

Comment on 'The Concise Formulation of Diffusive Sorption of Water in a Dry Soil' by Wilfried Brutsaert

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Recently, *Brutsaert* [1976] showed very effectively how the various approximate solutions for the sorption problem are related and which solutions are the most accurate. This is a valuable contribution to the theory of unsaturated soil water flow and is the type of information that hydrologists need from mathematicians and other theoreticians dealing with flow in porous media. I wholeheartedly agree that we need 'solutions that satisfy the dual requirement of physical consistency and computational simplicity.' His conclusion that 'for most practical problems the need for expensive numerical procedures and tedious perturbation and iterative analytical methods is probably obviated' is, unfortunately, true only for the sorptivity problem. Hopefully, some day this can be said for the hydrologic unsaturated soil water flow problem as a whole.

Brutsaert obviously considers as his primary audience hydrologists concerned with basin-wide application of soil water flow theory. To be successful in this, one must move from complex to more simple models. Whereas the bulk of his paper certainly does simplify without sacrificing physical principles, the philosophy expressed in the Introduction, Discussion, and Concluding Remarks is, in my opinion, exactly the opposite. That is, *Brutsaert* appears to advise hydrologists to obtain sorptivity values by his procedure, whereas sorptivity can be measured simply and directly [*Talsma*, 1969]. Why then should a hydrologist go through the very laborious procedures required to obtain 'experimental D data' and have to deal with confluent geometric functions? Or why should he go through the much harder task of determining $k = k_0 S^n$ and $S = A/[A + (-H)^B]$ and struggle with Γ functions? It makes much more sense to go the opposite route and determine diffusivity from sorptivity measurements as I have recently shown to be possible [*Dirksen*, 1975a]. Soil water diffusivity or hydraulic conductivity as a function of water content or pressure head can be calculated from sorptivity as a function of water content or pressure head at the absorption interface. Sorptivity measurements do not require any measurements below this interface. The linear relationship between cumulative absorption and the square root of time is, in general, quickly established. This makes it a very simple, fast method for determining soil water diffusivity functions that could be used basin-wide.

Sorptivity is not an intrinsic soil property but is dependent on both the initial water content θ_i (or pressure head H_i) and the water content θ_0 (or pressure head H_0) at the absorption interface. Obviously, *Brutsaert* had in mind absorption under ponding or from saturation when he developed his equations in terms of S (later in the paper, he identifies k_0 as the saturated hydraulic conductivity), but all his equations are also valid for $\theta_0 < \theta_{\text{sat}}$. That is, $S = 1$ does not necessarily mean that $\theta_0 = \theta_{\text{sat}}$. Therefore any one of the solutions in Table 1 can be used to derive $D(\theta)$ from $A_0(\theta_0, \theta_i = \text{const})$. I used [*Dirksen*,

1975a] the linearized solution with a weighted mean diffusivity with $\gamma = 0.67$, corresponding to (12), 'Crank's [1956] result.'

Brutsaert found that (33') results in a far smaller error compared with an exact solution than any of the other approximate solutions and then states that *Parlange* [1975] found the same equation to be superior to Crank's result. *Brutsaert* and *Parlange* used (33') to obtain only one value, the saturated sorptivity, from a complete $D[\theta]$ function. Both found a very small error for that value. In my reply to *Parlange* [*Dirksen*, 1975b], I pointed out that going in the opposite direction from $A_0(\theta_0, \theta_i = \text{const})$ to $D(\theta)$ the errors involved will generally vary with θ or $D(\theta)$. For three hypothetical soils with an exponential relationship between D and θ the average value $D(\theta)/D_{\text{exp}}$ for (33') was nearly perfect for each soil. But the variation of $D(\theta)/D_{\text{exp}}$ with θ (the 'slope' of a linear regression curve) was much larger for (33') than for (14) ($\gamma = 0.67$). This results in the same maximum error of about 3.5% for both cases. Figure 1 of *Dirksen* [1975b] shows that the absolute values of $D(\theta)$, the 'slope,' and the magnitude of the fluctuations all decrease with increasing θ . Results were best when γ was varied between 0.61 for a clay and 0.63 for a sand.

To evaluate the above results, one should remember that this reversed route involves a differentiation, which is inherently less accurate than the integration required in deriving A_0 from $D(\theta)$. Furthermore, the accuracy of these results depends on the accuracy of the areas under the curves of Figure 1 of *Gardner and Mayhugh* [1958]. To evaluate the latter would require a check of the original work of *Wagner* [1952] and *Philip* [1955]. The amount of time involved in doing this is not warranted, especially since the weighted mean diffusivity solution can be generalized and tested against the exact solution of (26), which *Brutsaert* used to evaluate all of his approximate solutions. Whereas, strictly speaking, 'Crank's result' refers to the weighted mean diffusivity solution with $\gamma = \frac{2}{3}$, (12) can be written in the general form

$$A_0 = \left[\frac{4(1 + \gamma)}{\pi} \cdot (\theta_0 - \theta_i) \int_0^1 S^\gamma D dS \right]^{1/2}$$

Substitution of the diffusivity for the exact solution (24) in this equation yields (27), with

$$F_1 = \left[\frac{2(1 + \gamma)(m + 1)}{m\pi} \cdot \left(\frac{1}{m + \gamma + 1} - \frac{1}{(m + 1)(2m + \gamma + 1)} \right) \right]^{1/2}$$

Setting this F_1 equal to $F_1 = (m + 1)^{-1/2}$ of the exact solution gives a quadratic equation $a\gamma^2 + b\gamma + c = 0$, with

$$a = (2/\pi)(m + 1) - 1$$

$$b = (4/\pi)m^2 + [(10/\pi) - 3]m + [(6/\pi) - 2]$$

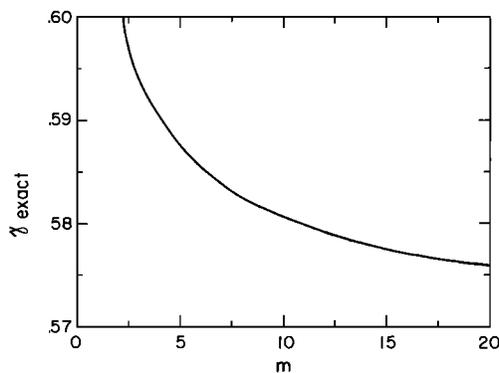


Fig. 1. Plot of γ_{exact} versus m .

$$c = (2/\pi)(2m^2 + 4m + 2) - (2m^2 + 3m + 1)$$

One root of this equation corresponds to the value of $\gamma = \gamma_{\text{exact}}$, for which the weighted mean diffusivity becomes exact. In Figure 1 of the present paper, γ_{exact} is plotted versus m . This result could be added to Brutsaert's Table 1. For $m = 3$ to $m = 20$, γ_{exact} varies from 0.594 to 0.576. These values are slightly lower than the best values for γ cited above for exponential diffusivities, and they show an opposite trend from coarse (large m) to fine (small m) soils. This shows that (33') is not superior to the weighted mean diffusivity solution if γ is allowed to vary rather than is held constant at $\gamma = \frac{1}{2}$.

In terms of Dirksen's [1975b] Figure 1 the 'slope' of (33') is the same as that for $\gamma = 0.50$, while all values for $D(\theta)$ are 0.954 times those for $\gamma = 0.50$. Thus (33') cannot satisfy the 'slope' for the values of γ_{exact} . However, it can be seen from this figure that a curve for $\gamma = 0.5825$, which is the value of γ_{exact} for the typical value of $m = 8$, and the curve for (33') will be so close together that any remaining differences will have no practical significance. The relationship between m and γ_{exact} in

Figure 1 is based on an exact analytical solution. Therefore if a weighted mean diffusivity, rather than (33'), is used to derive $D(\theta)$ from $A_0(\theta_0)$, γ can best be chosen from Figure 1 and the textural classification of the soil involved.

In conclusion, Brutsaert's paper is very helpful in evaluating the relative merits of the various approximate solutions of the soil water flow problem, but his practical application is misleading. Instead, the paper enhances the merits of determining soil water diffusivity functions from a series of sorptivity measurements. Finally, the generalized form of the weighted mean diffusivity solution can be made exact by a proper choice of γ , making it even more accurate than (33').

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