RESEARCH REPORT

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ANALYZING CROP SALT TOLERANCE DATA: MODEL DESCRIPTION AND USER'S MANUAL

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ABSTRACT

M. Th. van Genuchten.[1] 1983. Analyzing Crop Salt Tolerance Data: Model Description and User's Manual. Research Report No. 120, U.S. Salinity Laboratory, USDA/ARS, California, 50 p.

This report describes a computer program that can be used to analyze experimentally derived crop salt tolerance data. The program uses a non-linear least squares inversion method to find the unknown parameters in several salt tolerance response functions. One of three models included in the program is the familiar piecewise linear response function. Application of this function leads to estimates for the salinity threshold and the slope of the response curve. Two alternative types of salinity response functions are also considered. The report gives a detailed description of the computer model and the required input data. Application of the program is illustrated with several examples. A listing of the program is given in an appendix.

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1. INTRODUCTION

The presence or accumulation of excess soluble salts in the soil root zone and its negative effect on crop productivity is a widespread problem, especially in the arid and semiarid regions of the world. Although in some cases soil salinity can be controlled effectively by applying suitable water management schemes, high soil salinities often are difficult to prevent because of a lack of good quality irrigation water. In that case, an effective use of available soil and water resources dictates the production of agricultural (or other) crops that are relatively tolerant to high soil salinities. For this purpose, numerous field and laboratory experiments have been carried out to determine the salt tolerance of various crops. Results of these experiments are best analyzed in terms of an appropriate salt tolerance response function.

One popular way to express the relative salt tolerance of crops is by means of a piecewise linear response function (Maas and Hoffman, 1977). This function contains two independent parameters: the salinity threshold (c_t), being the maximum salinity without yield reduction as compared to the yield under nonsaline control conditions, and the slope (s) of the curve determining the fractional yield decline per unit increase in salinity beyond the threshold. In mathematical form:

$$Y_r = \begin{cases} 1 & 0 \leqslant c \leqslant c_t \\ 1 - s(c - c_t) & c_t < c \leqslant c_o \\ 0 & c > c_o \end{cases} \qquad [1]$$

where Y_r is the relative yield, c is the average rootzone salinity during the growing season, c_t is the threshold concentration, c_o is the concentration beyond which the yield is zero, and s is the absolute value of the

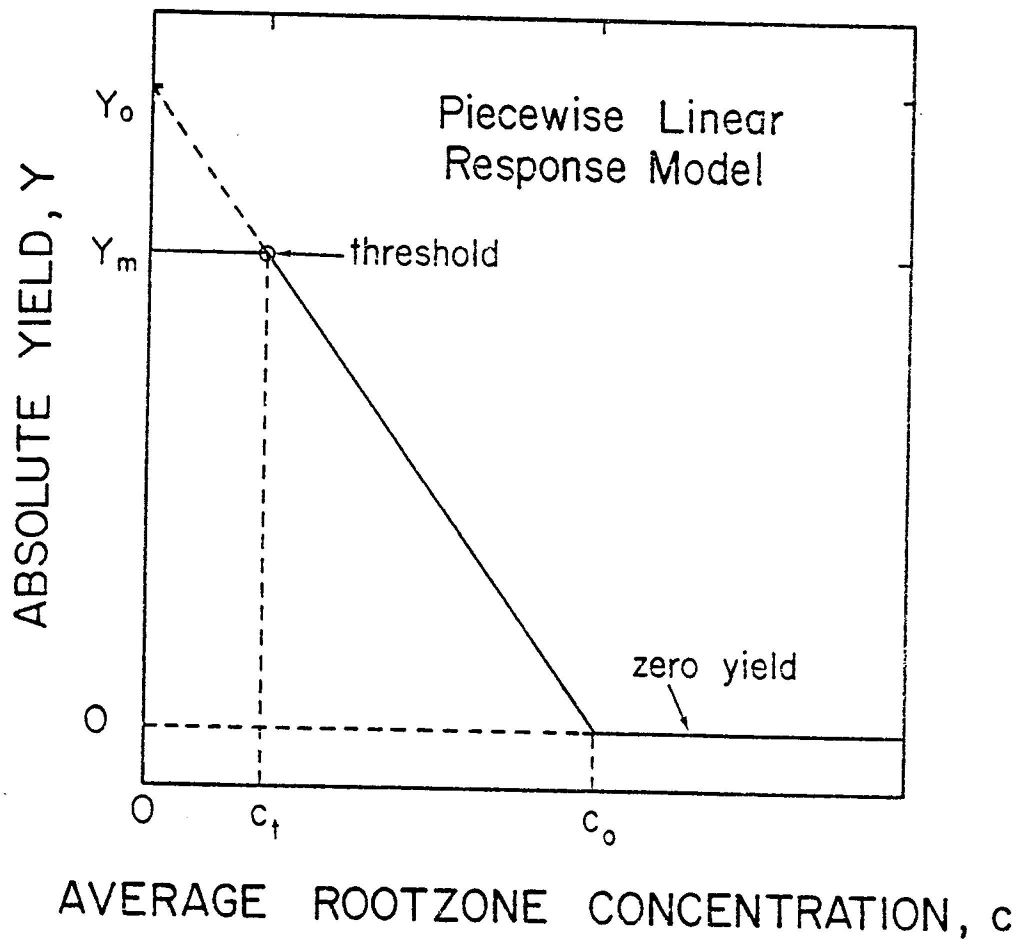

Figure 1. Graphical representation of the piecewise linear crop salt tolerance response function (Eq. 2).

to the left and at least three points to the right of the fitted threshold value. This makes the method less suitable for experiments with a limited number of data points. In this report we will use a more general non-linear least squares method. Appendix A gives a detailed description of the computer program (called "SALT"); the program itself is listed in Appendix D.

To allow for flexibility in analyzing different types of data sets, 20 different options have been included in the program. These options relate to the choice of a particular salt tolerance response function (Eq. [2] or alternative models), and to the type and number of model parameters that are fitted to the data. The different options are discussed briefly below. Specific examples are given in the next section.

Table 1 gives a list of the available options. A particular option in the program is chosen by specifying the input variable NOPT ("Option Number", see Table 1). When NOPT = 1, a simple linear regression analysis of the type

$$Y = Y_o - s_1 c \qquad [5]$$

with two unknown parameters (Y_o, s_1) will be carried out. Application of this method assumes that an independent estimate for Y_m is available, and hence, that the data already are normalized into relative yield fractions. It is important to realize that this method can be applied only to data points that are located between c_t and c_o (see Eq. 2). Once the regression based on Eq. [5] is carried out, the salinity threshold and slope can be calculated with the expressions

$$c_t = (Y_o - Y_m)/s_1 \qquad [6a]$$

and

$$s = s_1/Y_m. \qquad [6b]$$

When NOPT = 2, it is assumed that both Y_m and c_t are already known, thus leaving only the slope s to be calculated from the experimental data. In this study, s is obtained with the simple equation

$$s = \sum_{i=1}^{n} (Y_m - Y_i) / \sum_{i=1}^{n} (c_i - c_t) \qquad [7]$$

where (c_i, Y_i) represents the i-th data point $(1 \leq i \leq n)$, and n is the number of observed data points used in the analysis. An iterative procedure was built into the program such that only data points between c_t and c_o are considered. As will be shown later, Eq. [7] is especially useful when other methods based on Eq. [2] lead to salinity threshold values that are located to the left of the first measured data point (usually the non-saline control). As an alternative to Eq. [7], least squares techniques could have been used also to calculate s once Y_m and c_t are known. Because least squares techniques were found to give relatively more weight to data points that are far away from the threshold value (i.e., to data points associated with relatively low yields and high salinity levels), and because salt tolerance studies generally are concerned more with the region close to the threshold (i.e., with the higher yield values), it was decided to use only Eq. [7].

Nonlinear least squares techniques are used whenever NOPT $\geq$ 3. When NOPT = 3, the threshold c_t is assumed to be known beforehand and only s and Y_m are fitted to the data. When NOPT = 4, both c_t and s are calculated (Y_m is fixed), whereas for NOPT = 5 all three unknowns (Y_m, c_t and s) different management schemes (e.g., with varying leaching fractions or irrigation methods). One example of this type is considered in section 3.6.

Although Eq. [2] has been the more popular model for quantifying the salt tolerance of crops, two alternative formulations are also considered in this report. One expression is of the form

$$Y = \frac{Y_m}{1 + (c/c_{50})^p} \qquad [8]$$

where c_{50} is the salinity at which the yield is reduced by 50%, and where p is an empirical constant. Figure 2 gives a dimensionless plot of Y_r versus c/c_{50}. Equation [8] is used in the program for NOPT-values that run from 11 through 17 (see Table 1). As with Eq. [2], choice of a particular option depends on the number of unknown parameters in Eq. [8], and on the number of multiple Y_m-values available for different years or treatments. Examples based on Eq. [8] are shown in sections 3.3. and 3.5.

A second alternative salt tolerance model used in the computer program assumes an exponential relation between the yield and the average rootzone salinity:

$$Y = Y_m \exp(\alpha c - \beta c^2) \qquad [9]$$

where α and β are empirical constants. Figure 3 shows relative salt tolerance curves based on this equation, using three different combinations of α and β. Note that the curve for $\alpha > 0$ reaches a maximum at some positive value of the concentration; this maximum is located at $c = \alpha/2\beta$. When $\alpha = 0$, the initial slope of the response function is zero, and the curve is similar in shape to the curves shown in Fig. 2. Response functions based on Eq. [9] are used whenever $18 \leq NOPT \leq 20$ (Table 1). An example is given in section 3.2.

3. EXAMPLES

This section presents several examples illustrating the type of results that can be obtained with the optimization method. The examples, taken from the literature, were chosen such that various program options are clearly demonstrated. The concentration units for each example are the same as those used in the original publication. Appendix B lists the input data that were used in the calculations; computed results for these same input data are given in Appendix C.

3.1. Tall Fescue

This example considers the salt tolerance of tall fescue (Brown and Bernstein, 1953). Figure 4 compares two fitted curves with the observed data. Results for the solid line were obtained with NOPT = 5 (see Table 1), indicating that all three unknowns (c_t, s and Y_m) in Eq. [2] were fitted to the data. Note that only one point is located to the left of the threshold. This shows that the curve could have been calculated also with NOPT = 4, i.e., by fixing Y_m equal to the maximum observed yield and carrying out a two-parameter fit for c_t and s. However, in general it is impossible to know beforehand whether or not only one data point appears to the left of c_t, and hence it is always better to calculate all three unknowns simultaneously using NOPT = 5. Because the required computer time on an IBM 360/91 is in the order of a few seconds (or less), there is also no reason to limit the number of unknowns in the program by artificially fixing Y_m.

The dashed line in Fig. 4 is based on a linear regression fit of all data (NOPT = 1). Using this method and assuming that Y_m is equal to the control yield of the first data point, a drastically different threshold value is obtained: 2.50 for NOPT = 1 as compared to 4.53 for NOPT = 5. On the other hand, if the first data point was deleted from the data set, linear regression in this case would have generated exactly the same results as the complete three-parameter fit using NOPT = 5.

3.2. Perennial Rye

Results for the salt tolerance of perennial rye (Brown and Bernstein, 1953), shown in Fig. 5, are very similar to those of the previous example. Again, only one data point appears to the left of the threshold value, indicating that NOPT = 4 and NOPT = 5 would have produced exactly the same results. Also the use of linear regression techniques (NOPT = 1) would have lead to the same results, again provided that the first data point is deleted from the data set.

Figure 6 shows results for the same perennial rye data when Eq. [9], rather than Eq. [2], is fitted to the data. Note that the parameter α was found to be positive, causing the curve to acquire a maximum at $c = \alpha/2\beta = 2.5$ dS/m. Judging from Figs. 5 and 6, the exponential curve failed to produce better results than the piecewise linear model. This conclusion also follows from a comparison of the sum of the squared deviations of the observed (Y_i) versus the fitted (Y_i') yield values (SSQ):

$$SSQ = \sum_{i=1}^{n} (Y_i - Y_i')^2 \qquad [10]$$

Nearly identical values of SSQ were obtained for the two models: .0214 for the piecewise linear model and .0231 for the exponential model. Hence, both models are about equally successful in describing the salt tolerance data of perennial rye.

3.3. Tomato

Figure 7 shows salt tolerance data for tomato (Osawa, 1965). The dashed line represents the complete three-parameter fit based on Eq. [2] (NOPT = 5). Note that the threshold concentration appears to the left of the first data point. This situation leads to a unique (well-defined) value for the absolute slope ($Y_m s$). However, the fitted values of c_t and Y_m in this case are meaningless since no data points at the lower salinity values are available to fix these parameters. In fact, different initial estimates of the coefficients in the nonlinear least squares procedure would lead to different fitted values for c_t and s. There are two ways to resolve this problem. One method would be to assume that either c_t is known beforehand and equal to the salinity of the first data point, or that Y_m is known and coincides with the yield of that first point. Either assumption will fix the endpoint of the dashed curve in the upper left part of Fig. 7. One can accomplish this in the program by using either NOPT= 3 (c_t is fixed) or NOPT = 4 (Y_m is fixed), respectively. Unfortunately, this method still results in either a Y_m-value that is less than the yield associated with the first data point (NOPT = 3), or in a threshold salinity value that lies to the left of this point (NOPT = 4).

An alternative and more realistic approach would be to fix both c_t and Y_m by their values at the first data point in Fig. 7. In the program this is accomplished with a one-parameter fit for s based on Eq. [7] (NOPT = 2). Actually, the program switches automatically from NOPT = 5 to NOPT

= 2 whenever all observed data points are to the right of the fitted threshold value. The solid line in Fig. 7 was obtained with NOPT = 2. Note also that two data points appear to the right of c_o, the intersection between the fitted line and the concentration axis. The program uses an iterative procedure such that all points to the right of c_o are automatically discarded from the data set. In other words, no data points are included in the analysis whenever those points produce negative yield values as calculated with the fitted curve (see the computer output for this example in Appendix C).

For illustrative purposes, the same tomato data were analyzed also with Eq. [8]. Results are presented in Fig. 8. Clearly, Eq. [8] leads to a much better fit of the data than the piecewise linear response model, especially at higher salinity levels.

3.4. Grapefruit

Example 4 analyzes the salt tolerance of grapefruit using the same data as listed in a recent study by Feinerman et al. (1982). The data first were analyzed with NOPT = 5, i.e., for the three unknown parameters Y_m, c_t and s in Eq. 2. Figure 9 compares the fitted curve with the observed data points. Note that all data are located in the upper left part of the figure close to the threshold value. Because of a lack of observed data at the higher soil salinities, both the threshold and the slope of the curve have extremely large standard errors (see Appendix C). Actually, this was the only example that exhibited uniqueness problems during the inversion process. Uniqueness problems become apparent when different initial estimates in the computer program generate different values for the fitted parameters. The least squares method is based on the principle that the sum of squares (SSQ) of the deviations between the observed (Y_i) and calculated yields (Y_i') is minimized (see Eq. 10). In general, SSQ can be viewed as a three-dimensional function of the unknown parameters Y_m, c_t and s. In some cases, this function may manifest multiple minima to which the inversion method can converge. For the present example, several minima of SSQ were observed, one of which was located at $P_1 \equiv (Y_m, c_t, s)_1 = (103.95,\ 7.78,\ 0.0137)$, and one at $P_2 \equiv (Y_m, c_t, s)_2 = (102.76,\ 9.72,\ 0.0165)$. Figure 10 shows graphically the variation of SSQ along a straight line through these two points. Note that in actuality three minima with nearly identical SSQ's are present. The fitted line in Fig. 9 uses the parameter values associated with the lowest SSQ (P_1 in Fig. 10). From this figure, it must be clear that in this case little confidence can be attached to the accuracy of the fitted values. Example 1 was the only case encountered that exhibited this type of uniqueness problem. Nevertheless, it is recommended that the least squares inversion method be carried out with at least two different sets of initial estimates whenever the observed data are clustered around the threshold value such as was the case in this example. If the inversion results obtained with widely different initial estimates are identical, then it is probably safe to assume that the inversion is unique.

Results obtained here for grapefruit differ slightly from those obtained by Feinerman et al. (1982). This is because their regression technique differs somewhat from the nonlinear least squares method used in this study. In essence, the technique used by Feinerman et al. (1982) assumes unequal variances for the two line segments on either side of the threshold salinity, while the least squares technique used here assumes that the variances for the two lines are the same. For comparison, the

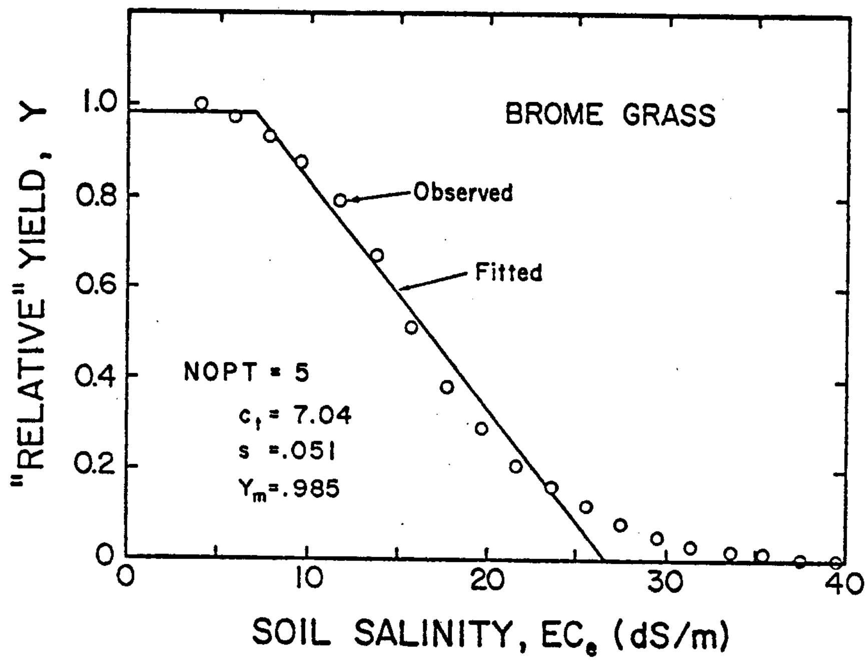

Figure 11. Observed and fitted salt tolerance response functions for bromegrass (data from McElgunn and Lawrence, 1973). The fitted curve was based on Eq. [2].

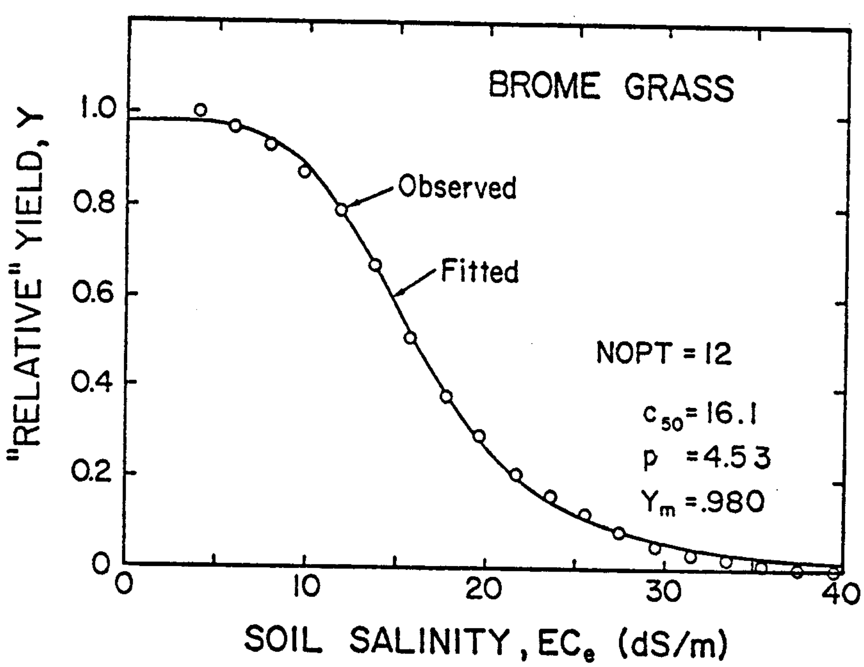

Figure 12. Observed and fitted salt tolerance response functions for bromegrass (data from McElgunn and Lawrence, 1973). The fitted curve was based on Eq. [8].

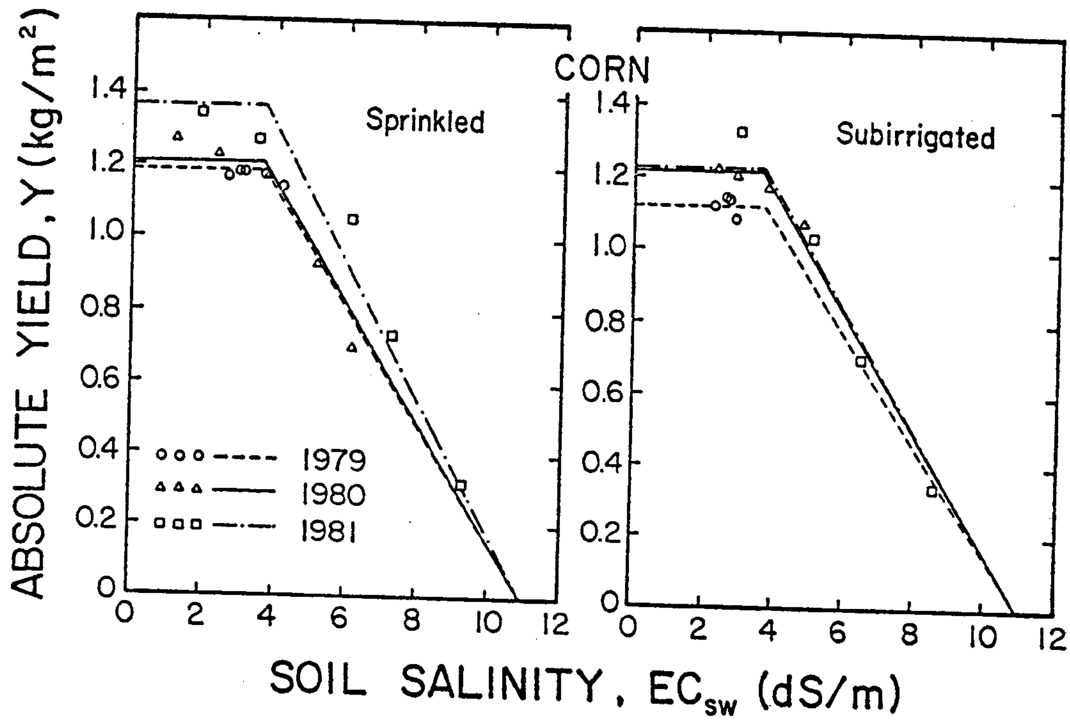

Figure 13. Observed and fitted salt tolerance response functions for corn (data from Hoffman et al., 1983).

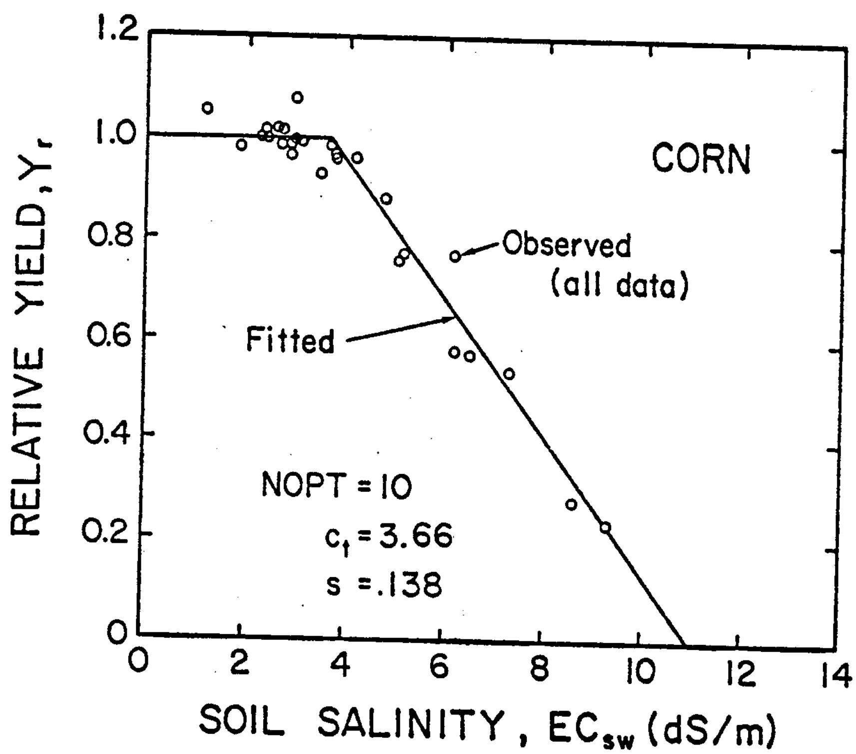

Figure 14. Plot of the relative salt tolerance function for corn as determined from the six fitted curves in Fig. 12.

In general, few uniqueness problems were observed when applying the nonlinear least squares inversion method. In one example, the observed data were found to be clustered in a relatively small portion of the salinity response curve. Data of this type can lead to large standard errors of the unknown coefficients. It is recommended that salt tolerance trials be carried out over a relatively broad range of salinity values with concomitant broad variations in observed crop yields. Such data lead to a better definition of the response function and produce smaller standard errors of the coefficients.

Osawa, T. 1965. Studies on the salt tolerance of vegetable crops with special reference to mineral nutrition. Bull. Univ. Osaka Pref., Series B, 16:13-57.

Shalhevet, J., and L. Bernstein. 1968. Effects of vertically heterogeneous soil salinity on plant growth and water uptake. Soil Sci., 106:85-93.

U.S. Salinity Laboratory Staff. 1954. Diagnosis and Improvement of Saline and Alkali Soils. USDA Handbook 60, Washington, D.C. 160 p.

van den Berg, C. 1950. The influence of salt in the soil on the yield of agricultural crops. Fourth Int. Congress Soil Sci. Trans., 1:411-413.

values of KNOB, NOPT, KBI and KB. If KNOB is positive, its value equals NOB, the number of observed data points. However, when KNOB is negative (or zero), the remaining information of that particular example still is read in, but the example is not executed. This feature allows one to set up a large file of observed data without having to execute again every case whenever some of the input parameters (e.g, NOPT) are modified. The input parameter KBI indicates whether new names of the unknown coefficients are read in for the example in question. For the first example (NCASE = 1), a card with the appropriate coefficient names must always be supplied. However, when NCASE > 1, the card with the coefficient names can be skipped if so desired. This is done by setting KBI = 0 (card 3, Table A2). In that case, the coefficient names of the previous example will be used. Finally, the input parameter KB specifies whether the initial estimates B(I) are specified in the input file (KB = 1), or are generated internally in the program (KB = 0). The option KB = 0 can be used only in connection with the piecewise linear response model (Eq. 2) and for NOPT $\leq$ 5. For all other NOPT-values, KB must be set equal to one, indicating than initial estimates of the various coefficients will be read in from the data file.

The fourth data card (Table A3) specifies the coefficient names and, as mentioned above, is needed only when either NCASE = 1 or KBI = 1 (or both). For NOPT $\leq$ 10, the first three coefficient names relate to the slope (s), the salinity threshold (c_t) and the control yield (Y_m or Y_m^1), respectively. For 11 $\leq$ NOPT $\leq$ 17, the first three coefficient names are related to c_{50}, p and Y_m, in that order. When NOPT = 18 or 19, the coefficient names are related to α, β and Y_m, again in that order. Finally, when NOPT = 20, the coefficient names are associated with β and Y_m,

	whether they are generated internally in the program (NB = 0). NB is set to one in the program for all NOPT-values exceeding 10.
NBI	Input parameter indicating whether the coefficient names are given in the input file (NBI = 1) or whether they are assumed to be the same as for the previous example. NBI is set to one in the program for all NOPT-values exceeding 10.
NC	Number of cases to be executed.
NIT	Iteration number during least squares analysis.
NOB	Number of data points (equal to the absolute value of KNOB).
NOPT	Option number indicating the type of response model and the number of unknown coefficients in that model (see Table 1).
NP	Number of unknown coefficients; generated internally as a function of the input parameter NOPT.
NPA(I)	Vector used to generate the value of NP as a function of NOPT.
SLOPE	Assumed or fitted value of the slope.
SR	Calculated correlation coefficient for the linear fit (NOPT = 1).
SREL	Value of the relative slope, s (NOPT = 1,2).
SSQ	Residual sum of squares.
STDA	Standard error of the fitted value of Y_o in Eq. [5] (NOPT = 1).
STDB	Standard error of the fitted value of s_1 in Eq. [5] (NOPT = 1).
STOPCR	Stop criterion. The iterative curve-fitting process is terminated when the relative change in the ratio of all coefficients becomes less than STOPCR. The value of STOPCR is arbitrarily set at .00001.
TITLE(I)	Vector containing the information of the title card (input label).
Y(I)	Vector of observed yields.
YMAX	Maximum of observed yields.
YMIN	Minimum of observed yields.
YZERO	Calculated value of Y_o.

7. APPENDIX B. Listing of Input Data

```
                  1         2         3         4         5         6         7         8
Column:  12345678901234567890123456789012345678901234567890123456789012345678901234567890
Card
  1          6
  2                    EXAMPLE 1: TALL FESCUE    (CR53)
  3          8    1    1    0
  4      SLOPE     THRESH    YM
  5             0.8      .508
  6             6.1      .397
  7             7.3      .470
  8             8.3      .380
  9            10.9      .248
 10            11.4      .293
 11            12.5      .238
 12            13.7      .157
 13                    EXAMPLE 2: PERENNIAL RYE    (CR53)
 14          8    2    0    0
 15             0.8      .467
 16             6.1      .496
 17             7.3      .433
 18             8.3      .245
 19            10.9      .317
 20            11.4      .293
 21            12.5      .208
 22            13.7      .179
 23                    EXAMPLE 3: TOMATO  (OSAWA, 1965)
 24          6    3    0    0
 25             2.1     2.133
 26             3.9     1.557
 27             5.6     1.557
 28             9.1      .917
 29            15.8      .384
 30            28.0      .085
 31                    EXAMPLE 4: GRAPEFRUIT (FEINERMAN ET AL., 1982)
 32         19    5    0    0
 33          5.069      100.7
 34          5.469      100.4
 35          5.476      107.4
 36          6.410      107.3
 37          7.980      103.9
 38          8.306      100.2
 39          9.346       99.4
 40         10.325      103.0
 41         10.358      101.9
 42         11.381      101.7
 43         11.467      100.7
 44         12.414       93.1
 45         13.411       93.8
 46         14.399       97.8
 47         14.461       90.3
 48         16.172       94.6
 49         16.457       92.6
 50         17.346       94.8
 51         19.822       81.9
```

8. APPENDIX C. Listing of Computer Output

```
*******************************************************************************
*                                                                             *
*          LEAST SQUARES ANALYSIS OF SALINITY RESPONSE CURVE          SALT    *
*                                                                             *
*          EXAMPLE 1: TALL FESCUE   (CR53)                                    *
*          NOPT =  1;  NP =  2                                                *
*                                                                             *
*******************************************************************************

LINEAR REGRESSION RESULTS FOR Y=YZERO-SLOPE*C
=============================================

YZERO =     0.5752 WITH STANDARD ERROR OF     0.0447
SLOPE =     0.0269 WITH STANDARD ERROR OF     0.0046
CORRELATION COEFFICIENT =     0.9222


CONTROL YIELD (YM) =     0.5080
THRESHOLD (CT) =     2.4985

SLOPE FOR ORIGINAL DATA (S*YM) =  0.026915
SLOPE FOR RELATIVE YIELD DATA (S) =  0.052983
INTERSECTION AT ZERO SALINITY (YZERO) =   0.57525
SALINITY EXTRAPOLATED TO ZERO YIELD (CZERO)=  21.37256

  CONC       Y-OBS      Y-FITTED    DEVIATION    REL YIELD     INDEX
  0.8000     0.5080     0.5537      -0.0457      1.0000         1
  6.1000     0.3970     0.4111      -0.0141      0.7815         1
  7.3000     0.4700     0.3788       0.0912      0.9252         1
  8.3000     0.3800     0.3519       0.0281      0.7480         1
 10.9000     0.2480     0.2819      -0.0339      0.4882         1
 11.4000     0.2930     0.2684       0.0246      0.5768         1
 12.5000     0.2380     0.2388      -0.0008      0.4685         1
 13.7000     0.1570     0.2065      -0.0495      0.3091         1
```

```
********************************************************************************
*                                                                              *
*         LEAST SQUARES ANALYSIS OF SALINITY RESPONSE CURVE          SALT      *
*                                                                              *
*         EXAMPLE 3: TOMATO  (OSAWA, 1965)                                     *
*         NOPT =  3; NP =  2                                                   *
*                                                                              *
********************************************************************************
```

NIT	SSQ	SLOPE	YM	THRESH
0	2.78435	0.03707	2.09034	7.00000
1	0.83345	0.04798	1.53612	7.00000
2	0.43518	0.09659	1.64509	7.00000
3	0.43342	0.09383	1.64622	7.00000
4	0.43342	0.09380	1.64623	7.00000
5	0.43342	0.09380	1.64623	7.00000

CORRELATION MATRIX
==================

	1	2
1	1.0000	
2	0.1858	1.0000

VARIABLE	VALUE	S.E.COEFF	T-VALUE	95% CONFIDENCE LIMITS LOWER	UPPER
SLOPE	0.093800	0.02249	4.170	0.0313	0.1563
YM	1.646223	0.17477	9.420	1.1610	2.1314

SLOPE FOR ORIGINAL DATA (S*YM) = 0.154416
SLOPE FOR RELATIVE YIELD DATA (S) = 0.093800
INTERSECTION AT ZERO SALINITY (YZERO) = 3.46285
SALINITY EXTRAPOLATED TO ZERO YIELD (CZERO)= 17.66096

CONC	Y-OBS	Y-FITTED	DEVIATION	REL YIELD	INDEX
2.1000	2.1330	1.6462	0.4868	1.2957	1
3.9000	1.5570	1.6462	-0.0892	0.9458	1
5.6000	1.5570	1.6462	-0.0892	0.9458	1
9.1000	0.9170	1.3220	-0.4050	0.5570	1
15.8000	0.3840	0.2874	0.0966	0.2333	1
28.0000	0.0850	0.0	0.0850	0.0516	1

```
****************************************************************************
*                                                                          *
*        LEAST SQUARES ANALYSIS OF SALINITY RESPONSE CURVE          SALT   *
*                                                                          *
*        EXAMPLE 5: BROME GRASS    (MCELGUNN AND LAWRENCE, 1973)           *
*        NOPT = 12; NP =  3                                                *
*                                                                          *
****************************************************************************
```

NIT	SSQ	C50	P-EXP	YM
0	0.31535	15.00000	2.00000	1.00000
1	0.11802	18.45212	3.77748	0.87088
2	0.01553	15.31569	4.18366	0.98748
3	0.00312	16.13985	4.48685	0.97950
4	0.00307	16.13974	4.53310	0.98035
5	0.00307	16.14027	4.53221	0.98036
6	0.00307	16.14027	4.53221	0.98036

CORRELATION MATRIX
=================

	1	2	3
1	1.0000		
2	0.5217	1.0000	
3	-0.6993	-0.5296	1.0000

VARIABLE	VALUE	S.E.COEFF	T-VALUE	95% CONFIDENCE LIMITS LOWER	UPPER
C50	16.140274	0.13079	123.408	15.8630	16.4175
P-EXP	4.532212	0.12099	37.459	4.2757	4.7887
YM	0.980359	0.00864	113.499	0.9620	0.9987

CONC	Y-OBS	Y-FITTED	DEVIATION	REL YIELD	INDEX
4.0000	1.0000	0.9786	0.0214	1.0200	1
5.9000	0.9700	0.9702	-0.0002	0.9894	1
7.9000	0.9300	0.9433	-0.0133	0.9486	1
9.8500	0.8700	0.8859	-0.0159	0.8874	1
11.8000	0.7900	0.7895	0.0005	0.8058	1
13.8000	0.6700	0.6572	0.0128	0.6834	1
15.7500	0.5100	0.5173	-0.0073	0.5202	1
17.7500	0.3800	0.3862	-0.0062	0.3876	1
19.7000	0.2900	0.2827	0.0073	0.2958	1
21.6500	0.2100	0.2049	0.0051	0.2142	1
23.6500	0.1600	0.1474	0.0126	0.1632	1
25.6000	0.1200	0.1079	0.0121	0.1224	1
27.5000	0.0800	0.0804	-0.0004	0.0816	1
29.5000	0.0500	0.0598	-0.0098	0.0510	1
31.5000	0.0300	0.0452	-0.0152	0.0306	1
33.5000	0.0200	0.0346	-0.0146	0.0204	1
35.5000	0.0100	0.0268	-0.0168	0.0102	1
37.5000	0.0000	0.0210	-0.0210	0.0000	1
39.5000	0.0000	0.0167	-0.0167	0.0000	1

9. Listing of Computer Program

```
                          MAIN
C
C     ******************************************************************
C     *                                                                *
C     *       NON-LINEAR LEAST-SQUARES ANALYSIS           SALT         *
C     *       OF SALINITY-RESPONSE CURVES                              *
C     *                                                                *
C     *       JANUARY 20, 1983                                         *
C     *                                                                *
C     ******************************************************************
C
C
      DIMENSION C(50),Y(50),F(50),R(50),DELZ(50,8),B(8),E(8),TH(8),P(8)
     1,PHI(8),Q(8),TB(8),A(8,8),BI(32),D(8,8),TITLE(20),IND(50),NPA(13)
      DATA MIT/20/,STOPCR/0.00001/,NPA/2,1,2,2,3,4,5,6,7,8,2,3,2/
C
C     ----- READ NUMBER OF CASES -----
      READ(5,1002) NC
      DO 120 NCASE=1,NC
C
C     ----- READ TITLE AND INPUT PARAMETERS -----
      READ(5,1000) TITLE
      READ(5,1002) KNOB,NOPT,KBI,KB
      IF(NOPT.GE.18) KBI=1
      IF(NOPT.GE.11) KB=1
      I=NOPT
      IF(I.GT.10) I=I-7
      NP=NPA(I)
      IF(KNOB.GT.0) WRITE(6,1003) TITLE,NOPT,NP
      NOB=IABS(KNOB)
C
C     ----- READ INITIAL ESTIMATES -----
      IF(NCASE.EQ.1.OR.KBI.EQ.1) READ(5,1006) (BI(I),I=17,32)
      IF(KB.EQ.1) READ(5,1008) (B(I),I=1,8)
      CMAX=0.0
      CMIN=10000.0
      YMAX=0.0
      YMIN=10000.0
C
C     ----- READ INPUT DATA -----
      DO 2 I=1,NOB
      READ(5,1010) C(I),Y(I),IND(I)
      IF(NP.LE.3) IND(I)=1
      CMIN=AMIN1(CMIN,C(I))
      YMIN=AMIN1(YMIN,Y(I))
      IF(Y(I).LT.0.001) GO TO 2
      CMAX=AMAX1(CMAX,C(I))
    2 YMAX=AMAX1(YMAX,Y(I))
      IF(KNOB.LT.0) GO TO 120
```

MAIN

```
      DO 16 I=1,NOB
      SA=SA+Y(I)-YM
   16 SB=SB+C(I)-CT
      SLOPE=-SA/SB
      SREL=SLOPE/YM
      YZERO=YM+SLOPE*CT
      CZERO=CT+YM/SLOPE
      WRITE(6,1014) YM,CT
      WRITE(6,1034) SLOPE,SREL,YZERO,CZERO
      TH(3)=YM
      DO 18 I=1,NOB
      F(I)=YM-SLOPE*(C(I)-CT)
   18 R(I)=Y(I)-F(I)
      IR=0
      DO 20 I=1,NOB
      IF(F(I).GE.0.) GO TO 20
      IR=1
      C(I)=CT
      Y(I)=YM
   20 CONTINUE
      IF(IR.GT.0) GO TO 14
      GO TO 109
C
C     ----- NONLINEAR LEAST-SQUARES ANALYSIS -----
   22 IF(KB.EQ.1) GO TO 26
      B(1)=(1.-YMIN/YMAX)/(CMAX-CMIN)
      B(2)=AMAX1(1.1*CMIN,0.25*CMAX)
      DO 24 I=3,8
   24 B(I)=0.98*YMAX
      IF(NOPT.EQ.4.OR.NOPT.EQ.11) B(3)=Y(1)
   26 IP=MAX0(NP,3)
      IF(NOPT.EQ.22) IP=2
      NP2=2*NP
      IP2=2*IP
      DO 28 I=1,16
   28 BI(I)=BI(I+16)
      NT=0
      B1=B(1)
      B2=B(2)
      B3=B(3)
      IF(NOPT.NE.3) GO TO 30
      B2=B(3)
      B(3)=B(2)
      B(2)=B2
      BI(3)=BI(21)
      BI(4)=BI(22)
      BI(5)=BI(19)
      BI(6)=BI(20)
```

MAIN

```
      DO 62 I=1,NP
      IF(TH(I)*TB(I)) 66,66,62
   62 CONTINUE
      SUMB=0
      CALL MODEL(TB,F,NOB,C,IND,NOPT)
      DO 64 I=1,NOB
      R(I)=Y(I)-F(I)
   64 SUMB=SUMB+R(I)*R(I)
   66 SUM1=0.0
      SUM2=0.0
      SUM3=0.0
      DO 68 I=1,NP
      SUM1=SUM1+P(I)*PHI(I)
      SUM2=SUM2+P(I)*P(I)
   68 SUM3=SUM3+PHI(I)*PHI(I)
      ANGLE=89.959-57.29578*ATAN(SUM1/SQRT(SUM2*SUM3-SUM1**2))
      DO 72 I=1,NP
      IF(TH(I)*TB(I)) 74,74,72
   72 CONTINUE
      IF(SUMB/SSQ-1.0) 80,80,74
   74 IF(ANGLE-30.0) 76,76,78
   76 STEP=0.5*STEP
      GO TO 56
   78 GA=10.*GA
      GO TO 50
C
C     ----- PRINT COEFFICIENTS AFTER EACH ITERATION -----
   80 CONTINUE
      DO 82 I=1,NP
   82 TH(I)=TB(I)
      WRITE(6,1018) NIT,SUMB,(TH(I),I=1,IP)
      IF(NOPT.GT.10.AND.NOPT.LT.18) GO TO 88
      IF(NOPT.EQ.3.OR.NOPT.EQ.20) GO TO 88
      IF(NOPT.LT.11) GO TO 85
      IF(ABS(TH(I)).GT.1.0E-06) GO TO 88
      IF(NT.EQ.1) GO TO 83
      NT=1
      B(1)=-B1
      B(2)=B2
      B(3)=B3
      GO TO 30
   83 B(1)=B(2)
      B(2)=B(3)
      DO 84 I=1,6
   84 BI(I)=BI(I+2)
      NOPT=20
      NP=2
      WRITE(1,1020)
```

MAIN

```
      IF(NOPT.EQ.3) SLOPE=TH(1)*TH(2)
      YZERO=TH(3)*(1.+TH(1)*TH(2))
      IF(NOPT.EQ.3) YZERO=B(2)*(1.+TH(1)*TH(3))
      CZERO=TH(2)+1./TH(1)
      IF(NOPT.EQ.3) CZERO=TH(3)+1./TH(1)
      WRITE(6,1034) SLOPE,TH(1),YZERO,CZERO
C
C     ----- PREPARE FINAL OUTPUT -----
  109 WRITE(6,1036)
      DO 118 I=1,NOB
      K=2+IND(I)
      IF(NOPT.EQ.3.OR.NOPT.EQ.20) K=2
      RY=Y(I)/TH(K)
      WRITE(6,1038) C(I),Y(I),F(I),R(I),RY,IND(I)
  118 CONTINUE
  119 IF(NOPT.NE.5) GO TO 120
      IF(TH(2).GT.CMIN) GO TO 120
      WRITE(6,1024)
      NP=1
      GO TO 10
  120 CONTINUE
C
C     ----- END OF PROBLEM -----
 1000 FORMAT(20A4)
 1002 FORMAT(6I5)
 1003 FORMAT(1H1,10X,82(1H*)/11X,1H*,80X,1H*/11X,1H*,10X,'LEAST SQUARES
     1ANALYSIS OF SALINITY RESPONSE CURVE',11X,'SALT',6X,1H*/11X,1H*,80X
     2,1H*/11X,1H*,20A4,1H*/11X,1H*,10X,'NOPT =',I3,': NP =',I3,52X,1H*/
     311X,1H*,80X,1H*/11X,82(1H*))
 1006 FORMAT(8(A4,A2,4X))
 1008 FORMAT(8F10.0)
 1010 FORMAT(2F10.0,I10)
 1012 FORMAT(//11X,'LINEAR REGRESSION RESULTS FOR Y=YZERO-SLOPE*C'/11X,4
     15(1H=)//11X,'YZERO =',F10.4,' WITH STANDARD ERROR OF',F10.4/11X,'S
     2LOPE =',F10.4,' WITH STANDARD ERROR OF',F10.4/11X,'CORRELATION COE
     3FFICIENT =',F10.4)
 1014 FORMAT(//11X,'CONTROL YIELD (YM) =',F10.4/11X,'THRESHOLD (CT) =',F
     110.4)
 1016 FORMAT(///11X,'NIT',6X,'SSQ',3X,10(4X,A4,A2))
 1018 FORMAT(10X,I3,F12.5,1X,10F10.5)
 1020 FORMAT(//11X,'ALPHA IS TOO SMALL, NEW NOPT =20'/11X,32(1H=))
 1022 FORMAT(//11X,'THRESHOLD IS TOO SMALL, USE ANOTHER NOPT'/11X,40(1H=
     1))
 1024 FORMAT(//11X,'CHANGED TO OPTION NUMBER 2')
 1025 FORMAT(//11X,'WARNING: THRESHOLD IS LESS THAN CMIN, USE ANOTHER NO
     1PT'/11X,55(1H=))
 1026 FORMAT(//13X,'CORRELATION MATRIX'/13X,18(1H=)/11X,10(3X,I2,4X))
 1028 FORMAT(9X,I4,10(1X,F7.4,1X))
```

MODEL

```
      SUBROUTINE MODEL(B,Y,NOB,C,IND,NOPT)
C
C     PURPOSE: TO CALCULATE Y(C)
C
      DIMENSION B(8),Y(50),C(50),IND(50)
C
C     ---------------
      IF(NOPT.GT.10) GO TO 10
    2 B2=B(2)
      IF(NOPT.EQ.3) B2=B(3)
      IF(NOPT.EQ.3) B3=B(2)
      DO 8 I=1,NOB
      IF(NOPT.NE.3) B3=B(2+IND(I))
      IF(C(I)-B2) 4,4,6
    4 Y(I)=B3
      GO TO 8
    6 Y(I)=B3-B(1)*B3*(C(I)-B2)
      Y(I)=AMAX1(Y(I),0.)
    8 CONTINUE
      RETURN
   10 IF(NOPT.GT.17) GO TO 14
      DO 12 I=1,NOB
   12 Y(I)=B(2+IND(I))/(1.+(C(I)/B(1))**B(2))
      RETURN
   14 IF(NOPT.EQ.20) GO TO 18
      DO 16 I=1,NOB
   16 Y(I)=B(3)*EXP(B(1)*C(I)-B(2)*C(I)**2)
      RETURN
   18 DO 20 I=1,NOB
   20 Y(I)=B(2)*EXP(-B(1)*C(I)**2)
      RETURN
      END
```