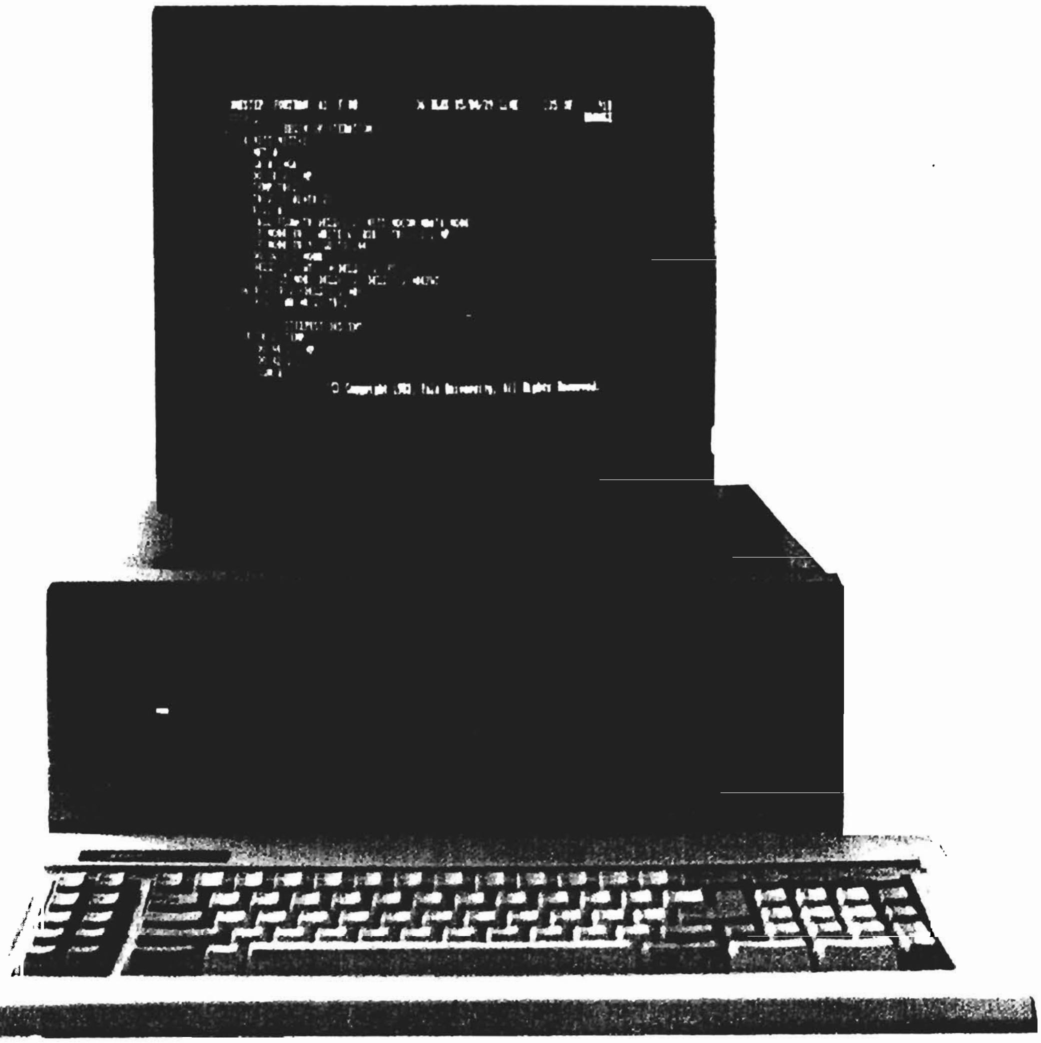
ONESTEP:

A Nonlinear Parameter Estimation Program for Evaluating Soil Hydraulic Properties from One-Step Outflow Experiments

J. B. Kool, J. C. Parker, and M. Th. van Genuchten



James R. Nichols, Dean and Director
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The Virginia Agricultural and Mechanical College came into being in 1872 upon acceptance by the Commonwealth of the provisions of the Morrill Act of 1862 "to promote the liberal and practical education of the industrial classes in the several pursuits and professions of life." Research and investigations were first authorized at Virginia's land-grant college when the Virginia Agricultural Experiment Station was established by the Virginia General Assembly in 1886.

The Virginia Agricultural Experiment Station received its first allotment upon passage of the Hatch Act by the United States Congress in 1887. Other related Acts followed, and all were consolidated in 1955 under the Amended Hatch Act which states "It shall be the object and duty of the State agricultural experiment stations . . . to conduct original and other researches, investigations and experiments bearing directly on and contributing to the establishment and maintenance of a permanent and effective agricultural industry of the United States, including the researches basic to the problems of agriculture and its broadest aspects and such investigations as have for their purpose the development and improvement of the rural home and rural life and the maximum contributions by agriculture to the welfare of the consumer . . . "

In 1962, Congress passed the McIntire-Stennis Cooperative Forestry Research Act to encourage and assist the states in carrying on a program of forestry research, including reforestation, land management, watershed management, rangeland management, wildlife habitat improvement, outdoor recreation, harvesting and marketing of forest products, and "such other studies as may be necessary to obtain the fullest and most effective use of forest resources."

In 1966, the Virginia General Assembly "established within the Virginia Polytechnic Institute a division to be known as the Research Division . . . which shall encompass the now existing Virginia Agricultural Experiment Station . . . "

ONESTEP: A Nonlinear Parameter Estimation Program for Evaluating Soil Hydraulic Properties from One-Step Outflow Experiments

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ABSTRACT

J. B. Kool, J. C. Parker, and M. Th. van Genuchten. 1985. ONESTEP: A nonlinear parameter estimation program for evaluating soil hydraulic properties from one-step outflow experiments. Bulletin 85-3. Virginia Agricultural Experiment Station, Blacksburg.

This report contains a description of the parameter estimation program ONESTEP. The program, written in FORTRAN IV, will estimate up to five unknown parameters in the van Genuchten soil hydraulic property model from measurements of cumulative outflow with time during one-step outflow experiments. The outflow data can be optionally supplemented with measurements of equilibrium moisture contents and pressure heads. The program combines a nonlinear optimization routine with a Galerkin finite element model for the one-dimensional flow equation. Results obtained for a silt loam soil are shown. Input instructions for the program, example input and output files, and a program listing are given in appendices.

ACKNOWLEDGMENTS

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INTRODUCTION

In recent years, interest has arisen in the feasibility of using parameter estimation methods to determine soil hydraulic properties from transient flow measurements. This approach can be summarized as follows:

1. Hydraulic properties are assumed to be described by a given model that contains a small number of unknown parameters.
2. Some flow-controlled attribute is measured at a number of times during transient flow.
3. The same flow process is simulated numerically using initial guesses for the unknown parameters.
4. Step 3 is repeated with adjusted parameter values at every step until an optimum match between observed and simulated response is obtained.
5. Back-substitution of final parameter values into the assumed model yields the soil hydraulic properties.

Different authors (Zachman et al, 1981, 1982; Hornung, 1983; Dane and Hruska, 1983) have considered a variety of experimental procedures for obtaining input data for the estimation problem. Kool et al. (1985) have presented a parameter estimation procedure which uses measurements of cumulative outflow against time from undisturbed soil cores, subjected to an instantaneous increase in pneumatic pressure. Experimentally, the procedure is identical to the familiar one-step method for determining soil water diffusivity. The procedure is experimentally simple, quickly executed, and applicable to soils of widely varying hydraulic properties. Theoretical results dealing with solution uniqueness and sensitivity to error, as well as actual results for a number of natural soils, are presented elsewhere (Kool et al., 1985; Parker et al., 1985).

This report describes the program ONESTEP, written in FORTRAN IV, that solves the parameter estimation problem. Input instructions, example input and output files, and a program listing are given in appendices.

DIRECT SOLUTION OF FLOW PROBLEM

The hydraulic system being considered consists of two layers -- a soil core and a porous plate. For vertical flow in the assumed nondeformable medium a solution to Richard's equation is required which may be written in the head formulation as:

$$C(h) \frac{\partial h}{\partial t} = \frac{\partial}{\partial x} [K(h)(\partial h/\partial x-1] \tag{1}$$

and subject to initial and boundary conditions:

$$h = h_o(x) \qquad t = 0,\ 0 \le x \le L \tag{2a}$$

$$\partial h/\partial x = 1 \qquad t > 0,\ x = 0 \tag{2b}$$

$$h = h_L - h^a \qquad t > 0,\ x = L \tag{2c}$$

where x is distance taken positive downward with x=0 located at the top of the soil core and x=L at the bottom of the porous plate, t is time, K is hydraulic conductivity, $C = d\theta/dh$ is the water capacity with θ the volumetric water content and h the pressure head, $h_o(x)$ is the initial pressure head distribution, h_L is the regulated pressure head at the bottom of the porous plate and $h^a = \Delta p/\rho g$ is the pneumatic head where Δp is the gauge gas pressure applied to the core, g is gravitational acceleration, and ρ is the density of water. The notation employed regards the pressure potential h as the component of total potential attributable to matric and hydrostatic components but excluding pneumatic contributions. On the assumption that gas pressure increments cause pneumatic potentials, h^a, to instantaneously propagate through the porous medium, h^a is effectively translated to the lower boundary condition.

Properties of the porous plate are independent of h with C=0 and $K=K_p$. For the soil, $\theta(h)$ is assumed to be described by van Genuchten's (1978a) expression

$$S_e = \begin{cases} [1 + |\alpha h|^n]^{-m} & h<0 \\ 1 & h\geq 0 \end{cases} \qquad (3a)$$

with the effective saturation S_e given by:

$$S_e = \frac{\theta - \theta_r}{\theta_s - \theta_r} \qquad (m = 1-1/n;\ n \geq 1 \qquad (3b)$$

where θ_s is the saturated water content, θ_r is a "residual" water content at which $dh/d\theta \rightarrow \infty$, and α and n are empirical coefficients. From (3) it follows immediately that

$$C = \alpha m(\theta_s-\theta_r)S_e^{1/m}(1-S_e^{1/m})^m(1-m)^{-1} \qquad (4)$$

and using Mualem's (1976) pore structure model one obtains:

$$K = K_s S_e^{1/2}[1-(1-S_e^{1/m})^m]^2 \qquad (5)$$

where K_s is the saturated hydraulic conductivity of the soil. Equations (3)-(5) define the required functions for C(h) and K(h) of the soil in terms of five parameters α, n, θ_r, θ_s, and K_s.

The solution of (1) and (2) using (3)-(5) for the soil hydraulic properties is obtained by a Galerkin finite element model UNSAT1 described by van Genuchten (1978b). Although this code can simulate saturated and unsaturated flow, for pressure outflow from initially saturated soil, the solution method is not very efficient. In such situations, very large hydraulic gradients occur initially across the soil core; these impose the need for very fine time and space discretizations to ensure convergence at each time increment. Because the first stage of outflow represents only a minor portion of the whole outflow process and because computational efficiency is of great importance when a numerical flow model is coupled with an inversion algorithm, the first stage of outflow from initally saturated soil is treated as a moving boundary problem. The moving boundary problem is solved by means of simple forward difference scheme that uses the same spatial discretization of the flow domain as applied to the finite element solution (Figure 1). From the specified initial condition (2a), it is first determined if part or all of the soil column is unsaturated. If the entire column is initially unsaturated, initial condition (2a) is simply passed on to the finite element routine to begin flow simulation. The criterion for determining whether the soil at node x_i is unsaturated is:

$$h_o(x_i) \leq h_e \tag{6}$$

where h_e is some value of the pressure head close to zero, which was arbitrarily fixed at:

$$h_e = h(0.99\,\theta_s) \tag{7}$$

On the other hand, if all or part of the soil is saturated, the solution proceeds in the following manner. Let x_i be the nodal

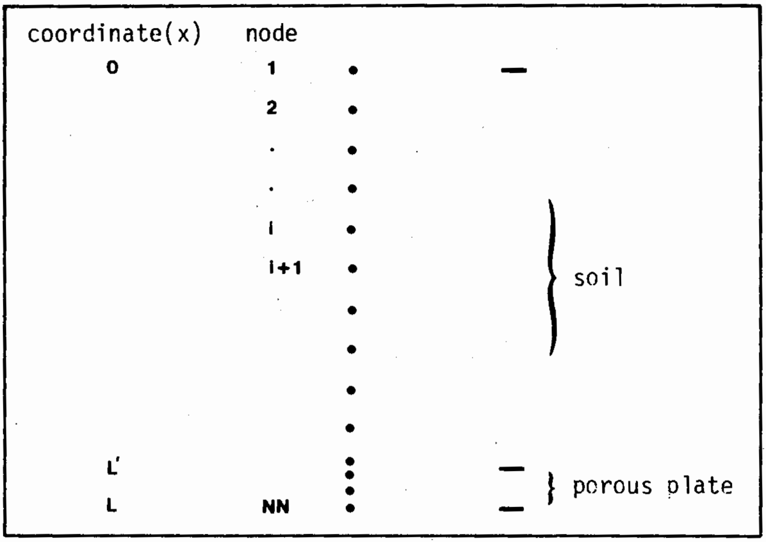

Figure 1. Nodal arrangement of the flow system.

coordinate at which the boundary between unsaturated and saturated zones is located, i.e. the soil is unsaturated at $0<x<x_i$ and is still saturated at $x_i \leq x \leq L'$ where L' is the length of the soil core. During the next time step the element between x_i and x_{i+1} becomes unsaturated. As the element becomes unsaturated, its moisture content is reduced from θ_s to 0.99 θ_s. The total volume of water removed from the element is thus given by:

$$Q_i = 0.01\ \theta_s\ A\ \Delta x \tag{8}$$

where $\Delta x = x_{i+1}-x_i$ and A is the core area perpendicular to flow. The time increment required for the element to become unsaturated follows from Darcy's law and is calculated as:

$$\Delta t_i = 0.01\ \theta_s\ \Delta x/(K_s dH/dx) \tag{9}$$

where H=h-x is the hydraulic head. The denominator in (9) represents the flux density across the lower boundary of the element. The flux across the upper element boundary is assumed to be negligible. The ratio of hydraulic head gradients across the saturated part of the soil and the porous plate is equal to the inverse ratio of saturated conductivities:

$$\frac{dH/dx\ (s)}{dH/dx\ (p)} = \frac{K_p}{K_s} \tag{10}$$

Combining (10) with the relationship for the average total hydraulic head drop, when the boundary between saturated and unsaturated zones is located between nodes x_i and x_{i+1}:

$$\begin{aligned}\Delta H &\approx h_e - h_L + \Delta P/\rho g + (L'-x)\\ &= (L'-x)dH/dx(s) + (L-L')dH/dx(p)\end{aligned} \tag{11}$$

where $x = (x_i + x_{i+1}/2)$, leads to the following approximation for the hydraulic head gradient across the saturated part of the soil:

$$\frac{dH}{dx}(s) \approx \frac{h_e + L + \Delta P/\rho g - h_L - \bar{x}}{L'-\bar{x} + (L-L')\ K_s/K_p} \tag{12}$$

By employing this scheme for the first stage of outflow from saturated cores, marked reductions in computational time were obtained for the problems that we have investigated, with negligible concomitant loss in accuracy.

After all elements become unsaturated, program control is passed to the finite element routine. Equations (1), (2b), and (2c) are then solved with the modified initial condition:

$$h(x,t_o) = h_e \qquad (13)$$

$$t_o = \sum_{j=i}^{M-1} \Delta t_j$$

Cumulative outflow at time t during the finite element simulation is obtained as:

$$Q(t_i) = A \int_0^L [\theta(x,t) - \theta(x,0)]\, dx \qquad (14)$$

Note that the equilibrium pressure head distribution $h_f(x)=h(x,t=\infty)$ and therefore $\theta(h_f)$ may be obtained directly from the lower boundary condition (2c). The corresponding equilibrium outflow can thus be obtained immediately without the need to solve (1) for large times.

The finite element routine uses a variable weighted finite difference model to approximate time derivatives (van Genuchten, 1978b, eq. 14), where the temporal weighting coefficient ω may vary between 0 and 1. Stability requires that $0.5 \leq \omega \leq 1$. Choosing $\omega = 0.5$ corresponds to a second-order correct time-centered model. When $\omega=1$ is used, a first-order correct, backward difference scheme results. Coefficients K and C in (6) are always evaluated at the half-time level, independent of the time weighting scheme used. Due to the strong nonlinearity of (1), an iterative procedure is used to obtain the new pressure head distribution $h_{t+\Delta t}$, at the end of Δt. The general iteration scheme is:

$$h^{k+1}_{t+\Delta t} = \varepsilon h^{k}_{t+\Delta t} + (1-\varepsilon) h^{k-1}_{t+\Delta t} \qquad (15)$$

where ε is a weighting coefficient ($0 \leq \varepsilon \leq 1$) and k denotes iteration number. Iteration continues until a specified degree of correspondence is obtained between pressure head distributions before and after a certain iteration step. As in the original code, convergence is determined by the program variables TOL1 and TOL2. In most cases choosing $\varepsilon=1$ will lead to fastest convergence. In some cases, however, notably when simulating flow through coarse materials, the above scheme with $\varepsilon=1$ fails to converge rapidly due to oscillation in successive iterations. These oscillations are most pronounced when a time-centered ($\omega=0.5$) scheme is used and often may be effectively removed by switching to a backward difference scheme ($\omega=1$). In severe cases, oscillations can be further damped by using $\varepsilon=0.5$ in (15). Time step size is controlled by the number of iterations required for convergence. When the iteration fails to converge in a given number of steps (controlled by variable NITMAX), the time step is halved and the iteration started anew. Slow convergence may greatly increase computer costs, due to both the greater computational effort required per time step and to the increased number of (smaller) time steps. To avoid excessive computer costs, the number of times the iterative procedure fails to converge within NITMAX timesteps is counted (variable NOCON) and execution is interrupted when NOCON exceeds a specified limit (controlled by variable NOMAX).

SOLUTION OF INVERSE PROBLEM

Values of the unknown parameters defining the soil hydraulic properties in (3)-(5) are desired which minimize the objective function:

$$E(b) = \sum_{i=1}^{N} [w_i\{Q(t_t)-\hat{Q}(b,t_i)\}]^2 + \sum_{j=1}^{M}[v_j\{\theta(h_j)-\hat{\theta}(b,h_j)\}]^2 \tag{16}$$

where $Q(t_i)$ is the measured cumulative outflow at times t_i $(i=1,\ldots N)$, $Q(b,t_i)$ represents the numerically calculated cumulative outflow computed for the trial parameter vector b, $\theta(h_j)$ is the water content corresponding to pressure head h_j for equilibrium desorption, $\theta(b,h_j)$ is the predicted water content for trial parameter vector b as calculated from (3), and w_i and v_j are weighting factors. The second term in the objective function allows available data from equilibrium desorption experiments to be included in the optimization process. As shown by Parker et al. (1985), this inclusion will reduce the likelihood of nonuniqueness·problems during the inversion process The differences $\theta(h_j)-\theta(b,h_j)$ are automatically pre-weighted by the factor:

$$a = M\sum_{i=1}^{N} Q(t_i) \;/\; N\sum_{j=1}^{M} \theta(h_j) \tag{17}$$

which gives $\theta(h_j)$-values approximately the same weight as the $Q(t_i)$-observations. Unless otherwise specified in the program input, weighting coefficients w_i are fixed at unity, and coefficients v_j are set equal to a.

Parameters are adjusted until the optimum set of values, b^o, for which E(b) has a minimum, is obtained. Assuming that the problem has a unique solution, that is that E(b) has only one (global) minimum, then backsubstitution of the parameter values b^o into (3)-(5) yields the soil hydraulic properties.

The optimization algorithm used to evaluate b^o is based on Marquardt's maximum neighborhood method (Marquardt, 1963) which is an optimum combination of the method of steepest descent and the Gauss-Newton Taylor series method. The computer code for implementing this method was adapted from Meeter (1964). For highly nonlinear problems this method is efficient and quite robust.

The value of any of the five parameters θ_s, θ_r, K_s, α, and n may be either optimized or kept constant during optimization. Since θ_s and K_s can be measured relatively easily, it is recommended that measured values be used for these two parameters. Limiting the number of parameters to be optimized will reduce problems of non-uniqueness and will also reduce computational effort. The optimization algorithm evaluates the partial derivatives $\partial E/\partial j$ for each fitted parameter j at every iteration of the optimization routine, thus requiring for each iteration at least J+1 solutions of the flow problem, where J is the number of fitted parameters.

Both upper and lower limits may be specified for the parameters to be optimized. When no limits are specified, unconstrained optimization is used. Note that both upper and lower limits must be specified for constrained optimization. For the three parameter model (α, n, θ_r), it was found in practice to be unnecessary to specify limits for α. The van Genuchten model (eqs. 3-5) requires $n \geq 1$. Using a lower limit of 1.1 for n will avoid numerical problems that may occur when n is very close to 1. Finally, it is recommended to specify an upper limit for θ_r to insure $\theta_r < \theta_s$ to avoid occasional run time errors. The upper limit for θ_r may be equaled to the final water content for the transient flow experiment or to the lowest equilibrium water content if these data are available.

PROGRAM DESCRIPTION

The FORTRAN IV program ONESTEP couples nonlinear regression analysis and a numerical solution of the direct flow problem. ONESTEP consists of a main program (MAIN) and seven subprograms (MATINV, FLOW, STAGE1, TOTALM, BANSOL, MATEQ, SPR). The main program first reads data from an input file (unit=5) which is echoed back to an output file (unit=6). Input instructions and a glossary of important program variables are given in Appendices A and B, respectively.

Input data consist of a number of program parameters, dimensions of soil core and porous plate, and initial values for the five soil parameters (α, n, θ_r, θ_s and K_s). Each of the parameters can be either held constant or optimized (controlled by the value of INDEX for each parameter). Input data also specify the initial pressure head distribution and nodal spacings. Material properties are node-centered so the boundary between plate and soil must be located between two nodes with different material indices. Observed Q_i and t_i data (FO(I) and HO(I), respectively) are read in along with the weighting factors, WT(I), if nonuniform weighting is desired. If equilibrium h-θ data are included in the optimization, they are stored in the same arrays FO(I) and HO(I), respectively, following the Q(t) data. The variable ITYPE distinguishes whether data are Q(t) data (ITYPE=0) or θ(h) data (ITYPE=1). If WT(I) values are omitted, weighting factors default to WT(I)=1 for all observations. With respect to eqs. (16) and (17), WT corresponds to W_i for Q(t) data and to V_j/a for θ(h) data.

Most of the parameter estimation routine is carried out in MAIN which also outputs results when the optimization is completed, either because the parameters have converged (controlled by variable STOPCR) or because the maximum number of iterations of the optimization routine is reached (controlled by variable MIT). Subroutine MATINV performs the matrix inversion for the optimization routine, whereas subroutine FLOW controls the solution of the flow-equation. Subroutine STAGE1 checks whether the soil is initially saturated or unsaturated, and in the former case solves for the first stage of the outflow process. Subroutine MATEQ performs the finite element simulation, and performs the necessary calculations for assembly of the global matrix equation, which is subsequently solved in subroutine BANSOL to obtain the updated values of the pressure heads and their gradients. Subroutine TOTALM calculates the amount of water present in the sample at desired times by spatially integrating the nodal moisture contents. The function SPR generates the different soil hydraulic functions [$\theta(h)$, $K(h)$, $C(h)$, and $h(\theta)$] according to (3)-(5).

Versions of the program are currently operating on an IBM 3084 mainframe, Prime 750 minicomputer and IBM PC microcomputer. Minimum hardware requirements to run the program on an IBM PC are 70 K of random access memory, 1 disk drive and an 8087 math coprocessor. Machine readable copies of source and compiled code are available from the first author on request.

EXAMPLE PROBLEM

As an example of the parameter estimation program, results are given for a silt loam soil. An undisturbed core sample (3.95 cm long, 5.4 cm diameter) was taken from the field and equilibrated at zero tension in a Tempe pressure cell (Figure 2). For comparison with the transient method, the moisture retention characteristic $\theta(h)$ was first determined by stepwise increases of pneumatic pressure on the sample up to a pressure of 98 kPa (h^a = 10 m) with water loss measured at every step. Moisture contents for h=-30 and -150 m were measured on disturbed samples of the same soil. After resaturating, the pneumatic pressure was increased instantaneously to h^a=10 m and cumulative outflow was recorded with time. To maintain a constant head lower boundary condition during the one-step outflow test, the position of the measuring burette was adjusted every time a reading was made. Entrapped air beneath the porous plate was flushed through tube D by applying a peristaltic action on the lower tube C (see Figure 2). Upon completion of the one-step test the porous plate was removed and the soil resaturated. Saturated hydraulic conductivities of the soil and porous plate were measured by falling head tests using leaching solutions of 0.01 M $CaCl_2$.

The three unknown parameters α, n and θ_r were estimated from the observed cumulative outflow versus time data and also the measured θ at h=-150 m. Input and output files for this problem are given in Appendices C and D, respectively. Note that in our example all lengths are given in centimeters and times in hours; in general any consistent set of units may be used. Figure 3 compares the $\theta(h)$-curve obtained by parameter estimation with the experimental $\theta(h)$-data. In Figure 4, the predicted diffusivity curve is plotted with $D(\theta)$ values that were determined independently from one-step outflow measurements by the method of Passioura (1976).

Running this example problem required 45 seconds of CPU time on an IBM 3084 mainframe. The same problem on an IBM PC (with 8087) took about 60 minutes.

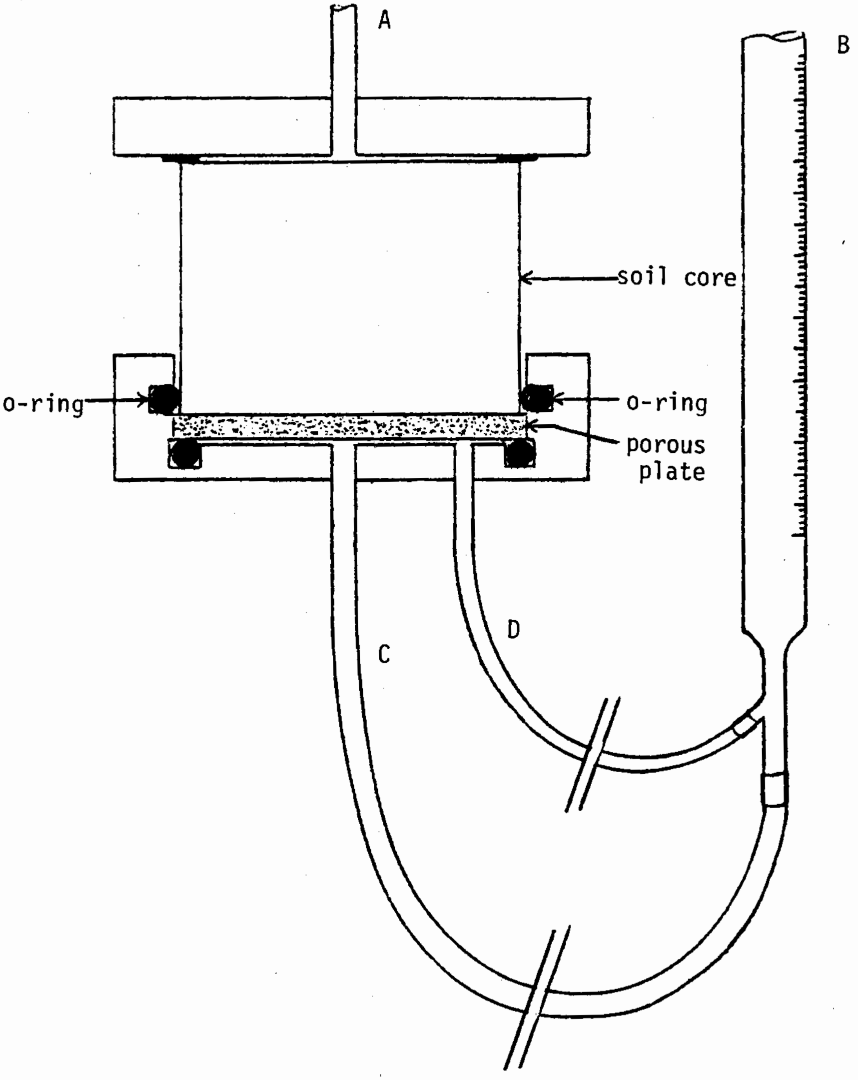

Figure 2. Cross-sectional view of Tempe pressure cell. Pneumatic pressure is applied through A. Outflow is collected and measured in burette B. Air is removed from beneath porous plate through the small diameter D via peristaltic action on tube C.

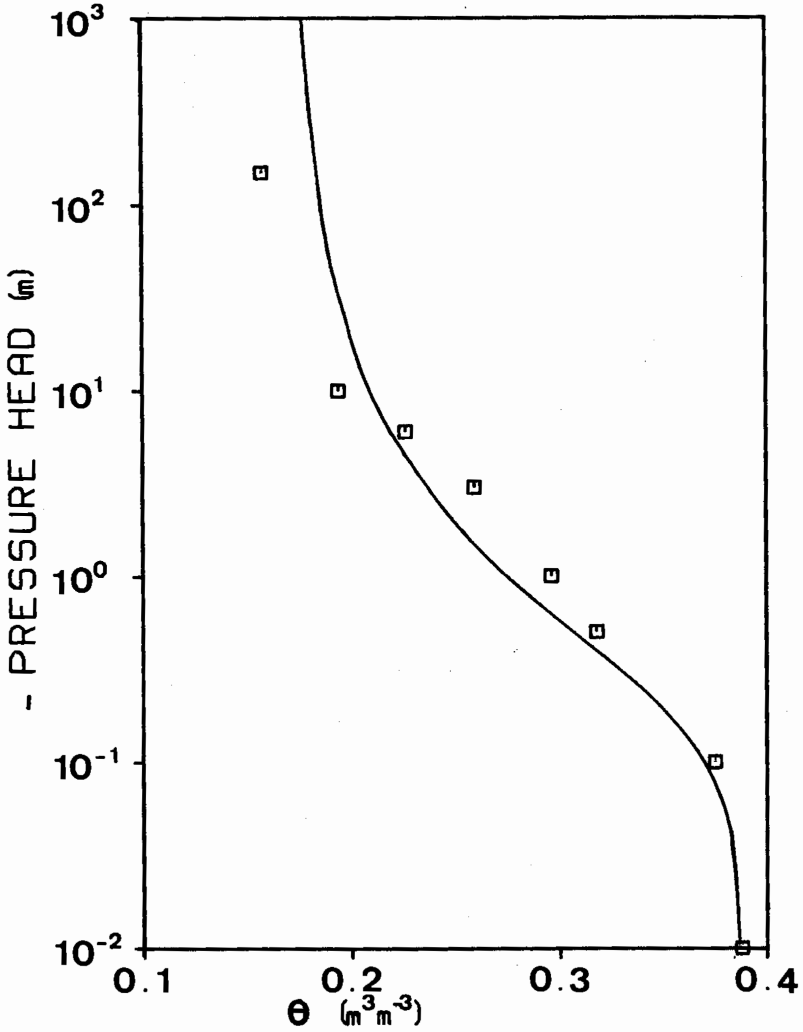

Figure 3. Observed (□) and predicted moisture characteristic for silt loam soil.

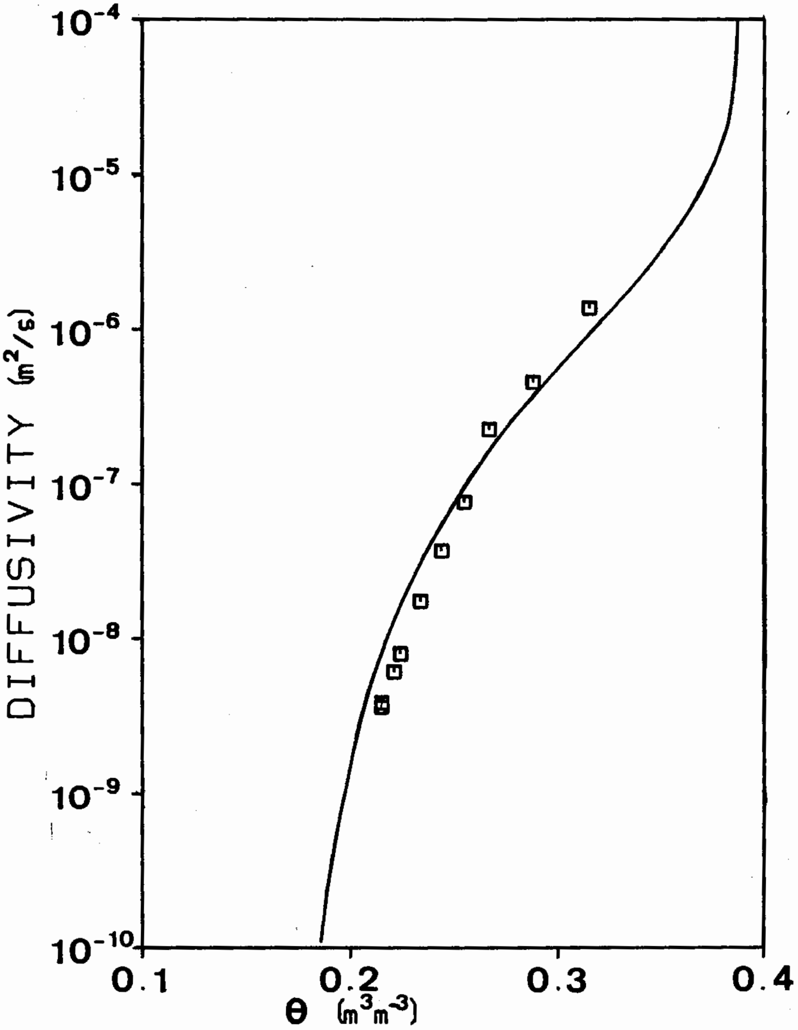

Figure 4. Hydraulic diffusivity calculated with method of Passioura (□) and predicted curve.

CONCLUSIONS AND FUTURE EXTENSIONS

Inversion of the unsaturated flow equation allows the water retention characteristic and hydraulic conductivity to be determined simultaneously from transient flow measurements. The one-step outflow method is a convenient procedure to obtain the necessary data from laboratory soil cores. Outflow versus time measurements, complemented with the measured 15 bar moisture content of the soil, in practice yield sufficient information to obtain a unique solution for the inverse problem. A requirement is that initial parameter estimates are sufficiently close to their final values. Reasonable initial estimates can be made on the basis of the texture of the sample. The FORTRAN IV program described in this bulletin specifically estimates the parameters in the van Genuchten model for hydraulic properties from outflow and/or equilibrium moisture content vs. pressure head data. Future work will involve the use of other input data for the inversion procedure. Measurements during infiltration into the soil can be used to obtain the wetting hydraulic properties of the soil. Used in conjunction with the outflow test, this may provide a convenient procedure for evaluating hysteresis. Also, extension of the method to use field measured infiltration and drainage data will be studied. Since most studies of soil-water characteristics are ultimately directed towards field-scale flow processes, in-situ measurements are more relevant than data obtained from generally small soil cores, the collection of which will unavoidably introduce some disturbance of the soil.

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APPENDIX A: Input instructions for ONESTEP

Card	Columns	Format	Variable	Comments
1	1-5	I5	NCASES	Number of cases considered. The remaining cards are read in for each case.
2	1-80	20A4	TITLE	Title
3	1-80	20A4	TITLE	Title
4				Blank card
5	1-5	I5	NN	Number of nodes
	6-10	I5	LNS	Number of nodes in soil
	11-20	E10.0	DNUL	Initial time step
	21-30	F10.0	PN1	Pressure head lower boundary
	31-40	F10.0	AIRP	Pneumatic head
	41-50	F10.0	EPS1	Temporal weighting coefficient
	51-60	F10.0	EPS2	Iteration weighting coefficient
6	1-10	F10.0	SLL	Length of soil core
	11-20	F10.0	PLL	Thickness of porous plate
	21-30	F10.0	DIAM	Diameter of soil core
	31-40	F10.0	CPLT	Conductivity of porous plate
	41-45	I5	NOBB	Number of observations
	46-50	I5	MDATA	Mode for input data
	51-55	I5	MODE	Mode for output
	56-60	I5	MIT	Maximum iterations of optimization routine
7				Blank card
8	1-10	F10.0	B(1)	Initial value of α
	11-20	F10.0	B(2)	Initial value of n
	21-30	F10.0	B(3)	Initial value of θ_r
	31-40	F10.0	B(4)	Initial value of θ_s

	41-50	F10.0	B(5)	Initial value of K_s
9	1-10	I10	INDEX(1)	Index for each
	11-20	I10	INDEX(2)	coefficient
	21-30	I10	INDEX(3)	
	31-40	I10	INDEX(4)	
	41-50	I10	INDEX(5)	
10	1-10	F10.0	BMIN(1)	Minimum value for
	11-20	F10.0	BMIN(2)	each coefficient
	21-30	F10.0	BMIN(3)	
	31-40	F10.0	BMIN(4)	
	41-50	F10.0	BMIN(5)	
11	1-10	F10.0	BMAX(1)	Maximum value for
	11-20	F10.0	BMAX(2)	each coefficient
	21-30	F10.0	BMAX(3)	
	31-40	F10.0	BMAX(4)	
	41-50	F10.0	BMAX(5)	
12[1]	1-5	I-5	K	Nodal number
	6-10	I5	MAT	Material index, 1=soil;2=plate
	11-20	F10.0	X(K)	Nodal coordinate
	21-30	F10.0	P(2K-1)	Initial pressure head
13+NN[2]	1-10	F10.0	HO(I)	Time (ITYPE=0) or pressure-head (ITYPE=1)
	11-20	F10.0	FO(I)	Outflow (ITYPE=0) or moisture content (ITYPE=1)
	21-30	F10.0	WT(I)	Weighting factor; omit for default weighting
	31-40	I5	ITYPE(I)	Data type: ITYPE=0 for Q(t), ITYPE=1 for θ(h)

---

[1] This card specifies the geometry and the initial conditions of the flow system, and is repeated for each of the NN nodes.

[2] This card contains the observed Q(t) and/or θ(h)-data and is repeated NOBB times.

APPENDIX B: List of most significant variables in ONESTEP

Units are given in brackets (L=length T=time), and name of the subprogram in which the variable is used is given in parentheses.

Variable	Definition
ALPHA	Parameter α $[L^{-1}]$ (SPR)
AEP	Air-entry value of h $[L]$ (STAGE1).
AIRP	Pneumatic head $[L]$.
AREA	Cross-sectional area of soil core $[L]$.
B(5)	Array containing initial values of parameters (MAIN); B(1)=α, B(2)=n, B(3)=θ_r, B(4)=θ_s, B(5)=K_s.
BI(10)	Array containing names of parameters (MAIN).
BMAX(5)	Array containing maximum permissible parameter values. See note on BMIN.
BMIN(5)	Array containing minimum permissible parameter values. If BMIN and BMAX are omitted for any parameter, their values will not be subject to any constraints.
BN(15),TH(15),TB(15)	Arrays containing present parameter values.
CONDS	Hydraulic conductivity $[LT^{-1}]$ (SPR).
CPLT	Hydraulic conductivity of porous plate $[LT^{-1}]$.

DELT	Timestep [T] (FLOW).
DELCH	Relative increase of DELT between two consecutive time steps (FLOW).
DELMAX	Maximum value for DELT [T] (FLOW).
DELMIN	Minimum value for DELT [T] (FLOW).
DIAM	Diameter of soil core [L] (MAIN).
DNUL	Initial time step [T] (FLOW).
EPS1	Temporal weighting coefficient, $0 \le EPS1 \le 1$ (FLOW) EPS1=0, 0.5 and 1 corresponds to forward, central and backward difference model respectively.
EPS2	Iteration weighting coefficient.
FC(25)	Array containing simulated results (FLOW).
FO(25)	Array containing observed results (FLOW).
HO(25)	Array containing observation times [T] or pressure heads [L].
INDEX(5)	Index for each parameter. If INDEX(I)=0, the parameter B(I) is known and kept constant; if INDEX(1)=1, B(I) represents only a initial guess and the best-fit value for the parameter is determined by optimization.
ISTEP	Time step counter (FLOW).
ITYPE	Flag for input data. ITYPE=0: Q(t) data, ITYPE=1: $\theta(h)$ data.
LNS	Number of nodes located in soil.

MAT	Material index; MAT=1: soil, MAT=2: porous plate.
MAXTRY	Maximum number of function evaluations within an iteration of the optimization routine to find new parameter values that decrease SSQ. Currently set to 20 in MAIN.
MDATA	Mode for outflow data. MDATA=1: transient outflow data only; MDATA=2: last Q(t) entry represents equilibrium outflow (t is dummy value).
MIT	Maximum number of iterations for optimization routine. (MAIN)
MODE	Mode for optimization routine. MODE=0: flow equation is solved only for initial parameter values. MODE=1: optimization process continues until parameter values converge or the number of iterations reaches MIT; all intermediate parameter values are printed. MODE=2: as MODE=1, except parameter values are only printed at end of every iteration. MODE=3: as MODE=2 but $\theta(h)$ and K(h) according to final parameter values are also printed. MODE=9: error condition, program continues with next case.
NN	Total number of nodes used in finite element solution. (FLOW)
NCASES	Number of cases considered (MAIN).
NIT	Iteration number within time step (FLOW).
NITMAX	Maximum for NIT (FLOW).
NITT	Iteration number for optimization routine (MAIN).
NOCON	Counter for number of times NIT exceeds NITMAX (FLOW).

NOB	Number of observations.
NOMAX	Maximum for NOCON (FLOW)
NP	Number of parameters to be optimized
NSTEPS	Maximum for ISTEP (FLOW).
P(NN2)	Array containing values of h and $\partial h/\partial x$ at previous time step (FLOW).
PE(NN2)	Array containing latest values of h and $\partial h/\partial x$ (FLOW).
PN1	Lower boundary condition during outflow process [L] (FLOW).
PLL	Thickness of porous plate (STAGE1).
QOUT	Cumulative outflow during first stage (STAGE1).
RELSAT	Relative saturation (θ/θ_s) at which the soil is assumed to be unsaturated (STAGE1).
SLL	Length of soil core [L] (STAGE1).
SPR(MAT,BN,N,PR)	Function to calculate the soil hydraulic properties. MAT: material index, MAT=1: soil, MAT=2: porous plate. NB: array containing parameter values. N: Index. N=1: $\theta(h)$ is calculated, N=2: K(h) is calculated. N=3: C(h) is calculated, N=4: $h(\theta)$ is calculated. PR: value of h (or θ when N=4).
STOPCR	Stop criterion for parameter optimization. Optimization stops when relative change in each parameter becomes less than STOPCR (MAIN).

SUMT	Cumulative time [T] (FLOW).
T(60)	Array containing intermediate values of h and $\partial h/\partial x$ (FLOW)
TE(60)	Array containing values of h and $\partial h/\partial x$ at previous iteration (FLOW).
TOL1,TOL2	Absolute and relative convergence criteria for iterative solution process (FLOW).
TMINIT	Initial amount of water in soil [L^3] (FLOW).
TMIN	Current amount of water in soil [L^3] (FLOW).
WCR	Parameter θ_r (SPR).
WCS	Parameter θ_s (SPR).
WT(25)	Array containing weights assigned to every observation.
X(30)	Array containing nodal coordinates [L].

APPENDIX C. INPUT FILE FOR EXAMPLE PROBLEM

```
    1
    SILT LOAM 8/04/84
    METHOD II INPUT DATA (PARKER ET AL, 1984)

   13   10    1.E-05   2.52    1000.0    1.0       1.0
  3.95     0.57      5.40      0.003       10    2    2   15

0.0250    1.50      0.200       0.388     5.400
         1         1         1         0         0
            1.1
           10.0        0.300
    1    1   0.0         -2.0
    2    1   0.5         -1.5
    3    1   1.0         -1.0
    4    1   1.5         -0.5
    5    1   2.0          0.0
    6    1   2.5          0.5
    7    1   3.0          1.0
    8    1   3.50         1.5
    9    1   3.75         1.75
   10    1   3.90         1.90
   11    2   4.00         2.00
   12    2   4.25         2.25
   13    2   4.52         2.52
   0.017    1.80
   0.033    3.70
   0.050    4.80
   0.167    7.80
   0.500   10.20
   1.033   11.40
   2.750   12.85
   5.417   13.59
 999.9     15.62
-15000.0    0.157        1
```

APPENDIX D. OUTPUT FILE FOR EXAMPLE PROBLEM

```
*******************************************************************************
*                                                                             *
*                                                                             *
*    SILT LOAM 8/04/84                                                        *
*    METHOD II INPUT DATA (PARKER ET AL, 1984)                                *
*                                                                             *
*******************************************************************************


PROGRAM PARAMETERS
==================
NUMBER OF NODES.....................(NN)................ 13
NODE AT SOIL-PLATE BOUNDARY.........(LNS)............... 10
INITIAL TIME STEP...................(DNUL)............ 0.10E-04
PRESSURE HEAD LOWER BOUNDARY........(PN1)............    2.520
PNEUMATIC PRESSURE..................(AIRP)............1000.000
TEMPORAL WEIGHTING COEFF............(EPS1)............   1.00
ITERATION WEIGHTING COEFF...........(EPS2)............   1.00
MAX. ITERATIONS.....................(MIT).............. 15
DATA MODE...........................(MDATA)............  2
NO. OF OBSERVATIONS.................(NOBB)............. 10


SOIL AND PLATE PROPERTIES
=========================
SOIL COLUMN LENGTH..................(SLL)............    3.950
COLUMN DIAMETER.....................(DIAM)...........    5.400
THICKNESS OF PLATE..................(PLL)............    0.570
PLATE CONDUCTIVITY..................(CONDS(2))......  .3000E-02
SATURATED MOISTURE CONTENT..........(WCS)............    0.388
RESIDUAL MOISTURE CONTENT...........(WCR)............    0.200
FIRST COEFFICIENT...................(ALPHA)..........    0.025
SECOND COEFFICIENT..................(N)..............    1.500
SATURATED CONDUCTIVITY SOIL.........(CONDS(1))......  .5400E+01


INITIAL CONDITIONS
==================
NODE  DEPTH  PRESSURE HEAD     MOISTURE CONTENT
  1    0.00     -2.000           0.3873
  2    0.50     -1.500           0.3875
  3    1.00     -1.000           0.3878
  4    1.50     -0.500           0.3879
  5    2.00      0.000           0.3880
  6    2.50      0.500           0.3880
  7    3.00      1.000           0.3880
  8    3.50      1.500           0.3880
  9    3.75      1.750           0.3880
 10    3.90      1.900           0.3880


OBSERVED OUTFLOW
================
   OBS      HRS      OUTFLOW      TYPE     WEIGHT
    1      0.017     1.800        0         1.0
    2      0.033     3.700        0         1.0
    3      0.050     4.800        0         1.0
    4      0.167     7.800        0         1.0
    5      0.500    10.200        0         1.0
    6      1.033    11.400        0         1.0
    7      2.750    12.850        0         1.0
```

```
    8     5.417     13.590          0         1.0
    9   999.900     15.620          0         1.0
   10-15000.000      0.157          1         1.0
ITERATION NO        SSQ             ALPHA           N              WCR
      0         14.3551178           0.0250        1.5000        0.2000
      1          8.6202688           0.0395        1.5927        0.1829
      2          5.1384296           0.0586        1.5318        0.1871
      3          4.3154287           0.0495        1.5358        0.1828
      4          4.0541239           0.0530        1.5177        0.1823
      5          3.8451042           0.0511        1.5077        0.1806
      6          3.6868916           0.0504        1.4988        0.1792
      7          3.5398378           0.0494        1.4891        0.1780
      8          3.4026499           0.0484        1.4813        0.1767
      9          3.2833033           0.0479        1.4737        0.1756
     10          3.1732512           0.0472        1.4667        0.1744
     11          3.0758791           0.0471        1.4610        0.1732

RSQUARE FOR REGRESSION OF PREDICTED VS OBSERVED =0.99930


CORRELATION MATRIX
==================
          1           2           3
   1    1.0000
   2    0.6918      1.0000
   3    0.5749      0.9510      1.0000


NON-LINEAR LEAST-SQUARES ANALYSIS: FINAL RESULTS
================================================
                                               95% CONFIDENCE LIMITS
VARIABLE         VALUE         S.E.COEFF.     LOWER         UPPER
  ALPHA         0.04705         0.0125        0.02         0.0767
  N             1.46097         0.1035        1.22         1.7058
  WCR           0.17321         0.0166        0.13         0.2124


--------OBSERVED & FITTED OUTFLOW --------
                                          RESI-
NO    TIME (HR)      OBS       FITTED     DUAL
 1      0.017      1.800      2.077     -0.277
 2      0.033      3.700      3.847     -0.147
 3      0.050      4.800      4.848     -0.048
 4      0.167      7.800      7.594      0.206
 5      0.500     10.200      9.975      0.225
 6      1.033     11.400     11.410     -0.010
 7      2.750     12.850     13.078     -0.228
 8      5.417     13.590     14.014     -0.424
 9    999.900     15.620     16.141     -0.521
10-15000.000      0.157      0.184     -0.027


FINAL CONDITIONS
================
                PRESSURE HEAD                MOISTURE CONTENT
NODE   DEPTH    FUNCTION     GRADIENT        FUNCTION    GRADIENT
  1    0.00    -1002.000       1.000          0.210       0.000
  2    0.50    -1001.500       1.000          0.210       0.000
  3    1.00    -1001.000       1.000          0.210       0.000
  4    1.50    -1000.500       1.000          0.210       0.000
  5    2.00    -1000.000       1.000          0.210       0.000
  6    2.50     -999.500       1.000          0.210       0.000
  7    3.00     -999.000       1.000          0.210       0.000
  8    3.50     -998.500       1.000          0.210       0.000
  9    3.75     -998.250       1.000          0.210       0.000
 10    3.90     -998.100       1.000          0.210       0.000


INITIAL AMOUNT OF MOISTURE IN SAMPLE       =   35.099 CM3
FINAL      ,,            ,,                =   18.910  ,,
```

APPENDIX E. PROGRAM LISTING

```
C
C
C     ****************************************************************
C     *                                                              *
C     *  EVALUATION OF SOIL HYDRAULIC PROPERTIES FROM                *
C     *  ONESTEP OUTFLOW DATA  BY PARAMETER ESTIMATION               *
C     *                                                              *
C     *                                   J.B. KOOL, VA TECH, 1984   *
C     ****************************************************************
C
C
      DIMENSION R(25),DELZ(25,5),BI(10),E(5),C(5),CHI(5),Q(5),TB(15),A(3
     15,5),D(5,5),TITLE(20),FC(25),TH(15),FO(25),BMIN(5),BMAX(5),W1(25),
     2B(5),TYPE(2)
      COMMON/AAA/X(30),P(60),ISPR(30),NN,AREA,LNS,PLL,SLL,PN1,HO(25),DNU
     1L,NOB,NOB2,TMINIT,EPS1,EPS2,INDEX(5),NVAR
      DATA STOPCR/.01/,TYPE(1)/'OUTF'/,TYPE(2)/'LOW '/,QRTPI/0.7853982/
      DATA TH(11),TH(12),TH(13),TH(14)/0.0001,8.0,0.19,0.2/
      DATA TB(11),TB(12),TB(13),TB(14)/0.0001,8.0,0.19,0.2/
      DATA BI(1),BI(2)/'ALPH','A   '/,BI(3),BI(4)/'N   ','    '/
      DATA BI(5),BI(6)/'WCR ','    '/,BI(7),BI(8)/'WCS ','    '/
      DATA BI(9),BI(10)/'COND','S   '/,MAXTRY/20/
      NVAR=5
C
C     ----- READ & WRITE TITLE AND PARAMETERS -----------
      READ(5,1000)NCASES
      DO 144 ICASE=1,NCASES
      WRITE(6,1002)
      DO 1 I=1,2
      READ(5,1004) TITLE
    1 WRITE(6,1006) TITLE
      WRITE(6,1008)
      READ(5,1014) NN,LNS,DNUL,PN1,AIRP,EPS1,EPS2
      READ(5,1016) SLL,PLL,DIAM,CPLT,NOBB,MDATA,MODE,MIT
      TH(15)=CPLT
      TB(15)=CPLT
      WRITE(6,1018) NN,LNS,DNUL,PN1,AIRP,EPS1,EPS2,MIT,MDATA,NOBB
      PN1=PN1-AIRP
C
C     ---- READ INITIAL VALUE OF COEFFICIENTS -----
      READ(5,1020) (B(I),I=1,NVAR)
      READ(5,1022) (INDEX(I),I=1,NVAR)
      READ(5,1020) (BMIN(I),I=1,NVAR)
      READ(5,1020) (BMAX(I),I=1,NVAR)
      WRITE(6,1026)SLL,DIAM,PLL,CPLT,B(4),B(3),B(1),B(2),B(5)
C
C     ---- READ INITIAL CONDITONS -----
      DO 6 I=1,NN
      READ(5,1028) K,MAT,X(K),Z1
      IF(K.EQ.I) GO TO 4
      WRITE(6,1030) I
      MODE=9
    4 ISPR(I)=MAT
      I2=2*I
      I1=I2-1
      P(I1)=Z1
      P(I2)=1.0
    6 CONTINUE
      AREA= QRTPI*DIAM**2
C
C     ------ REARRANGE PARAMETER ARRAY ------
```

```
      NU1=NVAR+1
      NU2=NVAR*2
      NP=0
      DO 8  I=NU1,NU2
      I1=I-NVAR
      TB(I)=B(I1)
      IF(INDEX(I1).EQ.0) GO TO 8
      NP=NP+1
      K=2*NP-1
      J=2*I1-1
      BI(K)=BI(J)
      BI(K+1)=BI(J+1)
      B(NP)=B(I1)
      TB(NP)=B(I1)
      TH(NP)=B(NP)
      BMIN(NP)=BMIN(I1)
      BMAX(NP)=BMAX(I1)
    8 TH(I)=B(I1)
C
C     ----- WRITE INITIAL CONDITIONS -----
      WRITE(6,1024)
      DO 10 L=1,LNS
      I=2*L-1
      I1=I+1
      Z1=SPR(1,TB,1,P(I))
      WRITE(6,1062) L,X(L),P(I),Z1
   10 CONTINUE
C
C     ----- READ & WRITE INPUT DATA -----
      WRITE(6,1032) TYPE(1),TYPE(2),TYPE(1),TYPE(2)
      N1=0
      DO 12 N=1,NOBB
      READ(5,1021) HO(N),FO(N),ITYPE,WT(N)
      IF(WT(N).EQ.0.0) WT(N)=1.0
      WRITE(6,1036) N,HO(N),FO(N),ITYPE,WT(N)
      IF(ITYPE.EQ.0) N1=N1+1
   12 CONTINUE
      IF(MODE.EQ.9) GO TO 144
      NOB=N1
      NOB2=NOBB-NOB
C
C     ----- DEFAULT WEIGHTING FOR THETA(H) DATA ----
      QAVE=0.0
      THAVE=0.0
      DEFWT=1.0
      IF(NOB.EQ.0.OR.NOB2.EQ.0) GO TO 20
      DO 14 I=1,NOB
   14 QAVE=QAVE+FO(I)
      QAVE=QAVE/NOB
      I1=NOB+1
      DO 16 I=I1,NOBB
   16 THAVE=THAVE+FO(I)
      THAVE=THAVE/NOB2
      DEFWT=QAVE/THAVE
C
C     ----- COMPUTE OUTFLOW FOR INITIAL PARAMETER VALUES ------
   20 GA=0.02
      NOCON=0
      NITT=0
      NP2=2*NP
      CALL FLOW(TH,FC,NITT,NOCON,MDATA,MODE)
      IF(MODE.EQ.9) GO TO 144
      SSQ=0.
      DO 32 I=1,NOBB
      R(I)=WT(I)*(FO(I)-FC(I))
      IF(I.GT.NOB)R(I)=R(I)*DEFWT
   32 SSQ=SSQ+R(I)*R(I)
      IF(NP.NE.0)WRITE(6,1040) (BI(I),I=1,NP2)
      IF(NP.EQ.0)WRITE(6,1040)
      IF(NP.NE.0)WRITE(6,1042) NITT,SSQ,(TH(I),I=1,NP)
      IF(NP.EQ.0)WRITE(6,1042) NITT,SSQ
      IF(MODE.EQ.0.OR.NP.EQ.0) GO TO 110
```

```
C
C      ----- BEGIN OF ITERATION -----
   34 NITT=NITT+1
      NET=0
      GA=0.1*GA
      DO 38 J=1,NP
      TEMP=TH(J)
      TH(J)=1.01*TH(J)
      Q(J)=0.
      CALL FLOW(TH,DELZ(1,J),NITT,NOCON,MDATA,MODE)
      IF(MODE.EQ.1)WRITE(6,1038) (TH(I),I=1,NP)
      IF(MODE.EQ.9) GO TO 144
      DO 36 I=1,NOBB
      DELZ(I,J)=WT(I)*(DELZ(I,J)-FC(I))
      IF(I.GT.NOB) DELZ(I,J)=DELZ(I,J)*DEFWT
   36 Q(J)=Q(J)+DELZ(I,J)*R(I)
      Q(J)=100.*Q(J)/TH(J)
C
C      ----- STEEPEST DESCENT -----
   38 TH(J)=TEMP
      DO 44 I=1,NP
      DO 42 J=1,I
      SUM=0.
      DO 40 K=1,NOBB
   40 SUM=SUM+DELZ(K,I)*DELZ(K,J)
      D(I,J)=10000.*SUM/(TH(I)*TH(J))
   42 D(J,I)=D(I,J)
C
C      ----- D = MOMENT MATRIX -----
   44 E(I)=SQRT(D(I,I))
   50 DO 52 I=1,NP
      DO 52 J=1,NP
   52 A(I,J)=D(I,J)/(E(I)*E(J))
C
C      ----- A IS THE SCALED MOMENT MATRIX -----
      DO 54 I=1,NP
      C(I)=Q(I)/E(I)
      CHI(I)=C(I)
   54 A(I,I)=A(I,I)+GA
      CALL MATINV(A,NP,C)
C
C      ----- C/E IS THE CORRECTION VECTOR -----
      STEP=1.0
   56 NET=NET+1
      DO 58 I=1,NP
      TB(I)=C(I)*STEP/E(I)+TH(I)
      IF(BMIN(I).EQ.BMAX(I)) GO TO 58
      IF(TB(I).LT.BMIN(I)) TB(I)=BMIN(I)
      IF(TB(I).GT.BMAX(I)) TB(I)=BMAX(I)
      C(I)=(TB(I)-TH(I))*E(I)/STEP
   58 CONTINUE
   60 DO 62 I=1,NP
      IF(TH(I)*TB(I))66,66,62
   62 CONTINUE
      SUMB=0.0
      CALL FLOW(TB,FC,NITT,NOCON,MDATA,MODE)
      IF(MODE.EQ.1)WRITE(6,1038) (TB(I),I=1,NP)
      IF(MODE.EQ.9) GO TO 144
      DO 64 I=1,NOBB
      R(I)=WT(I)*(FO(I)-FC(I))
      IF(I.GT.NOB)R(I)=R(I)*DEFWT
   64 SUMB=SUMB+R(I)*R(I)
   66 SUM1=0.0
      SUM2=0.0
      SUM3=0.0
      DO 68 I=1,NP
      SUM1=SUM1+C(I)*CHI(I)
      SUM2=SUM2+C(I)*C(I)
   68 SUM3=SUM3+CHI(I)*CHI(I)
      ARG=SUM1/SQRT(SUM2*SUM3)
      ANGLE=57.29578*ATAN2(SQRT(1.-ARG*ARG),ARG)
C
```

```
C     ----------
      DO 72 I=1,NP
      IF(TH(I)*TB(I))74,74,72
   72 CONTINUE
      IF(NET.GE.MAXTRY) GO TO 79
      IF(SUMB/SSQ-1.0)80,80,74
   74 IF(ANGLE-30.0)76,76,78
   76 STEP=STEP/2.0
      GO TO 56
   78 GA=10.*GA
      GO TO 50
   79 WRITE(6,1086)
      GO TO 96
C
C     ----- PRINT COEFFICIENTS AFTER EACH ITERATION -----
   80 CONTINUE
      DO 82 I=1,NP
   82 TH(I)=TB(I)
      WRITE(6,1042)NITT,SUMB,(TB(I),I=1,NP)
C
   90 DO 92 I=1,NP
      IF(ABS(C(I)*STEP/E(I))/(1.0E-20+ABS(TH(I)))-STOPCR) 92,92,94
   92 CONTINUE
      GO TO 96
   94 SSQ=SUMB
      IF(NITT.LT.MIT) GO TO 34
C
C     ----- END OF ITERATION LOOP -----
   96 CONTINUE
      CALL MATINV(D,NP,C)
C
C     ----- WRITE RSQUARE, CORRELATION MATRIX -----
      SUMO=0.0
      SUMC=0.0
      SUMO2=0.0
      SUMC2=0.0
      SUMOC=0.0
      DO 98 I=1,NOBB
      SUMO=SUMO+FO(I)
      SUMC=SUMC+FC(I)
      SUMO2=SUMO2+FO(I)*FO(I)
      SUMC2=SUMC2+FC(I)*FC(I)
   98 SUMOC=SUMOC+FO(I)*FC(I)
      RSQ=(SUMOC-SUMO*SUMC/NOBB)**2/((SUMO2-SUMO*SUMO/NOBB)*(SUMC2-SUMC*
     1SUMC/NOBB))
      WRITE(6,1050) RSQ
      DO 100 I=1,NP
  100 E(I)=SQRT(D(I,I))
      WRITE(6,1046) (I,I=1,NP)
      DO 104 I=1,NP
      DO 102 J=1,I
  102 A(J,I)=D(J,I)/(E(I)*E(J))
  104 WRITE(6,1048) I,(A(J,I),J=1,I)
C
C     ----- CALCULATE 95% CONFIDENCE INTERVAL -----
      Z=1./FLOAT(NOBB-NP)
      SDEV=SQRT(Z*SUMB)
      WRITE(6,1052)
      TVAR=1.96+Z*(2.3779+Z*(2.7135+Z*(3.187936+2.466666*Z**2)))
      DO 108 I=1,NP
      SECOEF= E(I)*SDEV
      TSEC=TVAR*SECOEF
      TMCOE=TH(I)-TSEC
      TPCOE=TH(I)+TSEC
      K=2*I
      J=K-1
  108 WRITE(6,1058) BI(J),BI(K),TH(I),SECOEF,TMCOE,TPCOE
C
C     ----- PREPARE FINAL OUTPUT -----
  110 WRITE(6,1066) TYPE(1),TYPE(2)
      DO 118 I=1,NOBB
      R(I)=FO(I)-FC(I)
  118 WRITE(6,1068) I,HO(I),FO(I),FC(I),R(I)
```

```
C
C      ------ FINAL PRESSURE HEAD AND MOISTURE CONTENT DISTRIBUTION -----
  122 TMIN= 0.
      WRITE(6,1080)
      NE=LNS-1
      DO 130 L=1,NE
      I=2*L-1
      I1=I+1
      I2=I+2
      I3=I+3
      EL=X(L+1)-X(L)
      P13=.7407407*P(I)+.2592593*P(I2)+.0740741*EL*(2.*P(I1)-P(I3))
      P23=.2592593*P(I)+.7407407*P(I2)+.0740741*EL*(P(I1)-2.*P(I3))
      Z1=SPR(1,TB,1,P(I))
      Z2=SPR(1,TB,3,P(I))*P(I1)
      Z3=SPR(1,TB,1,P13)
      Z4=SPR(1,TB,1,P23)
      Z5=SPR(1,TB,1,P(I2))
      WRITE(6,1060) L,X(L),P(I),P(I1),Z1,Z2
  130 TMIN=TMIN+EL*(0.5*(Z1+Z5)+Z3+Z4)/3.
      TMIN=TMIN*AREA
      Z2=SPR(1,TB,3,P(I2))*P(I3)
      WRITE(6,1060) LNS,X(LNS),P(I2),P(I3),Z5,Z2
      TMINIT=TMINIT-TH(14)*AREA*PLL
      WRITE(6,1082) TMINIT,TMIN
      IF(MODE.LE.2) GO TO 144
C
C      ----- WRITE SOIL HYDRAULIC PROPERTIES -----
      WRITE(6,1069)
      PRESS=1.18850
      SN1=0.0
      RKLN=1.0
      SALPHA=TB(6)
      SN=TB(7)
      SWCR=TB(8)
      SWCS=TB(9)
      SCONDS=TB(10)
      WRITE(6,1072) SN1,SWCS,RKLN,SCONDS
      DO 140 I=1,75
      IF(RKLN.LT.(-16.)) GO TO 142
      PRESS=1.18850*PRESS
      SM=1.-1./SN
      SN1=SM*SN
      RWC=1./(1.+(SALPHA*PRESS)**SN)**SM
      WC=SWCR+(SWCS-SWCR)*RWC
      TERM=1.-RWC*(SALPHA*PRESS)**SN1
      IF((TERM.LT.5.E-05).OR.(RWC.LT.0.06)) TERM = SM*RWC**(1./SM)
      RK=SQRT(RWC)*TERM*TERM
      TERM=SALPHA*SN1*(SWCS-SWCR)*RWC*RWC**(1./SM)*(SALPHA*PRESS)**
     1(SN-1.)
      AK=SCONDS*RK
      DIFFUS=AK/TERM
      PRLN=ALOG10(PRESS)
      AKLN=ALOG10(AK)
      RKLN=ALOG10(RK)
      DIFLN=ALOG10(DIFFUS)
  140 WRITE(6,1070) PRESS,PRLN,WC,RK,RKLN,AK,AKLN,DIFFUS,DIFLN
  142 CONTINUE
  144 CONTINUE
C
C      ----- END OF PROBLEM -----
 1000 FORMAT(I5)
 1002 FORMAT(1H1,10X, 82(1H*)/11X,1H*,80X,1H*/11X,1H*,80X,1H*)
 1004 FORMAT(20A4)
 1006 FORMAT(11X,1H*,20A4,1H*)
 1008 FORMAT(11X,1H*,80X,1H*/11X,82(1H*))
 1014 FORMAT(/2I5,E10.1,4F10.0)
 1016 FORMAT(4F10.0,4I5/)
 1018 FORMAT(//11X,'PROGRAM PARAMETERS'/11X,18(1H=)/11X,
     +'NUMBER OF NODES.....................(NN)...............',I3/11X,
     +'NODE AT SOIL-PLATE BOUNDARY.........(LNS)..............',I3/11X,
     +'INITIAL TIME STEP...................(DNUL).............',E9.2/11X,
     +'PRESSURE HEAD LOWER BOUNDARY........(PN1)..............',F8.3/11X,
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     +'PNEUMATIC PRESSURE..................(AIRP)............',F8.3/11X,
     +'TEMPORAL WEIGHTING COEFF............(EPS1)............',F7.2/11X,
     +'ITERATION WEIGHTING COEFF...........(EPS2)............',F7.2/11X,
     +'MAX. ITERATIONS.....................(MIT).............',I3/11X,
     +'DATA MODE...........................(MDATA)...........',I3/11X,
     +'NO. OF OBSERVATIONS.................(NOBB)............',I3)
1020 FORMAT(5F10.0)
1021 FORMAT(2F10.0,I5,F10.0)
1022 FORMAT(5I10)
1026 FORMAT(//11X,'SOIL AND PLATE PROPERTIES'/11X,25(1H=)/11X,
     +'SOIL COLUMN LENGTH..................(SLL).............',F8.3/11X,
     +'COLUMN DIAMETER.....................(DIAM)............',F8.3/11X,
     +'THICKNESS OF PLATE..................(PLL).............',F8.3/11X,
     +'PLATE CONDUCTIVITY..................(CONDS(2))......  ',E9.4/11X,
     +'SATURATED MOISTURE CONTENT..........(WCS).............',F8.3/11X,
     +'RESIDUAL MOISTURE CONTENT...........(WCR).............',F8.3/11X,
     +'FIRST COEFFICIENT...................(ALPHA)...........',F8.3/11X,
     +'SECOND COEFFICIENT..................(N)...............',F8.3/11X,
     +'SATURATED CONDUCTIVITY SOIL.........(CONDS(1))......  ',E9.4)
1024 FORMAT(//11X,'INITIAL CONDITIONS'/11X,18(1H=)/11X,'NODE',2X,'DEPTH
    1',2X,'PRESSURE HEAD',5X,'MOISTURE CONTENT')
1028 FORMAT(2I5,2F10.0)
1030 FORMAT(//5X,8(1H*),'ERROR ENCOUNTERED WHILE READING INITIAL CONDIT
    1IONS, CHECK NODE',I4,1X,'EXECUTION TERMINATED',9(1H*))
1032 FORMAT(//11X,'OBSERVED ',2A4/11X,16(1H=)/14X,'OBS',5X,'HRS',5X,2A4
    1,4X,'TYPE',4X,'WEIGHT')
1036 FORMAT(11X,I5,2F10.3,5X,I3,F10.1)
1038 FORMAT(42X,5(F8.4,3X))
1040 FORMAT(1H1,10X,'ITERATION NO',6X,'SSQ',5X,5(6X,A4,A2))
1042 FORMAT(15X,I2,5X,F12.7,8X,5(F8.4,3X))
1046 FORMAT(//11X,'CORRELATION MATRIX'/11X,18(1H=)/14X,10(4X,I2,5X))
1048 FORMAT(11X,I3,10(2X,F7.4,2X))
1050 FORMAT(//11X,'RSQUARE FOR REGRESSION OF PREDICTED VS OBSERVED =',
    1F7.5)
1052 FORMAT(//11X,'NON-LINEAR LEAST-SQUARES ANALYSIS: FINAL RESULTS'/
    111X,48(1H=)/53X,'95% CONFIDENCE LIMITS'/11X,'VARIABLE',8X,'VALUE',
    27X,'S.E.COEFF.',4X,'LOWER',8X,'UPPER')
1058 FORMAT(13X,A4,A2,4X,F10.5,5X,F9.4,5X,F6.2,4X,F9.4,5X,F9.4)
1060 FORMAT(10X,I4,1X,F7.2,2F11.3,3X,2F9.3)
1062 FORMAT(10X,I4,1X,F7.2,F11.3,8X,F9.4)
1066 FORMAT(//10X,8(1H-),'OBSERVED & FITTED ',2A4,8(1H-)/45X,'RESI-'/1
    10X,'NO',3X,'TIME (HR)',4X,'OBS',4X,'FITTED',4X,'DUAL')
1068 FORMAT(10X,I2,F10.3,1X,3F9.3)
1069 FORMAT(1H1,10X,'PRESSURE',4X,'LOG P',6X,'WC',7X,'REL K',5X,'LOG RK
    1',6X,'ABS K',4X,'LOG KA',5X,'DIFFUS',5X,'LOG D')
1070 FORMAT(10X,E10.3,F8.3,F10.4,3(E13.3,F8.3))
1072 FORMAT(10X,E10.3,8X,F10.4,E13.3,8X,E13.3)
1080 FORMAT(//11X,'FINAL CONDITIONS'/11X,16(1H=)/25X,'PRESSURE HEAD',12
    1X,'MOISTURE CONTENT'/11X,'NODE',2X,'DEPTH',3X,'FUNCTION',3X,'GRADI
    2ENT',6X,'FUNCTION',3X,'GRADIENT')
1082 FORMAT(//11X,'INITIAL AMOUNT OF MOISTURE IN SAMPLE     =',F8.3,' C
    1M3'/11X,'FINAL         ,,           ,,                =',F8.3,'  ,, ')
1086 FORMAT(//11X,'NO FURTHER REDUCTION IN SSQ OBTAINED, OPTIMIZATION S
    1TOPPED')
      STOP
      END
      SUBROUTINE MATINV(A,NP,B)
      DIMENSION A(5,5),B(5),INDX(5,2)
      DO 2 J=1,5
    2 INDX(J,1)=0
      I=0
    4 AMAX=-1.0
      DO 10 J=1,NP
      IF(INDX(J,1)) 10,6,10
    6 DO 10 K=1,NP
      IF(INDX(K,1)) 10,8,10
    8 P=ABS(A(J,K))
      IF(P.LE.AMAX) GO TO 10
      IR=J
      IC=K
      AMAX=P
   10 CONTINUE
```

```
      IF(AMAX) 30,30,14
   14 INDX(IC,1)=IR
      IF(IR.EQ.IC) GO TO 18
      DO 16 L=1,NP
      P=A(IR,L)
      A(IR,L)=A(IC,L)
   16 A(IC,L)=P
      P=B(IR)
      B(IR)=B(IC)
      B(IC)=P
      I=I+1
      INDX(I,2)=IC
   18 P=1./A(IC,IC)
      A(IC,IC)=1.0
      DO 20 L=1,NP
   20 A(IC,L)=A(IC,L)*P
      B(IC)=B(IC)*P
      DO 24 K=1,NP
      IF(K.EQ.IC) GO TO 24
      P=A(K,IC)
      A(K,IC)=0.0
      DO 22 L=1,NP
   22 A(K,L)=A(K,L)-A(IC,L)*P
      B(K)=B(K)-B(IC)*P
   24 CONTINUE
      GO TO 4
   26 IC=INDX(I,2)
      IR=INDX(IC,1)
      DO 28 K=1,NP
      P=A(K,IR)
      A(K,IR)=A(K,IC)
   28 A(K,IC)=P
      I=I-1
   30 IF(I) 26,32,26
   32 RETURN
      END
      SUBROUTINE FLOW(BN,FC,NITT,NOCON,MDATA,MODE)
      DIMENSION T(60),FC(25),PIN(60),PE(60),TE(60),BN(15)
      COMMON/AAA/X(30),P(60),ISPR(30),NN,AREA,LNS,PLL,SLL,PN1,HO(25),DNU
     1L,NOB,NOB2,TMINIT,EPS1,EPS2,INDEX(5),NVAR
      COMMON/ST1/IOBS,SMT1,ISTP1,IFLAG,OLDT
      DATA NITMAX/10/,TOL1/1.00/,TOL2/0.005/,NSTEPS/300/,DELMAX/0.5/
      DATA NOMAX/45/
C
C     ----- UPDATE PARAMETER ARRAY ----
      K=0
      NU1=NVAR+1
      NU2=NVAR*2
      DO 2 I=NU1,NU2
      IF(INDEX(I-NVAR).EQ.0) GO TO 2
      K=K+1
      BN(I)=BN(K)
    2 CONTINUE
C     ----- DEFINE INITIAL CONDITIONS & CALCULATE OUTFLOW
C                                  DURING SATURATED STAGE--------
      EPSM=1.-EPS2
      ISTP1=1
      IOBS=1
      QOUT=0.
      IFLAG=0
      SMT1=0
      NN2=2*NN
      IF(NITT.GT.0) GO TO 5
      DO 4 I=1,NN2
      PIN(I)=P(I)
    4 CONTINUE
      NOB1=NOB-1
      IF(MDATA.LE.1) NOB1=NOB
    5 DO 6 I=1,NN2
      P(I)=PIN(I)
    6 CONTINUE
      P(NN2-1)=PN1
```

```
C
C     ----- SOLVE FOR FIRST STAGE OF OUTFLOW -----
      CALL STAGE1(BN,FC,QOUT)
      DO 8 I=1,NN2
      PE(I)=P(I)
    8 CONTINUE
C
C     ------ DETERMINE AMOUNT OF WATER IN SAMPLE -----
      CALL TOTALM(P,BN,TMIN)
      TMINIT=TMIN+QOUT
      DELT=DNUL
      DELMIN=0.005*DNUL
      DIN=DELT
      SUMT=SMT1+DELT
      ISTEP=ISTP1
C
C     ----- DYNAMIC PART OF MODEL -----
      N1=NN2-1
    9 NIT=0
C     IF(MODE.EQ.0)WRITE(6,1002) NIT,DELT,ISTEP,SUMT,(PE(I),I=1,NN2)
   10 DO 12 I=1,NN2
      T(I)=PE(I)
   12 CONTINUE
      NIT=NIT+1
      CALL MATEQ(BN,EPS1,PE,DELT)
      IF(NIT.GT.1) GO TO 14
      DO 13 I=1,NN2
   13 TE(I)=PE(I)
C
C     ----- CHECK ITERATIVE PROCESS -----
   14 DO 15 I=1,NN2,2
      TOL=TOL1+TOL2*ABS(T(I))
      IF(ABS(PE(I)-T(I)).GT.TOL) GO TO 16
   15 CONTINUE
C     IF(MODE.EQ.0)WRITE(6,1002) NIT,DELT,ISTEP,SUMT,(PE(I),I=1,NN2)
      IF(DELT.LT.DELMIN) GO TO 19
      GO TO 28
   16 IF(NIT.GE.NITMAX) GO TO 18
      DO 17 I=1,NN2
      TEMP=EPS2*PE(I)+EPSM*TE(I)
      TE(I)=PE(I)
      PE(I)=TEMP
   17 CONTINUE
      GO TO 10
   18 NOCON=NOCON+1
      DELT=0.5*DELT
      IF(DELT.GE.DELMIN.AND.NOCON.LE.NOMAX) GO TO 22
      IF(NOCON.GT.NOMAX) WRITE(6,1007)
   19 IF(DELT.LT.DELMIN) WRITE(6,1008) DELT,DELMIN,SUMT,NITT
      WRITE(6,1009)
      DO 20 I=1,NN
      J=2*I-1
      WRITE(6,1010)I,X(I),P(J),P(J+1),PE(J),PE(J+1)
   20 CONTINUE
      MODE=9
      RETURN
   22 SUMT=SUMT-DELT
      DO 24 I=1,NN2
   24 PE(I)=0.5*(P(I)+PE(I))
      GO TO 9
C
C     ---- CALCULATE CUM. OUTFLOW IF IFLAG=1 ------
C
   28 IF(IFLAG.EQ.0) GO TO 34
      DELF=(SUMT-HO(IOBS))/DELT
      DO 30 I=1,NN2
      T(I)=PE(I)-DELF*(PE(I)-P(I))
   30 CONTINUE
      CALL TOTALM(T,BN,TMIN)
      FC(IOBS)=TMINIT-TMIN
      IOBS=IOBS+1
      IFLAG=0
C
```

```
C     ----- PREPARE FOR NEXT TIME STEP -----
   34 IF(IOBS.GT.NOB1.OR.ISTEP.GE.NSTEPS) GO TO 40
      DELCH=1.0
      IF(NIT.LE.2) DELCH=1.25
      IF(NIT.GE.6) DELCH=0.80
      DELCH=AMIN1(DELCH,DELMAX/DELT)
      DELT=DELT*DELCH
      DO 36 J=1,NN2
      PE1=PE(J)-P(J)
      P(J)=PE(J)
      PE(J)=P(J)+DELCH*PE1
   36 CONTINUE
      SUMT=SUMT+DELT
      IF(SUMT.GE.HO(IOBS)) IFLAG=1
      ISTEP=ISTEP+1
      GO TO 9
C
C     --------------------
   40 IF(ISTEP.GE.NSTEPS) WRITE(6,1014)NSTEPS,SUMT,NITT
      IF(MDATA.LE.1) GO TO 42
C
C     ---- COMPUTE EQUILIBRIUM OUTFLOW IF MDATA=2 ----
      XZERO=PLL+SLL-PN1
      DO 41 I=1,NN
      J=2*I-1
      J1=J+1
      PE(J)=X(I)-XZERO
      PE(J1)=1.0
   41 CONTINUE
      CALL TOTALM(PE,BN,TMIN)
      FC(IOBS)=TMINIT-TMIN
   42 DO 44 I=1,NN2
      P(I)=PE(I)
   44 CONTINUE
C
C     ------ CALCULATE THETA(H) IF NOB2 > 0 -----
      IF(NOB2.EQ.0) GO TO 60
      N1=NOB+1
      N2=NOB+NOB2
      DO 46 I=N1,N2
      FC(I)=SPR(1,BN,1,HO(I))
   46 CONTINUE
   60 RETURN
C
C     ----------
 1001 FORMAT(11X,4E10.3)
 1002 FORMAT(/11X,'PE(I) DURING ITERATION (NIT=',I3,' DELT=',E10.2,' IST
     1EP=',I4,' SUMT=',E10.3,')'/(10X,10F11.3))
 1003 FORMAT(11X,2F10.5)
 1007 FORMAT(//11X,'TROUBLE CONVERGING, START AGAIN WITH DIFFERENT INITI
     1AL VALUES ')
 1008 FORMAT(//11X,8(1H*),'DELT =',E11.4,', IS LESS THAN DELMIN (=',E11.
     14,'). EXECUTION TERMINATED AT TIME =',E11.4,' (NIT=)',I5)
 1009 FORMAT(//1X,' LAST CALCULATED VALUES'/11X,22(1H*)/11X,'NODE',5X,'D
     1EPTH',9X,'P(I)',6X,'GRADIENT',9X,'PE(I)',6X,'GRADIENT')
 1010 FORMAT(11X,I4,F10.2,2(3X,2F12.4))
 1014 FORMAT(/11X,'NO. OF STEPS EXCEEDS',I4,' AT TIME=',F10.3,' DURING
     1ITERATION',I4)
 1016 FORMAT(11X,I5,F10.3,2(2X,F10.3))
      END
      SUBROUTINE STAGE1(BN,FC,QOUT)
C
C     PURPOSE: SIMULATE FIRST (SATURATED) STAGE OF FLOW
C
      DIMENSION BN(15),FC(25)
      COMMON/AAA/X(30),P(60),ISPR(30),NN,AREA,LNS,PLL,SLL,PN1,HO(25)
      COMMON/ST1/IOBS,SUMT,ISTEP,IFLAG,OLDT
      DATA RELSAT/0.99/
      LPLUS=LNS+1
      UNSAT= RELSAT*BN(9)
      AEP=SPR(1,BN,4,UNSAT)
   10 DELWC=BN(9)-UNSAT
      DO 20 I=1,LNS
```

```
      J=2*I-1
      IF(P(J).GT.AEP) GO TO 21
   20 CONTINUE
      DELH=P(LNS)-PN1
      GO TO 37
   21 IF(I.GT.1) GO TO 22
      DELH=P(1)-PN1-X(NN)
      EL=X(2)-X(1)
      GRAD=BN(15)*DELH/(BN(10)*PLL+BN(15)*SLL)
      I=2
      GO TO 24
   22 IMIN=I-1
      DELH=AEP+X(IMIN)-PN1-X(NN)
      EL=X(I)-X(IMIN)
      TS=SLL-X(I)+0.5*EL
      GRAD=BN(15)*DELH/(BN(10)*PLL+BN(15)*TS)
   24 OLDT=SUMT
      QOLD=QOUT
      QOUT=QOUT+ DELWC*AREA*EL
      DELT=DELWC*EL/(BN(10)*GRAD)
      SUMT=SUMT+DELT
      IF(SUMT.GE.HO(IOBS)) IFLAG=1
      IF(IFLAG.EQ.0) GO TO 26
      FC(IOBS)=(SUMT-HO(IOBS))/DELT*QOLD+(HO(IOBS)-OLDT)/DELT*QOUT
      IOBS=IOBS+1
      IFLAG=0
   26 I=I+1
      ISTEP=ISTEP+1
      IF(I.GT.LNS) GO TO 34
      GO TO 22
   34 DO 36 I=1,LNS
      J=2*I-1
      P(J)=AEP
   36 P(J+1)=1.0
   37 DO 38 I=LPLUS,NN
      J=2*I-1
      P(J)=-DELH*(X(I)-SLL)/PLL + P(LNS)
   38 P(J+1)=-DELH/PLL
   42 CONTINUE
C
      RETURN
      END
C
C     -------------------
C
      SUBROUTINE TOTALM(P,BN,TMIN)
      DIMENSION P(60),BN(15)
      COMMON/AAA/X(30),EMTY(60),ISPR(30),NN,AREA
C
C     PURPOSE: TO CALCULATE THE AMOUNT OF WATER IN SAMPLE
C
      TMIN=0.
      NE=NN-1
      DO 10 L=1,NE
      MAT=ISPR(L)
      MAT1=ISPR(L+1)
      I=2*L-1
      I1=I+1
      I2=I+2
      I3=I+3
      EL=X(L+1)-X(L)
      P13=.7407407*P(I)+.2592593*P(I2)+.0740741*EL*(2.*P(I1)-P(I3))
      P23=.7407407*P(I2)+.2592593*P(I)+.0740741*EL*(P(I1)-2.*P(I3))
      Z1=SPR(MAT,BN,1,P(I))
      Z3=0.7407407*SPR(MAT,BN,1,P13)+0.2592593*SPR(MAT1,BN,1,P13)
      Z4=0.2592593*SPR(MAT,BN,1,P23)+0.7407407*SPR(MAT1,BN,1,P23)
      Z5=SPR(MAT1,BN,1,P(I2))
      TMIN=TMIN+AREA*EL*(0.5*(Z1+Z5)+Z3+Z4)/3.
   10 CONTINUE
      RETURN
      END
      SUBROUTINE BANSOL(NEQ)
```

```
C
C     PURPOSE: TO SOLVE THE GLOBAL MATRIX EQUATION
C
      COMMON /CCC/ S(60,4), F(60)
      N1=NEQ-1
      DO 4 I=1,N1
      J=I-1
      M=MIN0(4,NEQ-J)
      P=S(I,1)
      DO 4 L=2,M
      C=S(I,L)/P
      II=J+I
      JJ=0
      DO 2 K=L,M
      JJ=JJ+1
    2 S(II,JJ)=S(II,JJ)-C*S(I,K)
    4 S(I,L)=C
      DO 6 I=1,N1
      J=I-1
      M=MIN0(4,NEQ-J)
      C=F(I)
      F(I)=C/S(I,1)
      DO 6 L=2,M
      II=J+L
    6 F(II)=F(II)-S(I,L)*C
      F(NEQ)=F(NEQ)/S(NEQ,1)
      DO 8 I=1,N1
      II=NEQ-I
      J=II-1
      M=MIN0(4,NEQ-J)
      DO 8 K=2,M
      L=J+K
    8 F(II)=F(II)-S(II,K)*F(L)
      RETURN
      END
      SUBROUTINE MATEQ(BN,EPSI,PE,DELT)
C
C     PURPOSE: TO CALCULATE THE GLOBAL MATRIX EQUATION
C
      COMMON /AAA/ X(30),P(60),ISPR(30),NN
      COMMON /CCC/ S(60,4), F(60)
      DIMENSION PE(60),FE(4,3), DX(4,3), SR(10), T(60),BN(15)
      DATA FE(1,1),FE(3,3),FE(1,2),FE(3,2),FE(1,3),FE(3,1),DX(1,1),DX(1,
     13),DX(1,2),DX(3,2),DX(3,1),DX(3,3)/2*.9208488,2*0.5,2*.0791512,2*-
     2.4285714,-.75,.75,2*.4285714/
C
C     ----------
      NE=NN-1
      NN2=2*NN
      N1=NN2-1
      N2=NN2-2
      N3=NN2-3
      NEQ=N1
      EPSM=EPSI-1.
      DO 2 I=1,NN2
      T(I)=0.5*(PE(I)+P(I))
      DO 2 J=2,4
    2 S(I,J)=0.
C
C     ----- CONTRIBUTIONS OF NODAL INTEGRATION POINTS -----
      DO 4 I=1,NN
      LL=2*I-1
      L1=LL+1
      II=MAX0(I-1,1)
      JJ=MIN0(I+1,NN)
      EL1=0.05*(X(JJ)-X(II))
      MAT=ISPR(I)
      CAP1=SPR(MAT,BN,3,T(LL))*EL1/DELT
      COND1=SPR(MAT,BN,2,T(LL))*EL1
      S(LL,1)=CAP1
      S(L1,1)=EPSI*COND1
      F(LL)=CAP1*P(LL)
```

```
    4 F(L1)=(EPSM*P(L1)+1.)*COND1
C
C     ----- ELEMENT LOOP; CONSTRUCT GLOBAL MATRIX -----
      DO 12 L=1,NE
      LL=2*L-2
      L1=LL+1
      L2=LL+2
      L3=LL+3
      L4=LL+4
      EL=X(L+1)-X(L)
C
C     ----- CALCULATE HERMITIAN BASIS FUNCTIONS -----
      FE(2,1)=.1181895*EL
      FE(2,2)=.125*EL
      FE(2,3)=.0246676*EL
      DX(2,1)=.1993777*EL
      DX(2,2)=-.125*EL
      DX(2,3)=-.1279491*EL
      DO 6 K=1,3
      FE(4,K)=-FE(2,4-K)
    6 DX(4,K)=DX(2,4-K)
C
C     ----- CALCULATE MATERIAL PROPERTIES AT LOBATTO POINTS -----
      W1=FE(1,1)*T(L1)+FE(2,1)*T(L2)+FE(3,1)*T(L3)+FE(4,1)*T(L4)
      W2=FE(1,2)*T(L1)+FE(2,2)*T(L2)+FE(3,2)*T(L3)+FE(4,2)*T(L4)
      W3=FE(1,3)*T(L1)+FE(2,3)*T(L2)+FE(3,3)*T(L3)+FE(4,3)*T(L4)
      MAT1=ISPR(L)
      MAT2=ISPR(L+1)
      G2=.0861869
      G1=1.0027021
      COND1= (G1*SPR(MAT1,BN,2,W1)+G2*SPR(MAT2,BN,2,W1))/EL
      COND2=.7111111*(SPR(MAT1,BN,2,W2)+SPR(MAT2,BN,2,W2))/EL
      COND3=(G2*SPR(MAT1,BN,2,W3)+G1*SPR(MAT2,BN,2,W3))/EL
      EL1=.25*EL/DELT
      CAP1=(G1*SPR(MAT1,BN,3,W1)+G2*SPR(MAT2,BN,3,W1))*EL1
      CAP2=.7111111*(SPR(MAT1,BN,3,W2)+SPR(MAT2,BN,3,W2))*EL1
      CAP3=(G2*SPR(MAT1,BN,3,W3)+G1*SPR(MAT2,BN,3,W3))*EL1
C
C     ----- ADD ELEMENT CONTRIBUTIONS TO GLOBAL MATRIX -----
      K=0
      DO 10 I=1,4
      II=LL+I
      DO 10 J=1,4
      W1=DX(J,1)*DX(I,1)*COND1+DX(J,2)*DX(I,2)*COND2+DX(J,3)*DX(I,3)*
     1COND3
      W2=FE(J,1)*FE(I,1)*CAP1+FE(J,2)*FE(I,2)*CAP2+FE(J,3)*FE(I,3)*CAP3
      JJ=J+1-I
      K=K+1
      S(II,JJ)=S(II,JJ)+W1*EPSI+W2
   10 SR(K)=W1*EPSM+W2
C
C     ----- CONSTRUCT RHS VECTOR ----
      EL1=.2142857*EL*(COND1+1.75*COND2+COND3)
      F(L1)=F(L1)+SR(1)*P(L1)+SR(2)*P(L2)+SR(3)*P(L3)+SR(4)*P(L4)-EL1
      F(L2)=F(L2)+SR(2)*P(L1)+SR(5)*P(L2)+SR(6)*P(L3)+SR(7)*P(L4)+
     1(.0996889*COND1-.0625*COND2-.0639746*COND3)*EL*EL
      F(L3)=F(L3)+SR(3)*P(L1)+SR(6)*P(L2)+SR(8)*P(L3)+SR(9)*P(L4)+EL1
   12 F(L4)=F(L4)+SR(4)*P(L1)+SR(7)*P(L2)+SR(9)*P(L3)+SR(10)*P(L4)+
     1(.0996889*COND3-.0625*COND2-.0639746*COND1)*EL*EL
C
C     ----- INCLUDE BOUNDARY CONDITIONS -----
      S(2,1)=1.
      W1=AMIN1(0.5,100./(X(2)-X(1)))
C     W2=1.
      F(2)=(1.-W1)*PE(2)+W1
      F(1)=F(1)-S(1,2)*F(2)
      F(3)=F(3)-S(2,2)*F(2)
      F(4)=F(4)-S(2,3)*F(2)
      S(1,2)=0.
      S(2,2)=0.
      S(2,3)=0.
```

```
   24 CONTINUE
      F(N1)=F(NN2)-S(N1,2)*P(N1)
      F(N2)=F(N2)-S(N2,2)*P(N1)
      F(N3)=F(N3)-S(N3,3)*P(N1)
      S(N1,1)=S(NN2,1)
      S(N2,2)=S(N2,3)
      S(N3,3)=S(N3,4)
   30 CONTINUE
C
C     ----- SOLVE FOR UNKNOWNS -----
      CALL BANSOL(NEQ)
C
C     ----------
      DO 34 I=1,N2
   34 PE(I)=F(I)
      PE(NN2)=F(N1)
      RETURN
      END
      FUNCTION SPR(MAT,BN,N,PR)
C
C
C     PURPOSE: TO CALCULATE THE SOIL-HYDRAULIC PROPERTIES
C
      DIMENSION BN(15)
      DATA CONDM/1.E-08/,SS/1.E-07/
      K=MAT*5
      A=BN(K+1)
      R=BN(K+2)
      S=1.-1./R
      WCR=BN(K+3)
      WCS=BN(K+4)
      CONDS=BN(K+5)
      IF(PR)1,10,10
    1 P=ABS(PR)
      THETA=(1.+(A*P)**R)**(-S)
      IF(N-2) 2,4,6
    2 SPR=WCR+(WCS-WCR)*THETA
      RETURN
    4 T=1.-THETA*(A*P)**(R-1.)
      IF(THETA.LT.0.04) T=S*THETA**(1./S)
      COND=CONDS*SQRT(THETA)*T*T
      SPR=AMAX1(COND,CONDM)
      RETURN
    6 T=1.+(A*P)**R
      WC=WCR+(WCS-WCR)*THETA
      SPR=(WC-WCR)*(R-1.)*A*(A*P)**(R-1.)/T + WC*SS/WCS
      RETURN
   10 GO TO (12,14,16,18),N
   12 SPR=WCS
      RETURN
   14 SPR=CONDS
      RETURN
   16 SPR=SS
      RETURN
   18 THETA=(PR-WCR)/(WCS-WCR)
      S=R/(1.-R)
      IF(THETA.GT.0.999999) GO TO 20
      SPR=-(THETA**S-1.)**(1./R)/A
      RETURN
   20 SPR=0.
      RETURN
      END
```