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Yield Related Interpretations of Irrigation Uniformity and Efficiency Measures

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Summary. A number of irrigation performance parameters have been used to quantify irrigation uniformity and efficiency. These parameters are usually regarded as performance indices only, without a physically significant interpretation. Common irrigation uniformity and efficiency measures can be related to expected yields from non-uniform irrigation of certain hypothetical crops. Calculation of these measures is equivalent to, or closely related to, calculation of expected yields with non-uniform irrigation for the water-yield functions of Fig. 1. This relationship between performance measures and expected yield lends physical significance to irrigation uniformity measures, and suggests useful generalizations of traditional uniformity and efficiency measures that are directly related to crop yield.

Introduction

During the four decades since Christiansen (1942) first introduced his uniformity coefficient, several different coefficients have been defined and used to characterize the uniformity and efficiency of irrigations. Dabbous (1962), Su (1979) and Solomon (1983) presented comprehensive discussions of many such measures, and would provide the interested reader with a good introduction to the literature on this topic.

Norum (1961) has criticized the uniformity coefficient of Christiansen (U_C) and other similar measures as "incomplete" and "arbitrary" representations of the distribution of application depths, and for "lack of pertinent physical significance". Such measures are usually regarded as convenient quantitative indices of irrigation performance, but signifying no physical consequence of non-uniform irrigation. This is not to suggest that irrigation uniformity has no physical significance. Howell (1964), Seginer (1978) and Stern and Bresler (1983) have clearly shown that

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irrigation uniformity can influence crop yields. But the uniformity measures themselves are not usually taken to be physically significant. Some irrigation efficiency measures, notably the water storage efficiency concept of Hart and Reynolds (1965), can be interpreted physically as pertaining to volumes of water. While this is important to the farmer because of water costs, water volumes may be related only indirectly to the farmer's main concern: crop production.

The analysis presented here shows that common irrigation uniformity and efficiency measures can be interpreted as yield predictions for some special situations. This leads to natural generalizations of the uniformity and efficiency concepts that relate directly to crop responses to non-uniform irrigation.

Uniformity and Efficiency Measures

Uniformity measures are recipes for producing, from several estimates of irrigation depths at various locations throughout the irrigated area, a single number which varies with the uniformity of the irrigation depths. The usual approach is to compute some measure of the dispersion of the depth values, and express this in a dimensionless way by comparing it to the average value. Uniformity measures are a function of the variability of the irrigation depths, though as Cogels (1983) and Seginer (1979) point out, an adjustment depending on the plant spacing may be desirable.

Christiansen (1942) chose the mean deviation as his measure of dispersion. The mean deviation (δ) is the average value of the absolute differences between each value and the mean (μ). In decimal form, the uniformity coefficient of Christansen (U_C) is given by:

$$U_C = 1 - \delta/\mu \,. \tag{1}$$

Citing the utility of the standard deviation (σ) as a measure of dispersion in basic statistics, Wilcox and Swailes (1947) proposed their own uniformity coefficient (U_W):

$$U_W = 1 - \sigma/\mu \,. \tag{2}$$

This coefficient has been called the Wilcox-Swailes coefficient by some (Su 1979), and the statistical uniformity coefficient by others (Bralts et al. 1981 a, b).

Hart (1961) proposed another uniformity coefficient (U_H) which incorporates the standard deviation, and also recognizes an historical debt to Christiansen:

$$U_H = 1 - \sqrt{2\pi}\;(\sigma/\mu) \,. \tag{3}$$

If irrigation depths are normally distributed, then $U_H = U_C$. U_H is sometimes called the HSPA uniformity coefficient because Hart's work was sponsored by the Hawaiian Sugar Planters Association.

Another uniformity measure frequently used is the ratio of the average of the lowest quarter of the irrigation depths (μ_{LQ}) to the general mean depth (μ). Dabbous (1962) credits the Soil Conservation Service of the U.S. Department of Agriculture with originating this measure, which they called pattern efficiency. More recently the term distribution uniformity (U_D) has been used (Kruse 1978):

$$U_D = \mu_{LQ}/\mu \,. \tag{4}$$

In reference to trickle irrigation, Keller and Karmeli (1975) called this ratio emission uniformity. Hart (1961) also proposed a statistical version of distribution uniformity (U_{DH}) that is numerically equal to U_D whenever irrigation depths are normally distributed:

$$U_{DH} = 1 - 1.27\,(\sigma/\mu)\,. \tag{5}$$

The coefficient 1.27 stems from the fact that for a normal distribution, the mean of the low quarter of the values occurs approximately 1.27 standard deviations below the mean.

Efficiency measures quantify irrigation uniformity by computing some consequence of uniformity, given certain assumptions about how the irrigation system will be operated. Whereas uniformity measures depend only on the degree to which water is applied uniformly, efficiency measures depend on both uniformity and the mode of operation for the irrigation system. Though many efficiency terms have been defined (Hart et al. 1979; Kruse 1978; Merriam and Keller 1978), by far the most common one in general use is the water storage efficiency (E_{WS}) of Hart and Reynolds (1965).

Their definition of E_{WS} is based on the notion of adequate irrigation. An area is said to be adequately irrigated if the amount applied to that area meets or exceeds the irrigation requirement at the time of application. Due to non-uniformity, some portions of the irrigated area may be adequately irrigated while other portions are in deficit. The portion of the area adequately irrigated may be increased or decreased by applying more or less water. In adequately irrigated areas, the amount of applied water entering the soil in excess of the requirement is presumed lost to deep percolation and is not stored in the root zone. In the deficit area (the area not adequately irrigated), all infiltrated water is assumed stored in the root zone. E_{WS} is defined as that fraction of the water applied and infiltrated into the soil that is stored in the root zone. For a fixed uniformity of application, E_{WS} depends on the portion of the area (T) that is adequately irrigated, with E_{WS} generally decreasing as T is increased. The notation $E_{WS}(T)$ is used to indicate the water storage efficiency when the fraction of the area that is adequately irrigated is T.

Expected Yields

The response of a crop to water may be summarized in a water-yield function, relating yield to the seasonal amount of water made available to the crop. Available water includes moisture stored in the root zone at the beginning of the season, effective rainfall, and water applied as irrigation. Since absolute crop yield will depend on factors other than available water, it is convenient to express yield in relative terms. Relative yield (y) is defined as the ratio of actual yield to the yield expected under existing conditions if water is not limiting. Similarly, relative available water may be defined as the ratio of actual available water to the amount of available water which corresponds to maximum yields. Functions relating relative yield to relative available water are fairly general, and can be representative of more than one location or year. Solomon (1983) reviewed the literature on water-yield functions and presented typical functions for many agricultural crops.

Adjusting the yield function so that the water variable refers only to water applied by the irrigation system makes the significance of various irrigation options more apparent, though with some loss of generality. The shape of the yield function also depends on the timing of irrigations throughout the season, but yield functions are approximately valid for many cases where "reasonable" irrigation schedules are used. Vaux and Pruitt (1983) offer an excellent discussion of yield functions, including factors limiting their validity or transferability.

It is useful to define two distinct dimensionless irrigation variables. Let W be the variable denoting seasonal irrigation application, with units of depth, and W^* denote the value of W which corresponds to a relative yield of 1 (if $y=1$ over a range of W, let W^* be the smallest value of W for which this is so). Let ω be defined as W/W^*. The yield function expressing the relationship between relative yield and ω is $y(\omega)$. Now suppose that μ is the average (over space) of the non-uniform seasonal irrigation application, and define w as W/μ. The distribution function for the dimensionless irrigation depths applied is $f(w)$, and is assumed to be zero outside the interval bounded by the maximum and minimum dimensionless applied depths.

Note that it would be inappropriate to express the yield function in terms of w, since w depends on μ, which is under the control of the irrigator. Likewise, it is inappropriate to express the irrigation depth distribution function in terms of ω, since ω depends on W^*, which is crop and location dependent. While w and ω are distinct, they are related by a water management parameter $\varrho=\mu/W^*$. ϱ exceeds 1 whenever the irrigator chooses to apply a mean depth greater than W^*. Clearly $\omega=\varrho w$, and $y(\omega)=y(\varrho w)$.

Subject to a few assumptions discussed below, the relative yield expected from non-uniform irrigation is given by:

$$y=\int_{-\infty}^{\infty} y(\varrho w)\, f(w)\, dw\,. \tag{6}$$

It is assumed in (6) that the uniformity of applied water is indicative of the uniformity of moisture in the root zone, since presumably this is what will influence crop yields. In a 1972 study, Hart concluded that subsurface horizontal redistribution of water should occur in such a way as to increase the uniformity of moisture within the soil over time. On the other hand, Sinai and Zaslavsky (1977) have shown that various surface effects and anisotropy within the soil can cause even a uniform application of water to result in non-uniform distribution of moisture within the root zone. At least for the purposes of interpreting uniformity and efficiency measures, the stated assumption appears to be a reasonable simplification.

While it is possible to speak of the uniformity of seasonal irrigation depths, practical uniformity evaluations are most often made on individual irrigation events. In some instances, such as sprinkler systems where lateral line offsets are used or where there is significant wind variation over time, it is more realistic to apply uniformity performance measures to the combined result of multiple irrigation events. Using (6) in the interpretation of irrigation performance parameters will be most meaningful when the irrigation uniformity being considered is representative of conditions throughout the growing season.

Uniformity and Efficiency Measures as Expected Yields

U_C

Christiansen's uniformity coefficient is probably the most frequently used uniformity measure. It turns out that the calculation of U_C gives the same result as calculating expected yield with equation (6) for a special case. Consider the yield function $y(\omega) = 1 - |\omega - 1|$, shown in Fig. 1a as curve ABC. Segment DEG is drawn for an arbitrary value of ω. Note that the segment $DE = 1 - EG$, and that $EG = EF$. But $EF = |\omega - 1|$. Assume that the mean irrigation application coincides with W^* so that the water management parameter $\varrho = 1$, and $w = \omega$. If the non-uniformity of the irrigation application is characterized by $f(w)$, then the expected yield may be calculated from equation (6):

$$y = \int_{-\infty}^{\infty} (1 - |w - 1|) f(w)\, dw \tag{7}$$

$$y = \int_{-\infty}^{\infty} f(w)\, dw - \int_{-\infty}^{\infty} |w - 1| f(w)\, dw\,. \tag{8}$$

Because $f(w)$ is a distribution function, the first integral in (8) must equal unity, while the second is the mean deviation of w. Therefore,

$$y = 1 - \delta/\mu = U_C\,. \tag{9}$$

U_C may thus be interpreted as the fraction of maximum yield expected from non-uniform irrigation of a crop having the yield function $y(\omega) = 1 - |\omega - 1|$, when the irrigator causes μ to coincide with W^*. Whether or not U_C relates directly to the yield of a particular crop will depend on how realistically the yield function of Fig. 1a represents the crop and location in question. While this yield function would not generally be regarded as a realistic model of crop response to irrigation, it can provide the basis for a physically significant interpretation of U_C in some instances.

Crops often exhibit what might be termed a *water requirement threshold:* some amount of water must be made available to the crop before any marketable yield can be produced. In many instances, this threshold may be exceeded without irrigation. It may be that the threshold is very small (forage crops for example), or that effective rainfall in the local area (including moisture stored in the soil at the beginning of the season) is sufficient to exceed the threshold. This would usually be the case in areas where the crop of interest is grown on a dryland basis.

Whenever irrigation is not required for available water to exceed the threshold, $y(0) \geq 0$. In such situations, it is reasonable to state that the yield function of Fig. 1a exhibits a more drastic crop response to both under- and over-irrigation than would be expected for the actual crop and location. This suggests that yields under non-uniform irrigation predicted from $y(\omega) = 1 - |\omega - 1|$, assuming $\varrho = 1$, will be less than those actually expected. In other words, the relative yield expected from non-uniform irrigation ($\varrho = 1$) of the actual crop will not be less than the Christiansen's uniformity coefficient for the irrigation. So long as irrigation is not

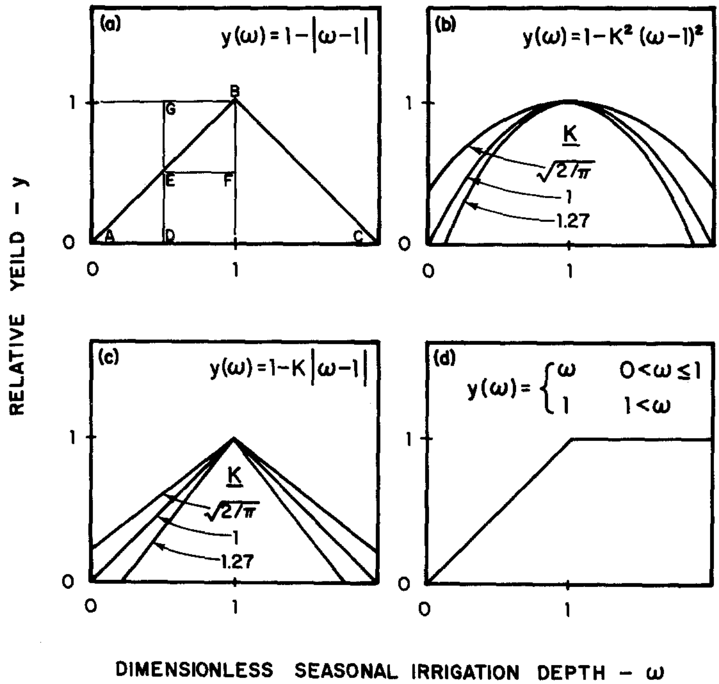


Fig. 1. Yield functions for interpreting irrigation performance measures

required to satisfy the threshold water requirement, U_C can be interpreted as a lower bound on the relative yield from non-uniform irrigation ($\varrho = 1$).

U_W, U_H, U_{DH}

These three uniformity measures are similar in that they all incorporate the standard deviation as a measure of the dispersion of irrigation depths. Let U_K be defined by

$$U_K = 1 - K(\sigma/\mu)\,. \tag{10}$$

For $K = 1$, $U_K = U_W$; for $K = \sqrt{2/\pi}$, $U_K = U_H$; for $K = 1.27$, $U_K = U_{DH}$. Now consider the yield function $y(\omega) = 1 - K^2(\omega - 1)^2$ as shown in Fig. 1 b. As before, it is assumed that $\varrho = 1$, so that $\omega = w$. Calculating the expected yield from (6) leads directly to:

$$y = 1 - K^2(\sigma/\mu)^2 \tag{11}$$

which implies

$$y = U_K(2 - U_K)\,. \tag{12}$$

Here, the expected yield is not exactly equal to the parameter U_K, but is completely determined by it. Again, the relevance of this observation depends on the extent to which the yield function $y(\omega) = 1 - K^2(\omega - 1)^2$ represents the crop and location of interest. Any quadratic water-yield function, when expressed in terms of the dimensionless variables y and ω will appear in this form, with the value of K being crop and location dependent. Quadratic yield functions have been offered for a number of crops and locations (Grimes et al. 1973; Larsen 1978; Shalhevet et al. 1981). Thus uniformity measures in the form of U_K will often be more directly related to crop yield than U_C, and the most physically significant measure from among $\{U_W, U_H, U_{DH}\}$ will depend on crop and location. Whenever a crop's water-field function is quadratic, a uniformity measure directly related to expected yield is U_K, with K taken from the yield function.

A relationship equivalent to (12) is:

$$U_K = 1 - \sqrt{1-y} \tag{13}$$

which relates U_K to the expected decrease in relative yield $(1-y)$ due to non-uniform irrigation of any crop responding as in Fig. 1 b. This suggests, for example, that a relative yield decrease of 0.05 is to be associated with non-uniform irrigation such that $U_K = 1 - \sqrt{0.95} = 0.78$ (assuming $\varrho = 1$). For $K = \sqrt{2/\pi}$, this implies irrigation for which $\sigma/\mu = 0.28$.

A similar, though more esoteric, connection exists between yield decreases and parameters of the U_K form, involving the yield function $y(\omega) = 1 - K \mid \omega - 1 \mid$ shown in Fig. 1 c. It is again assumed that $\varrho = 1$, and hence $\omega = w$. The yield decrease associated with any $w \neq 1$ is $K \mid w - 1 \mid$. The average yield decrease may be calculated as:

$$\int_{-\infty}^{\infty} K \mid w-1 \mid f(w)\, dw\,.$$

Suppose, however, that the word "average" is used in the RMS (root-mean-square) sense. The RMS average yield decrease is

$$\sqrt{\int_{-\infty}^{\infty} (K \mid w-1 \mid)^2 f(w)\, dw}\,,$$

and

$$K \sqrt{\int_{-\infty}^{\infty} (w-1)^2 f(w)\, dw} = K(\sigma/\mu)\,. \tag{14}$$

Thus for crops responding as in Fig. 1 c with $\varrho = 1$,

$$U_K = 1 - (\text{average yield decrease}) \tag{15}$$

where "average" is understood in the RMS sense. Though this formulation may appear more simple than the one derived from Fig. 1 b, it is probably less useful for interpretive purposes. The more realistic yield functions of Fig. 1 b and the more natural understanding of "average" argue in favor of interpretations of U_K based on $U_K = 1 - \sqrt{1-y}$.

U_D

Distribution uniformity may be related to expected yields through the yield function of Fig. 1 d. This yield function may be a fairly realistic representation of crop response to water, particularly when sufficient natural or man-made drainage exists to prevent waterlogging of the root zone and fertility is managed to counteract any nutrients leached by excess water application (Doorenbos and Kassam 1979; Seginer 1978). Suppose that the water management parameter ϱ is selected so that 75% of the area is adequately irrigated, and let ϱ' be the particular value of ϱ for which this is true. For any distribution $f(w)$, the expected yield is given by

$$y = \int_{-\infty}^{w'} \varrho' \, w \, f(w) \, dw + \int_{w'}^{\infty} f(w) \, dw \tag{16}$$

where w' is that value of w which separates the low quarter and the upper three quarters of the w values. Since 75% of the area is adequately irrigated, w' corresponds to $\omega = 1$. In general, $\omega = \varrho w$, so w' must be numerically equal to $1/\varrho'$. This choice of w' implies that the integral on the right of (16) is exactly equal to 0.75. The integral on the left is related to the average dimensionless application over the low quarter, which can be calculated as follows

$$\left[\int_{-\infty}^{w'} w \, f(w) \, dw \right] \Big/ \left[\int_{-\infty}^{w'} f(w) \, dw \right] = 4 \mu_{LQ}/\mu \, . \tag{17}$$

Combining these observations with the definition of U_D,

$$y = (\varrho'/4) \, [U_D] + (3/4) \tag{18}$$

which can be rearranged to give

$$U_D = (4y - 3)/\varrho' \, . \tag{19}$$

So U_D too can be expressed in terms of expected yields for a specific yield function. The numerator clearly involves the expected yields from non-uniform irrigation. The definition $\varrho = \mu/W^*$ suggests that ϱ' may be interpreted as the relative amount of overwatering necessary to ensure that 75% of the area is adequately irrigated. Thus U_D may be viewed as a measure of water use efficiency: the ratio of two terms having to do with the expected yield and the amount of water that must be applied to attain that yield (assuming, of course, yield response as in Fig. 1d, and an irrigation adequacy of 75%).

E_{WS}

Seen in this light, it is not surprising that yields predicted from Fig. 1d can also be related to the water storage efficiency concept. Let T be the portion of the area that is adequately irrigated, and let ϱ'' be the corresponding value of the management parameter ϱ. As before, the expected yield is calculated:

$$y = \int_{-\infty}^{w''} \varrho'' \, w \, f(w) \, dw + \int_{w''}^{\infty} f(w) \, dw \tag{19}$$

where w' separates $f(w)$ into an upper T fraction and a lower $(1 - T)$ fraction. Because ϱ'' is chosen so that T fraction of the area is adequately irrigated, w'' corresponds to $\omega = 1$. If the variable of integration is changed to ω, the yield calculation becomes:

$$y = \int_{-\infty}^{1} \omega \, f(\omega) \, d\omega + \int_{1}^{\infty} f(\omega) \, d\omega \, . \tag{20}$$

The left-hand integral in (20) is the total water (in terms of ω) applied in the deficit area, while the integral on the right equals T. However, as illustrated in Fig. 2, T can be interpreted as the water applied to the adequately irrigated area that does not exceed the requirement. Thus y can be interpreted as the portion of the water that is stored in the root zone when irrigation management satisfies a unit deficit over T fraction of the area.

It is easy to show that the total water (in terms of ω) applied over the area is equal to ϱ''. First realize that the total must numerically equal the average since the total area in Fig. 2 is 1. Since $\omega = W/W^*$, the average ω must equal $\mu/W^* = \varrho$, which in this case is ϱ''. Thus $E_{\mathrm{WS}}(T)$ can be computed as the ratio of y to ϱ'':

$$E_{\mathrm{WS}}(T) = y/\varrho'' \tag{21}$$

whenever the yield response is as in Fig. 1 d, and ϱ'' is chosen to adequately irrigate T portion of the area. Like U_D, $E_{\mathrm{WS}}(T)$ may be viewed as a measure of water use efficiency: yield per unit water required to produce that yield.

Special Purpose Measures

The foregoing has shown that traditional irrigation uniformity and efficiency measures can be related to the expected yields due to non-uniform irrigation for the hypothetical water-yield functions of Fig. 1. In this sense, the traditional irrigation performance parameters are seen to be special cases of the general yield prediction equation (6). It is natural to propose the calculation of expected yield itself as a way of defining special purpose irrigation performance parameters imbued with physical significance. Uniformity and efficiency measures that are appropriate to a particular crop and climate may be based on a relative yield function estimated for that crop and locale. A yield related uniformity coefficient (U_y) may be defined as:

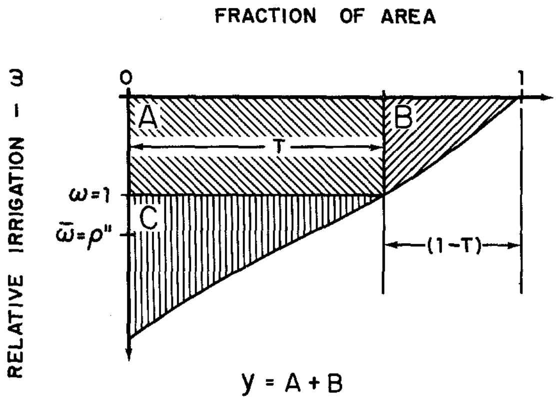
$$U_y = \int_{-\infty}^{\infty} y(\varrho w) f(w)\, dw\,. \tag{22}$$

U_y is a dimensionless, special purpose measure of the significance of irrigation system and management decisions. For fixed ϱ, U_y assesses the irrigation uniformity of $f(w)$ exactly as it should affect the crop to be grown at the site. If $f(w)$ is fixed, then U_y assesses the management choice of μ relative to W^* exactly as it should affect the crop to be grown at the site. A yield related efficiency measure (E_y) may also be defined:

$$E_y = (1/\varrho) \int_{-\infty}^{\infty} y(\varrho w) f(w)\, dw = U_y/\varrho\,. \tag{23}$$

E_y is an efficiency measure which assesses exactly the usefulness of the water application to the crop to be grown at the site.

It might be argued that, in practice, the use of U_y and E_y is not a viable approach, since there may be scant basis for the estimation of the yield function $y(\omega)$: better to compute traditional measures, which at least are objective, than to employ a subjectively estimated yield function. There are two fallacies in this argument. First, while computation of the traditional measures may be objective, the



$$y = A + B$$

$$\rho'' = A + B + C$$

$$E_{WS} = (A+B)/(A+B+C)$$

Fig. 2. Geometry of water storage efficiency

use of them is not. To use a performance measure requires employing that measure to make decisions or evaluate alternatives. How does one evaluate the significance of, say, $U_C = 0.75$ with respect to the crop, soil, rainfall pattern, etc. for the situation at hand? Would it not be easier to record one's judgment regarding crop response to physical inputs in an estimated yield function than to subjectively interpret a statistical parameter that has only indirect physical significance?

Second, none of the traditional measures are truely objective since each incorporates its own value system (which may easily go unrecognized). The calculation procedure for each measure, in effect, applies a certain penalty structure to irrigation amounts other than the mean. For example, U_C penalizes each amount in proportion to its deviation from the mean, while U_K assigns penalties proportional to the square of the deviation. Both measures treat deviations above and below the mean equally. The penalty structures for U_D and E_{WS} are more complex, but are inherent in their use none the less. The yield functions in Fig. 1 display the value system underlying the traditional irrigation performance parameters.

Whenever a water-yield function is known, it represents the value system most appropriate to evaluations and decisions regarding irrigation uniformity. The proposed measures U_y and E_y acknowledge this by explicitly incorporating $y(\omega)$ as their value system. Even an estimated yield function, based on observations of locally attainable dryland and maximum yields along with a judgment of yield susceptibility to excess water and nutrient leaching, may represent a more appropriate value system than any of those implied by traditional measures.

Conclusions

Though normally used as quantitative indices without physical significance, common irrigation uniformity measures can be shown to be special cases of the general problem of predicting the yield expected from non-uniform irrigation. For particular hypothetical yield functions and conditions, traditional uniformity and efficiency measures are numerically equal to, or are determined by, expected relative yields.

U_C appears to be a practical lower bound on the relative yield expected from non-uniform irrigation of agricultural crops whenever effective rainfall exceeds the water requirement threshold and μ coincides with W^*. Parameters of the form U_K may often be more closely related to expected relative yields than U_C. For suitable crop response curves, U_K is equal to 1 minus the square root of the expected yield decrease due to non-uniform irrigation. The shape of the crop response curve for which this is true varies with K, implying that the most physically significant value of K is crop and locale dependent. Both U_D and E_{WS} are shown to be analogs of water use efficiency: ratios of a yield related term to an indicator of the water required to produce this expected relative yield. The yield function for which this is true may be realistic under some circumstances.

The notion that traditional irrigation uniformity and efficiency measures are connected with expected yields suggests natural definitions for the yield related uniformity and efficiency measures U_y and E_y. Use of U_y or E_y requires an estimate for the water-yield function appropriate to the particular crop and site of interest. Providing such an estimate may be no more difficult than trying to evaluate the significance of more traditional measures. The use of U_y and E_y are recommended since they assess the significance of irrigation decisions exactly as they should affect the crop to be grown at the site.

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