# Mass Transport in Saturated-Unsaturated Porous Media: One-Dimensional Solutions

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# Mass Transport in Saturated-Unsaturated Porous Media: One-Dimensional Solutions

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*ABSTRACT*

Numerical solutions of the non-linear partial differential equations describing the simultaneous movement of water and solutes in a one-dimensional saturated-unsaturated and non-homogeneous soil profile are presented. The effects of linear adsorption and zero- and first-order decay are included in the governing transport equation. The numerical solutions are based upon one finite difference (FD) and three finite element schemes: linear finite elements with (MFE) and without mass-lumping (LFE), and Hermitian cubic finite elements (HFE).

The HFE-scheme always generated the most accurate solutions of both the moisture and solute fronts when simulating the infiltration of water and chloride in a 125-cm deep soil profile, but this occurred at the expense of somewhat more computation time. It is concluded that the FD- and MFE-schemes are preferred when infiltration in extremely dry soil needs to be simulated. The HFE-scheme seems more attractive for less extreme cases. One of the example problems compares results obtained with the Hermitian finite element scheme with those generated with a newly developed analytical solution for solute movement under steady-state flow conditions. The example was used to check the programming accuracy of the various decay terms in the transport equation.

The Hermitian finite element computer model (SUMATRA-1) is listed in an appendix of this report. As illustrated by an example, the model may be used to study water and solute movement in a one-dimensional, saturated-unsaturated and non-homogeneous soil profile. Both fairly abrupt layering and smoothly changing soil profile properties are considered in the model.

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*NOTATION*

| | |
|---|---|
| $a$ | Parameter in Eq. (47) ($L^{-1}$). |
| $a_j$ | Entries in unknown coefficient vector {X}. |
| [A] | Coefficient matrix in global matrix equation. |
| $b_i$ | Below diagonal entries of [P]. |
| [B] | Coefficient matrix of time derivative in global matrix equation. |
| $c$ | Concentration ($ML^{-3}$). |
| $c_i$ | Initial concentration ($ML^{-3}$). |
| $c_\ell$ | Concentration at $x = \ell$ ($ML^{-3}$). |
| $c_o$ | Concentration at $x = 0$, or of leaching solution ($ML^{-3}$). |
| $\hat{c}$ | Finite element approximation of c ($ML^{-3}$). |
| $\bar{c}$ | Laplace transform of c. |
| $C^*$ | Specific soil moisture capacity ($L^{-1}$). |
| $C_i$ | Values of c at nodes ($ML^{-3}$). |
| $d_i$ | Diagonal entries of [P]. |
| $D$ | Dispersion coefficient ($L^2T^{-1}$). |
| $D_o$ | Molecular diffusion coefficient ($L^2T^{-1}$). |
| $D^-$ | Dispersion coefficient corrected for numerical dispersion, to be evaluated at new time level ($L^2T^{-1}$). |
| $D^+$ | Dispersion coefficient corrected for numerical dispersion, to be evaluated at old time level ($L^2T^{-1}$). |
| $e_i$ | Above diagonal entries of [P]. |
| {F} | Right-hand side vector of global matrix equation. |
| $g$ | Parameter defined by Eq. (26a). |
| $h$ | Pressure head (L). |

*NOTATION (continued):*

| | |
|---|---|
| $h_i$ | Initial pressure head (L). |
| $h_\ell$ | Pressure head at $x = \ell$ (L). |
| $h_o$ | Pressure head at $x = 0$ (L). |
| $\hat{h}$ | Finite element approximation of h (L). |
| $H_i$ | Nodal values of pressure head (L). |
| k | Distribution coefficient ($M^{-1}L^3$). |
| K | Hydraulic conductivity ($LT^{-1}$). |
| $K_A, K_B$ | Hydraulic conductivity of soil types A and B ($LT^{-1}$). |
| $K_{cl}, K_{ls}$ | Hydraulic conductivity of clay loam and loamy sand ($LT^{-1}$). |
| $K_s$ | Saturated hydraulic conductivity ($LT^{-1}$). |
| $\hat{K}$ | Hydraulic conductivity distribution over an arbitrary element ($LT^{-1}$). |
| $\ell$ | Depth of soil profile (L). |
| $L_s$ | Operator on c as defined by Eq. (7). |
| $L_w$ | Operator on h as defined by Eq. (1). |
| m | Parameter defined by Eq. (47). |
| n | Parameter in Eq. (47); also used for the number of equations in the Galerkin solution of the transport equation. |
| $p_i$ | Constants in Eq. (A2). |
| [P] | Global coefficient matrix for new time level. |
| q | Volumetric flux ($LT^{-1}$). |
| $q_1$, $q_n$ | Volumetric flux at nodes 1 and n, respectively ($LT^{-1}$). |
| $q_\ell$ | Volumetric flux at $x = \ell$ ($LT^{-1}$). |
| $q_o$ | Volumetric flux at $x = 0$ ($LT^{-1}$). |
| $q_s$ | Solute flux ($ML^{-2}T^{-1}$). |

*NOTATION (continued):*

| | |
|---|---|
| $[Q]$ | Global coefficient matrix for old time level. |
| $R$ | Retardation factor. |
| $s_i$ | Entries of vector $\{S\}$. |
| $S$ | Adsorbed concentration. |
| $\{S\}$ | Right-hand vector of global matrix equation. |
| $S_s$ | Specific storage coefficient ($L^{-1}$). |
| $S_w$ | Degree of fluid saturation. |
| $t$ | Time (T). |
| $t_o$ | Pulse length (T). |
| $v$ | Average pore-water velocity ($LT^{-1}$). |
| $v^*$ | Parameter defined by Eq. (43c) ($LT^{-1}$). |
| $x$ | Vertical distance (L). |
| $\{X\}$ | Vector of unknown coefficients. |
| $\alpha$ | First-order liquid phase rate coefficient ($T^{-1}$). |
| $\beta$ | First-order solid phase rate coefficient ($T^{-1}$). |
| $\gamma$ | Zero-order liquid phase rate coefficient ($ML^{-3}T^{-1}$). |
| $\Delta t$ | Time increment (T). |
| $\Delta x$ | Nodal distance (L). |
| $\varepsilon$ | Porosity. |
| $\theta$ | Volumetric moisture content. |
| $\theta_i$ | Initial moisture content. |
| $\theta_\ell$ | Moisture content at $x = \ell$. |
| $\theta_o$ | Moisture content at $x = 0$. |
| $\theta_r$ | Residual moisture content. |
| $\theta_s$ | Saturated moisture content. |
| $\hat{\theta}$ | Moisture content distribution over an arbitrary element. |

*NOTATION (continued):*

| | |
|---|---|
| $\Theta$ | Dimensionless moisture content. |
| $\lambda$ | Dispersivity (L). |
| $\mu$ | General first-order decay coefficient ($T^{-1}$). |
| $\xi$ | Local coordinate. |
| $\xi_0$ | Parameter appearing in definition of basis functions. |
| $\rho$ | Bulk density ($ML^{-3}$). |
| $\tau$ | Tortuosity factor. |
| $\phi$ | General basis functions. |
| $\phi_j^0$ | Linear basis functions. |
| $\phi_{0j}^1, \phi_{1j}^1$ | Hermitian basis functions. |

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## 1. INTRODUCTION

The simultaneous transport of water and solutes under transient saturated-unsaturated conditions plays an important role in many branches of agriculture and engineering. In agriculture many chemicals such as fertilizers and pesticides, as well as those naturally present in irrigation waters, are routinely applied to the land. Some of these chemicals will remain in the root zone or will be taken up by plant roots, while others will be subject to leaching, thereby becoming potential contributors to the pollution of underlying groundwater systems. The need to limit such groundwater pollution, yet to let the plants make optimal use of applied nutrients, makes it necessary to have a clear understanding of the behavior of these chemicals in the unsaturated zone.

The increased use of land for the disposal of a wide variety of domestic and industrial wastes further accentuates the importance of understanding transport processes in the unsaturated zone. Without an adequate insight into these processes, the long-term consequences of such waste disposal practices are likely to result in a gradual and continuous deterioration of both soil and groundwater resources.

As a natural outgrowth of the desire of agricultural and environmental engineers to understand chemical behavior in the unsaturated zone, quantitative descriptions of the relevant transport mechanisms were attempted. Results of such research can be used to analyze alternative management practices for minimizing soil and groundwater pollution. This has led to an extensive body of literature dealing with the numerical simulation of chemical transport in the unsaturated zone. Most of the earlier studies on saturated-unsaturated transport have used finite difference techniques for solution of the governing transfer equations[1-17], although recently also several finite element solutions have been published[18-22].

It is the purpose of this report to identify and select a constructive model that may be used to simulate the simultaneous flow of water and solutes in a one-dimensional (vertical) soil profile under transient saturated-unsaturated conditions. The resulting model is based upon a Hermitian (cubic) finite element solution of the governing transport equations, and includes such processes as linear equilibrium adsorption and zero- and first-order decay. A fully documented listing of the computer program is given at the end of the report. Results obtained with the first-order continuous Hermitian finite element scheme are compared with those based upon finite differences and two linear finite element schemes. An example problem, furthermore, illustrates the use of the Hermitian finite element code for studying solute transport in unsaturated, non-homogeneous field soils.

## 2. THEORETICAL DEVELOPMENT

### 2.1. Governing equations.

The partial differential equation governing the one-dimensional vertical flow of water in a saturated-unsaturated medium is given by

$$L_w(h) \equiv \frac{\partial}{\partial x} \left(K \frac{\partial h}{\partial x} - K\right) - C^* \frac{\partial h}{\partial t} = 0 \qquad (1)$$

where

$C^*$ is the specific soil moisture capacity ($L^{-1}$),

$h$ is the pressure head ($L$),

$K$ is the hydraulic conductivity ($LT^{-1}$),

$x$ is the vertical distance (positive down) ($L$), and

$t$ is the time ($T$).

The specific moisture capacity ($C^*$) for a saturated-unsaturated medium is defined as

$$C^* = \frac{\theta}{\varepsilon} S_s + \varepsilon \frac{\partial S_w}{\partial h} \qquad (2)$$

where

$\theta$ is the volumetric moisture content ($L^0$),

$\varepsilon$ is the porosity,

$S_s$ is the specific storage coefficient ($L^{-1}$), and

$S_w$ is the degree of fluid saturation.

The second term on the right-hand side of (2) is zero for a completely saturated medium. The term containing $S_s$ in (2), on the other hand, will be insignificantly small compared to the second term when the soil becomes unsaturated. $C^*$ in that case will be closely approximated by the slope of the moisture content - pressure head curve, i.e.,

$$C^* \simeq \frac{\partial \theta}{\partial h} . \tag{3}$$

Equation (1) is highly non-linear in the unsaturated zone because both the moisture content and the hydraulic conductivity are functions of the pressure head. This study does not consider hysteresis in any of these functions.

The initial condition of the system is given by

$$h(x,0) = h_i(x) \quad . \tag{4}$$

One of the following two boundary conditions may be specified at the soil surface (x = 0)

$$h(0,t) = h_o(t) \tag{5a}$$

$$\left(-K \frac{\partial h}{\partial x} + K\right)\Bigg|_{x=0} = q_o(t) \tag{5b}$$

where $q_o(t)$ represents the actual (net) flux at the soil surface (precipitation + irrigation - evaporation). Similar boundary conditions may also be applied to the lower boundary of the soil profile ($x = \ell$), i.e.,

$$h(\ell,t) = h_\ell(t) \qquad (6a)$$

or

$$\left(-K \frac{\partial h}{\partial x} + K\right)\Big|_{x=\ell} = q_\ell(t) \qquad (6b)$$

where $q_\ell(t)$ represents the imposed drainage flux. For a free draining profile $q_\ell(t)$ equals K at $x = \ell$, and (6b) reduces to

$$\frac{\partial h}{\partial x}\Big|_{x=\ell} = 0 \qquad (6c)$$

The governing equation for chemical transport under transient flow conditions is taken as

$$L_s(c) = \frac{\partial}{\partial x}\left(\theta D \frac{\partial c}{\partial x} - qc\right) - \frac{\partial}{\partial t}(\theta c + \rho S) + \alpha\theta c + \beta\rho S + \gamma\theta \qquad (7)$$

where

- $c$ is the solution concentration ($ML^{-3}$),
- $D$ is the dispersion coefficient ($L^2T^{-1}$),
- $S$ is the adsorbed concentration,
- $q$ is the volumetric flux ($LT^{-1}$),
- $\alpha$ is a first-order rate constant (liquid phase) ($T^{-1}$),
- $\beta$ is a first-order rate constant (solid phase) ($T^{-1}$),
- $\gamma$ is a zero-order rate constant (liquid phase) ($ML^{-3}T^{-1}$), and
- $\rho$ is the bulk density ($ML^{-3}$).

The solution of (7) requires knowledge of both the moisture content ($\theta$) and the volumetric flux (q). The moisture content is assumed to be a unique function of the pressure head (h), and can therefore be obtained from solutions of Eq. (1). Also the volumetric flux follows immediately from solutions of (1) by making use of Darcy's law as follows

$$q = -K \frac{\partial h}{\partial x} + K \tag{8}$$

The dispersion coefficient (D) represents the effects of both molecular diffusion and mechanical dispersion, and is assumed to be adequately defined by

$$D = D_o \tau + \lambda |v| \tag{9}$$

where

$D_o$ is the molecular diffusion coefficient ($L^2T^{-1}$),

$\tau$ is the tortuosity factor,

$\lambda$ is the dispersivity (L), and

v (= $q/\theta$) is the average pore-water velocity ($LT^{-1}$).

The solution of (7) requires an expression relating the adsorbed concentration (S) with the solution concentration (c). Several models for adsorption or ion exchange are available for this purpose, such as equilibrium and kinetic models. In this study only single-ion equilibrium transport is considered, and the general adsorption isotherm is described by a linear (or linearized) isotherm of the form

$$S = k\ c \tag{10}$$

where k is an empirical constant ($M^{-1}L^3$). Substitution of (10) into (7) gives

$$L_s(c) = \frac{\partial}{\partial x}\left(\theta D \frac{\partial c}{\partial x} - qc\right) - \frac{\partial}{\partial t}(\theta R c) + (\alpha\theta + \beta\rho k)c + \gamma\theta \tag{11}$$

where the retardation factor R is defined as

$$R = 1 + \frac{\rho k}{\theta} . \tag{12}$$

The initial condition for the concentration is given by

$$c(x,0) = c_i(x) . \tag{13}$$

The following mixed (or third-type) boundary condition may be specified at the soil surface

$$\left(-\theta D \frac{\partial c}{\partial x} + qc\right)\Bigg|_{x=0} = \begin{cases} q_o(t)\ c_o(t) & \text{if } q_o(t) > 0 \\ 0 & \text{if } q_o(t) \leq 0 \end{cases} \tag{14a}$$

where $q_o(t)$ is the same as in (5b), and where $c_o(t)$ is the concentration of the infiltrating water. Note that the solute flux becomes zero during periods of evaporation ($q_o < 0$) and redistribution ($q_o = 0$). In some cases it may be necessary to specify a first-type boundary condition at the soil surface. In that case (14a) should be replaced by

$$c(0,t) = c_o(t) \ . \qquad (14b)$$

The following boundary condition at $x=\ell$ may be used when a free draining profile is considered

$$\left.\frac{\partial c}{\partial x}\right|_{x=\ell} = 0 \ . \qquad (15a)$$

In some cases it may be necessary to specify a first-type boundary condition at the lower boundary of the soil profile; for example when, during periods of excessive evaporation, the profile is in contact with saline groundwater. In that case one has

$$c(\ell,t) = c_\ell(t) \qquad (15b)$$

where $c_\ell(t)$ is the concentration of the groundwater. Equation (15b) holds only for upward flow of water at $x=\ell$, i.e., for $q_\ell(t) < 0$.

## *2.2. Numerical solution of the transport equation.*

Galerkin-finite element techniques were used to solve the governing flow and transport equations. The numerical solution of the flow equation was given in a separate report[23], and only the solution procedure for the transport equation will be given here.

In the finite element approach the dependent variable, c, is approximated by a finite series of the form

$$c(x,t) \simeq \hat{c}(x,t) = \sum_{j=1}^{n} \phi_j(x)\ a_j(t) \tag{16}$$

where the $\phi_j(x)$ are the selected basis (or shape) functions, and the $a_j(t)$ unknown, time dependent coefficients which represent solutions of (11) at specified points ("nodes") within the solution domain. The approximate solution $\hat{c}(x,t)$ will converge to the correct solution $c(x,t)$ when n increases to infinity. Because only a finite number of basis functions are used in (16), it must be evident that the residual $L_s(\hat{c})$, obtained by substituting (16) into (11), will not become zero but attain a certain non-zero value. The residual, however, may be minimized by making $L_s(\hat{c})$ orthogonal to a set of mutually independent weighting functions. In the Galerkin method these weighting functions are chosen to be identical to the basis functions $\phi_j(x)$ in (16), resulting in the following set of n equations in n unknowns

$$\int_0^{\ell} L_s \left[ \sum_{j=1}^{n} \phi_j(x)\ a_j(t) \right] \phi_i(x)\ dx = 0 \qquad i=1,2,\ldots,n \tag{17}$$

Substitution of (11) into (17) and further simplification will lead to a set of n ordinary differential equations of the form

$$[A]\ \{X\} + [B]\ \{\frac{dX}{dt}\} = \{F\} \qquad (18)$$

where the coefficients [A], [B] and {F} represent arrays which are functions of the spatial coordinate, and where {X} is a vector containing the unknown, time dependent coefficients $a_j(t)$.

The analysis leading to (18) is characteristic of most finite element solutions of (1), or similar transport equations, and shows that the Galerkin method is used only for approximation of the spatial derivatives while the time derivatives have been left intact. Although the finite element method could be easily extended to also include integration of the time derivatives in (11), such an approach would become unnecessarily complicated and probably also computationally inefficient[24]. The common approach, therefore, has been to solve (18) by finite difference methods[19,20,25]. For a centered-in-time, Crank-Nicolson, approach, for example, this will result in the following difference scheme

$$\frac{1}{2}[A]\ (\{X\}^{t+\Delta t} + \{X\}^{t}) + \frac{1}{\Delta t}\ [B]\ (\{X\}^{t+\Delta t} - \{X\}^{t}) = \{F\} \qquad (19)$$

where $\Delta t$ is the time increment used in the numerical calculations. Although the scheme based on (19) is second-order correct in time, it has been shown to frequently produce oscillations in computed concentration distributions, especially when a sharp concentration front is simulated[26,27]. This oscillatory behavior is generally most serious for large values of the dimensionless group $q\Delta x/(\theta D)$, in which $\Delta x$ represents the element size.

Instead of (19) one could also use a backward-in-time (implicit) finite difference scheme of the form

$$[A]\ \{X\}^{t+\Delta t} + \frac{1}{\Delta t}\ [B]\ (\{X\}^{t+\Delta t} - \{X\}^{t}) = \{F\}\ . \qquad (20)$$

This approach, leading to a first-order approximation of the time derivative, effectively removes the oscillations but unfortunately, often at the expense of a smeared concentration front[26,29].

Some of the problems associated with the occurrence of undesired oscillations and excessive numerical dispersion can be eliminated by making use of a higher-order scheme for approximation of the time derivative. Such a higher-order integration scheme will become especially attractive when, as in this study, higher-order basis functions are used in the finite element formulation. Earlier studies[27,28] have shown that very accurate solutions of the one-dimensional convective-dispersive equation (Eq. 21 below) can be obtained through the introduction of appropriate dispersion corrections.

$$\frac{\partial c}{\partial t} = D\ \frac{\partial^2 c}{\partial x^2} - v\ \frac{\partial c}{\partial x} \qquad (21)$$

It is shown in Appendix A that dispersion-corrections can be obtained also for the transient flow case (Eq. 11). For the present analysis this is accomplished by redefining (11) as follows (see Appendix A for the derivation)

$$L_s(c) \equiv \frac{(\theta Rc)^{t+\Delta t} - (\theta Rc)^{t}}{\Delta t}$$

$$- \frac{1}{2} \left[\frac{\partial}{\partial x} (\theta D^- \frac{\partial c}{\partial x} - qc) + (\alpha\theta + \beta\rho k)c + \gamma\theta\right]^{t+\Delta t}$$

$$- \frac{1}{2} \left[\frac{\partial}{\partial x} (\theta D^+ \frac{\partial c}{\partial x} - qc) + (\alpha\theta + \beta\rho k)c + \gamma\theta\right]^{t} = 0 \qquad (22)$$

where

$$D^- = D - \frac{q^2 \Delta t}{6\theta^2 R} \qquad (23a)$$

$$D^+ = D + \frac{q^2 \Delta t}{6\theta^2 R} \qquad (23b)$$

Equation 22 is essentially a centered-in-time, Crank-Nicolson type scheme with correction factors applied to the dispersion coefficients. Note that the correction factors are different for the old and new time levels.

The finite element analysis, starting with (22) instead of (11), proceeds along familiar lines. Substitution of (22) into (17) and integration by parts of the spatial derivatives yields the following matrix equation

$$[P]^{t+\Delta t} \{X\}^{t+\Delta t} = [Q]^{t} \{X\}^{t} + \{S\} \qquad (24)$$

with the different arrays given by

$$[P_{ij}] = \int_0^{\ell} \left[\frac{1}{2}(\theta D^- \frac{d\phi_j}{dx} - q\phi_j) \frac{d\phi_i}{dx} + (\frac{\theta R}{\Delta t} - g)\ \phi_j\ \phi_i\ \right]dx \qquad (25a)$$

$$[Q_{ij}] = \int_0^{\ell} [-\frac{1}{2}(\theta D^{+} \frac{d\phi_j}{dx} - q\phi_j) \frac{d\phi_i}{dx} + (\frac{\theta R}{\Delta t} + g) \phi_j \phi_i]dx \quad (25b)$$

$$\{S_i\} = \int_0^{\ell} \frac{1}{2}\gamma(\theta^{t+\Delta t} + \theta^{t}) \phi_i dx - \frac{1}{2}(q_s^{t+\Delta t} + q_s^{t})\phi_i \Big|_0^{\ell} \quad (25c)$$

$$\{X_i\} = a_i \quad (25d)$$

and where

$$g = \frac{1}{2}(\alpha\theta + \beta\rho k) \quad (26a)$$

$$q_s = -\theta D \frac{\partial \hat{c}}{\partial x} + q\hat{c} \quad (26b)$$

Equation (24) shows that the coefficient matrices [P] and [Q] are evaluated at the new and old time levels, respectively, but that the vector {S} (Eq. 25c) contains parameters which have to be evaluated at both time levels. The unknown coefficients $a_i$ are obtained by evaluating the integrals in (25) and subsequently solving matrix Eq. (24). The approximate solution $\hat{c}(x,t)$ follows then immediately upon substitution of these coefficients into (16).

## 2.3. *Basis Functions*

To facilitate evaluation of the integrals appearing in (25), the soil profile may be subdivided into an assemblage of subdomains or "elements". The basis functions $\phi_j(x)$ are then used to spatially approximate the unknown function over each element separately. Several sets of basis functions are available for this purpose, such as the zero-order continuous linear, quadratic or cubic basis functions. These functions characteristically attain a unit value at one nodal point of the element, and a zero value at the remaining nodes, while they are identical to zero outside the element considered. From this definition it follows immediately that the integrals in (25) only need to be evaluated once over each single element, and that the unknown coefficients $a_j(t)$ now coincide with the values of the dependent variable, c, at the node for which the basis function was defined. For a linear, one-dimensional element, for example, Eq. (16) reduces to

$$\hat{c}(x,t) = C_1(t)\ \phi_1^0(x) + C_2(t)\ \phi_2^0(x) \tag{27}$$

where $C_1(t)$ and $C_2(t)$ represent the unknown concentration values of the two corner nodes of the element. The basis functions $\phi_j^0$ in (27) can be written in terms of a local ($\xi$) coordinate system, as follows[30]

$$\phi^0 = \frac{1}{2}(1+\xi_0\xi) \tag{28}$$

where $\xi_0=\pm 1$. The local coordinate $\xi$ is defined in terms of the global coordinate system x, as

$$\xi = \frac{2(x-x_1)}{\Delta x} - 1 \qquad (x_1 < x < x_2) \qquad (29)$$

where $\Delta x = (x_2 - x_1)$ represents the nodal distance of the element considered.

A special class of basis functions is based on Hermitian polynomials. If these are used, one not only solves for the values of the function itself, but also for the values of the spatial derivatives. For example, the approximating function $\hat{c}$, when used on conjunction with the one-dimensional, first-order continuous cubic (Hermitian) basis functions, becomes

$$\hat{c}(x,t) = \sum_{j=1}^{2} \left[ \phi^1_{0j}(x)\ C_j(t) + \phi^1_{1j}(x)\ \frac{dC_j}{dx}(t) \right] \qquad (30)$$

with the Hermitian basis functions, in terms of the $\xi$-coordinate, given by[31]

$$\phi^1_0 = -\frac{1}{4}(\xi+\xi_0)^2(\xi\xi_0-2) \qquad (\xi_0 = \pm 1) \qquad (31a)$$

$$\phi^1_1 = \frac{\Delta x}{8}\ \xi_0(\xi+\xi_0)^2(\xi\xi_0-1) \qquad (\xi_0 = \pm 1) \qquad (31b)$$

Figure 1 gives a graphical representation of the four basis functions defined by (31).

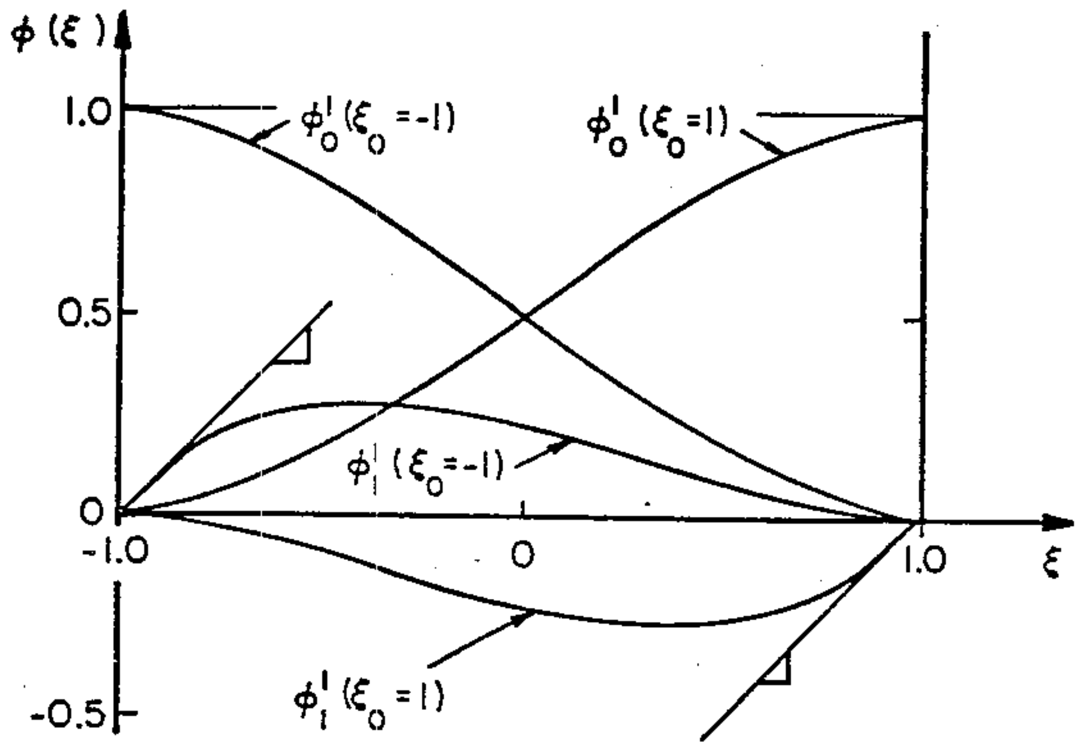



Fig. 1. First-order continuous Hermitian basis functions[32].

### *2.4. Numerical implementation*

Several approaches are possible for evaluating the integrals in (25). One approach would be to expand the different coefficients under the integral signs in terms of the basis functions and the values of the coefficients at the element corners, analogous to (27) and (30) for linear and Hermitian elements, respectively. For linear elements this is accomplished as follows

$$\hat{f} = f_1\phi_1^0(\xi) + f_2\phi_2^0(\xi) \tag{32}$$

where $\hat{f}$ represents spatial approximations for any of the coefficients (or group of coefficients) in (25): $(\theta D)$, $q$, $(\theta R)$, $g$, or $(\gamma\theta)$, and where $f_1$ and $f_2$ are the values of these coefficients at the corners of the element. The advantage of this approach is that the integrations need to be carried out only once, thereby minimizing computational effort. Application of (32) to each of the terms in $[P_{ij}]$ (Eq. 25a) and direct integration of the spatial integrals leads to a tri-diagonal matrix of the form

$$[P] = \begin{bmatrix} d_1 & e_1 & & 0 \\ b_2 & d_2 & e_2 & \\ & & & \\ & b_{n-1} & d_{n-1} & e_{n-1} \\ 0 & & b_n & d_n \end{bmatrix}. \tag{33}$$

The entries $b_i$, $d_i$, and $e_i$ of [P] are given in Table 1. A comparison of (25a) and (25b) shows that the entries of [Q] will become nearly identical to those of [P] in Table 1, except that in the equations $D^-$, q, and g have to be replaced by $(-D^+)$, (-q), and (-g), respectively. For the entries $s_i$ of the vector {S} (Eq. 25c) one obtains:

$$s_1 = 2r_1 + r_2 + \frac{1}{2}[q_s(0,t+\Delta t) + q_s(0,t)] \qquad (34a)$$

$$s_i = r_{i-1} + 4r_i + r_{i+1} \qquad (i = 2,\ldots,n-1) \qquad (34b)$$

$$s_n = r_{n-1} + 2r_n - \frac{1}{2}[q_s(\ell,t+\Delta t) + q_s(\ell,t)] \qquad (34c)$$

where

$$r_i = \frac{\Delta x}{12} \gamma_i (\theta_i^{t+\Delta t} + \theta_i^t) \ . \qquad (35)$$

The expansions above are obtained with the assumption that the nodal distance ($\Delta x$) is identical for all elements. The derivation of the coefficient matrices remains essentially the same for varying $\Delta x$, except that the final expressions will become more complicated.

It is also possible to evaluate the integrals in (25) numerically, for example by using Gaussian quadrature. The pressure head distribution ($\hat{h}$) over each element is, for this purpose, first obtained from the solutions of the flow equation by making use of an expansion similar to (27) for linear, and (30) for Hermitian elements. This leads directly to estimates of the soil moisture content ($\theta$) and the hydraulic conductivity (K) at

*Table 1.* Values of $e_i$, $d_i$, and $b_i$ in [P] (Eq. 33) for linear finite elements (LFE).

$$e_i = -\frac{1}{4\Delta x}(\theta_i D_i^- + \theta_{i+1}D_{i+1}^-) + \frac{1}{12}(q_i + 2q_{i+1}) + \frac{\Delta x}{12\Delta t}(\theta_i R_i + \theta_{i+1}R_{i+1}) - \frac{\Delta x}{12}(g_i + g_{i+1})$$

$$d_1 = \frac{1}{4\Delta x}(\theta_1 D_1^- + \theta_2 D_2^-) + \frac{1}{12}(2q_1 + q_2) + \frac{\Delta x}{12\Delta t}(3\theta_1 R_1 + \theta_2 R_2) - \frac{\Delta x}{12}(3g_1 + g_2)$$

$$d_i = \frac{1}{4\Delta x}(\theta_{i-1}D_{i-1}^- + 2\theta_i D_i^- + \theta_{i+1}D_{i+1}^-) + \frac{1}{12}(q_{i+1} - q_{i-1}) + \frac{\Delta x}{12\Delta t}(\theta_{i-1}R_{i-1} + 6\theta_i R_i + \theta_{i+1}R_{i+1})$$

$$- \frac{\Delta x}{12}(g_{i-1} + 6g_i + g_{i+1}) \qquad (i = 2,\ldots,n-1)$$

$$d_n = \frac{1}{4\Delta x}(\theta_{n-1}D_{n-1}^- + \theta_n D_n^-) - \frac{1}{12}(q_{n-1} + 2q_n) + \frac{\Delta x}{12\Delta t}(\theta_{n-1}R_{n-1} + 3\theta_n R_n) - \frac{\Delta x}{12}(g_{n-1} + 3g_n)$$

$$b_i = -\frac{1}{4\Delta x}(\theta_{i-1}D_{i-1}^- + \theta_i D_i^-) - \frac{1}{12}(2q_{i-1} + q_i) + \frac{\Delta x}{12\Delta t}(\theta_{i-1}R_{i-1} + \theta_i R_i) - \frac{\Delta x}{12}(g_{i-1} + g_i)$$

each numerical integration point. Differentiation of $\hat{h}$ with respect to x, furthermore, gives an estimate of the pressure gradient distribution over each element, information which in turn is used to calculate the volumetric flux (q) at each integration point. The dispersion coefficient subsequently follows from (9). For non-homogeneous soils it is also necessary to obtain estimates of the bulk density ($\rho$), the distribution constant (k), the dispersivity ($\lambda$) and the rate coefficients $\alpha$, $\beta$, and $\gamma$ at each integration point.

An obvious disadvantage of numerical integration is that the integrals in (25) need to be integrated every time step. This is because $\theta$, D, and q are functions of time. The first approach, direct integration, was consequently followed in the case of linear basis functions since this approach resulted in an efficient and fairly accurate scheme. Poor results, however, were obtained when this approach was used in the case of Hermitian basis functions, especially for solution of the flow equation (Eq. 1)[23]. The second approach, numerical integration, was hence used in conjunction with Hermitian elements. A five-point Lobatto integration scheme was used for that purpose (see Eq. 25.4.32 of Abramowitz and Stegun[33]). Lobatto integration has a distinct advantage over Gaussian integration in that this method locates some of the integration points at the element corners. Inspection of Fig. 1 shows that the Hermitian basis function or their gradients become zero at least at one, and in some cases at both corner nodes. Several of the terms in (25) hence reduce to zero for the nodal integration points, thereby reducing computational effort and leading to a more efficient numerical integration method[23]).

An alternative linear finite element scheme may be obtained by applying mass lumping to the time derivative. This approach was first used by Neuman[34] to enhance numerical convergence when simulating infiltration in extremely dry soil. Mass lumping in time was achieved here by approximating the last terms in (25a) and (25b) by

$$\int_0^{\ell} \left(\frac{\theta R}{\Delta t} \pm g\right)\phi_j\phi_i dx = \int_0^{\ell} \frac{\theta R}{\Delta t}\,\phi_i dx \pm \int_0^{\ell} g\phi_j\phi_i dx \qquad (36)$$

Substitution of (36) into (25a) and (25b) and subsequent direct integration of the spatial integrals analogous to the procedure outlined earlier leads to slightly different equations for the entries $b_i$, $d_i$, and $e_i$ in [P] (Eq. 33). Table 2 gives appropriate formulas for these entries. The matrix [Q] will become the same as the new matrix [P] provided that $D^-$, q, and g are replaced by $(-D^+)$, $(-q)$, and $(-g)$. The vector {S}, of course, is not affected by mass lumping.

Another scheme considered in this study is based on a finite difference approximation of the transport equation. Applying standard finite difference techniques to the derivatives in (11) and including the same dispersion corrections as before (Eq. 23) leads to a final matrix equation of the type given by (24). The matrix [P] in this case becomes slightly different than for mass-lumped linear finite elements (see Table 3). The vector {S} remains the same as before, while [Q] follows again from [P] by replacing $D^-$, q, and g by $(-D^+)$, $(-q)$, and $(-g)$, respectively.

*Table 2.* Values of $e_i$, $d_i$, and $b_i$ in [P] (Eq. 33) for mass-lumped finite elements (MFE).

$$e_i = -\frac{1}{4\Delta x}(\theta_i D_i^- + \theta_{i+1} D_{i+1}^-) + \frac{1}{12}(q_i + 2q_{i+1}) - \frac{\Delta x}{12}(g_i + g_{i+1})$$

$$d_1 = \frac{1}{4\Delta x}(\theta_1 D_1^- + \theta_2 D_2^-) + \frac{1}{12}(2q_1 + q_2) + \frac{\Delta x}{6\Delta t}(2\theta_1 R_1 + \theta_2 R_2) - \frac{\Delta x}{12}(3g_1 + g_2)$$

$$d_i = \frac{1}{4\Delta x}(\theta_{i-1} D_{i-1}^- + 2\theta_i D_i^- + \theta_{i+1} D_{i+1}^-) + \frac{1}{12}(q_{i+1} - q_{i-1}) + \frac{\Delta x}{6\Delta t}(\theta_{i-1} R_{i-1} + 4\theta_i R_i + \theta_{i+1} R_{i+1})$$

$$- \frac{\Delta x}{12}(g_{i-1} + 6g_i + g_{i+1}) \qquad (i = 2,\ldots,n-1)$$

$$d_n = \frac{1}{4\Delta x}(\theta_{n-1} D_{n-1}^- + \theta_n D_n^-) - \frac{1}{12}(q_{n-1} + 2q_n) + \frac{\Delta x}{6\Delta t}(\theta_{n-1} R_{n-1} + 2\theta_n R_n) - \frac{\Delta x}{12}(g_{n-1} + 3g_n)$$

$$b_i = -\frac{1}{4\Delta x}(\theta_{i-1} D_{i-1}^- + \theta_i D_i^-) - \frac{1}{12}(2q_{i-1} + q_i) - \frac{\Delta x}{12}(g_{i-1} + g_i)$$

*Table 3.* Values of $e_i$, $d_i$, and $b_i$ in [P] (Eq. 33) for finite differences (FD).

$$e_i = -\frac{1}{4\Delta x}(\theta_i D_i^- + \theta_{i+1} D_{i+1}^-) + \frac{1}{12}(q_i + 2q_{i+1}) - \frac{\Delta x}{12}(g_i + g_{i+1})$$

$$d_1 = \frac{1}{4\Delta x}(\theta_1 D_1^- + \theta_2 D_2^-) + \frac{1}{12}(2q_1 + q_2) + \frac{\Delta x}{2\Delta t}(\theta_1 R_1) - \frac{\Delta x}{12}(3g_1 + g_2)$$

$$d_i = \frac{1}{4\Delta x}(\theta_{i-1} D_{i-1}^- + 2\theta_i D_i^- + \theta_{i+1} D_{i+1}^-) + \frac{1}{12}(q_{i+1} - q_{i-1}) + \frac{\Delta x}{\Delta t}(\theta_i R_i) - \frac{\Delta x}{12}(g_{i-1} + 6g_i + g_{i+1})$$

$$(i = 2, \ldots, n-1)$$

$$d_n = \frac{1}{4\Delta x}(\theta_{n-1} D_{n-1}^- + \theta_n D_n^-) - \frac{1}{12}(q_{n-1} + 2q_n) + \frac{\Delta x}{2\Delta t}(\theta_n R_n) - \frac{\Delta x}{12}(g_{n-1} + 3g_n)$$

$$b_i = -\frac{1}{4\Delta x}(\theta_{i-1} D_{i-1}^- + \theta_i D_i^-) - \frac{1}{12}(2q_{i-1} + q_i) - \frac{\Delta x}{12}(g_{i-1} + g_i)$$

## *3. RESULTS*

Three example problems will be presented in this section. The first one considers the often quoted field infiltration experiment of Warrick *et al.*[35][see also Bresler[5], Ungs *et al.*[12], Gureghian *et al.*[15], and Segol[21]]. The example is used to study the accuracy of the four numerical schemes discussed in the previous section (i.e., finite differences, linear finite elements, mass-lumped linear finite elements, and Hermitian finite elements). The second example considers the movement of a chemical undergoing adsorption and decay in a one-dimensional soil profile under steady-state flow and constant moisture content conditions (q and $\theta$ are constants). Numerical results are compared with those based on an analytical solution, leading to a verification of the different rate terms in the transport equation. The third example, finally, illustrates how the Hermitian finite element code SUMATRA-1, listed in Appendix C, can be used to study water and solute movement in a non-homogeneous and layered field soil.

### *3.1. The infiltration experiment of Warrick et al.*

In this example (Warrick *et al.*[35]) water and chloride are allowed to infiltrate into a 125-cm deep, homogeneous soil profile having the following hydraulic properties:

$$\theta(h) = \begin{cases} 0.6829 - 0.09524 \ln(|h|) & h \leq -29.484 \\ 0.4531 - 0.02732 \ln(|h|) & -29.484 < h \leq -14.495 \end{cases} \tag{37a}$$

$$K(h) = \begin{cases} 19.34\ 10^4\ |h|^{-3.4095} & h \leq -29.484 \\ 516.8\ |h|^{-0.97814} & -29.484 < h \leq -14.495 \end{cases} \quad (37b)$$

where the hydraulic conductivity is given in cm/day, and the pressure head in cm. The initial and boundary conditions are as follows:

$$\theta(x,0) = \begin{cases} 0.1500 + 0.0008333\ x & 0 < x \leq 60 \\ 0.2000 & 60 < x \leq 125 \end{cases} \quad (38a)$$

$$c(x,0) = 0. \quad (38b)$$

$$h(0,t) = -14.495 \qquad (\theta_o = 0.38) \quad (38c)$$

$$c(0,t) = \begin{cases} 209 & 0 < t \leq 0.11667 \\ 0. & t > 0.11667 \end{cases} \quad (38d)$$

$$h(125,t) = -159.19 \qquad (\theta_\ell = 0.20) \quad (38e)$$

$$\frac{\partial c}{\partial x}(125,t) = 0. \quad (38f)$$

where the concentration is given in meq/liter. The initial condition (38a) defines a linear increase in the soil moisture content to a depth of 60 cm. The equivalent pressure distribution at t=0 follows directly from (38a) by making use of the soil moisture retention curve (Eq. 37a).

The present example was used to evaluate the accuracy of the four numerical schemes discussed above, i.e., finite differences (FD), linear finite elements (LFE), mass-lumped linear finite elements (MFE), and Hermitian finite elements (HFE). Figures 2 and 3 show the computed moisture distributions versus depth, while the corresponding solute distributions are presented in Fig. 4 and 5. The solid line in each figure represents the assumed "correct" solution, and was obtained by using increasingly smaller spatial and time increments until all four schemes (FD, LFE, MFE, and HFE) generated the same results. A constant spatial increment ($\Delta x$) of 2.5 cm was used for the three zero-order continuous schemes (FD, LFE, and MFE), while the Hermitian finite element scheme used a $\Delta x$ of 5 cm.

Figure 2 shows that the FD and LFE-schemes generate moisture distributions which deviate slightly from the correct one, especially near the toe of the wetting front. Some small oscillations are also present near the toe of the moisture front for the LFE-scheme. These oscillations are characteristic of those cases where sharp moisture (and pressure) fronts are simulated. Figure 2 shows that the oscillations are only significant during the early stages of infiltration, and that they gradually disappear in time. It appears that, at least from a practical point of view, these oscillations are of minor importance, especially if one is aware of the numerical reason for their presence. The magnitude of the oscillations can be decreased by using small spatial and time increments.

Results obtained with the MFE-scheme (Fig. 3) are nearly identical to those obtained with the LFE-scheme, except near the toe of the moisture front. No oscillations were observed when mass-lumping was applied to the linear finite element scheme. Mass-lumping, hence, is very effective in removing the undesired oscillations.

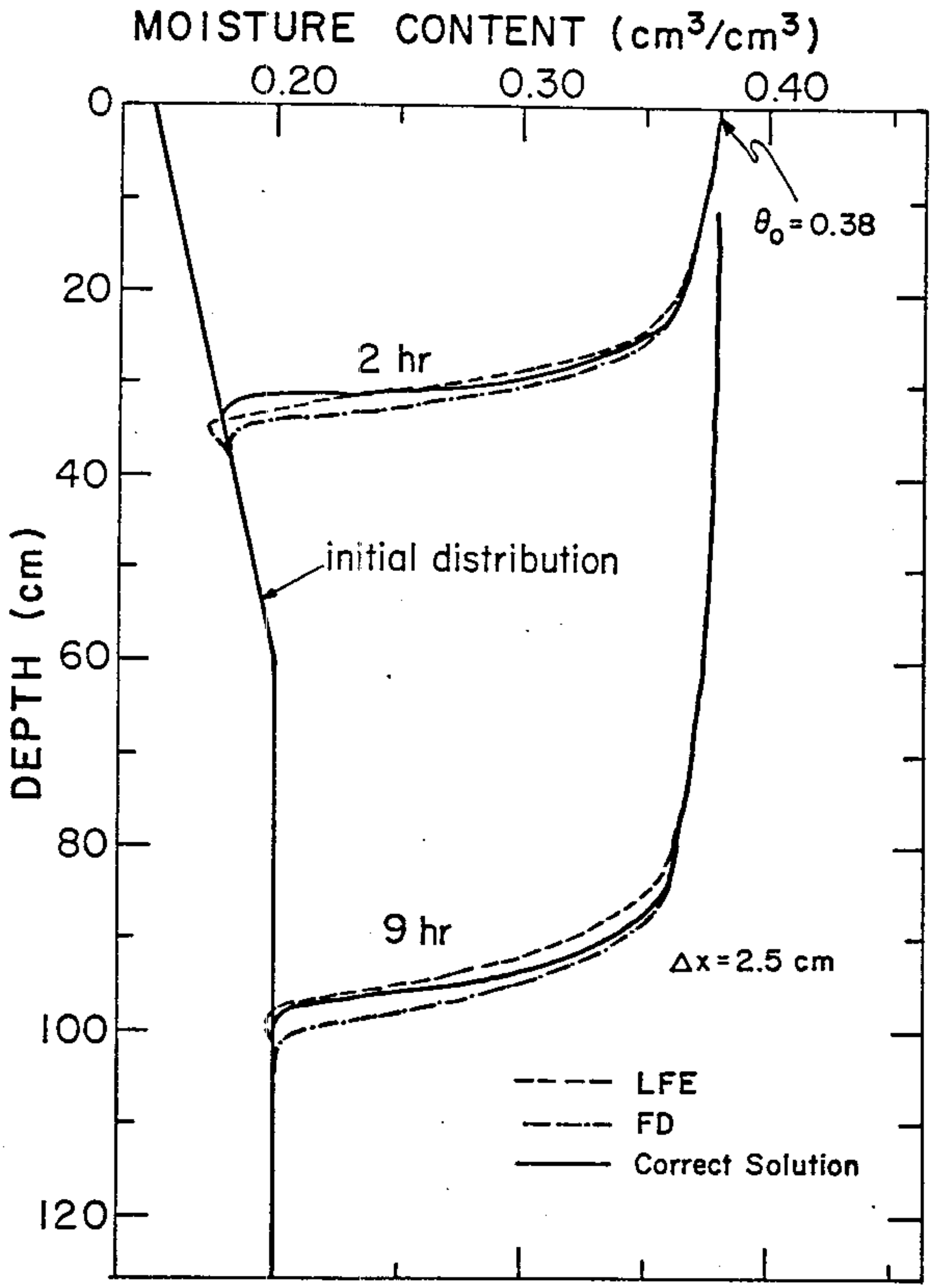



Fig. 2. Moisture content profiles obtained with finite differences (FD) and linear finite elements (LFE)

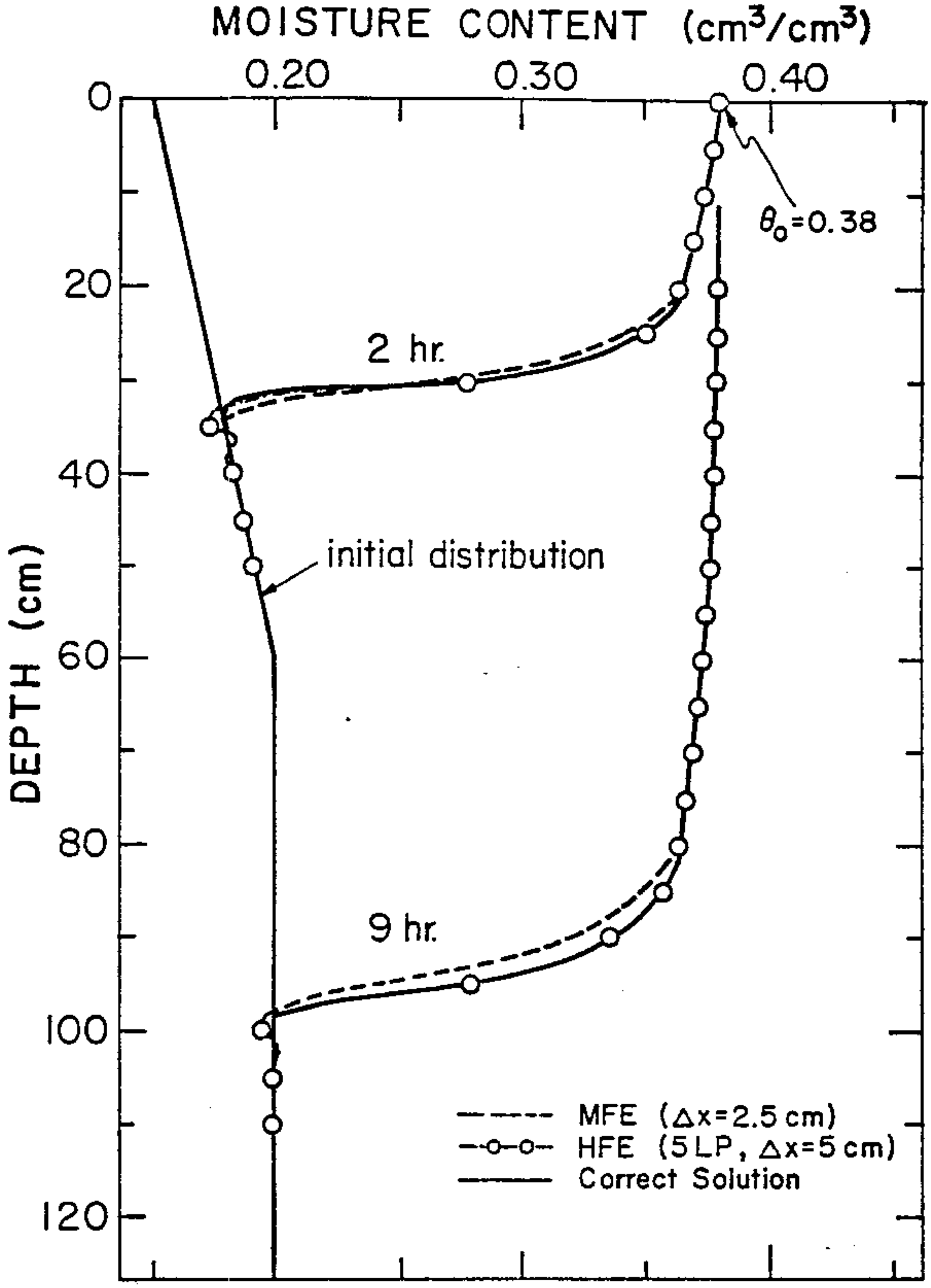



Fig. 3. Moisture content profiles obtained with mass-lumped linear (MFE) and Hermitian finite elements (HFE)

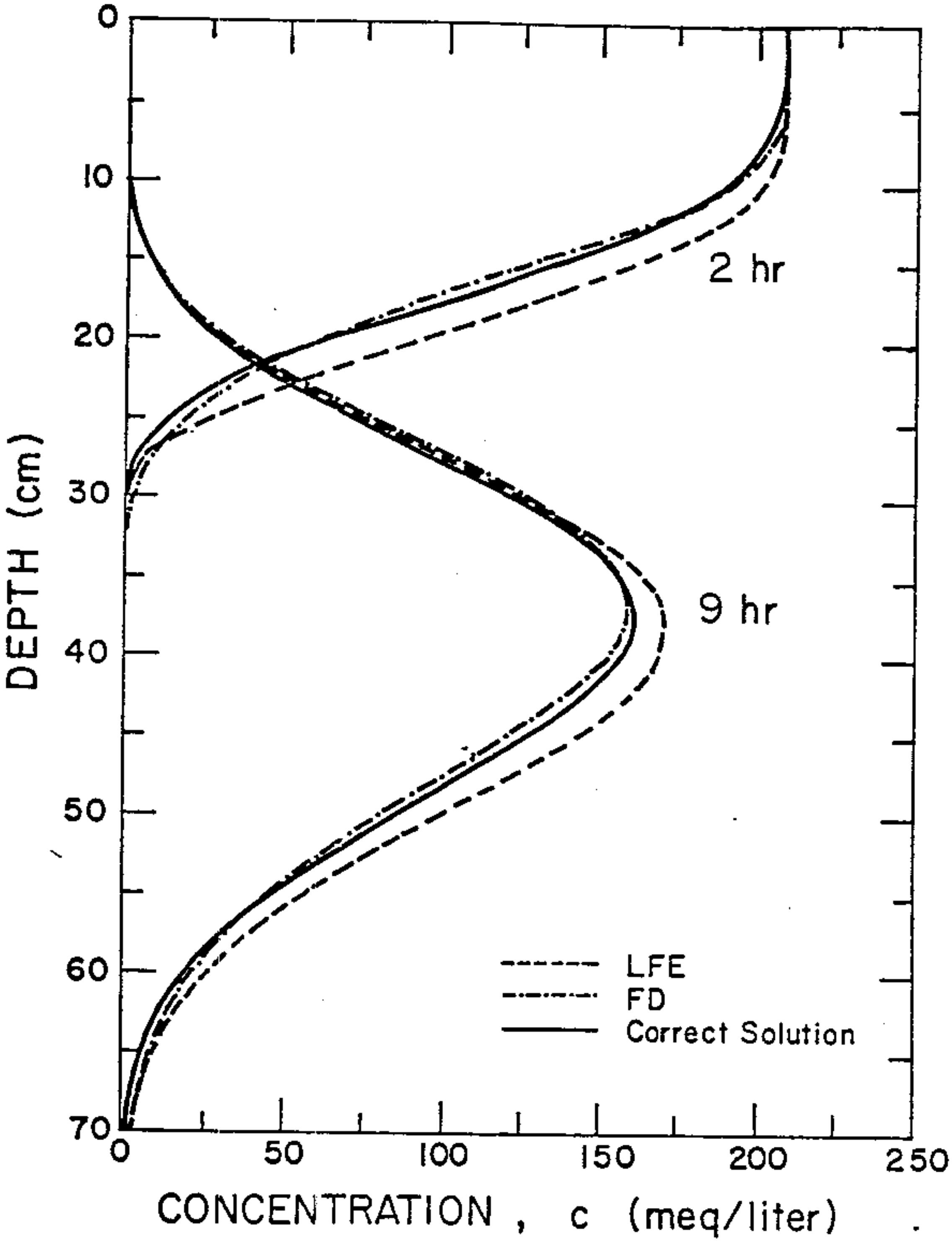



Fig. 4. Chloride distributions versus depth obtained with finite differences (FD) and linear finite elements (LFE)

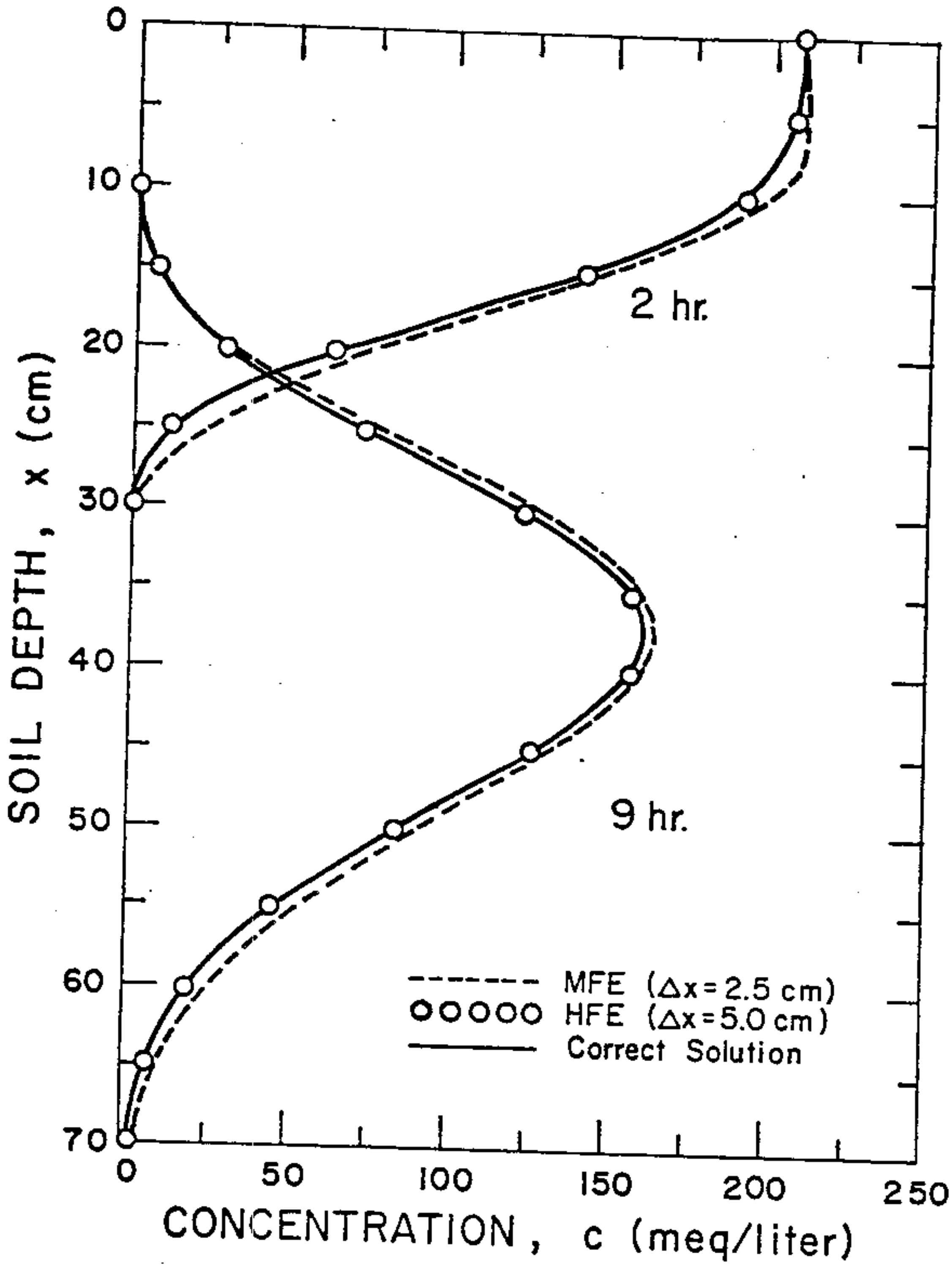



Fig. 5. Chloride distributions versus depth obtained with mass-lumped linear (MFE) and Hermitian finite elements (HFE)

The most accurate results were obtained with the Hermitian finite element scheme (Fig. 3). The numerical solution, obtained with a 5-point Lobatto integration scheme, is nearly identical to the correct solution, except again for a few oscillations near the wetting front. Approximately the same results were obtained with a 4-point Gaussian quadrature scheme. Application of lower-order integration schemes, on the other hand, such as 4-point Lobatto and 3-point Gaussian quadrature, resulted in computed distributions which lagged slightly behind the correct solution. A more extensive discussion of Fig. 2 and 3 is given in an earlier report[23].

Compuated chloride distributions versus depth are shown in Fig. 4 and 5. The most accurate results were again obtained with the HFE-scheme. Also the FD- and MFE-schemes generated fairly accurate results, albeit less than for the HFE-scheme. Results obtained with the LFE-scheme were considerably less accurate than those for the other schemes. It appears that most of the observed deviations between the LFE-results and the correct solution were generated during the early stages of infiltration (Fig. 4). Some serious oscillations in computed solute distributions were observed at that time, resulting in a deeper penetration of the solute front after two hours.

Figures 2-5 demonstrate that the first-order continuous Hermitian finite element scheme generates solutions which are superior to those obtained with the three zero-order continuous schemes. Unfortunately, this was accomplished with approximately two times as much computer time. Several reasons account for this. First, the HFE-scheme generates matrix equations for both flow and transport which have a bandwidth of 7, while the three zero-order continuous schemes produce matrix equations which have a bandwidth of only 3. Hence, much more time is needed for solution of the Hermitian global matrix equation. Second, numerical integration techniques were used to evaluate the integrals of Eq. (25) for the HFE-scheme. This in itself already leads to more computer time, but, in addition, forces one to a more time-consuming element by element assembly of the global matrix equations. The HFE-scheme hence does not immediately present an attractive alternative to the other schemes, unless its relative accuracy does not change dramatically with an increase in the element size $\Delta x$. To study this, several computer runs were made with different spatial increments. Doubling the element size, for example, still led to fairly accurate descriptions of the moisture and solute fronts, although now some more serious oscillations did appear in the computed moisture distributions[23]. The computer time in this case, however, was reduced by a factor of about three, and became roughly equal to the computer time needed for execution of the various zero-order continuous schemes.

The various curves in Fig. 2, 3, 4, and 5 were obtained with first-type, constant pressure and concentration boundary conditions imposed at the soil surface (Eq. 38c, 38d). Several computer runs were also made for the case where a constant flux is imposed, i.e. Eq. (5b) and (14a) with $q_o = 37.8$ cm/day (equal to the hydraulic conductivity at a moisture content of 0.38 $cm^3/cm^3$) and $c_o = 209$ meq/l. Considerably better and nearly identical results were now obtained with the three zero-order continuous schemes (FD, LFE, and MFE). The HFE-scheme, however, remained the most accurate one of the four. Required computer times, when using a flux-type boundary condition, were approximately 25% less than when a first-type boundary condition was used (see also van Genuchten[23]).

From the examples given here and several other numerical experiments it is concluded tht the FD- and MFE-schemes will generate the most stable solutions when a steep moisture front is present, e.g., during infiltration into a very dry, coarse soil. The resulting solutions, however, may diverge somewhat from the correct ones when the simulation progresses in time, while the calculated wetting fronts could become more dispersed (less steep) as compared to the correct fronts. The HFE-scheme seems superior in locating the correct spatial location of the moisture and solute fronts, although some oscillations may develop near the toe of the wetting front, especially when large elements are used. If some minor oscillations are permitted, the HFE-scheme becomes very competitive with the FD- and MFE-schemes. In many cases no sharp concentration fronts need to be simulated. In that case the Hermitian finite element scheme becomes very attractive because much larger spatial and time increments are allowed with this method. For the more extreme cases, however, it appears that finite difference and mass-lumped finite elements will generate the best results.

## *3.2. Effects of production and decay on solute transport.*

This example considers the effects of the different rate coefficients in the transport equation (Eq. 11) on solute movement under steady flow and constant moisture content conditions. The example is included to verify the accuracy of the Hermitian finite element code listed in Appendix C.

For steady flow (q is constant) and a profile at constant moisture content ($\theta$ is constant), the transport equation (Eq. 11) reduces to

$$R \frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2} - v \frac{\partial c}{\partial x} - \mu c + \gamma \tag{39}$$

where $\mu$ represents the combined effect of linear decay in the solution and absorbed phases:

$$\mu = - (\alpha + \frac{\beta \rho k}{\theta}) \ . \tag{40}$$

Note that $\mu$ here is taken to be positive for decay and negative for production.

Equation (39) will be solved for a semi-infinite profile and the following initial and boundary conditions

$$c(x,0) = c_i \tag{41a}$$

$$(-D \frac{\partial c}{\partial x} + vc)\bigg|_{x=0} = \begin{cases} vc_o & 0 < t \leq t_o \\ 0 & t > t_o \end{cases} \ . \tag{41b}$$

where $c_i$ and $c_o$ are assumed to be constant. The analytical solution, derived in Appendix B, is given by

$$c(x,t) = \begin{cases} (c_o - \frac{\gamma}{\mu})\; c_1(x,t) + c_2(x,t) & 0 < t \le t_o \\ (c_o - \frac{\gamma}{\mu})\; c_1(x,t) + c_2(x,t) - c_o\, c_1(x,t-t_o) & t > t_o \end{cases} \qquad (42)$$

where

$$\begin{aligned} c_1(x,t) = {} & \frac{v}{(v+v^*)} \exp\left[\frac{(v-v^*)x}{2D}\right] \operatorname{erfc}\left[\frac{Rx - v^*t}{2(DRt)^{\frac{1}{2}}}\right] \\ & + \frac{v}{(v-v^*)} \exp\left[\frac{(v+v^*)x}{2D}\right] \operatorname{erfc}\left[\frac{Rx + v^*t}{2(DRt)^{\frac{1}{2}}}\right] \\ & + \frac{v^2}{2\mu D} \exp\left(\frac{vx}{D} - \frac{\mu t}{R}\right) \operatorname{erfc}\left[\frac{Rx + vt}{2(DRt)^{\frac{1}{2}}}\right] \end{aligned} \qquad (43a)$$

$$\begin{aligned} c_2(x,t) = {} & \left(\frac{\gamma}{\mu} - c_i\right) \exp\left(-\frac{\mu t}{R}\right) \left\{ \frac{1}{2} \operatorname{erfc}\left[\frac{Rx - vt}{2(Dt)^{\frac{1}{2}}}\right] \right. \\ & + \left(\frac{v^2 t}{\pi RD}\right)^{1/2} \exp\left[-\frac{(Rx - vt)^2}{4DRt}\right] \\ & \left. - \frac{1}{2}\left(1 + \frac{vx}{D} + \frac{v^2 t}{DR}\right) \exp\left(\frac{vx}{D}\right) \operatorname{erfc}\left[\frac{Rx + vt}{2(DRt)^{\frac{1}{2}}}\right] - 1 \right\} \\ & + \frac{\gamma}{\mu} \end{aligned} \qquad (43b)$$

and

$$v^* = (v^2 + 4\,\pi D)^{1/2} \qquad (43c)$$

Analytical and Hermitian finite element results for two specific cases are compared in Fig. 6 and 7. Figure 6 shows solute distributions for a chemical (e.g., a pesticide) undergoing linear adsorption ($R > 0$) and linear decay ($\mu > 0$) in a 100-cm deep soil profile. Parameter values for this example were, rather arbitrarily, chosen as follows

| | |
|---|---|
| $v = 25$ cm/day | $D = 37.5$ cm$^2$/day |
| $\rho = 1.4$ g/cm$^3$ | $\theta = 0.30$ cm$^3$/cm$^3$ |
| $k = 0.5$ | $\alpha = -0.10$ 1/day |
| $\beta = -0.05$ 1/day | $\gamma = 0.0$ meq/liter/day |
| $c_i = 0.0$ meq/liter | $c_o = 1.0$ meq/liter |
| $t_o = 5.0$ days | |

It is evident from Fig. 6 that analytical and numerical results are essentially the same. The dashed curve was obtained without decay ($\alpha = \beta = \gamma = 0$), and is included to accentuate the effects of the first-order decay terms on the calculated distributions. Equations (41) and (42) for no decay reduce to[36]

$$c(x,t) = \begin{cases} c_i + (c_o - c_i)\; c_1(x,t) & 0 < t \leq t_o \\ \\ c_i + (c_o - c_i)\; c_1(x,t) - c_o\; c_1(x,t-t_o) & t > t_o \end{cases} \qquad (44)$$

where

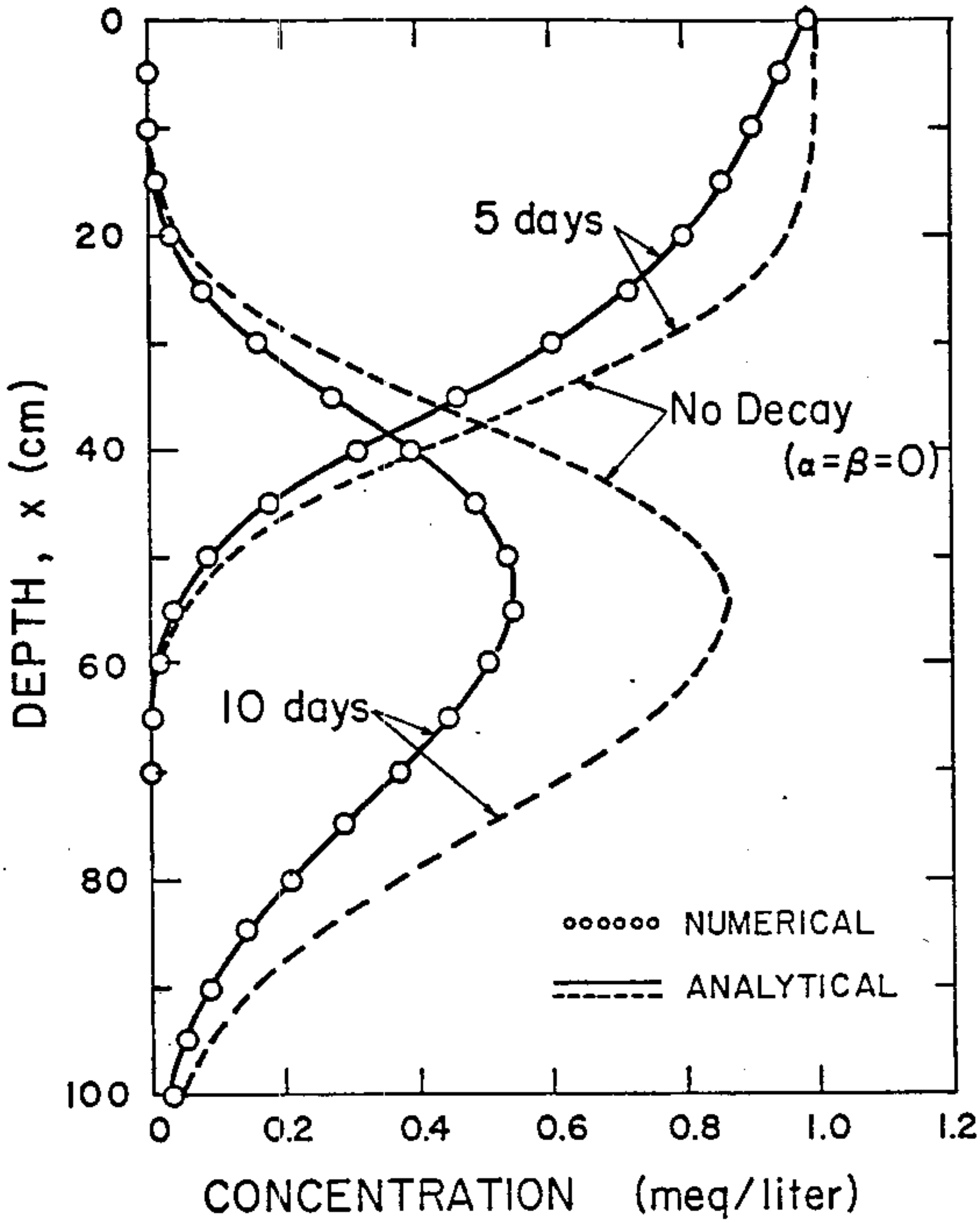



Fig. 6. Calculated concentration distributions versus depth for a chemical undergoing linear adsorption and linear decay during steady leaching through a 100 cm deep soil profile. The dashed line was obtained without decay. Values of the different parameters used in the calculations are given in the text.

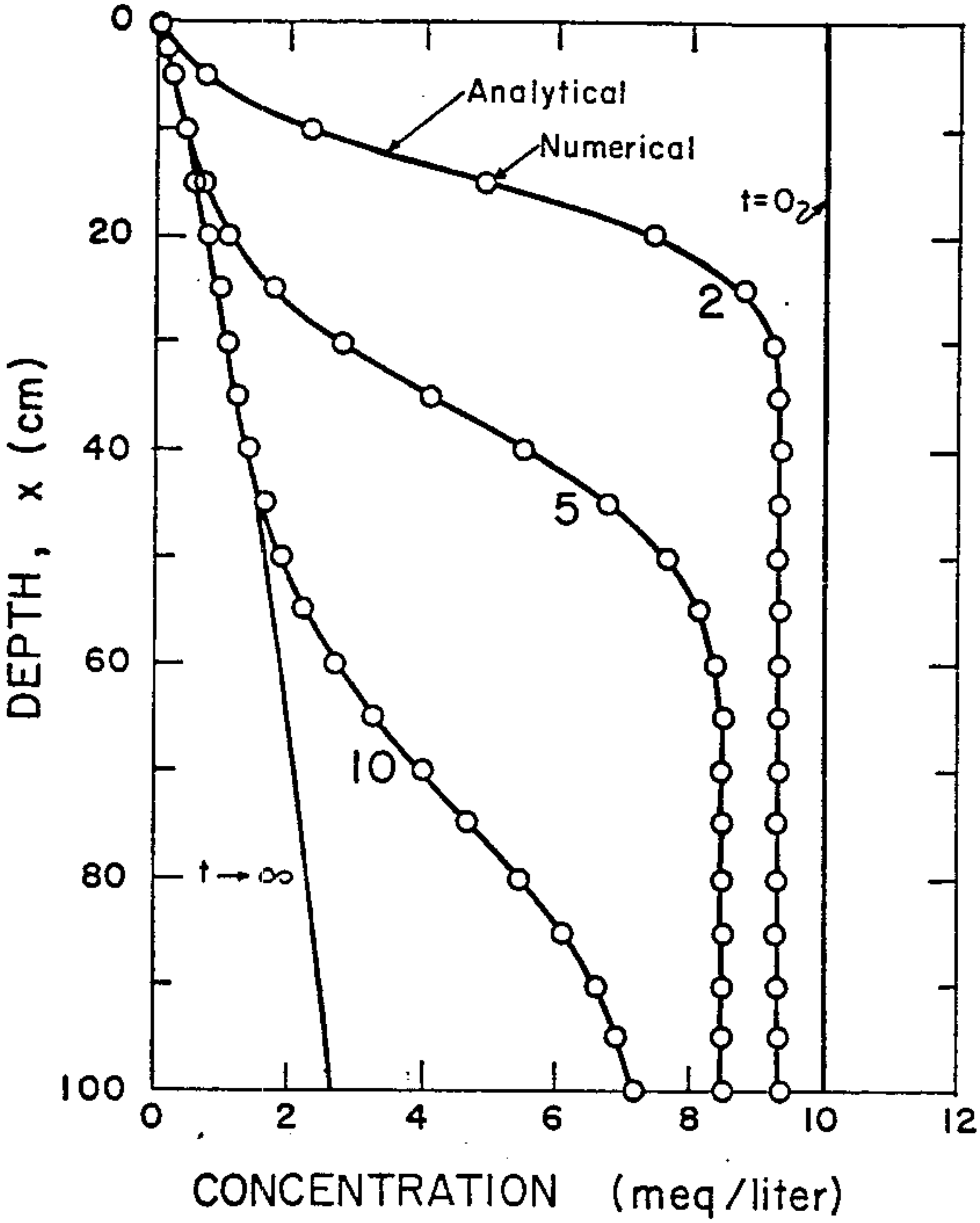



Fig. 7. Calculated concentration distributions for a chemical undergoing linear adsorption, linear decay, and zero-order production during steady leaching through a 100 cm deep soil profile. Numbers at the curves indicate times in days after leaching was initiated. Values for the different parameters used in the calculations are given in the text.

$$c_1(x,t) = \frac{1}{2}\,\text{erfc}\left[\frac{Rx - vt}{2(DRt)^{\frac{1}{2}}}\right] + \left(\frac{v^2t}{\pi DR}\right)^{\frac{1}{2}} \exp\left[\frac{(Rx - vt)^2}{4DRt}\right]$$

$$- \frac{1}{2}\exp(vx/D)\,\left(1 + \frac{vx}{D} + \frac{v^2t}{DR}\right)\,\text{erfc}\left[\frac{Rx + vt}{2(DRt)^{\frac{1}{2}}}\right] \qquad (45)$$

Figure 7 presents a more general case where, in addition to linear adsorption and linear first-order decay, also a first-order production term is present ($\gamma > 0$). The profile, initially having a concentration of 10 meq/liter is leached continuously ($t_o$ large) with solute-free irrigation water until a steady-state solute distribution is obtained. The steady-state distribution follows immediately from (42) and (43) by letting t go to infinity:

$$c_1(x,t) = 2\left(c_o - \frac{\gamma}{\mu}\right)\frac{v}{(v+v^*)}\,\exp\left[\frac{(v-v^*)x}{2D}\right] + \frac{\gamma}{\mu} \qquad (46)$$

The curves in Fig. 7 were obtained with the same parameter values as before, except for the following changes

$\gamma$ = 1.0 meq/liter/day $\qquad$ $c_i$ = 10.0 meq/liter

$t_o \rightarrow \infty$ $\qquad$ $c_o$ = 0.0 meq/liter

It is emphasized here that Eq. (11) and (39), or appropriate simplifications thereof, have found widespread application in soil science, environmental engineering and water resources. Some of the known applications include the movement of ammonium and nitrate in soils[37,38,39], pesticide movement[9,40], the transport of radioactive waste materials[19,41], the fixation of certain iron and zinc chelates[42], and the precipitation or dissolution of gypsum[43,44,45] or other salts[46]. The Hermitian finite element code (Appendix C), hence, may find useful application to any of these problems.

## *3.3. Water and solute movement in a non-homogeneous soil profile.*

This example considers the infiltration of a pulse of water and a dissolved chemical into a non-homogeneous and layered soil profile. Following infiltration, water and solute are allowed to redistribute under the influence of gravity and a small evaporation rate at the soil surface. The flow part of this example was discussed in detail in a previous report[23], and only a brief description of the numerical experiment will be given here.

Figure 8 gives a schematic cross-section of the assumed soil profile. A clay loam layer is present immediately below the soil surface to a depth of 25 cm, where it changes fairly abruptly to a loamy sand. Between 25 and 75 cm the soil changes smoothly from a loamy sand to a sand. A dense (restricting) layer is located between 75 and 87 cm. The dense layer, in turn, is underlain by sand down to a depth of 170 cm. The following models were used to describe the hydraulic properties of the different soil materials[23,47]

$$\theta = \theta_r + \frac{(\theta_s - \theta_r)}{[1+(a|h|)^n]^m} \qquad (m = 1-1/n) \qquad (47)$$

$$K = K_s\, \Theta^{\frac{1}{2}} \left[1 - (1-\Theta^{1/m})^m\right]^2 \qquad (48)$$

where $\theta_r$ and $\theta_s$ are the residual and saturated moisture contents, respectively, $\alpha$, $n$, and $m = (1-1/n)$ are parameters characteristic of the particular soil material, $K_s$ is the saturated hydraulic conductivity, and $\Theta$ is the dimensionless soil moisture content, defined as

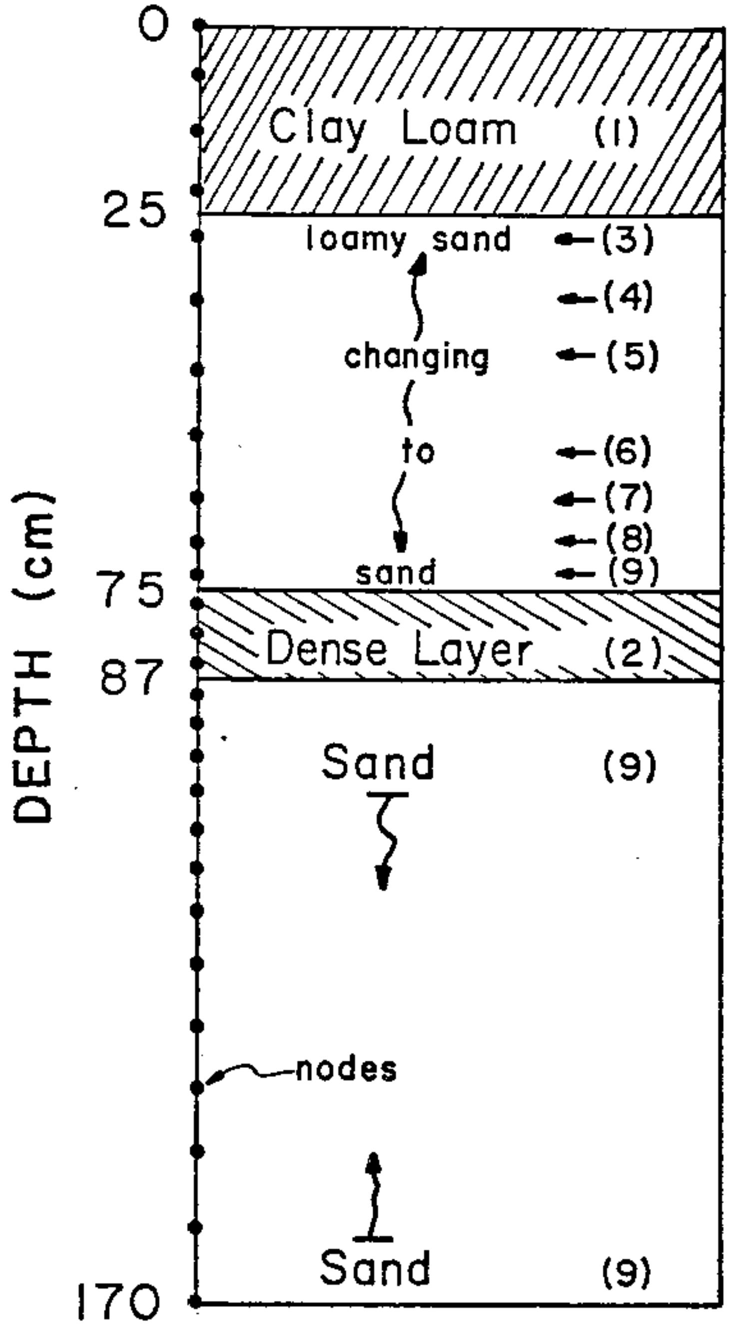



Fig. 8. Schematic cross-section of the soil profile used in example 3.

$$\Theta = \frac{\theta - \theta_r}{\theta_s - \theta_r} \tag{49}$$

The equation for the hydraulic conductivity was obtained by substituting (47) into the predictive conductivity model of Mualem[48] (Eq. 21 of Mualem). An extensive discussion of Eq. (47) and (48) is given by van Genuchten[47]. Values for the different parameters appearing in three equations are given in Table 4 for each soil type used. Figures 9 and 10 present graphical interpretations of some of the resulting curves.

*Table 4.* Constants used to describe the soil-hydraulic properties of the nine soils of example 3.

| | $\theta_r$ | $\theta_s$ | $\alpha$ | n | $K_s$ | $S_s$ |
|---|---|---|---|---|---|---|
| Soil No. | ($cm^3/cm^3$) | ($cm^3/cm^3$) | ($cm^{-1}$) | (-) | (cm/day) | ($cm^{-1}$) |
| 1. (Clay Loam) | .20 | .54 | .008 | 1.8 | 25. | $4.10^{-7}$ |
| 2. (Dense Layer) | .25 | .40 | .009 | 3. | 10. | $5.10^{-8}$ |
| 3. (Loamy Sand) | .17 | .47 | .010 | 2. | 75. | $1.10^{-7}$ |
| 4. | .1611 | .4611 | .01036 | 2.178 | 132.8 | $1.10^{-7}$ |
| 5. | .15 | .45 | .0108 | 2.4 | 205. | $1.10^{-7}$ |
| 6. | .14 | .44 | .0112 | 2.6 | 270. | $1.10^{-7}$ |
| 7. | .1311 | .4311 | .01156 | 2.778 | 327.8 | $1.10^{-7}$ |
| 8. | .1244 | .4244 | .01182 | 2.911 | 371.1 | $1.10^{-7}$ |
| 9. (Sand) | .12 | .42 | .012 | 3.0 | 400. | $1.10^{-7}$ |

Values for the parameters needed in the solute transport part of the model are given in Table 5. Figure 11 shows how some of these parameters change with depth. Note that the distributions remain continuous, even

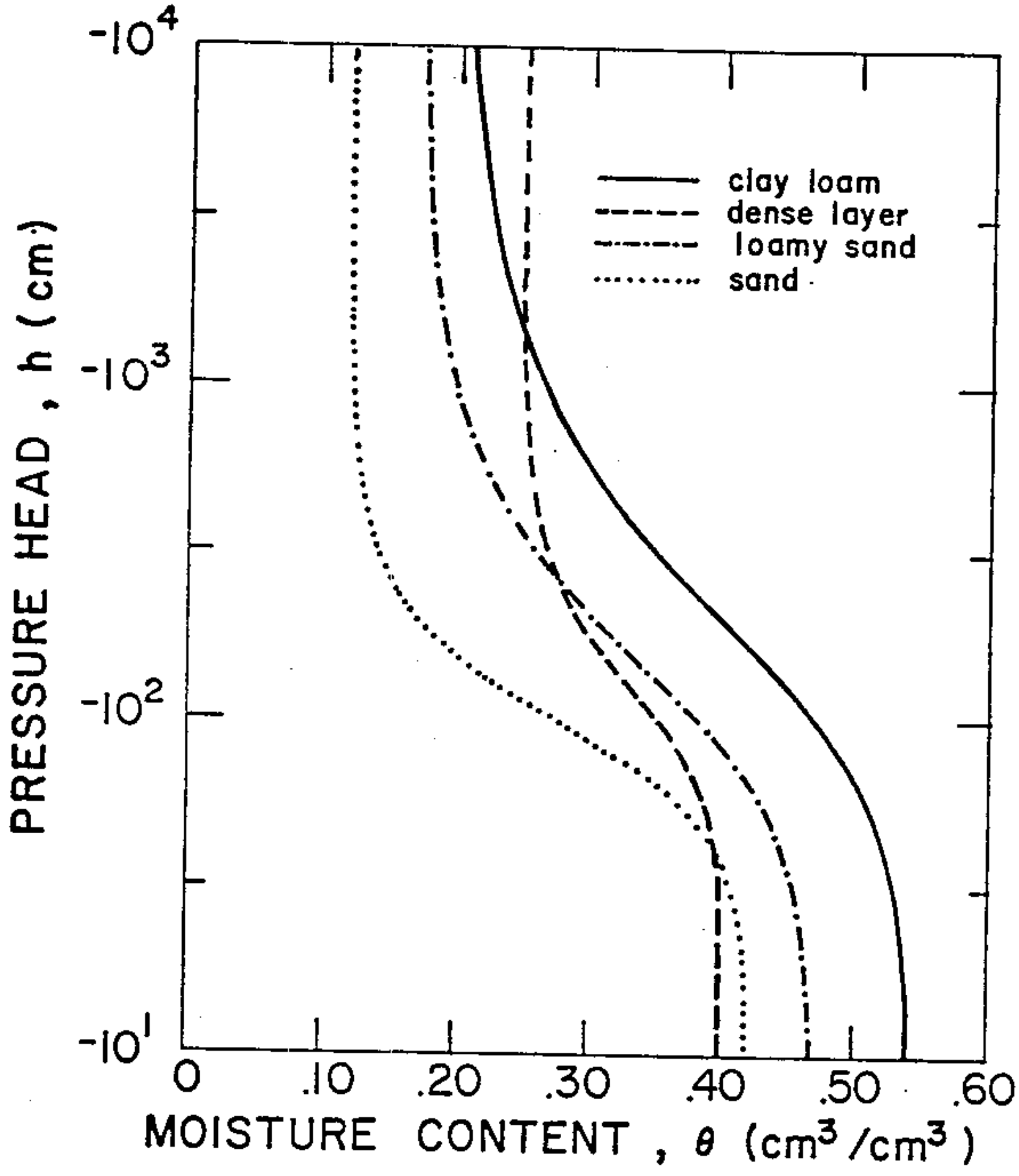



Fig. 9. Soil moisture retention curves of the main soil types used in example 3.

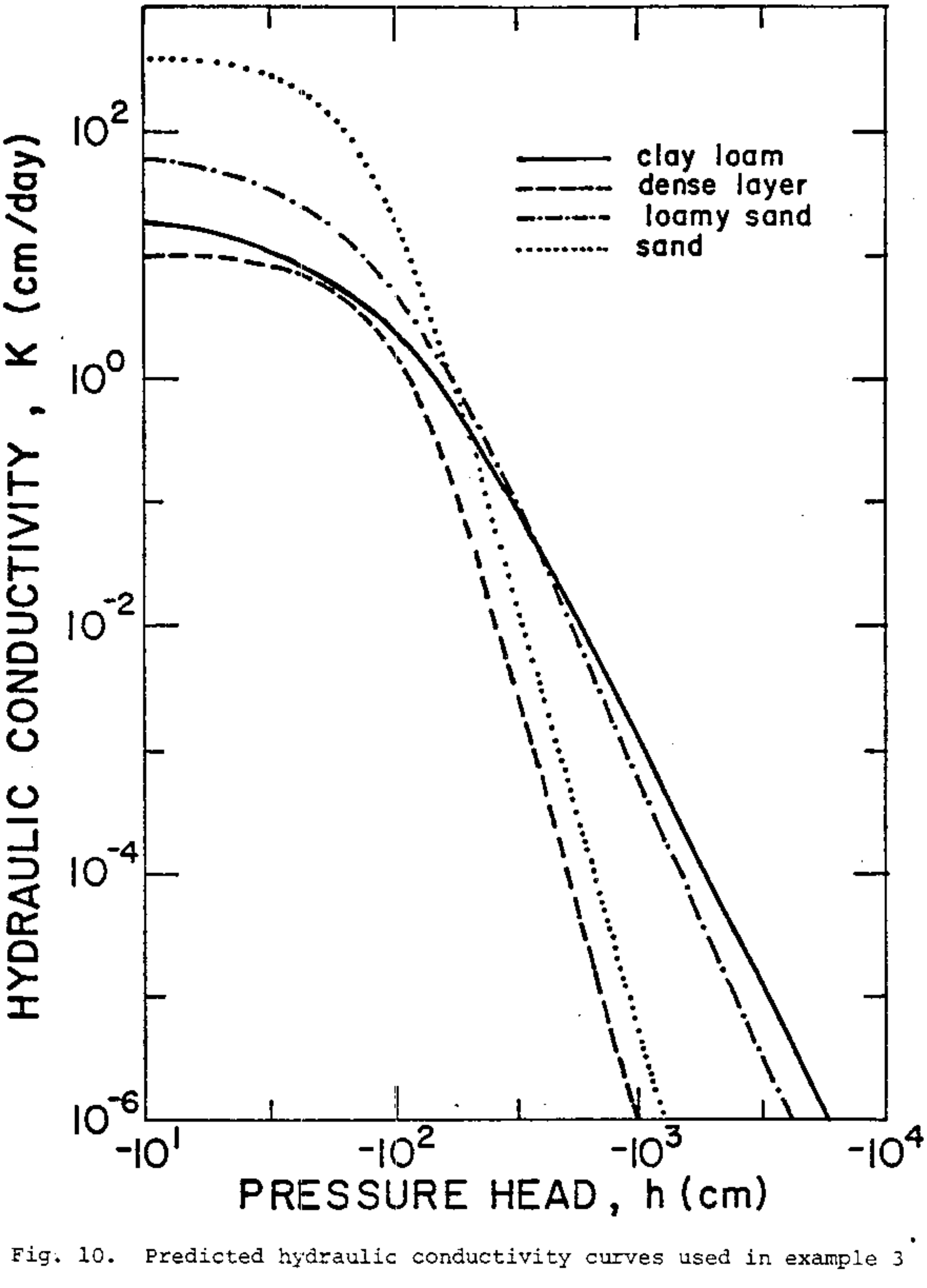



Fig. 10. Predicted hydraulic conductivity curves used in example 3

*Table 5.* Soil-physical and soil-chemical data of the nine soils used in example 3.

| Soil No. | $\rho_b$ $(g/cm^3)$ | $D_o\tau$ $(cm^2/day)$ | $\lambda$ (cm) | k $(cm^3/g)$ | $\alpha$ (1/day) | $\beta$ (1/day) | $\gamma$ $(g/cm^3/day)$ |
|---|---|---|---|---|---|---|---|
| 1. (Clay Loam) | 1.22 | 0.67 | 3.50 | 0.500 | -0.100 | -0.050 | 1.000 |
| 2. (Dense Layer) | 1.60 | 0.67 | 2.00 | 0.200 | 0.0 | 0.0 | 0.0 |
| 3. (Loamy Sand) | 1.41 | 0.67 | 3.00 | 0.300 | -0.092 | -0.046 | 0.800 |
| 4. | 1.43 | 0.67 | 2.73 | 0.247 | -0.072 | -0.036 | 0.450 |
| 5. | 1.46 | 0.67 | 2.40 | 0.180 | -0.048 | -0.024 | 0.150 |
| 6. | 1.49 | 0.67 | 2.10 | 0.120 | -0.024 | -0.012 | 0.030 |
| 7. | 1.51 | 0.67 | 1.83 | 0.067 | -0.009 | -0.0045 | 0.002 |
| 8. | 1.53 | 0.67 | 1.63 | 0.027 | -0.002 | -0.001 | 0.0 |
| 9. (Sand) | 1.54 | 0.67 | 1.50 | 0.0 | 0.0 | 0.0 | 0.0 |

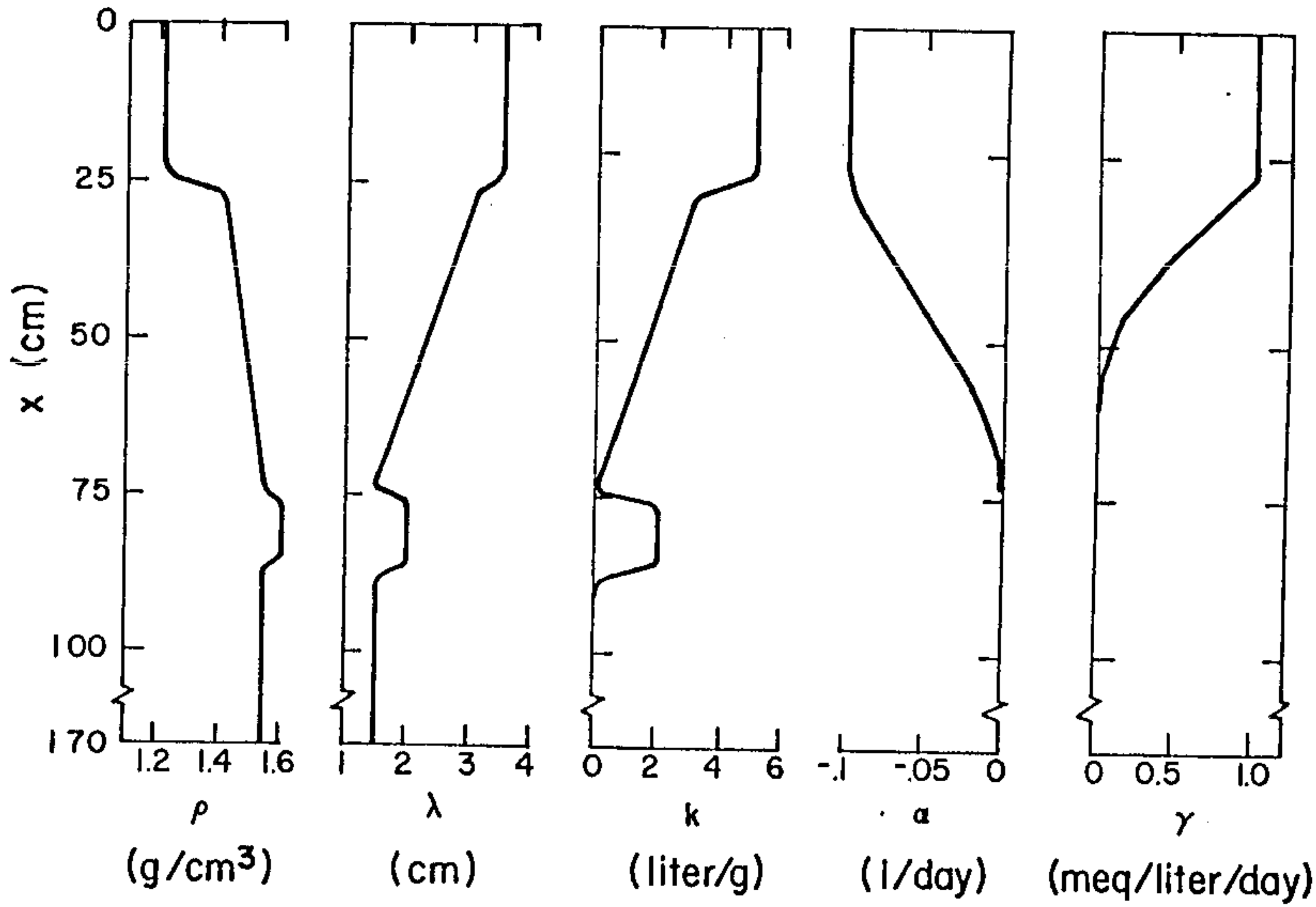


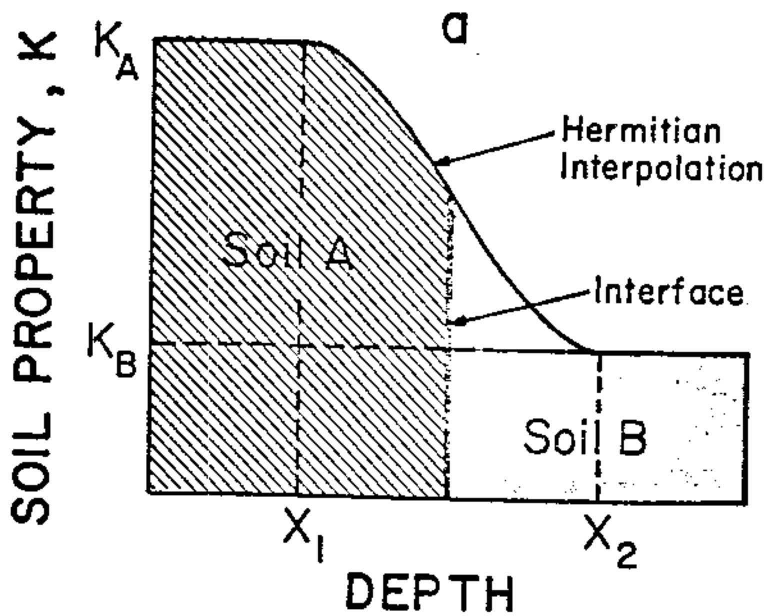

Fig. 11. Spatial distributions of the parameters ρ, λ, k, α and γ as used in example 3

across the boundaries between two different soil types. This continuous change of the soil properties versus depth is a direct consequence of the fact that Hermitian basis functions are used in the simulation. The following example may clarify this. Suppose that the soil would change suddenly from a clay loam to a loamy sand at x=25 cm (Fig. 8). Continuity of the soil moisture flux, q, across the interface requires that

$$q = K_{cl} \ (1 - \frac{dh}{dx}) = K_{ls} \ (1 - \frac{dh}{dx}) \qquad (50)$$

where the subscripts cl and ls indicate evaluation in the clay loam and loamy sand layers, respectively. Because the Hermitian finite element scheme generates a continuous pressure gradient distribution versus depth, it follows immediately from (50) that also the hydraulic conductivity has to be continuous across the interface between the two layers, i.e., that $K_{cl}=K_{ls}$ for all pressure heads. It is clear that this can be the case only when the soil changes in a continuous manner from clay loam to loamy sand As a result of this also the soil moisture content, or any other soil property will change continuously across the interface.

Several approaches are possible for modeling the continuous change of one soil material into another one. In this study Hermitian basis functions are used to interpolate soil properties between two consecutive nodes when an "abrupt" (but continuous) boundary is present, such as is the case between the clay loam and loamy sand layers at x=25 cm in Fig. 8. Suppose, for example, that the hydraulic conductivity, K, needs to be evaluated over a certain element having corner nodes at $x=x_1$ and $x=x_2$ (Fig. 12a). Let the conductivity-pressure head curve at $x_1$ be given by $K_A(h)$, and at $x_2$ by $K_B(h)$. The pressure head distribution between $x_1$ and $x_2$ is given by



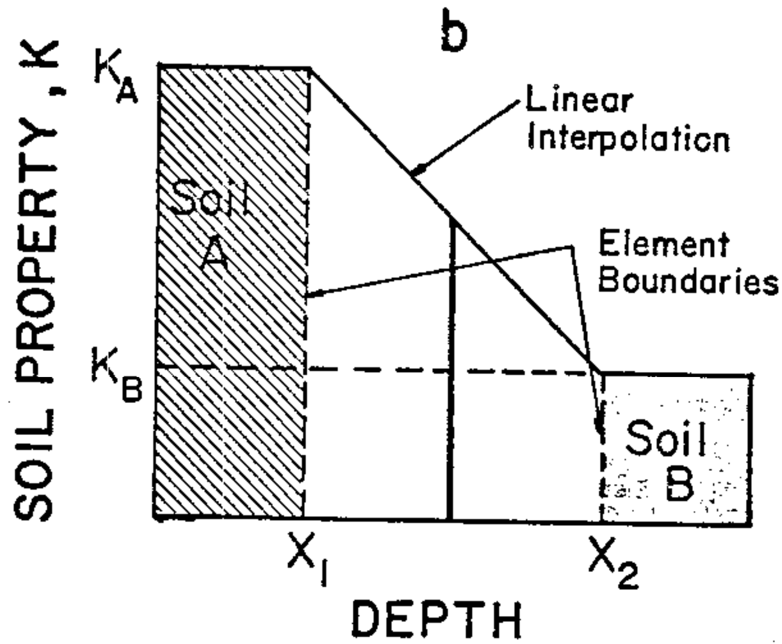




Fig. 12. Schematic representation of the two interpolation schemes used in example 3: a. Restricted Hermitian interpolation for simulation of an "abrupt" boundary; b. Linear interpolation for simulation of smoothly changing soil properties.

$$\hat{h}(x) = \phi^1_{01}(\xi)\ H_1 + \phi^1_{11}(\xi)\ \frac{dH_2}{dx} + \phi^1_{02}(\xi)\ H_2 + \phi^1_{12}(\xi)\ \frac{dH_2}{dx} \qquad (51)$$

where the basis functions $\phi^1_{ij}(\xi)$ are given by (31) and $\xi$ by (29). The hydraulic conductivity distribution between $x_1$ and $x_2$ is now approximated by

$$\hat{K}(x) = \phi^1_{01}\ (\xi)\ K_A(\hat{h}) + \phi^1_{02}\ (\xi)\ K_B(\hat{h}) \qquad (52)$$

Relations similar to (52) are also used for the soil-moisture content, $\Theta(x)$, the soil-moisture capacity, $\hat{C}^*(x)$, and for any of the parameters listed in Table 5.

A slightly different approach is followed when the soil properties change smoothly with depth, such as is the case between x=25 and x=75 cm in Fig. 8. Except for the three rate coefficients ($\alpha$, $\beta$, and $\gamma$), all parameters in Tables 4 and 5 are assumed to change linearly with depth between x=28 cm (the first node located in the loamy sand) and x=73 cm (the node located in the sand). Because the parameters a, n, $\theta_r$, $\theta_s$, and $K_s$ change linearly with depth in this interval, and because of the non-linear properties of Eq. (45) and (46), it is not expected that the resulting K(h) and $\theta$(h) curves also will change in a linear fashion with depth. The suggested linear change of the different parameters in (45) and (46) is used here only as a first approximation, and may not be valid in every case where the soil properties change gradually with depth. In fact, when observed curves are available for any point between 28 and 73 cm, they can be used directly in the calculations. As a consequence of this approach each node between the loamy sand and the sand will have its own parameter values, including its own characteristic $\theta(h)$ and $K(h)$ curves (Soil No. 4-8 in Figure 6 and Tables 4 and 5). Figure 11 shows that the three rate coefficients ($\alpha$, $\beta$, and $\gamma$) do not change linearly with depth in the interval between 25 and 75 cm. It was reasoned that production and decay would be most active near the soil surface, irrespective of the type of soil present, because of more favorable conditions (organic matter, oxygen, and possibly moisture content and temperature).

The gradual change of each soil parameter with depth, whether linear or slightly non-linear, was used only to interpolate each parameter from node to node and does not apply to the interpolation over each element *between* two consecutive nodes in the depth interval between 25 and 75 cm. A linear interpolation is used to obtain the distribution of each parameter over each element in this particular depth interval. For example, for the hydraulic conductivity this results in (Fig. 11B)

$$\hat{K}(x) = \phi_1^0 \ (\xi) \ K_A(\hat{h}) + \phi_2^0 \ (\xi) \ K_B(\hat{h}) \tag{53}$$

where the $\phi_j^0(\xi)$ now represent the linear basis functions (Eq. 28).

Initial and boundary conditions for the present example are given by Eq. (4), (5b), and (6c) for the flow equation, and by (13), (14a), and (15a) for the mass transport equation. Values for $h_i(x)$, $q_o(t)$, $c_i(x)$, and $c_o(t)$ in these equations are as follows:

$$h_i(x) = -350 \qquad 0 \leq x \leq 170 \tag{54a}$$

$$q_o(t) = \begin{cases} 25.0 & 0 < t \leq 1.0 \\ -0.5 & t > 1.0 \end{cases} \tag{54b}$$

$$c_i(x) = \frac{\gamma(x)}{\mu(x)} \qquad 0 \leq x \leq 170 \tag{55a}$$

$$c_o(t) = \begin{cases} 20 & 0 < t \leq 0.50 \\ 0 & t > 0.50 \end{cases} \tag{55b}$$

The initial condition (55a) is an approximation and follows from (39) by assuming that the profile reached an equilibrium state before leaching was initiated (v and D are small, $\partial c/\partial t$ approaches zero).

Calculated pressure head distributions versus depth during infiltration and redistribution are presented in Fig. 13 and 14, respectively. The calculated curves remain fairly smooth until the front reaches the dense layer. The distributions become very steep here, while the pressure gradient changes its sign from negative to positive immediately above the dense layer. The effect of the dense layer on the calculated curves remains clearly visible until the later stages of redistribution (Fig. 14).

As expected, the moisture content shows a much more irregular distribution versus depth than the pressure head (Fig. 15 and 16). Note that the moisture content changes rapidly with depth around x=25 cm (the clay loam - loamy sand boundary) and in the vicinity of the dense layer, but that the distributions remain continuous at all times. Figures 15 and 16 also show the distributions of $\theta_r$ and $\theta_s$ with depth. The calculated moisture content distributions always fall between these two curves.

Calculated solute distributions versus depth are presented in Fig. 17 and 18 for infiltration and redistribution, respectively. Note that the solute front moves much slower down into the profile. This is partly because of mixing of the invading solution with the water already present in the

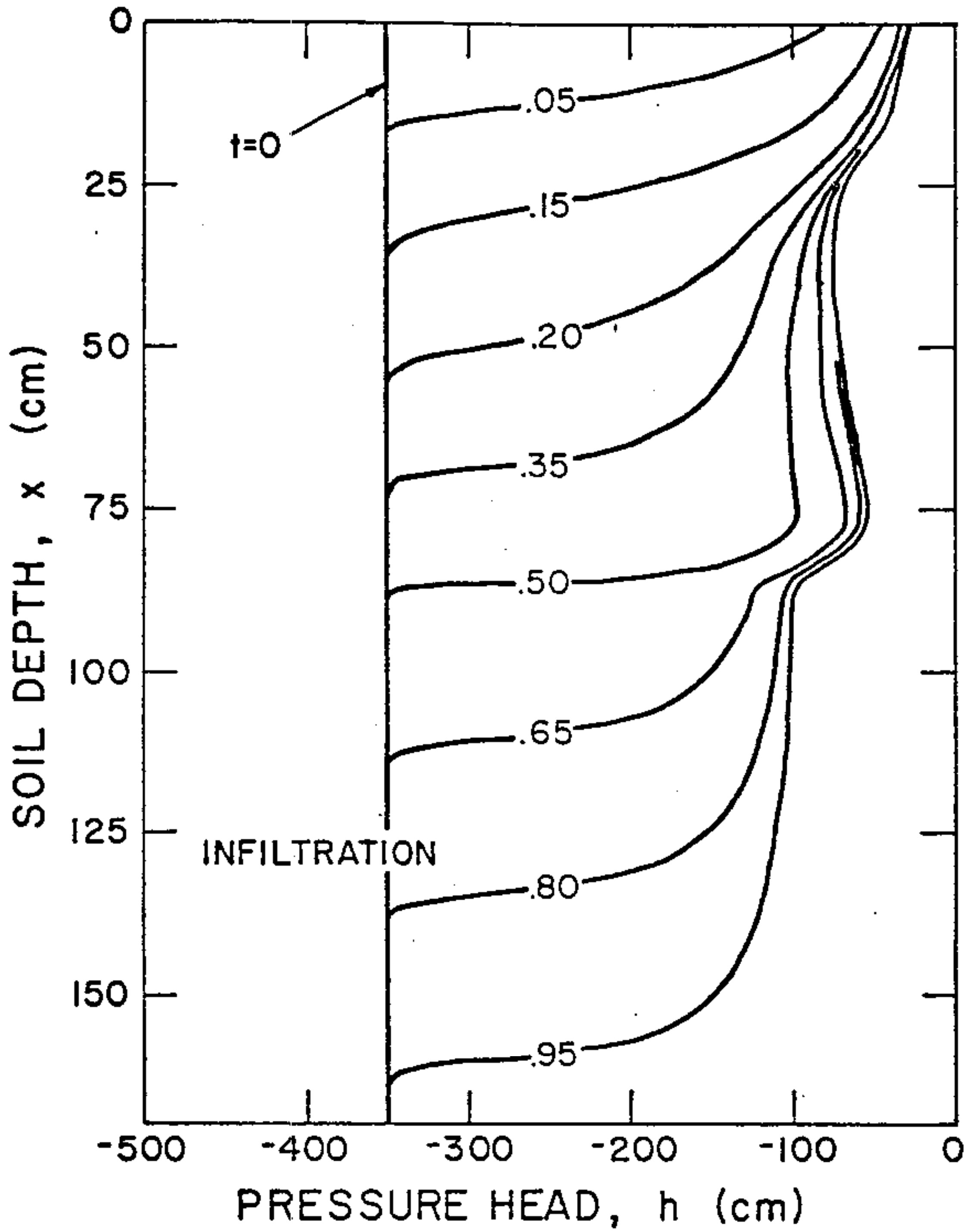



Fig. 13. Calculated pressure distributions during infiltration. Numbers on the curves indicate time in days from the start of the infiltration experiment.

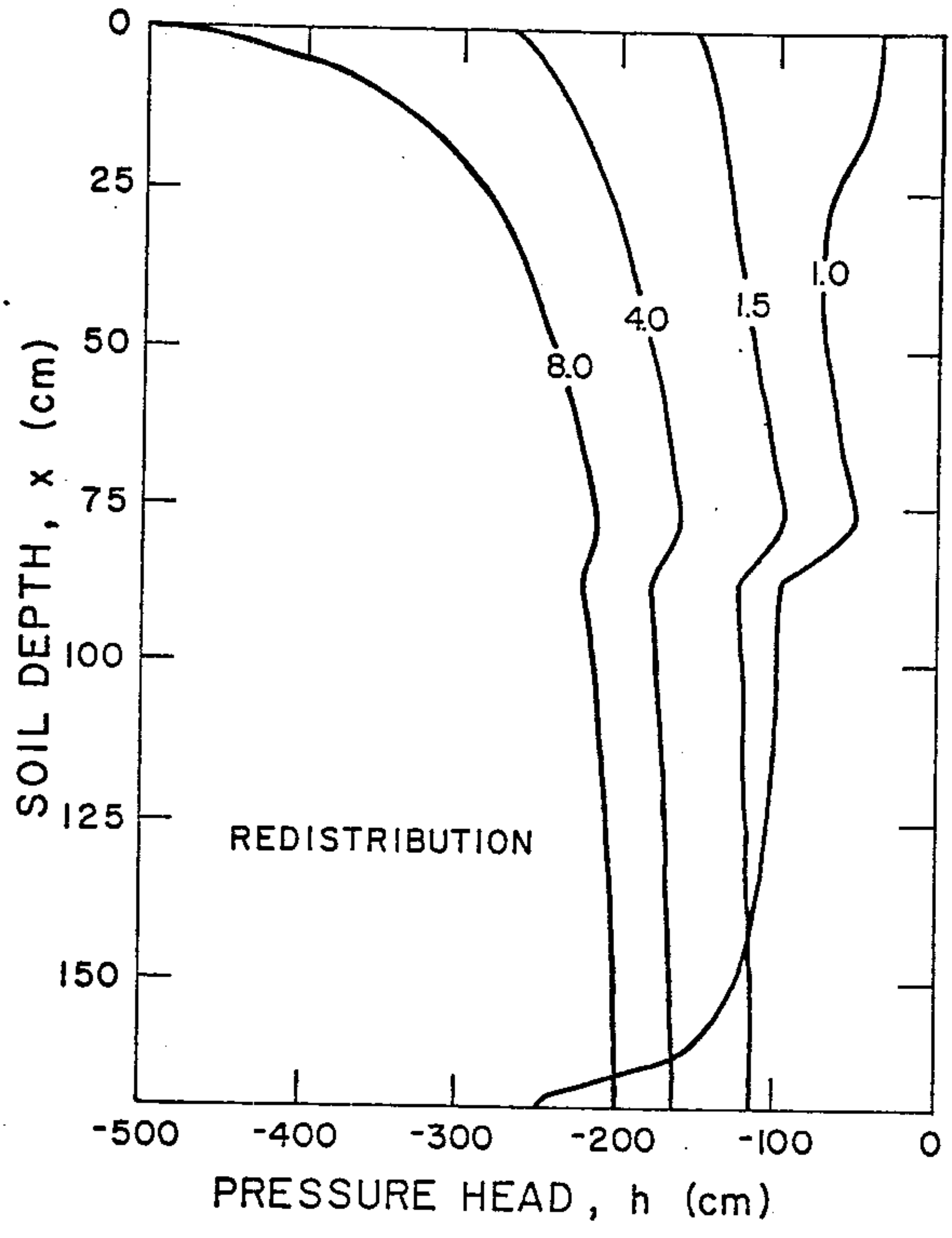



Fig. 14. Calculated pressure distributions during redistribution. Numbers on the curves indicate time in days from the start of the infiltration experiment.

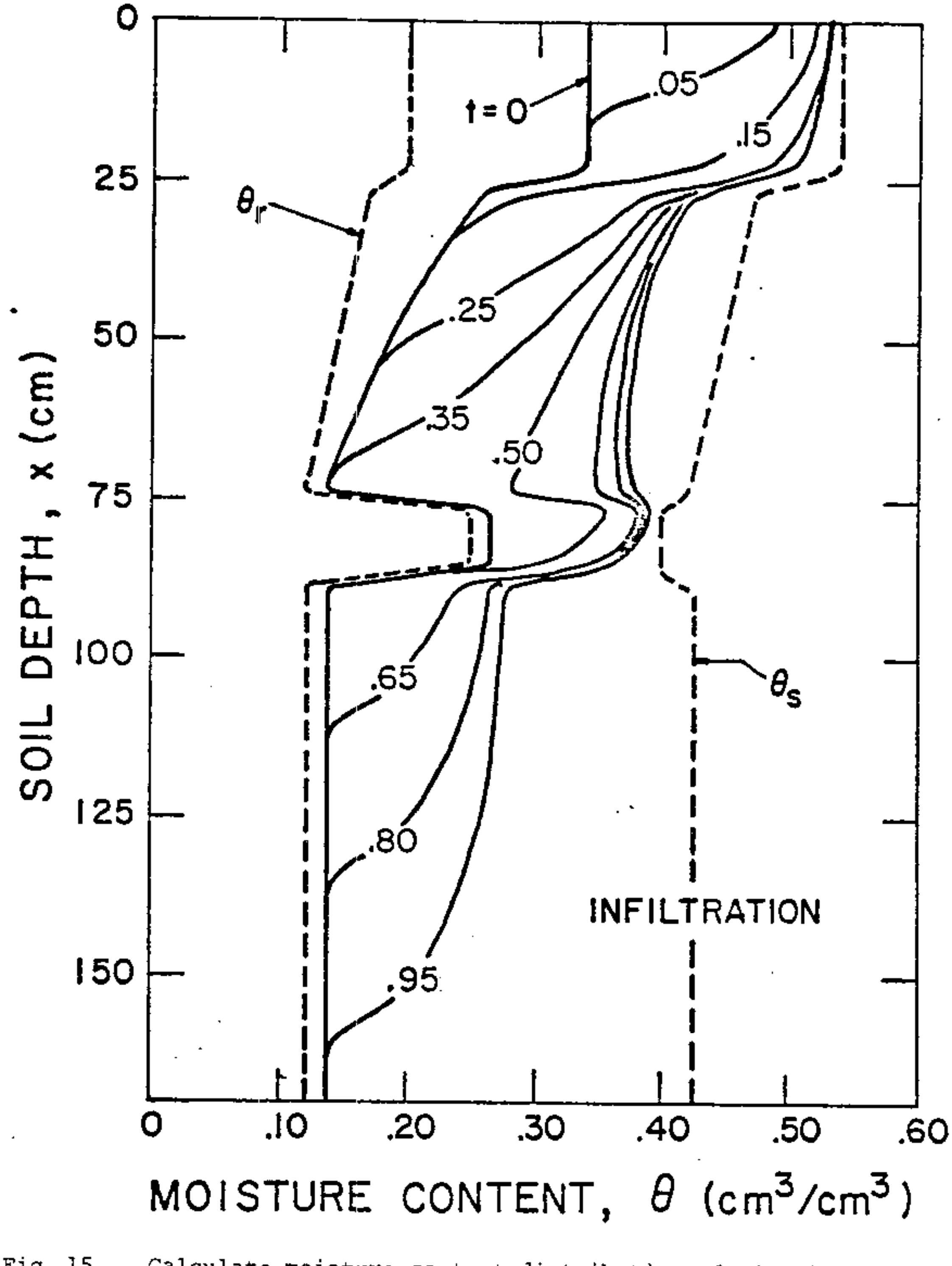



Fig. 15. Calculate moisture content distributions during infiltration. Numbers on the curves indicate time in days from the start of the infiltration experiment.

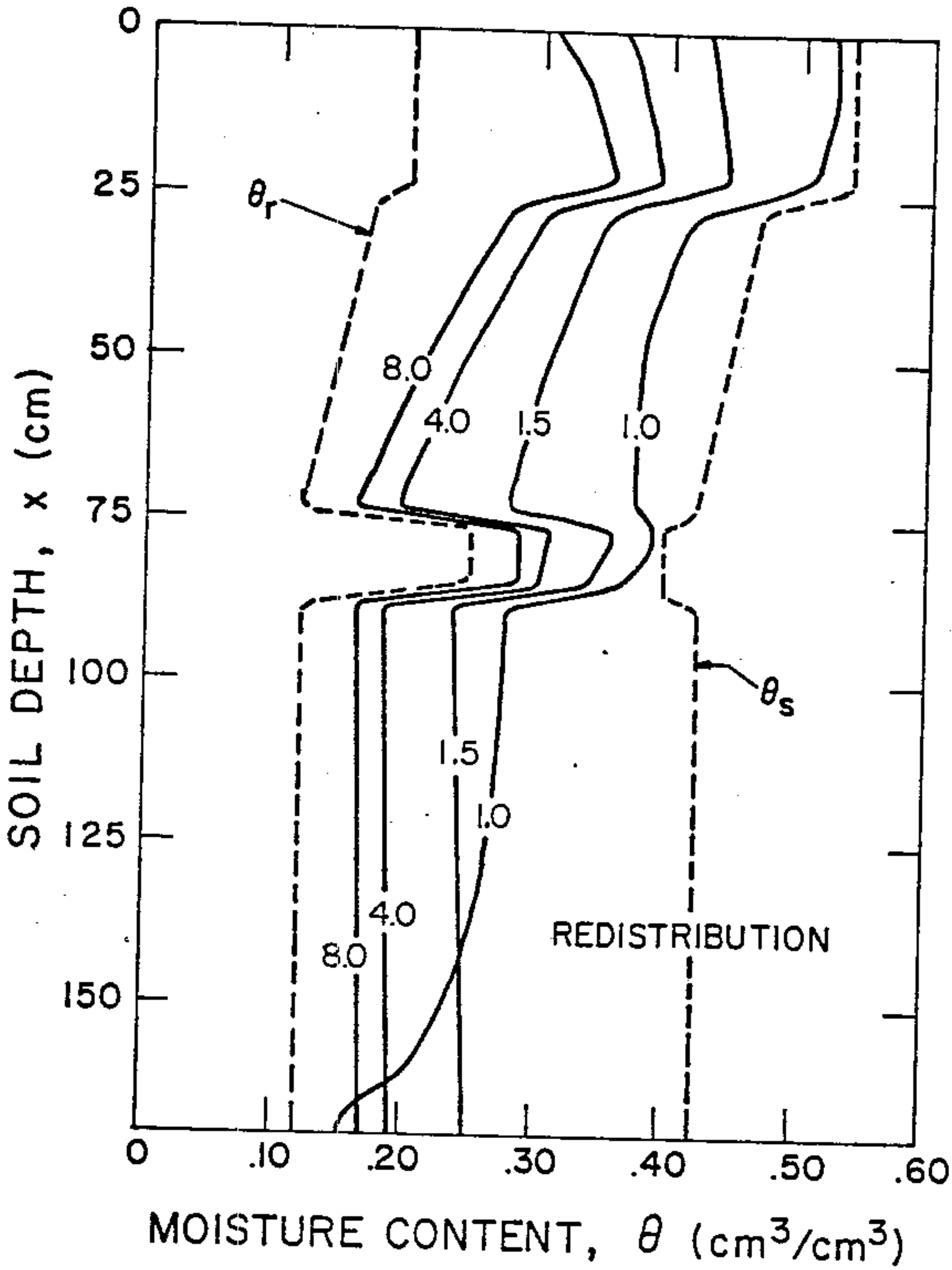
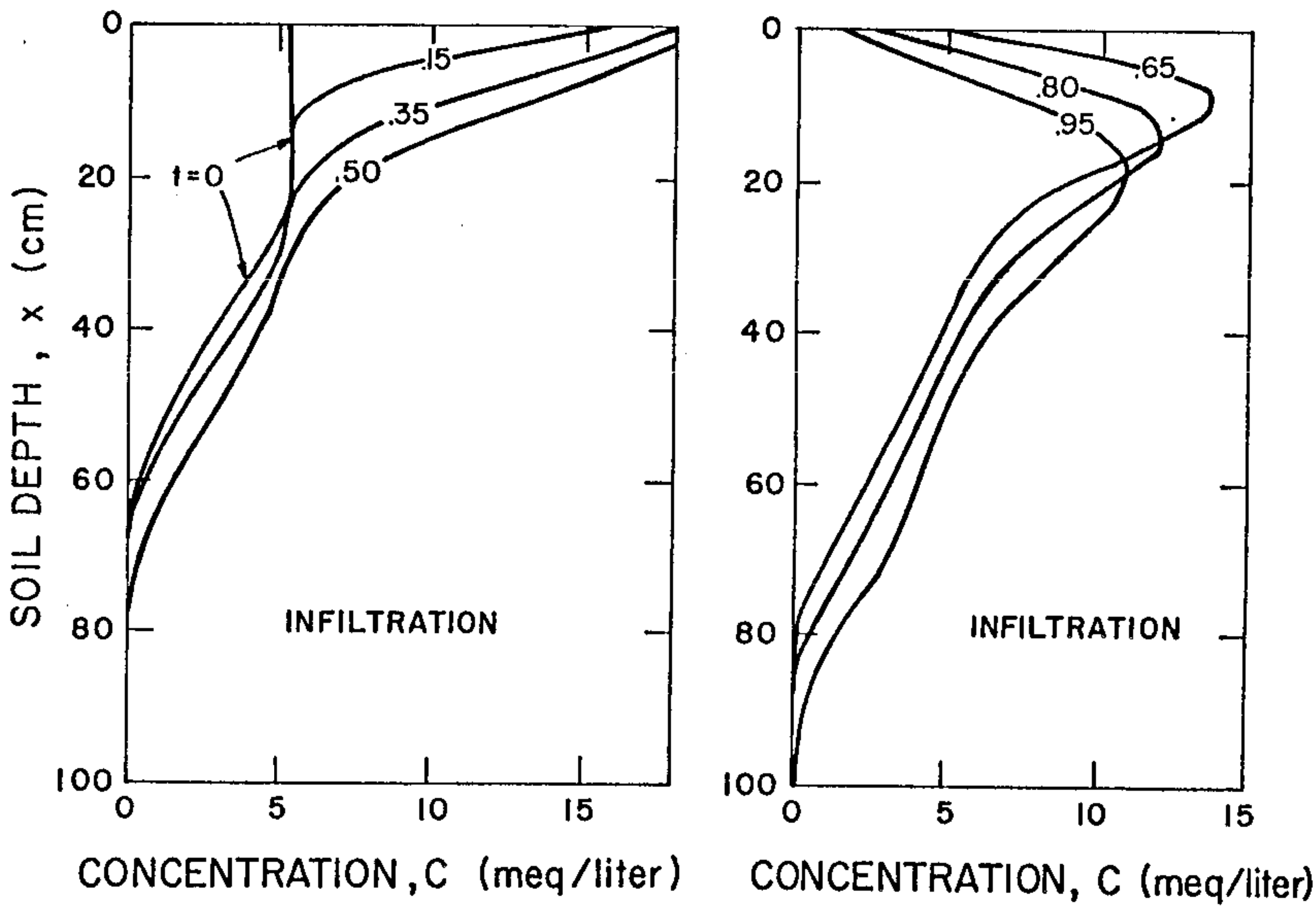



Fig. 16. Calculated moisture content distributions during redistribution. Numbers on the curves indicate time in days from the start of the infiltration experiment.




17. Calculated concentration distributions during infiltation. Numbers on the curves indicate time in days from the start of the infiltration experiment

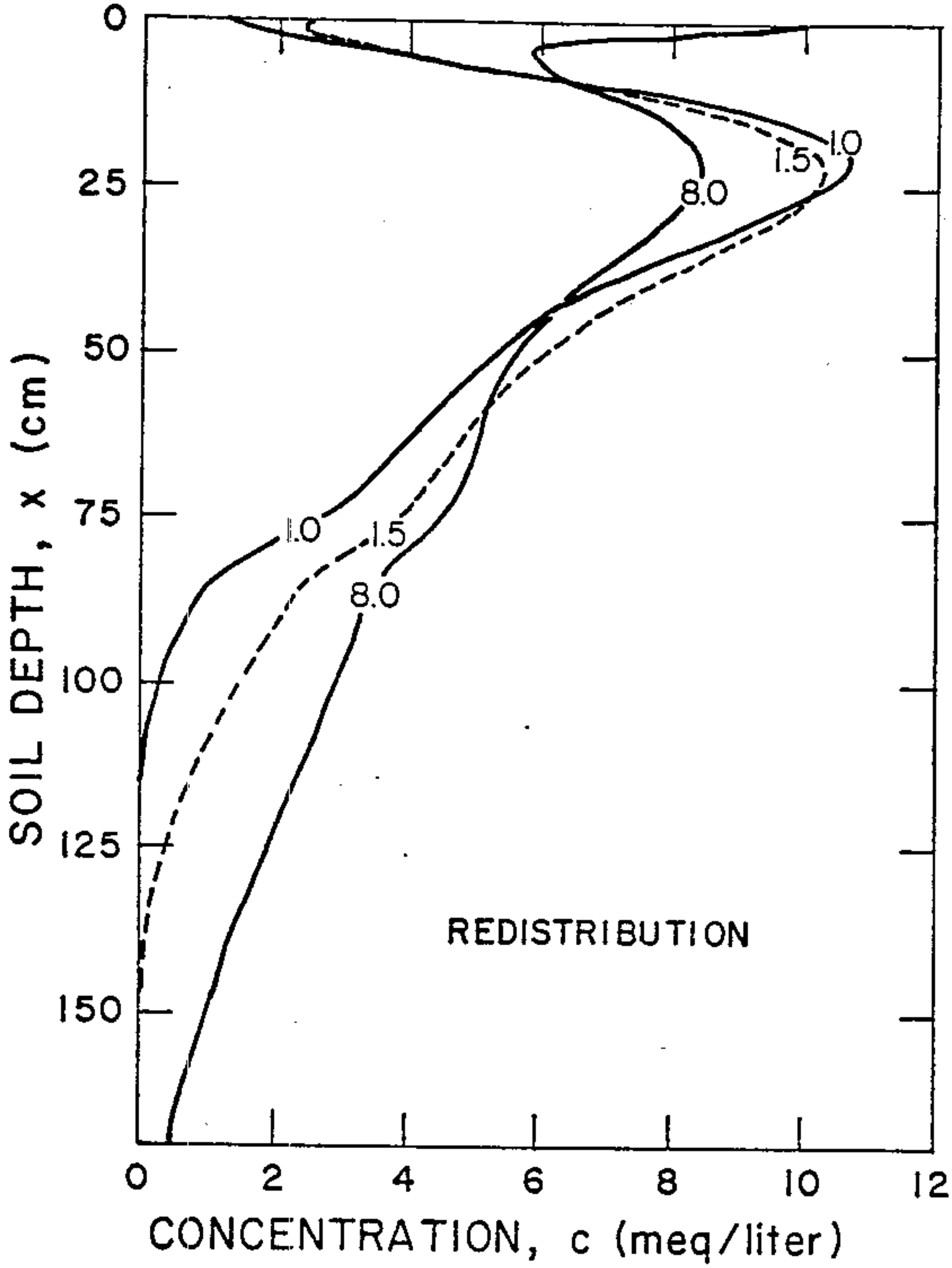



Fig. 18. Calculated concentration distributions during redistribution. Numbers on the curves indicate time in days from the start of the infiltration experiment.

profile, and partly because of the significant adsorption reactions in the upper 25 - 30 cm. The solute distributions remain very smooth, both during infiltration and redistribution, although some effects of the dense layer on the calculated distribution can be observed at a depth of approximately 80 cm. Note also that, because of the imposed small evaporation rate, some solute is moving towards the soil surface during redistribution, resulting in an increase in concentration with time at x=0.

## *REFERENCES*


1. Tanji, K. K., G. R. Dutt, J. L. Paul, and L. D. Doneen. 1967. Quality of percolating water, II. A computer method for predicting salt concentrations in soils at variable moisture contents. Hilgardia 38: 307-318.

2. Bresler, E., and R. J. Hanks. 1969. Numerical method for estimating simultaneous flow of water and salt in unsaturated soils. Soil Sci. Soc. Am. Proc. 33: 827-832.

3. Dutt, G. R., M. J. Shaffer, and W. J. Moore. 1972. Computer simulation model of dynamic bio-physicochemical processes in soils. Techn. Bull. No. 196, Agric. Exp. Sta., University of Arizona, Tucson, Arizona.

4. King, L. G., and R. J. Hanks. 1973. Irrigation management for the control of quality of irrigation return flow. EPA-R2-73-265. Office of Research and Development, U.S. Environmental Protection Agency, Washington, D.C.

5. Bresler, E. 1973. Simultaneous transport of solutes and water under transient unsaturated flow conditions. Water Resour. Res. 9(4): 975-986.

6. Kirda, C., D. R. Nielsen, and J. W. Biggar. 1973. Simultaneous transport of chloride and water during infiltration. Soil Sci. Soc. Am. Proc. 37(3): 339-345.

7. Perez, A. I., W. C. Huber, J. P. Heaney, and E. E. Pyatt. 1974. A water quality model for a conjunctive surface-groundwater system. EPA-600/5-74-013. Office of Research and Development, U.S. Environmental Protection Agency, Washington, D.C.

8. Shah, D. B., G. A. Coulman, L. T. Novak, and B. G. Ellis. 1975. A mathematical model for phosphorus movement in soils. J. Env. Qual. 4(1): 87-92.

9. Wood, A. L., and J. M. Davidson. 1975. Fluometuron and water content distributions during infiltration: measured and calculated. Soil Sci. Soc. Am. Proc. 39(5): 820-825.

10. Bresler, E. 1975. Two-dimensional transport of solutes during non-steady infiltration from a trickle source. Soil Sci. Soc. Am. Proc. 39(4): 604-613.

11. Hildebrand, M. A., and D. M. Himmelbau. 1977. Transport of nitrate ion in unsteady, unsaturated flow in porous media. A.I. Ch. E. J. 23(3): 326-335.

12. Ungs, M., R. W. Cleary, L. Boersma, and S. Yingjajaval. 1976. The quantitative description of transfer of water and chemicals through soils. In Land as a Waste Management Alternative. R.C. Loehr (ed.). Ann Arbor Science, Ann Arbor, Michigan, pp. 109-137.

13. Selim, H. M., R. S. Mansell, and A. Elzeftawy. 1976. Distributions of 2,4-D and water in soil during infiltration and redistribution. Soil Sci. 121: 176-183.

14. Hillel, D., H. Talpaz, and H. van Keulen. 1976. A macroscopic-scale model of water uptake by a nonuniform root system and of water and salt movement in the soil profile. Soil Sci. 121(4): 242-255.

15. Gureghian, A. B., R. W. Cleary, and D. S. Ward. 1977. One-dimensional modeling of unsaturated pollutant transport. Final report 208 project, Nassau-Suffolk County Regional Planning Board, Hauppauge, New York.

16. Shaffer, M. J., R. W. Ribbens, and C. W. Huntley. 1977. Prediction of mineral quality of irrigation return flow. V. Detailed return flow salinity and nutrient simulation model. EPA-600/2-77-179e. Office of Research and Development, U.S. Environmental Protection Agency, ADA, Oklahoma.

17. Wierenga, P. J. 1977. Solute distribution profiles computed with steady-state and transient water movement models. Soil Sci. Soc. Am. J. 41(6): 1050-1055.

18. Sykes, J. F. 1975. Transport phenomena in variably saturated porous media, Ph.D. thesis. Department of Civil Engineering, University of Waterloo.

19. Duguid, J. O., and M. Reeves. 1976. Material transport through porous media: a finite-element Galerkin model. Oak Ridge National Laboratory, ORNL-4928.

20. Segol, G. 1976. A three-dimensional Galerkin finite element model for the analysis of contaminant transport in variably saturated porous media. User's guide. Department of Earth Sciences, University of Waterloo.

21. Segol, G. 1977. A three-dimensional Galerkin-finite element model for the analysis of contaminant transport in saturated-unsaturated porous media. In Finite Elements in Water Resources, W.G. Gray *et al.* (eds.), Pentech Press, London, pp. 2.123-2.144.

22. van Genuchten, M. Th., G. F. Pinder, and W. F. Saukin. 1977. Modeling of leachate and soil interactions in an aquifer. In Management of Gas and Leachate in Landfills. S.K. Banerji (ed.), EPA-600/9-77-026. Office of Research and Development, U.S. Environmental Protection Agency, Cincinnati, Ohio, pp. 95-103.

23. van Genuchten, M. Th. 1978. Numerical solutions of the one-dimensional saturated-unsaturated flow equation. Research Report 78-WR-9, Water Resources Program, Department of Civil Engineering, Princeton University, Princeton, New Jersey.

24. Gray, W. G., and G. F. Pinder. 1974. Galerkin approximation of the time derivative in the finite element analysis of groundwater flow. Water Resour. Res. 10(4): 821-828.

25. Pinder, G. F. 1973. A Galerkin-finite element simulation of groundwater contamination on Long Island, New York. Water Resour. Res. 9(6): 1657-1669.

26. Gray, W. G., and G. F. Pinder. 1976. An analysis of the numerical solution of the transport equation. Water Resour. Res. 12: 547-555, 1976.

27. van Genuchten, M. Th. 1977. On the accuracy and efficiency of several numerical schemes for solving the convective-dispersive equation. In Finite Elements in Water Resources, W.G. Gray *et al.* (eds.), Pentech Press, London, pp. 1.71-1.90.

28. van Genuchten, M. Th., and W. G. Gray. 1978. Analysis of some dispersion corrected numerical schemes for solution of the transport equation. Int. J. Numer. Methods Eng. 12(3): 387-404.

29. Pinder, G. F., and W. G. Gray. 1977. Finite Element Simulation in Surface and Subsurface Hydrology. Academic Press, New York.

30. Ergatoudis, I., B. M. Irons, and O. C. Zienkiewicz. 1968. Curves isoparamatric quadrilateral elements for finite element analysis. Int. J. Solids Struct. 4: 31-42.

31. Oden, J. T. 1972. Finite Elements of Nonlinear Continua. McGraw-Hill, New York.

32. Zienkiewicz, O. C. 1971. The Finite Element Method in Engineering Science, McGraw-Hill, New York.

33. Abramowitz, M., and I. A. Stegun. 1970. Handbook of Mathematical Functions. Dover Publ., New York.

34. Neuman, S. P. 1972. Finite element computer programs for flow in saturated-unsaturated porous media. Second Annual Report, No. A10-SWC-77, Hydraul. Eng. Lab., Technion, Haifa, Israel.

35. Warrick, A. W., J. W. Biggar, and D. R. Nielsen. 1971. Simultaneous solute and water transfer for an unsaturated soil. Water Resour. Res. 7(5): 1216-1225.

36. Lindstrom, F. T., R. Haque, V. H. Freed, and L. Boersma. 1967. Theory on the movement of some herbicides in soils. Environ. Sci. Tech. 1(7): 561-565.

37. Cassel, D. K., M. Th. van Genuchten, and P. J. Wierenga. 1975. Predicting anion movement in disturbed and undisturbed soils. Soil Sci. Soc. Am. Proc. 39(6): 1015-1019.

38. Reddy, K. R., W. H. Patrick, Jr., and R. E. Phillips. 1976. Ammonium diffusion as a factor in nitrogen loss from flooded soils. Soil Sci. Soc. Am. J. 40(4): 528-533.

39. Misra, C., and B. K. Mishra. 1977. Miscible displacement of nitrate and chloride under field conditions. Soil Sci. Soc. Am. J. 41(3): 496-499.

40. van Genuchten, M. Th., and P. J. Wierenga. 1974. Simulation of one-dimensional solute transfer in porous media. Agr. Exp. Sta. Bull. 628, New Mexico State University, Las Cruces, New Mexico.

41. Arnett, R. C., R. A. Deju, R. W. Nelson, D. R. Cole, and R. E. Gephart. 1976. Conceptual and mathematical modeling of the Hanford groundwater flow regime. Report No. ARH-ST-140, Atlantic Richfield Hanford Company, Richland, Washington.

42. Lahav, N., and M. Hochberg. 1975. Kinetics of fixation of iron and zinc applied as FeEDTA, FeHDDHA and ZnEDTA in the soil. Soil Sci. Soc. Am. Proc. 39(1): 55-58.

43. Kemper, W. D., J. Olsen, and C. J. DeMooy. 1975. Dissolution rate of gypsum in flowing water. Soil Sci. Soc. Am. Proc. 39(3): 458-463.

44. Glas, T. K., and D. B. McWhorter. 1976. Salt transport in soil profiles with application to irrigation return flow. The dissolution and transport of gypsum in soils. Completion Rep. Series, No. 71, Environ. Resour. Center, Colorado State University, Fort Collins, Colorado.

45. Keisling, T. C., P. S. C. Rao, and R. E. Jessup. 1978. Pertinent criteria for describing the dissolution of gypsum beds in flowing water. Soil Sci. Soc. Am. J. 42(2): 234-236.

46. Melamed, D., R. J. Hanks, and L. S. Willardson. 1977. Model of salt flow in soil with a source-sink term. Soil Sci. Soc. Am. J. 41(1): 29-33.

47. van Genuchten, Rien. 1978. Calculating the unsaturated hydraulic conductivity with a new, closed form analytical model. Research Report 78-WR-08, Water Resources Program, Department of Civil Engineering, Princeton University, Princeton, New Jersey.

48. Mualem, Y. 1976. A new model for predicting the hydraulic conductivity of unsaturated porous media. Water Resour. Res. 12(3): 513-522.

49. Carslaw, H. S., and J. C. Jaeger. 1959. Conduction of Heat in Solids, 2nd ed., Oxford University Press, London.

*APPENDIX A.* A higher-order approximation of the time derivative in the transport equation.

In this Appendix a higher-order approximation for the time derivative in the transport equation will be derived. The derivation is based on the following third-order accurate difference equation in time[27]:

$$\frac{f^{t+\Delta t} - f^{t}}{\Delta t} = \frac{1}{2}\left[\frac{\partial f}{\partial t} - \frac{\Delta t}{6}\frac{\partial^2 f}{\partial t^2}\right]^{t+\Delta t} + \frac{1}{2}\left[\frac{\partial f}{\partial t} + \frac{\Delta t}{6}\frac{\partial^2 f}{\partial t^2}\right]^{t} \qquad \text{(A1)}$$

where the superscripts 't' and 't+Δt' indicate evaluation at times t and t+Δt, respectively, and where Δt represents the time increment used in the numerical calculations. One may easily verify that (A1) holds for any third-degree polynomial in time, i.e., for any function f of the form

$$f = p_o + p_1 t + p_2 t^2 + p_3 t^3 \qquad \text{(A2)}$$

for all $p_i$ (i=0,3) and t. Equation (A1) may be written for any arbitrary function f, hence also for f=(θRc), i.e.,

$$\frac{(\theta Rc)^{t+\Delta t} - (\theta Rc)^{t}}{\Delta t} = \frac{1}{2}\left[\frac{\partial(\theta Rc)}{\partial t} - \frac{\partial^2(\theta Rc)}{\partial t^2}\right]^{t+\Delta t}$$

$$+ \frac{1}{2}\left[\frac{\partial(\theta Rc)}{\partial t} - \frac{\partial^2(\theta Rc)}{\partial t^2}\right]^{t} \qquad \text{(A3)}$$

The first-order time derivatives of (A3) are given directly by the general transport equation (Eq. 7):

$$\frac{\partial(\theta Rc)}{\partial t} = \frac{\partial}{\partial x}\left(\theta D \frac{\partial c}{\partial x}\right) - \frac{\partial(qc)}{\partial x} + (\alpha\theta + \beta\rho k)c + \gamma\theta \quad \text{(A4)}$$

An approximation for the second-order derivative in (A3) may be obtained by considering the following expansion

$$\frac{\partial^2(\theta Rc)}{\partial t^2} = \frac{\partial}{\partial t}\left[\frac{\partial(\theta Rc)}{\partial t}\right]$$

$$\simeq - \frac{\partial}{\partial t}\left[\frac{\partial(qc)}{\partial x}\right] \quad \text{(A5)}$$

Equation (A5) assumes that dispersion and degradation can be neglected as higher-order effects. Reversing the order of differentiation in (A5) and further expansion gives:

$$\frac{\partial^2(\theta Rc)}{\partial t^2} \simeq - \frac{\partial}{\partial x}\left[\frac{\partial(qc)}{\partial t}\right]$$

$$= - \frac{\partial}{\partial x}\left(q \frac{\partial c}{\partial t} + c \frac{\partial q}{\partial t}\right) \quad \text{(A6)}$$

The second term on the right-hand side of (A6) will be neglected. This term will be significant only when the solute front coincides with the moisture front during infiltration. This will generally not be the case because the moisture front usually propagates much faster than the solute front[6,35]. Obviously, some small errors may be introduced in this way during the initial stages of the infiltration process, but these are taken for granted. With the above assumption, Eq. (A6) becomes

$$\frac{\partial^2(\theta Rc)}{\partial t^2} = - \frac{\partial}{\partial x}\left(q \frac{\partial c}{\partial t}\right) \quad \text{(A7)}$$

To obtain an estimate for $\partial c/\partial t$ in (A7), consider again the following approximation of (A4):

$$\frac{\partial(\theta R c)}{\partial t} \approx - \frac{\partial(qc)}{\partial x} \qquad \text{(A8)}$$

Expansion of this equation gives

$$\theta R \frac{\partial c}{\partial t} + c \frac{\partial(\theta R)}{\partial t} = -q \frac{\partial c}{\partial x} - c \frac{\partial q}{\partial x} \qquad \text{(A9)}$$

Because

$$\theta R = \theta + \rho k \qquad \text{(A10)}$$

and since $\rho$ and $k$ are independent of time, Eq. (A9) becomes

$$\theta R \frac{\partial c}{\partial t} + c \frac{\partial \theta}{\partial t} = - q \frac{\partial c}{\partial x} - c \frac{\partial q}{\partial x} \qquad \text{(A11)}$$

Consider, furthermore, the continuity equation for moisture flow:

$$\frac{\partial \theta}{\partial t} = - \frac{\partial q}{\partial x} \qquad \text{(A12)}$$

Substitution of (A12) into (A11) leads to

$$\theta R \frac{\partial c}{\partial t} = -q \frac{\partial c}{\partial x} \qquad \text{(A13)}$$

Solving this equation for $\partial c/\partial t$ and substitution into (A7) gives then the following approximation for the second-order derivatives in (A3)

$$\frac{\partial^2(\theta Rc)}{\partial t^2} \simeq \frac{\partial}{\partial x}\left(\frac{q^2}{\theta R}\frac{\partial c}{\partial x}\right) \tag{A14}$$

Substituting Eq. (A4) and (A14) into (A3) results in the following dispersion-corrected finite difference approximation of the transport equation

$$\frac{(\theta Rc)^{t+\Delta t} - (\theta Rc)^{t}}{\Delta t} = \frac{1}{2}\left[\frac{\partial}{\partial x}(\theta D^{-}\frac{\partial c}{\partial x} - qc) + (\alpha\theta + \beta\rho k)c + \gamma\theta\right]^{t+\Delta t} + \frac{1}{2}\left[\frac{\partial}{\partial x}(\theta D^{+}\frac{\partial c}{\partial x} - qc) + (\alpha\theta + \beta\rho k)c + \gamma\theta\right]^{t} \tag{A15}$$

where

$$D^{-} = D - \frac{q^2\Delta t}{6\theta^2 R} \qquad D^{+} = D + \frac{q^2\Delta t}{6\theta^2 R} \tag{A16}$$

*APPENDIX B.* Analytical solution for convective-dispersion with simultaneous adsorption, zero-order production, and first-order decay.

In this appendix an analytical solution will be derived for the transport equation

$$D \frac{\partial^2 c}{\partial x^2} - v \frac{\partial c}{\partial x} - R \frac{\partial c}{\partial t} = \mu c - \gamma. \qquad \text{(B1)}$$

Initial and boundary conditions imposed on (B1) are

$$c(x,0) = c_i \qquad \text{(B2)}$$

$$\left(-D \frac{\partial c}{\partial x} + vc\right)\Bigg|_{x=0} = \begin{cases} vc_o & 0 < t \leq t_o \\ 0 & t > t_o \end{cases} \qquad \text{(B3)}$$

$$\left|\frac{\partial c}{\partial x}\right| \leq M \qquad x \rightarrow \infty. \qquad \text{(B4)}$$

Boundary condition (B4) stipulates that the concentration gradient, $\partial c/\partial x$, remains finite ($M < \infty$) when x approaches infinity. The solution of Eq. (B1) - (B4) can be obtained by application of the Laplace transform. The Laplace transform of c with respect to time is defined by

$$\bar{c} = \bar{c}(x,s) = \int_0 e^{-st}\, c(x,t)\, dt \qquad \text{(B5)}$$

The transform of (B1) which satisfies the initial condition (B2) is

$$\frac{D}{R}\frac{\partial^2 \bar{c}}{\partial x^2} - \frac{v}{R}\frac{\partial \bar{c}}{\partial x} - (s + \frac{\mu}{R})\ \bar{c} = -\frac{\gamma}{Rs} - c_i \tag{B6}$$

The transform of (B3) takes the form (see for example Eq. 29.3.64 of Abramowitz and Stegun[33]).

$$-D\frac{\partial \bar{c}}{\partial x} + v\bar{c} = \frac{vc_o}{s}\left[1-\exp(-t_o s)\right] \qquad (x=0) \tag{B7}$$

The solution of (B6) and (B7), which is consistent with (B4), is

$$\bar{c}(x,s) = \frac{\frac{v}{D}\left(c_o - \frac{\gamma}{\mu}\right)\exp\left[\frac{vx}{2D} - x\left(\frac{v^2}{4D^2} + \frac{s + \mu/R}{D/R}\right)^{\frac{1}{2}}\right]}{s\left[\frac{v}{2D} + \left(\frac{v^2}{4D^2} + \frac{s + \mu/R}{D/R}\right)^{\frac{1}{2}}\right]}$$

$$- \frac{\frac{vc_o}{D}\ \exp(-t_o s)\ \exp\left[\frac{vx}{2D} - x\left(\frac{v^2}{4D^2} + \frac{s + \mu/R}{D/R}\right)^{\frac{1}{2}}\right]}{s\left[\frac{v}{2D} + \left(\frac{v^2}{4D^2} + \frac{s + \mu/R}{D/R}\right)^{\frac{1}{2}}\right]}$$

$$+ \frac{\frac{v}{D}\left(\frac{\gamma}{\mu} - c_i\right)\ \exp\left[\frac{vx}{2D} - x\left(\frac{v^2}{4D^2} + \frac{s + \mu/R}{D/R}\right)^{\frac{1}{2}}\right]}{(s + \mu/R)\ \left[\frac{v}{2D} + \left(\frac{v^2}{4D^2} + \frac{s + \mu/R}{D/R}\right)^{\frac{1}{2}}\right]}$$

$$+ \frac{\gamma/R}{s(s + \mu/R)} + \frac{c_i}{s + \mu/R} \tag{B8}$$

The inverse Laplace transform of the first term of (B8) can be obtained by first letting $p = s$, $h - v/(2D)$, $\kappa = D/R$, and $\alpha = \mu/R +$

$v^2/(4DR)$ in Eq. (31) of Appendix A of Carslaw and Jaeger[49], and subsequently using $a = -\mu/R - v^2/(4DR)$ in Eq. (29.2.12) of Abramowitz and Stegun[33]. The following expression was obtained

$$I_1(x,t) = (c_o - \frac{\gamma}{\mu})\left\{\frac{v}{(v+v^*)} \exp\left[\frac{(v-v^*)x}{2D}\right] \operatorname{erfc}\left[\frac{Rx - v^*t}{2(DRt)^{\frac{1}{2}}}\right] + \frac{v}{(v-v^*)} \exp\left[\frac{(v+v^*)x}{2D}\right] \operatorname{erfc}\left[\frac{Rx + v^*t}{2(DRt)^{\frac{1}{2}}}\right] + \frac{v^2}{2\mu D} \exp\left(\frac{vx}{D} - \frac{\mu t}{R}\right) \operatorname{erfc}\left[\frac{Rx + vt}{2(DRt)^{\frac{1}{2}}}\right]\right\} \quad \text{(B9)}$$

where

$$v^* = v\left(1 + \frac{4\mu D}{v^2}\right)^{\frac{1}{2}}. \quad \text{(B10)}$$

The inverse of the second term of (B8) follows immediately from the inverse of the first term by making use of Eq. (29.2.15) of Abramowitz and Stegun[33]:

$$I_2(x,t) = -c_o\left\{\frac{v}{(v+v^*)} \exp\left[\frac{(v-v^*)x}{2D}\right] \operatorname{erfc}\left[\frac{Rx - v^*(t-t_o)}{2(DRt)^{\frac{1}{2}}}\right] + \frac{v}{(v-v^*)} \exp\left[\frac{(v+v^*)x}{2D}\right] \operatorname{erfc}\left[\frac{Rx + v^*(t-t_o)}{2(DRt)^{\frac{1}{2}}}\right] + \frac{v^2}{2\mu D} \exp\left[\frac{vx}{D} - \frac{\mu(t-t_o)}{R}\right] \operatorname{erfc}\left[\frac{Rx + v(t-t_o)}{2(DRt)^{\frac{1}{2}}}\right]\right\}. \quad \text{(B11)}$$

The inverse transform of the third term of (B8) may be obtained by first considering the transport equation (B1) without the two rate terms, i.e.,

$$D\frac{\partial^2 c}{\partial x^2} - v\frac{\partial c}{\partial x} - R\frac{\partial c}{\partial t} = 0 \quad \text{(B12)}$$

The Laplace transform solution of this equation subject to the same initial and boundary conditions as before, but with $c_i = 0$, $c_o = 1$, and $t_o \to \infty$ (i.e., a continuous feed solution), is given by

$$\bar{c} = \frac{\frac{v}{D} \exp\left[\frac{vx}{2D} - x\left(\frac{v^2}{4D^2} + \frac{s + \mu/R}{D/R}\right)^{\frac{1}{2}}\right]}{s\left[\frac{v}{2D} + \left(\frac{v^2}{4D^2} + \frac{s}{D/R}\right)^{\frac{1}{2}}\right]} \qquad \text{(B13)}$$

The direct solution of (B12) itself is[36]

$$c = \frac{1}{2}\,\text{erfc}\left[\frac{Rx - vt}{2(DRt)^{\frac{1}{2}}}\right] + \left(\frac{v^2 t}{\pi RD}\right)^{\frac{1}{2}} \exp\left[-\frac{(Rx - vt)^2}{4DRt}\right]$$

$$- \frac{1}{2}\left(1 + \frac{vx}{D} + \frac{v^2 t}{DR}\right) \exp(vx/D)\ \text{erfc}\left[\frac{Rx + vt}{2(DRt)^{\frac{1}{2}}}\right]. \qquad \text{(B14)}$$

Equation (B14) is, hence, the inverse transform of (B13). Application of Eq. (29.2.12) of Abramowitz and Stegun[33] to Eq. (B13) and (B14) leads directly to the Laplace inverse of the third term of (B8):

$$I_3(x,t) = \left(\frac{\gamma}{\mu} - c_i\right) \exp\left(-\frac{\mu t}{R}\right) \left\{ \frac{1}{2}\,\text{erfc}\left[\frac{Rx - vt}{2(DRt)^{\frac{1}{2}}}\right]\right.$$

$$+ \left(\frac{v^2 t}{\pi RD}\right)^{\frac{1}{2}} \exp\left[-\frac{(Rx - vt)^2}{4DRt}\right]$$

$$\left. - \frac{1}{2}\left(1 + \frac{vx}{D} + \frac{v^2 t}{DR}\right) \exp(vx/D)\ \text{erfc}\left[\frac{Rx + vt}{2(DRt)^{\frac{1}{2}}}\right]\right\}. \qquad \text{(B15)}$$

The inverse transforms of the fourth and fifth terms of (B8) follow immediately from Eq. (29.3.12) and (29.3.8) of Abramowitz and Stegun[33]:

$$I_4(x,t) = \frac{\gamma}{\mu}\,(1 - e^{-\mu t}) \tag{B16}$$

$$I_5(x,t) = c_i\ e^{-\mu t} \tag{B17}$$

The inverse transform of (B8), which is the solution of the present problem, is hence given by see also Eq. (42) and (43)

$$c(x,t) = I_1(x,t) + I_2(x,t) + I_3(x,t) + I_4(x,t) + I_5(x,t) \tag{B19}$$

## *APPENDIX C*

SUMATRA-1

A COMPUTER PROGRAM FOR CALCULATING
SIMULTANEOUS WATER AND SOLUTE TRANSFER
IN A ONE-DIMENSIONAL, SATURATED-UNSATURATED
AND NON-HOMOGENEOUS SOIL PROFILE.

This appendix gives a brief description and listing of SUMATRA-1, a computer model for simulating simultaneous water and solute transfer in a one-dimensional, saturated-unsaturated and non-homogeneous soil profile. The program consists of a main program (MAIN), six subroutines (DATAIN, MATEQ, BANSOL, MATSO, SOLVE, and PRINT), and three functions (BC, SPR, and SPS). The main program controls the sequence of calculations, as shown schematically by the flow chart in Fig. A1. The subroutine DATAIN is first used to read the input data and to define the geometry and initial conditions of the system. In addition, DATAIN may be used to obtain a listing of the different physical and chemical soil properties used in the simulation (controlled by the output code KOD3).

The subroutine MATEQ performs the necessary calculations for assembly of the global matrix equation for flow, while the subroutine BANSOL subsequently solves this equation for the updated values of the pressure head (PE). A check on the iterative solution process is then carried out in MAIN. If convergence is not met, the iterative process either continues (if NIT < NITMAX), restarts with a smaller timestep DELT (if NIT = NITMAX), or stops altogether (if DELT < DELMIN). The simulation, on the other hand, proceeds in time when the imposed convergence criteria are met. The global matrix equation for solute transport is then assembled in MATSO and subsequently solved in SOLVE.

Possible print-out is provided by the subroutine PRINT. This subroutine also computes a cumulative mass balance for the soil profile. The simulation proceeds in time until either the maximum simulation time is exceeded (SUMT > TMAX), or a given number of time steps is executed (ISTEP = NSTEPS).

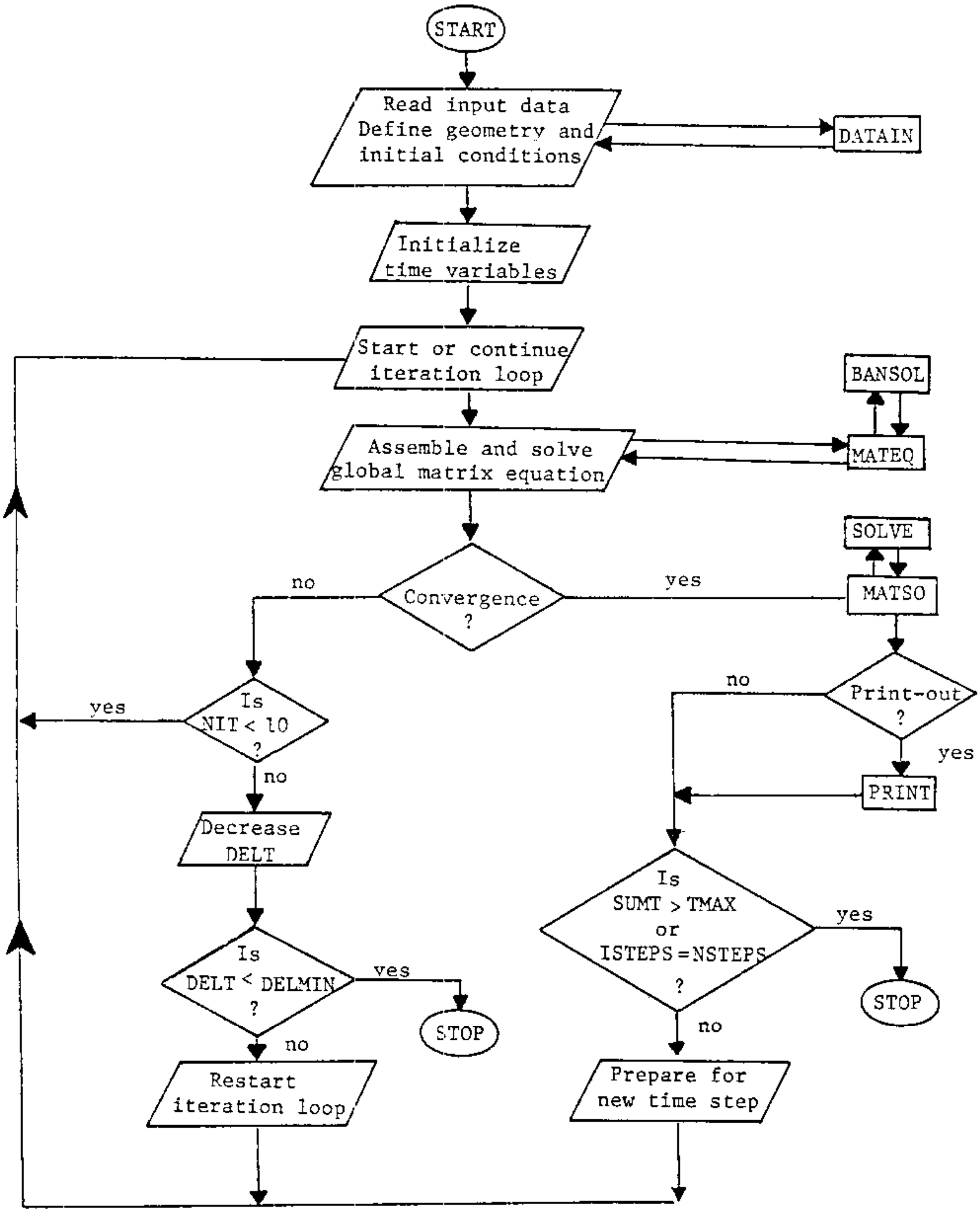



Fig. 11. Generalized flow chart of SUMATRA-1.

The functions BC, SPR, and SPS are problem-dependent. BC supplies the transient data for the boundary conditions at the soil surface. The function SPR defines the hydraulic functions [$\theta(h)$, $K(h)$, $C(h)$, and $h(\theta)$] for each soil type present. In the present version these functions are given by means of analytical expressions. If necessary they can also be supplied in table form. The function SPS, finally, specifies those soil-physical and chemical parameters which are relevant for the solute transport part of the program, i.e., the bulk density ($\rho$), the porous matrix diffusion coefficient ($D_o\tau$), the dispersivity ($\lambda$), the adsorption constant ($k$), and the three rate coefficients ($\alpha$, $\beta$, and $\gamma$). These parameters are given for each soil type present.

Table C1 gives a list of the most significant variables in SUMATRA-1. Instructions for preparing the input data cards are given in Table C2, while Table C3 specifies the actual data cards used for example 3. Part of the computer output for this example is given in Table C4, while the listing of the program itself is given in Table C5.

*Table C1.* Definition of the main program variables of SUMATRA-1. If the variable appears in only one function or subroutine, the name of that function or subroutine is given after the definition. If the variable represents an array, the maximum dimension of that array is also specified.

| *VARIABLE* | *DEFINITION* |
|---|---|
| A(5) | Vector used to calculate the soil moisture retention curve of each soil type for possible print-out (DATAIN). |
| ADC(NMAT) | Values of the distribution coefficient (k) for each soil type (SPS). |
| ALPHA(NMAT) | Values of the coefficient a in Eq. (45) for each soil type (SPR). |
| AN1,AN2,AN3 | Values of $(D^{-}\theta)/\Delta x$ at the three interior Lobatto integration points, evaluated at the new time level (MATSO). |
| AO1,AO2,AO3 | Values of $(D^{+}\theta)/\Delta x$ at the three interior Lobatto integration points, evaluated at the told time level (MATSO). |
| B(7) | Vector used to calculate the K(h) curve of each soil type for possible print-out (DATAIN). |
| BC(K,SUMT) | Function used to define the boundary condition at the soil surface as a function of time:<br>If K=1, $h_o(t)$ in Eq. (5a) is specified;<br>if K=2, $q_o(t)$ in Eq. (5b) is specified;<br>if K=3, $c_o(t)$ in Eq. (14b) is specified;<br>if K=4, $c_o(t)$ in Eq. (14c) is specified. |
| BN1,BN2,BN3 | Values of q/2 at the three interior integration points, evaluated at the new time level (MATSO). |
| BO1,BO2,BO3 | Values at q/2 at the three interior integration points, evaluated at the old time level (MATSO). |
| C(NN2) | Vector of nodal concentrations and their gradients. |
| CAP1,CAP2,CAP3. | Functions of the soil moisture capacity at the nodal or interior Lobatto integration points (MATEQ) |
| COND1,COND2,COND3. | Functions of the hydraulic conductivity at the nodal or interior Lobatto integration points (MATEQ). |
| CONDM | Minimum value of the unsaturated hydraulic conductivity (SPR) |

*Table C1 (continued):*

| *VARIABLE* | *DEFINITION* |
|---|---|
| CONDS(NMAT) | Values of the saturated hydraulic conductivity for each soil type (SPR). |
| DELCH | Change in DELT between two consecutive time steps (MAIN). |
| DELMAX | Maximum value of DELT during execution. |
| DELMIN | Minimum value of DELT permitted during execution. |
| DELT | Time increment. |
| DIF | Values of $(D_o\tau)$ at nodal or interior integration point (MATSO). |
| DIF(NMAT) | Values of $(D_o\tau)$ for each soil type (SPS). |
| DLO(NMAT) | Values of $\gamma$ for each soil type (SPSO. |
| DL1 | Value of $\alpha$ at nodal or interior integration point (MATSO). |
| DL1(NMAT) | Values of $\alpha$ for each soil type (SPS). |
| DRAIN | Drainage rate. |
| DS1 | Value of $\beta$ at nodal or interior integration point (MATSO). |
| DS1(NMAT) | Values of $\beta$ for each soil type (SPS). |
| DSP | Value of $\lambda$ at nodal or interior integration point (MATSO). |
| DSP(NMAT) | Values of $\lambda$ for each soil type (SPS). |
| DX(4,3) | Derivatives of the four basis functions, evaluated at the three interior Lobatto integration points. |
| EL | Nodal distance ($\Delta x$). |
| EPSI | Weighting coefficient for finite difference approximation of the time derivative in flow equation. |
| EPSM | = EPSI - 1. |
| F(NN2) | Right-hand side vector of global matrix equation for both the flow (MATEQ) and solute transport equations (MATSO). |
| FE(4,3) | Values of the four basis functions, evaluated at the three interior Lobatto integration points. |

*Table C1* (continued:

| *VARIABLE* | *DEFINITION* |
|---|---|
| INT(NN) | Array indicating type of material interpolation between two consecutive nodes. If INT=1, the soil properties are assumed to change linearly; if INT=0, a restricted Hermitian interpolation is used. |
| ISPR(NN) | Soil property index, defining soil type for each node. |
| ISTEP | Number of times steps since start of simulation (MAIN). |
| KDRAIN | Drainage code for lower boundary:<br>=0 if the pressure head is specified.<br>=1 if the pressure gradient is specified.<br>=2 if the drainage rate (DRAIN) is specified.<br>=3 if both the pressure head and its gradient are specified. |
| KOD1 | Output code for printing pressure heads and concentrations after each iteration and time step:<br>=0, no output is given;<br>=1, prints PE(I) after each iteration and each time step;<br>=2, prints PE(I) after each iteration and C(I) after each time step;<br>=3, prints C(I) after each time step. |
| KOD2 | Input code. If KOD2=0, initial moisture contents are read in and converted to pressure heads. If KOD2=1, pressure head values are read in (DATAIN). |
| KOD3 | Output code. If KOD3=N (N $\leq$ MAT), the soil-hydraulic properties of the first N soil-types ae printed. If KOD3=0, no such output is given (DATAIN). |
| KOD4 | Specifies the type of equation to be solved:<br>=0, solves both flow and solute transport equations;<br>=1, solves only the flow equation;<br>=2, solves only the transport equation. |
| KRAIN | Rainfall code for soil surface boundary:<br>=0 if the pressure head is specified;<br>=1 if the infiltration rate (RAIN) is specified. |
| L | Element number. |

*Table C1 (continued):*

| *VARIABLE* | *DEFINITION* |
|---|---|
| MAT,MAT1,MAT2 | Material index of certain node or integration point. |
| N1 | =NN2-1. |
| N2 | =NN2-2. |
| N3 | =NN2-3. |
| NE | Number of elements: NE = NN-1. |
| NIT | Number of iterations during particular time step (MAIN). |
| NITMAX | Maximum number of iterations (set to 10)(MAIN). |
| NITT | Total number of iterations since start of simulation. |
| NMAT | Number of different soil types (dummy variable). |
| NN | Number of nodes. |
| NN2 | =2*NN. |
| NSTEPS | Maximum number of time steps permitted. |
| P(NN2) | Vector of pressure heads and gradients at old time level. |
| PE(NN2) | Estimated or calculated vector of pressure heads and gradients for new time level. |
| PRDEL | Time increment for printed output. |
| PRTIME | Dummy variable, used to calculate output for every PRDEL (MAIN). |
| PULSE | Irreversible time switch for changing from a first-type to a flux-type boundary condition. |
| Q0(I) | Data points for transient first-type boundary condition of solute transport equation (BC). |
| Q1(I) | Data points for transient third-type (flux) boundary condition of solute transport equation (BC). |
| QN | Nodal value of volumetric flux (q) at new time level (MATSO). |
| QO | Nodal value of volumetric flux (q) at old time level (MATSO). |
| R0(I) | Data points for transient first-type boundary condition of flow equation (BC). |

*Table C1 (continued):*

| VARIABLE | DEFINITION |
|---|---|
| R1(I) | Data points for transient second-type (flux) boundary condition of flow equation (BC). |
| RAIN | Transient rain or evaporation rate. |
| RHO(NMAT) | Values of $\xi$ for each soil type (SPS). |
| RN(NMAT) | Values of the exponent of n in Eq. (45) for each soil type (SPR). |
| RTK | Value of ($\rho$k) at nodal or interior integration point (MATSO). |
| S(NN2,4), S(NN2,7). | Global matrices for flow (MATEQ) and mass transport (MATSO), respectively. |
| SPR(MAT,N.P) | Function to calculate the soil hydraulic properties of material MAT:<br>if N=1, the soil moisture content is given as a function of the pressure head P;<br>if N=2, the hydraulic conductivity is given as a function of the pressure head P;<br>if N=3, the soil moisture capacity is given as a function of the pressure head P;<br>if N=4, the pressure head is given as a function of the soil moisture content. |
| SPS(MAT,N) | Function used to specify the various soil-physical and soil-chemical properties of soil material MAT:<br>N=1 gives the soil bulk density ($\rho$);<br>N=2 gives the effective diffusion coefficient ($D_o\tau$);<br>N=3 gives the dispersivity ($\lambda$);<br>N=4 gives the adsorption constant (k);<br>N=5 gives the rate coefficient $\gamma$;<br>N=6 gives the rate coefficient $\alpha$;<br>N=7 gives the rate coefficient $\beta$. |
| SR(10) | Vector containing contributions of the three interior integration points to the right-hand side vector of each element matrix equation during assembly of the global matrix equation for flow (MATEQ). |
| SS(NMAT) | Values of $S_s$ for each soil type (SPR). |

*Table C1 (continued):*

| *VARIABLE* | *DEFINITION* |
|---|---|
| SUMT | Elapsed time since start of simulation, in days. |
| SUMT1,SUMT2 | Elapsed time since start of simulation, in hours and minutes, respectively (PRINT). |
| T(NN2) | Temporary storage vector for pressure heads and gradients during iterative solution process of flow equation (MAIN). |
| TO(I) | Vector used to specify the transient first-order boundary condition for the flow equation (BC). |
| T1(I) | Vector used to specify the transient second-type (flux) boundary condition for the flow equation (BC). |
| T1,T2,T3 | Functions of ($\gamma\theta$), evaluated at the three interior integration points (MATSO). |
| THETA | Dimensionless soil moisture content, $\Theta$ (SPR). |
| TITLE(20) | Array containing information of program title cards (DATAIN). |
| TMAX | Maximum simulation time. |
| TMDIFF | Error in the material balance calculations: TMDIFF=TMINF-TMINCR (PRINT). |
| TMIN | Total amount of moisture in soil profile ($cm^3$). |
| TMINCR | Increase in stored moisture since start of simulation: TMINCR=TMIN-TMINIT (PRINT). |
| TMINF | Cumulative uptake of moisture by profile since start of simulation, i.e., cumulative infiltration minus cumulative drainage. |
| TMINIT | Total amount of moisture in soil profile at start of simulation. |
| TOL | Convergence criterion for iterative solution process (MAIN). |
| TOL1 | Absolute convergence criterion. |
| TOL2 | Relative convergence criterion. |
| WC | Variable generally defining the soil moisture content. |
| WCN | Value of $\theta$ at nodal or interior integration point, evaluated at new time level (MATSO). |

*Table C1 (continued):*

| *VARIABLE* | *DEFINITION* |
|---|---|
| WCO | Value of $\theta$ at nodal or interior integration point, evaluated at old time level (MATSO). |
| WCR(NMAT) | Residual moisture content for each soil type (SPR). |
| WCS(NMAT) | Saturated moisture content for each soil type (SPR). |
| X(NN) | Nodal coordinates. |

*Table C2.* Input data instructions for SUMATRA-1.

| CARDS | COLUMNS | FORMAT | VARIABLE | COMMENT |
|---|---|---|---|---|
| 1,2,3 | 1-80 | 20(A4) | TITLE | |
| 4 | 1-5 | I5 | NN | Number of nodes. |
| | 6-10 | I5 | NSTEPS | Maximum number of time steps. |
| | 11-20 | F10.0 | DELT | Initial time step (days). |
| | 21-30 | F10.0 | DELMIN | Minimum permitted time step (days). |
| | 31-40 | F10.0 | DELMAX | Maximum time step (days). |
| | 41-50 | F10.0 | TMAX | Maximum simulation time (days). |
| | 51-60 | F10.0 | PRDEL | Time step for print-out (days). |
| | 61-70 | F10.0 | PULSE | Time switch for soil surface boundary condition (days). |
| | 71-80 | F10.0 | DRAIN | Drainage rate (cm/day). |
| 5 | 1-5 | I5 | KRAIN | Rainfall code. |
| | 6-10 | I5 | KDRAIN | Drainage code. |
| | 11-15 | I5 | KOD1 | Output code. |
| | 16-20 | I5 | KOD2 | Input code. |
| | 21-25 | I5 | KOD3 | Output code. |
| | 26-30 | I5 | KOD4 | Specifies equation to be solved. |
| | 31-40 | F10.0 | EPSI | Weighting coefficient for flow equation. |
| | 41-50 | F10.0 | TOL1 | Absolute convergence criterion (cm). |
| | 51-60 | F10.0 | TOL2 | Relative convergence criterion. |
| 6, etc. | 1-5 | I5 | I | Nodal number. |
| | 6-10 | I5 | ISPR(I) | Soil property index of node I. |
| | 11-20 | F10.0 | X(I) | Coordinate of node I (cm). |
| | 21-30 | F10.0 | P(2I-1) | Initial pressure head if KOD2=1, initial moisture content if KOD2=0. |
| | 31-40 | F10.0 | P(2I) | Initial pressure gradient if KOD2=1, initial moisture gradient if KOD2=0. |
| | 41-50 | F10.0 | C(2I-1) | Initial concentration. |
| | 51-60 | F10.0 | C(2I) | Initial concentration gradient. |
| | 61-65 | I5 | INT(I) | Interpolation index for element I. |

*Table C3.* Data input for example 3.

```
     SUMATRA-1

     EXAMPLE 3 OF REPORT
27 5000   .0005    .00005     0.5      8.0    .05     0.50    0.0
 1    1    0    1    9    0  0.500    0.500  0.000
 1    1    0.0    -350.0      0.0      5.270    0.0
 2    1    6.0    -350.0      0.0      5.270    0.0
 3    1   14.0    -350.0      0.0      5.270    0.0
 4    1   22.0    -350.0      0.0      5.270    0.0
 5    3   28.0    -350.0      0.0      4.731   -.159    1
 6    4   36.0    -350.0      0.0      3.500   -.159    1
 7    5   46.0    -350.0      0.0      1.869   -.135    1
 8    6   55.0    -350.0      0.0      0.824   -.110    1
 9    7   63.0    -350.0      0.0      0.168   -.055    1
10    8   69.0    -350.0      0.0                       1
11    9   73.0    -350.0      0.0
12    2   77.0    -350.0      0.0
13    2   81.0    -350.0      0.0
14    2   85.0    -350.0      0.0
15    9   89.0    -350.0      0.0
16    9   93.0    -350.0      0.0
17    9   97.0    -350.0      0.0
18    9  102.0    -350.0      0.0
19    9  107.0    -350.0      0.0
20    9  112.0    -350.0      0.0
21    9  118.0    -350.0      0.0
22    9  125.0    -350.0      0.0
23    9  133.0    -350.0      0.0
24    9  141.0    -350.0      0.0
25    9  150.0    -350.0      0.0
26    9  160.0    -350.0      0.0
27    9  170.0    -350.0      0.0
```

*Table C4.* Partial output for example 3.

```
*****************************************************************************
*                                                                           *
*        ONE-DIMENSIONAL UNSATURATED TRANSPORT                              *
*                                                                           *
*        SUMATRA-1                                                          *
*                                                                           *
*        EXAMPLE 3 OF REPORT                                                *
*                                                                           *
*****************************************************************************
```

INPUT PARAMETERS

```
NUMBER OF NODES.......................(NN)...............    27
MAXIMUM NUMBER OF TIME STEPS.......(NSTEPS).............. 5000
INITIAL TIME STEP..................(DELT)...........    0.00050
MINIMUM ALLOWABLE TIME STEP........(DELMIN)..........    0.00005
MAXIMUM ALLOWABLE TIME STEP........(DELMAX)..........    0.50000
MAXIMUM SIMULATION TIME............(TMAX)...........    8.00000
PRINT DELT FOR OUTPUT..............(PRDEL)..........    0.05000
PULSE LENGTH FOR 1ST-TYPE BC.......(PULSE)..........    0.50000
WEIGHTING COEFFICIENT..............(EPSI)...........    0.50000
ITERATION TOLERANCE................(TOL1)...........    0.50000
ITERATION TOLERANCE................(TOL2)...........    0.0
KRAIN.........................(RAINFALL CODE)..........      1
KDRAIN........................(DRAINAGE CODE)..........      1
KOD1..............(OUTPUT FOR EVERY ITERATION)..........      0
KOD2............(INPUT VARIABLE IS PRESSURE HEAD).......      1
KOD3..............(WRITE MATERIAL PROPERTIES)..........      9
KOD4...........(SOLVE ONLY FOR FLOW OR TRANSPORT).......      0
```

REDEFINED SURFACE VALUES

| ITERATION | MOIST. CONT. | PRESSURE | GRADIENT |
|---|---|---|---|
| 1 | 0.3946 | -208.337 | -67.8547 |
| 2 | 0.3854 | -226.467 | -88.6986 |
| 3 | 0.3842 | -228.800 | -91.7099 |
| 4 | 0.3842 | -228.989 | -91.9566 |

INITIAL CONDITIONS

| | | PRESSURE HEAD | | | | MOISTURE CONTENT | | | CONCENTRATION | | | | INDICES | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| NODE | DEPTH | FUNCTN | GRAD | F(1/3) | F(2/3) | FUNCTN | F(1/3) | F(2/3) | FUNCTN | GRAD | F(1/3) | F(2/3) | MAT | INT |
| 1 | 0.0 | -228.99 | -91.96 | -342.10 | -359.50 | 0.3842 | 0.3421 | 0.3373 | 8.536 | -3.266 | 4.786 | 4.665 | 1 | 0 |
| 2 | 6.0 | -350.00 | 0.0 | -350.00 | -350.00 | 0.3398 | 0.3398 | 0.3398 | 5.270 | 0.0 | 5.270 | 5.270 | 1 | 0 |
| 3 | 14.0 | -350.00 | 0.0 | -350.00 | -350.00 | 0.3398 | 0.3398 | 0.3398 | 5.270 | 0.0 | 5.270 | 5.270 | 1 | 0 |
| 4 | 22.0 | -350.00 | 0.0 | -350.00 | -350.00 | 0.3398 | 0.3172 | 0.2751 | 5.270 | 0.0 | 5.201 | 5.012 | 1 | 0 |
| 5 | 28.0 | -350.00 | 0.0 | -350.00 | -350.00 | 0.2524 | 0.2432 | 0.2340 | 4.731 | -0.159 | 4.318 | 3.913 | 3 | 1 |
| 6 | 36.0 | -350.00 | 0.0 | -350.00 | -350.00 | 0.2248 | 0.2151 | 0.2053 | 3.500 | -0.159 | 2.942 | 2.374 | 4 | 1 |
| 7 | 46.0 | -350.00 | 0.0 | -350.00 | -350.00 | 0.1955 | 0.1881 | 0.1806 | 1.669 | -0.135 | 1.491 | 1.152 | 5 | 1 |
| 8 | 55.0 | -350.00 | 0.0 | -350.00 | -350.00 | 0.1731 | 0.1673 | 0.1616 | 0.824 | -0.110 | 0.556 | 0.338 | 6 | 1 |

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 9 | 63.0 | -350.00 | 0.0 | -350.00 | -350.00 | 0.1558 | 0.1519 | 0.1480 | 0.168 | -0.055 | 0.076 | 0.019 | 7 | 1 |
| 10 | 69.0 | -350.00 | 0.0 | -350.00 | -350.00 | 0.1441 | 0.1417 | 0.1393 | 0.0 | 0.0 | 0.0 | 0.0 | 8 | 1 |
| 11 | 73.0 | -350.00 | 0.0 | -350.00 | -350.00 | 0.1369 | 0.1700 | 0.2316 | 0.0 | 0.0 | 0.0 | 0.0 | 9 | 0 |
| 12 | 77.0 | -350.00 | 0.0 | -350.00 | -350.00 | 0.2648 | 0.2648 | 0.2646 | 0.0 | 0.0 | 0.0 | 0.0 | 2 | 0 |
| 13 | 81.0 | -350.00 | 0.0 | -350.00 | -350.00 | 0.2648 | 0.2648 | 0.2648 | 0.0 | 0.0 | 0.0 | 0.0 | 2 | 0 |
| 14 | 85.0 | -350.00 | 0.0 | -350.00 | -350.00 | 0.2648 | 0.2316 | 0.1700 | 0.0 | 0.0 | 0.0 | 0.0 | 2 | 0 |
| 15 | 89.0 | -350.00 | 0.0 | -350.00 | -350.00 | 0.1369 | 0.1369 | 0.1369 | 0.0 | 0.0 | 0.0 | 0.0 | 9 | 0 |
| 16 | 93.0 | -350.00 | 0.0 | -350.00 | -350.00 | 0.1369 | 0.1369 | 0.1369 | 0.0 | 0.0 | 0.0 | 0.0 | 9 | 0 |
| 17 | 97.0 | -350.00 | 0.0 | -350.00 | -350.00 | 0.1369 | 0.1369 | 0.1369 | 0.0 | 0.0 | 0.0 | 0.0 | 9 | 0 |
| 18 | 102.0 | -350.00 | 0.0 | -350.00 | -350.00 | 0.1369 | 0.1369 | 0.1369 | 0.0 | 0.0 | 0.0 | 0.0 | 9 | 0 |
| 19 | 107.0 | -350.00 | 0.0 | -350.00 | -350.00 | 0.1369 | 0.1369 | 0.1369 | 0.0 | 0.0 | 0.0 | 0.0 | 9 | 0 |
| 20 | 112.0 | -350.00 | 0.0 | -350.00 | -350.00 | 0.1369 | 0.1369 | 0.1369 | 0.0 | 0.0 | 0.0 | 0.0 | 9 | 0 |
| 21 | 118.0 | -350.00 | 0.0 | -350.00 | -350.00 | 0.1369 | 0.1369 | 0.1369 | 0.0 | 0.0 | 0.0 | 0.0 | 9 | 0 |
| 22 | 125.0 | -350.00 | 0.0 | -350.00 | -350.00 | 0.1369 | 0.1369 | 0.1369 | 0.0 | 0.0 | 0.0 | 0.0 | 9 | 0 |
| 23 | 133.0 | -350.00 | 0.0 | -350.00 | -350.00 | 0.1369 | 0.1369 | 0.1369 | 0.0 | 0.0 | 0.0 | 0.0 | 9 | 0 |
| 24 | 141.0 | -350.00 | 0.0 | -350.00 | -350.00 | 0.1369 | 0.1369 | 0.1369 | 0.0 | 0.0 | 0.0 | 0.0 | 9 | 0 |
| 25 | 150.0 | -350.00 | 0.0 | -350.00 | -350.00 | 0.1369 | 0.1369 | 0.1369 | 0.0 | 0.0 | 0.0 | 0.0 | 9 | 0 |
| 26 | 160.0 | -350.00 | 0.0 | -350.00 | -350.00 | 0.1369 | 0.1369 | 0.1369 | 0.0 | 0.0 | 0.0 | 0.0 | 9 | 0 |
| 27 | 170.0 | -350.00 | 0.0 | | | 0.1369 | | | 0.0 | 0.0 | | | 9 | |

INITIAL INFILTRATION RATE 25.0000
INITIAL DRAINAGE RATE 0.0075
INITIAL AMOUNT OF MOISTURE IN PROFILE 32.5541
INITIAL MASS IN SOLUTION 65.6285

SOIL-HYDRAULIC PROPERTIES (MOISTURE CONTENT AND HYDRAULIC CONDUCTIVITY)

| PRESSURE HEAD | SOIL 1 WC | SOIL 1 COND | SOIL 2 WC | SOIL 2 COND | SOIL 3 WC | SOIL 3 COND | SOIL 4 WC | SOIL 4 COND | SOIL 5 WC | SOIL 5 COND |
|---|---|---|---|---|---|---|---|---|---|---|
| -0.600E 00 | 0.5400 | 0.243E 02 | 0.4000 | 0.100E 02 | 0.4700 | 0.741E 02 | 0.4611 | 0.132E 03 | 0.4500 | 0.205E 03 |
| -0.720E 00 | 0.5400 | 0.242E 02 | 0.4000 | 0.100E 02 | 0.4700 | 0.739E 02 | 0.4611 | 0.132E 03 | 0.4500 | 0.205E 03 |
| -0.864E 00 | 0.5400 | 0.241E 02 | 0.4000 | 0.100E 02 | 0.4700 | 0.737E 02 | 0.4611 | 0.132E 03 | 0.4500 | 0.204E 03 |
| -0.104E 01 | 0.5400 | 0.239E 02 | 0.4000 | 0.100E 02 | 0.4700 | 0.735E 02 | 0.4611 | 0.132E 03 | 0.4500 | 0.204E 03 |
| -0.124E 01 | 0.5400 | 0.238E 02 | 0.4000 | 0.100E 02 | 0.4700 | 0.731E 02 | 0.4611 | 0.131E 03 | 0.4500 | 0.204E 03 |
| -0.149E 01 | 0.5399 | 0.236E 02 | 0.4000 | 0.100E 02 | 0.4700 | 0.728E 02 | 0.4611 | 0.131E 03 | 0.4500 | 0.204E 03 |
| -0.179E 01 | 0.5399 | 0.234E 02 | 0.4000 | 0.999E 01 | 0.4700 | 0.723E 02 | 0.4611 | 0.130E 03 | 0.4500 | 0.203E 03 |
| -0.215E 01 | 0.5399 | 0.231E 02 | 0.4000 | 0.999E 01 | 0.4699 | 0.716E 02 | 0.4611 | 0.130E 03 | 0.4500 | 0.203E 03 |
| -0.258E 01 | 0.5399 | 0.228E 02 | 0.4000 | 0.999E 01 | 0.4699 | 0.712E 02 | 0.4610 | 0.129E 03 | 0.4500 | 0.202E 03 |
| -0.310E 01 | 0.5398 | 0.225E 02 | 0.4000 | 0.998E 01 | 0.4699 | 0.704E 02 | 0.4610 | 0.128E 03 | 0.4499 | 0.201E 03 |
| -0.372E 01 | 0.5397 | 0.221E 02 | 0.4000 | 0.998E 01 | 0.4698 | 0.695E 02 | 0.4610 | 0.127E 03 | 0.4499 | 0.200E 03 |
| -0.446E 01 | 0.5396 | 0.216E 02 | 0.4000 | 0.997E 01 | 0.4697 | 0.684E 02 | 0.4609 | 0.126E 03 | 0.4499 | 0.199E 03 |
| -0.535E 01 | 0.5395 | 0.211E 02 | 0.4000 | 0.995E 01 | 0.4696 | 0.672E 02 | 0.4608 | 0.124E 03 | 0.4498 | 0.197E 03 |
| -0.642E 01 | 0.5393 | 0.206E 02 | 0.4000 | 0.993E 01 | 0.4694 | 0.656E 02 | 0.4607 | 0.122E 03 | 0.4497 | 0.195E 03 |
| -0.770E 01 | 0.5390 | 0.199E 02 | 0.4000 | 0.990E 01 | 0.4691 | 0.638E 02 | 0.4604 | 0.120E 03 | 0.4496 | 0.192E 03 |
| -0.924E 01 | 0.5386 | 0.191E 02 | 0.3999 | 0.986E 01 | 0.4687 | 0.617E 02 | 0.4601 | 0.116E 03 | 0.4493 | 0.189E 03 |
| -0.111E 02 | 0.5381 | 0.183E 02 | 0.3999 | 0.980E 01 | 0.4682 | 0.592E 02 | 0.4597 | 0.113E 03 | 0.4489 | 0.184E 03 |
| -0.133E 02 | 0.5374 | 0.173E 02 | 0.3998 | 0.971E 01 | 0.4674 | 0.563E 02 | 0.4590 | 0.108E 03 | 0.4483 | 0.178E 03 |
| -0.160E 02 | 0.5363 | 0.163E 02 | 0.3997 | 0.958E 01 | 0.4662 | 0.529E 02 | 0.4579 | 0.103E 03 | 0.4475 | 0.171E 03 |
| -0.192E 02 | 0.5350 | 0.151E 02 | 0.3995 | 0.940E 01 | 0.4646 | 0.490E 02 | 0.4564 | 0.960E 02 | 0.4461 | 0.162E 03 |
| -0.230E 02 | 0.5331 | 0.138E 02 | 0.3991 | 0.914E 01 | 0.4624 | 0.446E 02 | 0.4542 | 0.882E 02 | 0.4440 | 0.150E 03 |
| -0.276E 02 | 0.5305 | 0.124E 02 | 0.3985 | 0.877E 01 | 0.4592 | 0.397E 02 | 0.4510 | 0.792E 02 | 0.4408 | 0.136E 03 |
| -0.331E 02 | 0.5270 | 0.109E 02 | 0.3974 | 0.826E 01 | 0.4548 | 0.343E 02 | 0.4464 | 0.691E 02 | 0.4361 | 0.120E 03 |
| -0.397E 02 | 0.5224 | 0.933E 01 | 0.3956 | 0.756E 01 | 0.4488 | 0.286E 02 | 0.4399 | 0.580E 02 | 0.4292 | 0.101E 03 |
| -0.477E 02 | 0.5163 | 0.782E 01 | 0.3926 | 0.663E 01 | 0.4408 | 0.231E 02 | 0.4311 | 0.465E 02 | 0.4193 | 0.809E 02 |
| -0.572E 02 | 0.5084 | 0.630E 01 | 0.3877 | 0.548E 01 | 0.4304 | 0.177E 02 | 0.4192 | 0.353E 02 | 0.4057 | 0.605E 02 |
| -0.687E 02 | 0.4985 | 0.488E 01 | 0.3802 | 0.416E 01 | 0.4173 | 0.128E 02 | 0.4041 | 0.250E 02 | 0.3879 | 0.417E 02 |
| -0.824E 02 | 0.4863 | 0.361E 01 | 0.3694 | 0.282E 01 | 0.4015 | 0.873E 01 | 0.3856 | 0.165E 02 | 0.3660 | 0.262E 02 |
| -0.989E 02 | 0.4717 | 0.254E 01 | 0.3551 | 0.166E 01 | 0.3833 | 0.557E 01 | 0.3643 | 0.100E 02 | 0.3408 | 0.149E 02 |
| -0.119E 03 | 0.4550 | 0.170E 01 | 0.3382 | 0.831E 00 | 0.3633 | 0.333E 01 | 0.3412 | 0.564E 01 | 0.3140 | 0.774E 01 |
| -0.142E 03 | 0.4364 | 0.108E 01 | 0.3205 | 0.357E 00 | 0.3424 | 0.187E 01 | 0.3175 | 0.295E 01 | 0.2875 | 0.367E 01 |
| -0.171E 03 | 0.4167 | 0.658E 00 | 0.3039 | 0.134E 00 | 0.3215 | 0.999E 00 | 0.2946 | 0.145E 01 | 0.2627 | 0.162E 01 |
| -0.205E 03 | 0.3964 | 0.381E 00 | 0.2899 | 0.453E-01 | 0.3015 | 0.506E 00 | 0.2733 | 0.677E 00 | 0.2409 | 0.678E 00 |
| -0.246E 03 | 0.3763 | 0.213E 00 | 0.2758 | 0.142E-01 | 0.2829 | 0.249E 00 | 0.2543 | 0.304E 00 | 0.2224 | 0.272E 00 |
| -0.295E 03 | 0.3569 | 0.115E 00 | 0.2705 | 0.427E-02 | 0.2662 | 0.119E 00 | 0.2379 | 0.132E 00 | 0.2071 | 0.106E 00 |
| -0.354E 03 | 0.3386 | 0.602E-01 | 0.2644 | 0.125E-02 | 0.2515 | 0.552E-01 | 0.2239 | 0.564E-01 | 0.1948 | 0.403E-01 |
| -0.425E 03 | 0.3219 | 0.309E-01 | 0.2601 | 0.356E-03 | 0.2387 | 0.253E-01 | 0.2123 | 0.237E-01 | 0.1850 | 0.152E-01 |
| -0.510E 03 | 0.3067 | 0.156E-01 | 0.2571 | 0.101E-03 | 0.2277 | 0.115E-01 | 0.2027 | 0.983E-02 | 0.1772 | 0.566E-02 |
| -0.612E 03 | 0.2930 | 0.781E-02 | 0.2549 | 0.284E-04 | 0.2183 | 0.514E-02 | 0.1948 | 0.405E-02 | 0.1712 | 0.210E-02 |
| -0.735E 03 | 0.2810 | 0.386E-02 | 0.2534 | 0.796E-05 | 0.2105 | 0.230E-02 | 0.1884 | 0.166E-02 | 0.1664 | 0.776E-03 |
| -0.882E 03 | 0.2703 | 0.190E-02 | 0.2524 | 0.223E-05 | 0.2038 | 0.102E-02 | 0.1832 | 0.680E-03 | 0.1628 | 0.286E-03 |
| -0.106E 04 | 0.2610 | 0.928E-03 | 0.2517 | 0.624E-06 | 0.1982 | 0.452E-03 | 0.1769 | 0.277E-03 | 0.1599 | 0.105E-03 |
| -0.127E 04 | 0.2529 | 0.452E-03 | 0.2511 | 0.174E-06 | 0.1936 | 0.200E-03 | 0.1755 | 0.113E-03 | 0.1577 | 0.387E-04 |
| -0.152E 04 | 0.2458 | 0.219E-03 | 0.2508 | 0.487E-07 | 0.1896 | 0.884E-04 | 0.1727 | 0.459E-04 | 0.1559 | 0.142E-04 |
| -0.183E 04 | 0.2396 | 0.106E-03 | 0.2506 | 0.136E-07 | 0.1864 | 0.390E-04 | 0.1705 | 0.186E-04 | 0.1546 | 0.522E-05 |
| -0.219E 04 | 0.2343 | 0.515E-04 | 0.2504 | 0.100E-07 | 0.1837 | 0.172E-04 | 0.1687 | 0.758E-05 | 0.1536 | 0.191E-05 |
| -0.263E 04 | 0.2296 | 0.249E-04 | 0.2503 | 0.100E-07 | 0.1814 | 0.758E-05 | 0.1672 | 0.308E-05 | 0.1528 | 0.703E-06 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| -0.316E 04 | 0.2256 | 0.120E-04 | 0.2502 | 0.100E-07 | 0.1795 | 0.334E-05 | 0.1660 | 0.125E-05 | 0.1521 | 0.258E-06 |
| -0.379E 04 | 0.2222 | 0.581E-05 | 0.2501 | 0.100E-07 | 0.1779 | 0.147E-05 | 0.1651 | 0.508E-06 | 0.1517 | 0.946E-07 |
| -0.455E 04 | 0.2192 | 0.281E-05 | 0.2501 | 0.100E-07 | 0.1766 | 0.648E-06 | 0.1643 | 0.206E-06 | 0.1513 | 0.347E-07 |
| -0.546E 04 | 0.2166 | 0.135E-05 | 0.2501 | 0.100E-07 | 0.1755 | 0.285E-06 | 0.1637 | 0.837E-07 | 0.1510 | 0.127E-07 |
| -0.655E 04 | 0.2143 | 0.653E-06 | 0.2500 | 0.100E-07 | 0.1746 | 0.126E-06 | 0.1632 | 0.340E-07 | 0.1508 | 0.100E-07 |
| -0.766E 04 | 0.2124 | 0.315E-06 | 0.2500 | 0.100E-07 | 0.1738 | 0.553E-07 | 0.1628 | 0.138E-07 | 0.1506 | 0.100E-07 |
| -0.944E 04 | 0.2107 | 0.152E-06 | 0.2500 | 0.100E-07 | 0.1732 | 0.244E-07 | 0.1625 | 0.100E-07 | 0.1505 | 0.100E-07 |

SOIL-HYDRAULIC PROPERTIES (MOISTURE CONTENT AND HYDRAULIC CONDUCTIVITY)

| PRESSURE | SOIL 6 | | SOIL 7 | | SOIL 8 | | SOIL 9 | | SOIL 10 | |
|---|---|---|---|---|---|---|---|---|---|---|
| HEAD | WC | COND | WC | COND | WC | COND | WC | COND | WC | COND |
| -0.600E 00 | 0.4400 | 0.270E 03 | 0.4311 | 0.328E 03 | 0.4244 | 0.371E 03 | 0.4200 | 0.400E 03 | | |
| -0.720E 00 | 0.4400 | 0.270E 03 | 0.4311 | 0.328E 03 | 0.4244 | 0.371E 03 | 0.4200 | 0.400E 03 | | |
| -0.864E 00 | 0.4400 | 0.270E 03 | 0.4311 | 0.328E 03 | 0.4244 | 0.371E 03 | 0.4200 | 0.400E 03 | | |
| -0.104E 01 | 0.4400 | 0.270E 03 | 0.4311 | 0.328E 03 | 0.4244 | 0.371E 03 | 0.4200 | 0.400E 03 | | |
| -0.124E 01 | 0.4400 | 0.269E 03 | 0.4311 | 0.327E 03 | 0.4244 | 0.371E 03 | 0.4200 | 0.400E 03 | | |
| -0.149E 01 | 0.4400 | 0.269E 03 | 0.4311 | 0.327E 03 | 0.4244 | 0.371E 03 | 0.4200 | 0.400E 03 | | |
| -0.179E 01 | 0.4400 | 0.269E 03 | 0.4311 | 0.327E 03 | 0.4244 | 0.371E 03 | 0.4200 | 0.400E 03 | | |
| -0.215E 01 | 0.4400 | 0.269E 03 | 0.4311 | 0.327E 03 | 0.4244 | 0.370E 03 | 0.4200 | 0.399E 03 | | |
| -0.258E 01 | 0.4400 | 0.268E 03 | 0.4311 | 0.327E 03 | 0.4244 | 0.370E 03 | 0.4200 | 0.399E 03 | | |
| -0.310E 01 | 0.4400 | 0.268E 03 | 0.4311 | 0.326E 03 | 0.4244 | 0.370E 03 | 0.4200 | 0.399E 03 | | |
| -0.372E 01 | 0.4400 | 0.267E 03 | 0.4311 | 0.325E 03 | 0.4244 | 0.369E 03 | 0.4200 | 0.398E 03 | | |
| -0.446E 01 | 0.4399 | 0.266E 03 | 0.4310 | 0.324E 03 | 0.4244 | 0.368E 03 | 0.4200 | 0.398E 03 | | |
| -0.535E 01 | 0.4399 | 0.264E 03 | 0.4310 | 0.323E 03 | 0.4243 | 0.367E 03 | 0.4199 | 0.397E 03 | | |
| -0.642E 01 | 0.4398 | 0.262E 03 | 0.4310 | 0.321E 03 | 0.4243 | 0.366E 03 | 0.4199 | 0.395E 03 | | |
| -0.770E 01 | 0.4397 | 0.259E 03 | 0.4309 | 0.319E 03 | 0.4242 | 0.363E 03 | 0.4198 | 0.393E 03 | | |
| -0.924E 01 | 0.4395 | 0.256E 03 | 0.4307 | 0.315E 03 | 0.4241 | 0.360E 03 | 0.4197 | 0.390E 03 | | |
| -0.111E 02 | 0.4392 | 0.251E 03 | 0.4305 | 0.311E 03 | 0.4239 | 0.356E 03 | 0.4195 | 0.386E 03 | | |
| -0.133E 02 | 0.4387 | 0.244E 03 | 0.4300 | 0.304E 03 | 0.4235 | 0.349E 03 | 0.4192 | 0.379E 03 | | |
| -0.160E 02 | 0.4379 | 0.236E 03 | 0.4294 | 0.295E 03 | 0.4229 | 0.340E 03 | 0.4186 | 0.370E 03 | | |
| -0.192E 02 | 0.4367 | 0.225E 03 | 0.4282 | 0.283E 03 | 0.4218 | 0.328E 03 | 0.4176 | 0.358E 03 | | |
| -0.230E 02 | 0.4347 | 0.211E 03 | 0.4264 | 0.267E 03 | 0.4200 | 0.311E 03 | 0.4159 | 0.340E 03 | | |
| -0.276E 02 | 0.4316 | 0.193E 03 | 0.4233 | 0.246E 03 | 0.4171 | 0.287E 03 | 0.4129 | 0.315E 03 | | |
| -0.331E 02 | 0.4268 | 0.171E 03 | 0.4185 | 0.219E 03 | 0.4122 | 0.257E 03 | 0.4061 | 0.282E 03 | | |
| -0.397E 02 | 0.4193 | 0.145E 03 | 0.4109 | 0.186E 03 | 0.4044 | 0.218E 03 | 0.4001 | 0.240E 03 | | |
| -0.477E 02 | 0.4087 | 0.115E 03 | 0.3993 | 0.147E 03 | 0.3922 | 0.172E 03 | 0.3875 | 0.189E 03 | | |
| -0.572E 02 | 0.3935 | 0.850E 02 | 0.3826 | 0.107E 03 | 0.3743 | 0.124E 03 | 0.3668 | 0.135E 03 | | |
| -0.687E 02 | 0.3732 | 0.569E 02 | 0.3599 | 0.698E 02 | 0.3499 | 0.789E 02 | 0.3430 | 0.845E 02 | | |
| -0.824E 02 | 0.3481 | 0.342E 02 | 0.3318 | 0.400E 02 | 0.3194 | 0.436E 02 | 0.3111 | 0.454E 02 | | |
| -0.989E 02 | 0.3195 | 0.182E 02 | 0.3001 | 0.200E 02 | 0.2856 | 0.207E 02 | 0.2758 | 0.208E 02 | | |
| -0.119E 03 | 0.2897 | 0.868E 01 | 0.2680 | 0.880E 01 | 0.2520 | 0.855E 01 | 0.2413 | 0.822E 01 | | |
| -0.142E 03 | 0.2611 | 0.375E 01 | 0.2383 | 0.348E 01 | 0.2217 | 0.315E 01 | 0.2109 | 0.289E 01 | | |
| -0.171E 03 | 0.2356 | 0.150E 01 | 0.2128 | 0.126E 01 | 0.1967 | 0.107E 01 | 0.1863 | 0.935E 00 | | |
| -0.205E 03 | 0.2142 | 0.564E 00 | 0.1922 | 0.434E 00 | 0.1770 | 0.342E 00 | 0.1674 | 0.285E 00 | | |
| -0.246E 03 | 0.1968 | 0.204E 00 | 0.1763 | 0.143E 00 | 0.1623 | 0.105E 00 | 0.1535 | 0.841E-01 | | |
| -0.295E 03 | 0.1831 | 0.718E-01 | 0.1642 | 0.461E-01 | 0.1515 | 0.318E-01 | 0.1435 | 0.242E-01 | | |
| -0.354E 03 | 0.1725 | 0.248E-01 | 0.1552 | 0.146E-01 | 0.1436 | 0.943E-02 | 0.1364 | 0.689E-02 | | |
| -0.425E 03 | 0.1644 | 0.847E-02 | 0.1486 | 0.457E-02 | 0.1380 | 0.277E-02 | 0.1315 | 0.194E-02 | | |
| -0.510E 03 | 0.1583 | 0.287E-02 | 0.1438 | 0.143E-02 | 0.1340 | 0.811E-03 | 0.1280 | 0.545E-03 | | |
| -0.612E 03 | 0.1537 | 0.970E-03 | 0.1403 | 0.442E-03 | 0.1312 | 0.237E-03 | 0.1255 | 0.153E-03 | | |
| -0.735E 03 | 0.1503 | 0.326E-03 | 0.1378 | 0.137E-03 | 0.1292 | 0.690E-04 | 0.1239 | 0.427E-04 | | |
| -0.862E 03 | 0.1477 | 0.109E-03 | 0.1359 | 0.424E-04 | 0.1278 | 0.201E-04 | 0.1227 | 0.119E-04 | | |
| -0.106E 04 | 0.1457 | 0.368E-04 | 0.1346 | 0.131E-04 | 0.1268 | 0.535E-05 | 0.1219 | 0.333E-05 | | |
| -0.127E 04 | 0.1443 | 0.123E-04 | 0.1336 | 0.405E-05 | 0.1261 | 0.170E-05 | 0.1213 | 0.931E-06 | | |
| -0.152E 04 | 0.1432 | 0.413E-05 | 0.1329 | 0.125E-05 | 0.1256 | 0.495E-06 | 0.1209 | 0.260E-06 | | |
| -0.183E 04 | 0.1424 | 0.138E-05 | 0.1324 | 0.387E-06 | 0.1252 | 0.144E-06 | 0.1206 | 0.722E-07 | | |
| -0.219E 04 | 0.1418 | 0.464E-06 | 0.1321 | 0.120E-06 | 0.1250 | 0.418E-07 | 0.1204 | 0.203E-07 | | |
| -0.263E 04 | 0.1413 | 0.155E-06 | 0.1318 | 0.369E-07 | 0.1248 | 0.121E-07 | 0.1203 | 0.100E-07 | | |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| -0.316E 04 | 0.1410 | 0.520E-07 | 0.1316 | 0.114E-07 | 0.1247 | 0.100E-07 | 0.1202 | 0.100E-07 |
| -0.379E 04 | 0.1407 | 0.174E-07 | 0.1315 | 0.100E-07 | 0.1246 | 0.100E-07 | 0.1201 | 0.100E-07 |
| -0.455E 04 | 0.1406 | 0.100E-07 | 0.1314 | 0.100E-07 | 0.1245 | 0.100E-07 | 0.1201 | 0.100E-07 |
| -0.546E 04 | 0.1404 | 0.100E-07 | 0.1313 | 0.100E-07 | 0.1245 | 0.100E-07 | 0.1201 | 0.100E-07 |
| -0.655E 04 | 0.1403 | 0.100E-07 | 0.1312 | 0.100E-07 | 0.1245 | 0.100E-07 | 0.1200 | 0.100E-07 |
| -0.786E 04 | 0.1402 | 0.100E-07 | 0.1312 | 0.100E-07 | 0.1245 | 0.100E-07 | 0.1200 | 0.100E-07 |
| -0.944E 04 | 0.1402 | 0.100E-07 | 0.1312 | 0.100E-07 | 0.1244 | 0.100E-07 | 0.1200 | 0.100E-07 |

PHYSICAL AND CHEMICAL SOIL PROPERTIES

| MATERIAL INDEX | BULK DENSITY | DIFFUSION COEFFICIENT | DISPERSIVITY | ADSORPTION CONSTANT | DECAY COEFFICIENTS 0-TH(LIQUID) | 1-ST(LIQUID) | 1-ST(SOLID) |
|---|---|---|---|---|---|---|---|
| 1 | 1.2200 | 0.6700 | 3.5000 | 0.5000 | 1.0000 | -0.1000 | -0.0500 |
| 2 | 1.6000 | 0.6700 | 2.0000 | 0.2000 | 0.0 | 0.0 | 0.0 |
| 3 | 1.4100 | 0.6700 | 3.0000 | 0.3000 | 0.8000 | -0.0920 | -0.0460 |
| 4 | 1.4300 | 0.6700 | 2.7300 | 0.2470 | 0.4500 | -0.0720 | -0.0360 |
| 5 | 1.4600 | 0.6700 | 2.4000 | 0.1800 | 0.1500 | -0.0480 | -0.0240 |
| 6 | 1.4900 | 0.6700 | 2.1000 | 0.1200 | 0.0300 | -0.0240 | -0.0120 |
| 7 | 1.5100 | 0.6700 | 1.8300 | 0.0670 | 0.0020 | -0.0090 | -0.0045 |
| 8 | 1.5300 | 0.6700 | 1.6300 | 0.0270 | 0.0 | -0.0020 | -0.0010 |
| 9 | 1.5400 | 0.6700 | 1.5000 | 0.0 | 0.0 | 0.0 | 0.0 |

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ELAPSED TIME — DAYS 0.0500 — HOURS 1.2002 — MINUTES 0.7201E 02 — DELT 0.4657E-02 — ISTEP 27 — NIT 2 — NITT 70

| NODE | DEPTH | PRESSURE HEAD FUNCTN | PRESSURE HEAD GRAD | PRESSURE HEAD F(1/3) | PRESSURE HEAD F(2/3) | MOISTURE CONTENT FUNCTN | MOISTURE CONTENT F(1/3) | MOISTURE CONTENT F(2/3) | CONCENTRATION FUNCTN | CONCENTRATION GRAD | CONCENTRATION F(1/3) | CONCENTRATION F(2/3) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.0 | -78.30 | -5.32 | -90.60 | -106.36 | 0.4900 | 0.4790 | 0.4653 | 12.867 | -2.030 | 9.121 | 6.403 |
| 2 | 6.0 | -125.83 | -10.70 | -170.38 | -237.06 | 0.4492 | 0.4170 | 0.3804 | 5.309 | -0.042 | 5.253 | 5.265 |
| 3 | 14.0 | -311.00 | -27.23 | -350.61 | -350.00 | 0.3516 | 0.3397 | 0.3398 | 5.276 | -0.005 | 5.269 | 5.270 |
| 4 | 22.0 | -347.83 | -3.73 | -351.26 | -350.07 | 0.3405 | 0.3169 | 0.2751 | 5.270 | -0.001 | 5.200 | 5.012 |
| 5 | 28.0 | -349.36 | -0.61 | -350.16 | -349.94 | 0.2526 | 0.2432 | 0.2340 | 4.732 | -0.159 | 4.319 | 3.914 |
| 6 | 36.0 | -349.69 | -0.03 | -349.67 | -349.54 | 0.2249 | 0.2151 | 0.2054 | 3.501 | -0.159 | 2.943 | 2.375 |
| 7 | 46.0 | -349.47 | -0.01 | -349.46 | -349.42 | 0.1956 | 0.1882 | 0.1807 | 1.870 | -0.135 | 1.492 | 1.152 |
| 8 | 55.0 | -349.44 | -0.02 | -349.47 | -349.48 | 0.1732 | 0.1674 | 0.1616 | 0.825 | -0.110 | 0.557 | 0.339 |
| 9 | 63.0 | -349.52 | -0.03 | -349.56 | -349.57 | 0.1558 | 0.1519 | 0.1480 | 0.168 | -0.055 | 0.076 | 0.019 |
| 10 | 69.0 | -349.59 | -0.01 | -349.59 | -349.54 | 0.1441 | 0.1417 | 0.1393 | 0.000 | -0.000 | -0.000 | -0.000 |
| 11 | 73.0 | -349.41 | 0.14 | -349.19 | -349.13 | 0.1369 | 0.1701 | 0.2317 | 0.000 | -0.000 | -0.000 | -0.000 |
| 12 | 77.0 | -349.56 | -0.59 | -349.98 | -349.98 | 0.2648 | 0.2648 | 0.2648 | 0.000 | -0.000 | -0.000 | 0.000 |
| 13 | 81.0 | -349.98 | -0.12 | -349.99 | -349.98 | 0.2648 | 0.2648 | 0.2648 | 0.000 | -0.000 | -0.000 | 0.000 |
| 14 | 85.0 | -350.40 | -0.57 | -350.80 | -350.69 | 0.2648 | 0.2316 | 0.1700 | 0.000 | -0.000 | -0.000 | 0.000 |
| 15 | 89.0 | -350.38 | 0.23 | -350.15 | -350.04 | 0.1368 | 0.1368 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 16 | 93.0 | -350.01 | 0.02 | -349.99 | -349.98 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 17 | 97.0 | -349.98 | -0.00 | -349.98 | -349.98 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 18 | 102.0 | -349.98 | -0.00 | -349.98 | -349.98 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 19 | 107.0 | -349.98 | -0.00 | -349.98 | -349.98 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 20 | 112.0 | -349.98 | -0.00 | -349.98 | -349.98 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 21 | 118.0 | -349.98 | -0.00 | -349.98 | -349.98 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 22 | 125.0 | -349.98 | -0.00 | -349.98 | -349.98 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 23 | 133.0 | -349.98 | -0.00 | -349.98 | -349.98 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 24 | 141.0 | -349.98 | -0.00 | -349.98 | -349.98 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 25 | 150.0 | -349.98 | -0.00 | -349.98 | -349.98 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 26 | 160.0 | -349.98 | -0.00 | -349.99 | -349.99 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 27 | 170.0 | -349.99 | 0.0 | | | 0.1369 | | | 0.000 | 0.0 | | |

INFILTRATION RATE................ 25.000
DRAINAGE RATE.................... 0.008
TOTAL MOISTURE IN PROFILE........ 33.793

MOISTURE ADDED TO PROFILE............ 1.250
MOISTURE INCREASE IN PROFILE......... 1.239
TOTAL MASS IN SOLUTION............... 80.126

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ELAPSED TIME — DAYS 0.1013 — HOURS 2.4313 — MINUTES 0.1459E 03 — DELT 0.9095E-02 — ISTEP 35 — NIT 3 — NITT 92

| NODE | DEPTH | PRESSURE HEAD FUNCTN | PRESSURE HEAD GRAD | PRESSURE HEAD F(1/3) | PRESSURE HEAD F(2/3) | MOISTURE CONTENT FUNCTN | MOISTURE CONTENT F(1/3) | MOISTURE CONTENT F(2/3) | CONCENTRATION FUNCTN | CONCENTRATION GRAD | CONCENTRATION F(1/3) | CONCENTRATION F(2/3) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.0 | -54.64 | -2.73 | -60.49 | -67.30 | 0.5106 | 0.5057 | 0.4997 | 14.733 | -1.499 | 11.687 | 8.691 |
| 2 | 6.0 | -75.37 | -4.40 | -88.72 | -106.58 | 0.4926 | 0.4807 | 0.4651 | 6.861 | -0.737 | 5.592 | 5.284 |
| 3 | 14.0 | -130.87 | -10.55 | -169.07 | -224.79 | 0.4452 | 0.4179 | 0.3862 | 5.287 | -0.021 | 5.262 | 5.273 |
| 4 | 22.0 | -294.11 | -28.06 | -332.28 | -344.93 | 0.3573 | 0.3220 | 0.2763 | 5.274 | -0.007 | 5.199 | 5.013 |
| 5 | 28.0 | -347.63 | -1.46 | -349.81 | -349.78 | 0.2529 | 0.2433 | 0.2341 | 4.734 | -0.160 | 4.319 | 3.915 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6 | 36.0 | -349.42 | -0.04 | -349.37 | -349.15 | 0.2249 | 0.2152 | 0.2055 | 3.502 | -0.159 | 2.944 | 2.377 |
| 7 | 46.0 | -348.99 | 0.01 | -348.93 | -348.88 | 0.1957 | 0.1863 | 0.1808 | 1.871 | -0.135 | 1.493 | 1.153 |
| 8 | 55.0 | -348.89 | -0.02 | -348.93 | -348.96 | 0.1733 | 0.1675 | 0.1617 | 0.825 | -0.110 | 0.557 | 0.339 |
| 9 | 63.0 | -349.03 | -0.04 | -349.09 | -349.12 | 0.1559 | 0.1520 | 0.1481 | 0.169 | -0.055 | 0.076 | 0.020 |
| 10 | 69.0 | -349.13 | 0.00 | -349.10 | -349.01 | 0.1442 | 0.1418 | 0.1394 | 0.000 | -0.000 | -0.000 | -0.000 |
| 11 | 73.0 | -348.83 | 0.18 | -348.54 | -348.48 | 0.1370 | 0.1702 | 0.2318 | 0.000 | -0.000 | -0.000 | -0.000 |
| 12 | 77.0 | -349.11 | -0.64 | -349.80 | -349.94 | 0.2649 | 0.2648 | 0.2648 | 0.000 | -0.000 | -0.000 | 0.000 |
| 13 | 81.0 | -349.97 | -0.10 | -350.01 | -350.13 | 0.2648 | 0.2648 | 0.2648 | 0.000 | -0.000 | -0.000 | 0.000 |
| 14 | 85.0 | -350.78 | -0.78 | -351.33 | -351.18 | 0.2647 | 0.2315 | 0.1699 | 0.000 | -0.000 | -0.000 | 0.000 |
| 15 | 89.0 | -350.76 | 0.30 | -350.43 | -350.22 | 0.1368 | 0.1368 | 0.1368 | 0.000 | -0.000 | -0.000 | 0.000 |
| 16 | 93.0 | -350.10 | 0.07 | -350.03 | -350.00 | 0.1363 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 17 | 97.0 | -349.98 | 0.01 | -349.98 | -349.97 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 18 | 102.0 | -349.97 | -0.00 | -349.97 | -349.97 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 19 | 107.0 | -349.97 | -0.00 | -349.97 | -349.97 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 20 | 112.0 | -349.97 | -0.00 | -349.97 | -349.97 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 21 | 118.0 | -349.97 | -0.00 | -349.97 | -349.97 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 22 | 125.0 | -349.97 | -0.00 | -349.97 | -349.97 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 23 | 133.0 | -349.97 | -0.00 | -349.98 | -349.97 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 24 | 141.0 | -349.97 | -0.00 | -349.97 | -349.97 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 25 | 150.0 | -349.98 | -0.00 | -349.98 | -349.98 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 26 | 160.0 | -349.98 | -0.00 | -349.98 | -349.98 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | -0.000 |
| 27 | 170.0 | -349.98 | 0.0 | | | 0.1369 | | | 0.000 | 0.0 | | |

INFILTRATION RATE................ 25.000
DRAINAGE RATE.................... 0.008
TOTAL MOISTURE IN PROFILE........ 35.071

MOISTURE ADDED TO PROFILE............ 2.532
MOISTURE INCREASE IN PROFILE......... 2.517
TOTAL MASS IN SOLUTION............... 95.581

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| ELAPSED TIME | DAYS | HOURS | MINUTES | DELT | ISTEP | NIT | NITT |
|---|---|---|---|---|---|---|---|
| | 0.1460 | 3.5227 | 0.2114E 03 | 0.9095E-02 | 40 | 3 | 107 |

| | | ---------PRESSURE HEAD-------- | | | | ---MOISTURE CONTENT--- | | | --------CONCENTRATION--------- | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| NODE | DEPTH | FUNCTN | GRAD | F(1/3) | F(2/3) | FUNCTN | F(1/3) | F(2/3) | FUNCTN | GRAD | F(1/3) | F(2/3) |
| 1 | 0.0 | -42.69 | -1.85 | -46.59 | -50.98 | 0.5202 | 0.5172 | 0.5136 | 15.769 | -1.204 | 13.219 | 10.638 |
| 2 | 6.0 | -56.00 | -2.68 | -63.74 | -73.20 | 0.5095 | 0.5028 | 0.4945 | 8.405 | -0.967 | 6.473 | 5.575 |
| 3 | 14.0 | -85.25 | -5.11 | -100.34 | -121.00 | 0.4838 | 0.4705 | 0.4531 | 5.318 | -0.025 | 5.277 | 5.269 |
| 4 | 22.0 | -151.17 | -13.59 | -180.48 | -215.55 | 0.4301 | 0.3859 | 0.3208 | 5.260 | -0.007 | 5.202 | 5.044 |
| 5 | 28.0 | -258.68 | -23.95 | -310.56 | -339.78 | 0.2782 | 0.2526 | 0.2362 | 4.771 | -0.168 | 4.336 | 3.918 |
| 6 | 36.0 | -348.37 | 0.40 | -348.13 | -346.87 | 0.2252 | 0.2154 | 0.2055 | 3.502 | -0.158 | 2.946 | 2.378 |
| 7 | 46.0 | -348.78 | 0.33 | -348.32 | -348.45 | 0.1958 | 0.1884 | 0.1809 | 1.872 | -0.135 | 1.494 | 1.154 |
| 8 | 55.0 | -348.48 | 0.08 | -348.41 | -348.52 | 0.1734 | 0.1676 | 0.1618 | 0.826 | -0.110 | 0.558 | 0.340 |
| 9 | 63.0 | -348.60 | -0.01 | -348.65 | -348.70 | 0.1559 | 0.1520 | 0.1481 | 0.169 | -0.055 | 0.077 | 0.020 |
| 10 | 69.0 | -348.70 | 0.03 | -348.64 | -348.53 | 0.1442 | 0.1418 | 0.1394 | 0.000 | -0.001 | -0.000 | -0.000 |
| 11 | 73.0 | -348.33 | 0.19 | -348.03 | -347.99 | 0.1370 | 0.1702 | 0.2318 | 0.000 | -0.000 | -0.000 | -0.000 |
| 12 | 77.0 | -348.73 | -0.97 | -349.59 | -349.87 | 0.2649 | 0.2648 | 0.2648 | 0.000 | -0.000 | -0.000 | 0.000 |
| 13 | 81.0 | -349.97 | -0.11 | -350.06 | -350.31 | 0.2648 | 0.2648 | 0.2648 | 0.000 | -0.000 | -0.000 | 0.000 |
| 14 | 85.0 | -351.08 | -0.88 | -351.70 | -351.53 | 0.2647 | 0.2315 | 0.1699 | 0.000 | -0.000 | -0.000 | 0.000 |
| 15 | 89.0 | -351.06 | 0.34 | -350.68 | -350.40 | 0.1368 | 0.1368 | 0.1368 | 0.000 | -0.000 | -0.000 | 0.000 |
| 16 | 93.0 | -350.22 | 0.11 | -350.11 | -350.04 | 0.1368 | 0.1368 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 17 | 97.0 | -350.01 | 0.02 | -349.98 | -349.97 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 18 | 102.0 | -349.97 | 0.00 | -349.97 | -349.97 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 19 | 107.0 | -349.97 | -0.00 | -349.97 | -349.96 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |

| NODE | DEPTH | FUNCTN | GRAD | F(1/3) | F(2/3) | FUNCTN | F(1/3) | F(2/3) | FUNCTN | GRAD | F(1/3) | F(2/3) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | 112.0 | -349.96 | -0.00 | -349.97 | -349.97 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 21 | 118.0 | -349.97 | -0.00 | -349.97 | -349.97 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 22 | 125.0 | -349.97 | -0.00 | -349.97 | -349.97 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 23 | 133.0 | -349.97 | -0.00 | -349.97 | -349.97 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 24 | 141.0 | -349.97 | -0.00 | -349.97 | -349.97 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 25 | 150.0 | -349.97 | -0.00 | -349.97 | -349.98 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 26 | 160.0 | -349.98 | -0.00 | -349.98 | -349.98 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | -0.000 |
| 27 | 170.0 | -349.98 | 0.0 | | | 0.1369 | | | 0.000 | 0.0 | | |

INFILTRATION RATE................ 25.000
DRAINAGE RATE.................... 0.008
TOTAL MOISTURE IN PROFILE........ 36.209

MOISTURE ADDED TO PROFILE............ 3.668
MOISTURE INCREASE IN PROFILE......... 3.655
TOTAL MASS IN SOLUTION............... 109.323

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| ELAPSED TIME | DAYS | HOURS | MINUTES | DELT | ISTEP | NIT | NITT |
|---|---|---|---|---|---|---|---|
| | 0.2013 | 4.8324 | 0.2899E 03 | 0.9095E-02 | 46 | 3 | 125 |

| | | ---------PRESSURE HEAD-------- | | | | ---MOISTURE CONTENT--- | | | --------CONCENTRATION--------- | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| NODE | DEPTH | FUNCTN | GRAD | F(1/3) | F(2/3) | FUNCTN | F(1/3) | F(2/3) | FUNCTN | GRAD | F(1/3) | F(2/3) |
| 1 | 0.0 | -33.73 | -1.32 | -36.50 | -39.56 | 0.5266 | 0.5247 | 0.5225 | 16.628 | -0.960 | 14.539 | 12.278 |
| 2 | 6.0 | -42.99 | -1.82 | -46.15 | -54.26 | 0.5200 | 0.5159 | 0.5109 | 10.096 | -1.030 | 7.791 | 6.317 |
| 3 | 14.0 | -61.73 | -3.11 | -70.65 | -82.14 | 0.5046 | 0.4968 | 0.4865 | 5.598 | -0.138 | 5.355 | 5.280 |
| 4 | 22.0 | -98.15 | -7.09 | -111.85 | -125.08 | 0.4724 | 0.4371 | 0.3813 | 5.249 | -0.018 | 5.193 | 5.078 |
| 5 | 28.0 | -138.60 | -6.96 | -159.09 | -185.34 | 0.3455 | 0.3209 | 0.2941 | 4.876 | -0.127 | 4.488 | 4.035 |
| 6 | 36.0 | -220.15 | -15.01 | -277.68 | -330.41 | 0.2656 | 0.2327 | 0.2091 | 3.574 | -0.168 | 2.982 | 2.364 |
| 7 | 46.0 | -348.67 | 2.66 | -345.68 | -347.95 | 0.1958 | 0.1888 | 0.1809 | 1.673 | -0.132 | 1.498 | 1.154 |
| 8 | 55.0 | -348.46 | 0.90 | -347.45 | -347.98 | 0.1734 | 0.1677 | 0.1618 | 0.826 | -0.106 | 0.560 | 0.340 |
| 9 | 63.0 | -348.19 | 0.22 | -348.03 | -348.16 | 0.1560 | 0.1521 | 0.1482 | 0.170 | -0.054 | 0.078 | 0.021 |
| 10 | 69.0 | -348.18 | 0.08 | -348.08 | -347.96 | 0.1443 | 0.1419 | 0.1395 | 0.001 | -0.000 | 0.000 | -0.000 |
| 11 | 73.0 | -347.77 | 0.20 | -347.46 | -347.44 | 0.1371 | 0.1703 | 0.2319 | -0.000 | 0.000 | 0.000 | 0.000 |
| 12 | 77.0 | -348.28 | -1.10 | -349.32 | -349.75 | 0.2649 | 0.2649 | 0.2643 | -0.000 | 0.000 | 0.000 | -0.000 |
| 13 | 81.0 | -349.96 | -0.16 | -350.15 | -350.52 | 0.2648 | 0.2648 | 0.2648 | -0.000 | 0.000 | 0.000 | -0.000 |
| 14 | 85.0 | -351.41 | -0.95 | -352.08 | -351.89 | 0.2647 | 0.2315 | 0.1699 | -0.000 | 0.000 | 0.000 | -0.000 |
| 15 | 89.0 | -351.39 | 0.36 | -350.96 | -350.63 | 0.1367 | 0.1368 | 0.1368 | -0.000 | 0.000 | 0.000 | -0.000 |
| 16 | 93.0 | -350.40 | 0.14 | -350.24 | -350.13 | 0.1368 | 0.1368 | 0.1368 | -0.000 | 0.000 | 0.000 | -0.000 |
| 17 | 97.0 | -350.06 | 0.04 | -350.01 | -349.98 | 0.1368 | 0.1369 | 0.1369 | -0.000 | 0.000 | 0.000 | -0.000 |
| 18 | 102.0 | -349.97 | 0.00 | -349.97 | -349.96 | 0.1369 | 0.1369 | 0.1369 | -0.000 | 0.000 | 0.000 | -0.000 |
| 19 | 107.0 | -349.96 | -0.00 | -349.96 | -349.96 | 0.1369 | 0.1369 | 0.1369 | -0.000 | 0.000 | 0.000 | -0.000 |
| 20 | 112.0 | -349.96 | -0.00 | -349.96 | -349.96 | 0.1369 | 0.1369 | 0.1369 | -0.000 | 0.000 | 0.000 | -0.000 |
| 21 | 118.0 | -349.96 | -0.00 | -349.96 | -349.96 | 0.1369 | 0.1369 | 0.1369 | -0.000 | 0.000 | 0.000 | -0.000 |
| 22 | 125.0 | -349.96 | -0.00 | -349.96 | -349.96 | 0.1369 | 0.1369 | 0.1369 | -0.000 | 0.000 | 0.000 | -0.000 |
| 23 | 133.0 | -349.96 | -0.00 | -349.96 | -349.96 | 0.1369 | 0.1369 | 0.1369 | -0.000 | 0.000 | 0.000 | -0.000 |
| 24 | 141.0 | -349.96 | -0.00 | -349.96 | -349.96 | 0.1369 | 0.1369 | 0.1369 | -0.000 | 0.000 | 0.000 | -0.000 |
| 25 | 150.0 | -349.97 | -0.00 | -349.97 | -349.97 | 0.1369 | 0.1369 | 0.1369 | -0.000 | 0.000 | 0.000 | -0.000 |
| 26 | 160.0 | -349.98 | -0.00 | -349.98 | -349.98 | 0.1369 | 0.1369 | 0.1369 | -0.000 | 0.000 | 0.000 | 0.000 |
| 27 | 170.0 | -349.98 | 0.0 | | | 0.1369 | | | -0.000 | 0.0 | | |

INFILTRATION RATE................ 25.000
DRAINAGE RATE.................... 0.008
TOTAL MOISTURE IN PROFILE........ 37.576

MOISTURE ADDED TO PROFILE............ 5.032
MOISTURE INCREASE IN PROFILE......... 5.022
TOTAL MASS IN SOLUTION............... 125.386

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| ELAPSED TIME | DAYS | HOURS | MINUTES | DELT | ISTEP | NIT | NITT |
|---|---|---|---|---|---|---|---|
| | 0.2514 | 6.0329 | 0.3620E 03 | 0.1137E-01 | 51 | 4 | 141 |

| | | ---------PRESSURE HEAD-------- | | | | ---MOISTURE CONTENT--- | | | --------CONCENTRATION--------- | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| NODE | DEPTH | FUNCTN | GRAD | F(1/3) | F(2/3) | FUNCTN | F(1/3) | F(2/3) | FUNCTN | GRAD | F(1/3) | F(2/3) |
| 1 | 0.0 | -29.68 | -1.11 | -32.02 | -34.59 | 0.5292 | 0.5277 | 0.5260 | 17.197 | -0.798 | 15.437 | 13.457 |
| 2 | 6.0 | -37.47 | -1.52 | -41.77 | -46.81 | 0.5240 | 0.5209 | 0.5170 | 11.429 | -0.998 | 9.031 | 7.227 |
| 3 | 14.0 | -52.92 | -2.54 | -60.16 | -69.31 | 0.5121 | 0.5059 | 0.4979 | 6.115 | -0.275 | 5.583 | 5.351 |
| 4 | 22.0 | -81.84 | -5.51 | -91.90 | -100.48 | 0.4868 | 0.4553 | 0.4046 | 5.263 | -0.026 | 5.201 | 5.105 |
| 5 | 28.0 | -108.22 | -3.77 | -118.45 | -129.31 | 0.3736 | 0.3562 | 0.3379 | 4.957 | -0.091 | 4.652 | 4.250 |
| 6 | 36.0 | -141.16 | -4.68 | -158.48 | -182.89 | 0.3187 | 0.2935 | 0.2650 | 3.795 | -0.175 | 3.174 | 2.533 |
| 7 | 46.0 | -219.92 | -13.55 | -273.65 | -328.80 | 0.2334 | 0.2049 | 0.1842 | 1.951 | -0.158 | 1.523 | 1.160 |
| 8 | 55.0 | -348.32 | 3.55 | -344.83 | -347.61 | 0.1734 | 0.1681 | 0.1619 | 0.828 | -0.110 | 0.560 | 0.341 |
| 9 | 63.0 | -348.16 | 1.28 | -347.09 | -347.66 | 0.1560 | 0.1522 | 0.1482 | 0.170 | -0.055 | 0.078 | 0.021 |
| 10 | 69.0 | -347.80 | 0.38 | -347.51 | -347.43 | 0.1443 | 0.1419 | 0.1395 | 0.301 | -0.001 | 0.000 | -0.000 |
| 11 | 73.0 | -347.27 | 0.23 | -346.94 | -346.96 | 0.1371 | 0.1703 | 0.2319 | -0.000 | 0.000 | 0.000 | 0.000 |
| 12 | 77.0 | -347.89 | -1.18 | -349.05 | -349.62 | 0.2650 | 0.2649 | 0.2648 | -0.000 | 0.000 | 0.000 | -0.000 |
| 13 | 81.0 | -349.94 | -0.23 | -350.23 | -350.71 | 0.2648 | 0.2648 | 0.2647 | -0.000 | 0.000 | 0.000 | -0.000 |
| 14 | 85.0 | -351.68 | -1.00 | -352.38 | -352.19 | 0.2647 | 0.2314 | 0.1698 | -0.000 | 0.000 | 0.000 | -0.000 |
| 15 | 89.0 | -351.66 | 0.38 | -351.21 | -350.84 | 0.1367 | 0.1367 | 0.1368 | -0.000 | 0.000 | 0.000 | -0.000 |
| 16 | 93.0 | -350.57 | 0.17 | -350.37 | -350.22 | 0.1368 | 0.1368 | 0.1368 | -0.000 | 0.000 | 0.000 | -0.000 |
| 17 | 97.0 | -350.12 | 0.06 | -350.05 | -350.00 | 0.1368 | 0.1369 | 0.1369 | -0.000 | 0.000 | 0.000 | -0.000 |
| 18 | 102.0 | -349.98 | 0.01 | -349.97 | -349.96 | 0.1369 | 0.1369 | 0.1369 | -0.000 | 0.000 | 0.000 | -0.000 |
| 19 | 107.0 | -349.96 | 0.00 | -349.96 | -349.95 | 0.1369 | 0.1369 | 0.1369 | -0.000 | 0.000 | 0.000 | -0.000 |
| 20 | 112.0 | -349.95 | -0.00 | -349.95 | -349.95 | 0.1369 | 0.1369 | 0.1369 | -0.000 | 0.000 | 0.000 | -0.000 |
| 21 | 118.0 | -349.96 | -0.00 | -349.96 | -349.96 | 0.1369 | 0.1369 | 0.1369 | -0.000 | 0.000 | 0.000 | -0.000 |
| 22 | 125.0 | -349.96 | -0.00 | -349.96 | -349.96 | 0.1369 | 0.1369 | 0.1369 | -0.000 | 0.000 | 0.000 | -0.000 |
| 23 | 133.0 | -349.96 | -0.00 | -349.96 | -349.95 | 0.1369 | 0.1369 | 0.1369 | -0.000 | 0.000 | 0.000 | -0.000 |
| 24 | 141.0 | -349.96 | -0.00 | -349.96 | -349.96 | 0.1369 | 0.1369 | 0.1369 | -0.000 | 0.000 | 0.000 | -0.000 |
| 25 | 150.0 | -349.96 | -0.00 | -349.96 | -349.97 | 0.1369 | 0.1369 | 0.1369 | -0.000 | 0.000 | 0.000 | -0.000 |
| 26 | 160.0 | -349.97 | -0.00 | -349.97 | -349.97 | 0.1369 | 0.1369 | 0.1369 | -0.000 | 0.000 | 0.000 | 0.000 |
| 27 | 170.0 | -349.97 | 0.0 | | | 0.1369 | | | -0.000 | 0.0 | | |

INFILTRATION RATE.................. 25.000
DRAINAGE RATE...................... 0.008
TOTAL MOISTURE IN PROFILE.......... 38.825

MOISTURE ADDED TO PROFILE............ 6.282
MOISTURE INCREASE IN PROFILE......... 6.271
TOTAL MASS IN SOLUTION............... 139.323

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| ELAPSED TIME | DAYS | HOURS | MINUTES | DELT | ISTEP | NIT | NITT |
|---|---|---|---|---|---|---|---|
| | 0.2968 | 7.1243 | 0.4275E 03 | 0.1137E-01 | 55 | 4 | 157 |

| | | ---------PRESSURE HEAD-------- | | | | ---MOISTURE CONTENT--- | | | --------CONCENTRATION--------- | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| NODE | DEPTH | FUNCTN | GRAD | F(1/3) | F(2/3) | FUNCTN | F(1/3) | F(2/3) | FUNCTN | GRAD | F(1/3) | F(2/3) |
| 1 | 0.0 | -27.92 | -1.03 | -30.08 | -32.45 | 0.5303 | 0.5290 | 0.5275 | 17.603 | -0.682 | 16.082 | 14.328 |
| 2 | 6.0 | -35.09 | -1.40 | -39.04 | -43.65 | 0.5257 | 0.5229 | 0.5195 | 12.464 | -0.938 | 10.100 | 8.133 |
| 3 | 14.0 | -49.21 | -2.30 | -55.76 | -63.96 | 0.5151 | 0.5097 | 0.5026 | 6.744 | -0.390 | 5.940 | 5.512 |
| 4 | 22.0 | -75.06 | -4.86 | -83.68 | -90.56 | 0.4928 | 0.4631 | 0.4148 | 5.321 | -0.045 | 5.233 | 5.138 |
| 5 | 28.0 | -96.37 | -2.75 | -103.56 | -110.53 | 0.3860 | 0.3718 | 0.3573 | 5.016 | -0.071 | 4.771 | 4.431 |
| 6 | 36.0 | -117.41 | -2.58 | -126.39 | -136.68 | 0.3426 | 0.3236 | 0.3032 | 4.021 | -0.164 | 3.433 | 2.801 |
| 7 | 46.0 | -149.11 | -4.13 | -163.25 | -184.14 | 0.2610 | 0.2597 | 0.2354 | 2.183 | -0.178 | 1.090 | 1.265 |
| 8 | 55.0 | -216.67 | -13.33 | -271.06 | -331.21 | 0.2085 | 0.1825 | 0.1642 | 0.885 | -0.122 | 0.586 | 0.344 |

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 9 | 63.0 | -349.26 | 7.10 | -343.70 | -347.32 | 0.1559 | 0.1526 | 0.1483 | 0.170 | -0.051 | 0.080 | 0.021 |
| 10 | 69.0 | -347.99 | 2.42 | -346.48 | -346.89 | 0.1443 | 0.1420 | 0.1396 | 0.001 | 0.001 | 0.001 | -0.000 |
| 11 | 73.0 | -346.97 | 0.64 | -346.41 | -346.55 | 0.1371 | 0.1704 | 0.2319 | -0.000 | 0.001 | 0.000 | 0.000 |
| 12 | 77.0 | -347.57 | -1.14 | -348.78 | -349.50 | 0.2650 | 0.2649 | 0.2648 | -0.000 | 0.000 | 0.000 | -0.000 |
| 13 | 81.0 | -349.94 | -0.27 | -350.3C | -350.87 | 0.2648 | 0.2648 | 0.2647 | -0.000 | 0.000 | 0.000 | -0.000 |
| 14 | 85.0 | -351.91 | -1.03 | -352.63 | -352.43 | 0.2646 | 0.2314 | 0.1698 | -0.000 | 0.000 | 0.000 | -0.000 |
| 15 | 89.0 | -351.89 | 0.39 | -351.41 | -351.02 | 0.1367 | 0.1367 | 0.1368 | -0.000 | 0.000 | 0.000 | -0.000 |
| 16 | 93.0 | -350.72 | 0.20 | -350.49 | -350.32 | 0.1368 | 0.1368 | 0.1368 | -0.000 | 0.000 | 0.000 | -0.000 |
| 17 | 97.0 | -350.20 | 0.08 | -350.10 | -350.03 | 0.1368 | 0.1368 | 0.1369 | -0.000 | 0.000 | 0.000 | -0.000 |
| 18 | 102.0 | -350.00 | 0.02 | -349.98 | -349.97 | 0.1369 | 0.1369 | 0.1369 | -0.000 | 0.000 | 0.000 | -0.000 |
| 19 | 107.0 | -349.96 | 0.00 | -349.96 | -349.95 | 0.1369 | 0.1369 | 0.1369 | -0.000 | 0.000 | 0.000 | -0.000 |
| 20 | 112.0 | -349.95 | -0.00 | -349.95 | -349.95 | 0.1369 | 0.1369 | 0.1369 | -0.000 | 0.000 | 0.000 | -0.000 |
| 21 | 118.0 | -349.95 | -0.00 | -349.95 | -349.95 | 0.1369 | 0.1369 | 0.1369 | -0.000 | 0.000 | 0.0C0 | -0.000 |
| 22 | 125.0 | -349.95 | -0.00 | -349.95 | -349.95 | 0.1369 | 0.1369 | 0.1369 | -0.000 | 0.000 | 0.000 | -0.000 |
| 23 | 133.0 | -349.95 | -0.00 | -349.95 | -349.95 | 0.1369 | 0.1369 | 0.1369 | -0.000 | 0.000 | 0.000 | -0.000 |
| 24 | 141.0 | -349.95 | -0.00 | -349.95 | -349.95 | 0.1369 | 0.1369 | 0.1369 | -0.000 | 0.000 | 0.000 | -0.000 |
| 25 | 150.0 | -349.96 | -0.00 | -349.96 | -349.96 | 0.1369 | 0.1369 | 0.1369 | -0.000 | 0.000 | 0.000 | -0.000 |
| 26 | 160.0 | -349.97 | -0.00 | -349.97 | -349.97 | 0.1369 | 0.1369 | 0.1369 | -0.000 | 0.000 | 0.000 | 0.000 |
| 27 | 170.0 | -349.98 | 0.0 | | | 0.1369 | | | -0.000 | 0.0 | | |

INFILTRATION RATE.................. 25.000
DRAINAGE RATE...................... 0.008
TOTAL MOISTURE IN PROFILE.......... 39.964

MOISTURE ADDED TO PROFILE............. 7.419
MOISTURE INCREASE IN PROFILE.......... 7.410
TOTAL MASS IN SOLUTION................ 151.395

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| ELAPSED TIME | DAYS | HOURS | MINUTES | DELT | ISTEP | NIT | NITT |
|---|---|---|---|---|---|---|---|
| | 0.3537 | 8.4885 | 0.5093E 03 | 0.1137E-01 | 60 | 5 | 178 |

| | | PRESSURE HEAD | | | | MOISTURE CONTENT | | | CONCENTRATION | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| NODE | DEPTH | FUNCTN | GRAD | F(1/3) | F(2/3) | FUNCTN | F(1/3) | F(2/3) | FUNCTN | GRAD | F(1/3) | F(2/3) |
| 1 | 0.0 | -26.77 | -0.98 | -28.81 | -31.05 | 0.5310 | 0.5298 | 0.5284 | 18.005 | -0.568 | 16.725 | 15.214 |
| 2 | 6.0 | -33.54 | -1.32 | -37.27 | -41.60 | 0.5267 | 0.5242 | 0.5210 | 13.556 | -0.851 | 11.313 | 9.272 |
| 3 | 14.0 | -46.80 | -2.15 | -52.91 | -60.48 | 0.5170 | 0.5121 | 0.5057 | 7.654 | -0.499 | 6.554 | 5.862 |
| 4 | 22.0 | -70.64 | -4.43 | -78.34 | -84.17 | 0.4908 | 0.4682 | 0.4216 | 5.492 | -0.089 | 5.336 | 5.210 |
| 5 | 28.0 | -88.87 | -2.19 | -94.39 | -99.36 | 0.3942 | 0.3821 | 0.3701 | 5.091 | -0.062 | 4.869 | 4.616 |
| 6 | 36.0 | -103.93 | -1.65 | -109.36 | -114.84 | 0.3582 | 0.3432 | 0.3278 | 4.274 | -0.141 | 3.755 | 3.170 |
| 7 | 46.0 | -120.61 | -1.80 | -126.22 | -132.78 | 0.3117 | 0.2960 | 0.2804 | 2.567 | -0.179 | 2.047 | 1.567 |
| 8 | 55.0 | -141.10 | -3.16 | -149.75 | -161.44 | 0.2625 | 0.2461 | 0.2279 | 1.140 | -0.133 | 0.799 | 0.496 |
| 9 | 63.0 | -180.10 | -8.79 | -199.23 | -234.62 | 0.2054 | 0.1901 | 0.1705 | 0.251 | -0.079 | 0.119 | 0.035 |
| 10 | 69.0 | -305.97 | -47.95 | -342.51 | -344.87 | 0.1497 | 0.1425 | 0.1398 | 0.002 | -0.004 | -0.001 | 0.000 |
| 11 | 73.0 | -343.89 | -5.73 | -347.10 | -345.81 | 0.1375 | 0.1703 | 0.2320 | 0.000 | -0.001 | -0.000 | -0.000 |
| 12 | 77.0 | -346.59 | -2.99 | -348.92 | -349.31 | 0.2651 | 0.2649 | 0.2649 | 0.000 | -0.000 | -0.000 | 0.000 |
| 13 | 81.0 | -349.73 | -0.84 | -350.50 | -351.05 | 0.2648 | 0.2648 | 0.2647 | 0.000 | -0.000 | -0.000 | 0.000 |
| 14 | 85.0 | -352.12 | -1.17 | -352.94 | -352.72 | 0.2646 | 0.2314 | 0.1698 | 0.000 | -0.000 | -0.000 | 0.000 |
| 15 | 89.0 | -352.15 | 0.39 | -351.67 | -351.25 | 0.1367 | 0.1367 | 0.1367 | 0.000 | -0.000 | -0.000 | 0.000 |
| 16 | 93.0 | -350.91 | 0.22 | -350.65 | -350.45 | 0.1368 | 0.1368 | 0.1368 | 0.000 | -0.000 | -0.000 | 0.000 |
| 17 | 97.0 | -350.30 | 0.10 | -350.17 | -350.08 | 0.1368 | 0.1368 | 0.1368 | 0.000 | -0.000 | -0.000 | 0.000 |
| 18 | 102.0 | -350.03 | 0.02 | -349.99 | -349.97 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 19 | 107.0 | -349.96 | 0.00 | -349.96 | -349.95 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 20 | 112.0 | -349.95 | 0.00 | -349.95 | -349.95 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 21 | 118.0 | -349.95 | -0.00 | -349.95 | -349.95 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 22 | 125.0 | -349.95 | -0.00 | -349.95 | -349.95 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |

| NODE | DEPTH | PRESSURE HEAD FUNCTN | GRAD | F(1/3) | F(2/3) | MOISTURE CONTENT FUNCTN | F(1/3) | F(2/3) | CONCENTRATION FUNCTN | GRAD | F(1/3) | F(2/3) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 23 | 133.0 | -349.95 | -0.00 | -349.95 | -349.94 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 24 | 141.0 | -349.95 | -0.00 | -349.95 | -349.95 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 25 | 150.0 | -349.95 | -0.00 | -349.96 | -349.96 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 26 | 160.0 | -349.96 | -0.00 | -349.97 | -349.97 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 27 | 170.0 | -349.97 | 0.0 | | | 0.1369 | | | 0.000 | 0.0 | | |

INFILTRATION RATE................. 25.000
DRAINAGE RATE..................... 0.008
TOTAL MOISTURE IN PROFILE........ 41.388

MOISTURE ADDED TO PROFILE............ 8.840
MOISTURE INCREASE IN PROFILE......... 8.834
TOTAL MASS IN SOLUTION................ 165.975

*********************************************************************************************

| ELAPSED TIME | DAYS | HOURS | MINUTES | DELT | ISTEP | NIT | NITT |
|---|---|---|---|---|---|---|---|
| | 0.4014 | 9.6345 | 0.5781E 03 | 0.9095E-02 | 65 | 6 | 204 |

| NODE | DEPTH | PRESSURE HEAD FUNCTN | GRAD | F(1/3) | F(2/3) | MOISTURE CONTENT FUNCTN | F(1/3) | F(2/3) | CONCENTRATION FUNCTN | GRAD | F(1/3) | F(2/3) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.0 | -26.20 | -0.95 | -28.19 | -30.37 | 0.5313 | 0.5301 | 0.5288 | 18.274 | -0.491 | 17.160 | 15.824 |
| 2 | 6.0 | -32.79 | -1.28 | -36.41 | -40.60 | 0.5272 | 0.5248 | 0.5218 | 14.329 | -0.777 | 12.220 | 10.190 |
| 3 | 14.0 | -45.63 | -2.08 | -51.52 | -58.80 | 0.5179 | 0.5132 | 0.5071 | 8.466 | -0.561 | 7.174 | 6.273 |
| 4 | 22.0 | -68.50 | -4.22 | -75.76 | -81.12 | 0.4987 | 0.4707 | 0.4249 | 5.734 | -0.137 | 5.497 | 5.319 |
| 5 | 28.0 | -85.32 | -1.92 | -90.11 | -94.28 | 0.3982 | 0.3871 | 0.3762 | 5.177 | -0.066 | 4.983 | 4.745 |
| 6 | 36.0 | -97.93 | -1.28 | -102.06 | -105.96 | 0.3655 | 0.3524 | 0.3391 | 4.452 | -0.122 | 3.997 | 3.467 |
| 7 | 46.0 | -109.71 | -1.11 | -113.15 | -116.84 | 0.3257 | 0.3132 | 0.3003 | 2.699 | -0.173 | 2.385 | 1.893 |
| 8 | 55.0 | -120.84 | -1.40 | -124.84 | -129.60 | 0.2867 | 0.2741 | 0.2607 | 1.442 | -0.141 | 1.079 | 0.750 |
| 9 | 63.0 | -135.50 | -2.47 | -140.53 | -146.84 | 0.2460 | 0.2348 | 0.2227 | 0.467 | -0.095 | 0.293 | 0.156 |
| 10 | 69.0 | -156.05 | -5.61 | -162.24 | -169.68 | 0.2085 | 0.1997 | 0.1906 | 0.064 | -0.033 | 0.029 | 0.010 |
| 11 | 73.0 | -184.07 | -14.83 | -196.37 | -224.26 | 0.1780 | 0.2029 | 0.2520 | 0.002 | -0.002 | 0.000 | -0.000 |
| 12 | 77.0 | -313.50 | -101.38 | -372.90 | -348.32 | 0.2683 | 0.2631 | 0.2649 | 0.000 | -0.000 | -0.000 | 0.000 |
| 13 | 81.0 | -341.14 | -26.48 | -356.81 | -350.88 | 0.2656 | 0.2643 | 0.2647 | 0.000 | -0.000 | -0.000 | 0.000 |
| 14 | 85.0 | -349.85 | -7.68 | -354.64 | -353.04 | 0.2648 | 0.2312 | 0.1698 | 0.000 | -0.000 | -0.000 | 0.000 |
| 15 | 89.0 | -351.83 | -0.94 | -352.15 | -351.43 | 0.1367 | 0.1367 | 0.1367 | 0.000 | -0.000 | -0.000 | 0.000 |
| 16 | 93.0 | -350.96 | -0.06 | -350.85 | -350.57 | 0.1368 | 0.1368 | 0.1368 | 0.000 | -0.000 | -0.000 | -0.000 |
| 17 | 97.0 | -350.36 | 0.05 | -350.25 | -350.13 | 0.1368 | 0.1368 | 0.1368 | 0.000 | -0.000 | -0.000 | 0.000 |
| 18 | 102.0 | -350.05 | 0.02 | -350.02 | -349.98 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 19 | 107.0 | -349.97 | 0.00 | -349.96 | -349.95 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 20 | 112.0 | -349.95 | -0.00 | -349.94 | -349.94 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 21 | 118.0 | -349.94 | -0.00 | -349.94 | -349.94 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 22 | 125.0 | -349.94 | -0.00 | -349.94 | -349.94 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 23 | 133.0 | -349.94 | -0.00 | -349.94 | -349.94 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 24 | 141.0 | -349.94 | -0.00 | -349.94 | -349.95 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 25 | 150.0 | -349.95 | -0.00 | -349.95 | -349.95 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 26 | 160.0 | -349.96 | -0.00 | -349.96 | -349.97 | 0.1369 | 0.1369 | 0.1369 | 0.000 | -0.000 | -0.000 | 0.000 |
| 27 | 170.0 | -349.97 | 0.0 | | | 0.1369 | | | 0.000 | 0.0 | | |

INFILTRATION RATE................. 25.000
DRAINAGE RATE..................... 0.008
TOTAL MOISTURE IN PROFILE........ 42.589

MOISTURE ADDED TO PROFILE............ 10.033
MOISTURE INCREASE IN PROFILE......... 10.034
TOTAL MASS IN SOLUTION................ 178.040

*Table C5.* Listing of SUMATRA-1.

```
C
C
C         ****************************************************************
C         *                                                              *
C         *            ONE-DIMENSIONAL UNSATURATED MASS TRANSPORT        *
C         *            NOVEMBER 1978                              SLP    *
C         *                                                              *
C         *            SUMATRA-1                                         *
C         *                                                              *
C         ****************************************************************
C
C
C
      COMMON /ONE/ X(30), ISPR(30), INT(30), P(60), PE(60), C(60)
      DIMENSION T(60)
      DATA NITMAX/10/
C
C     ----- OBTAIN INITIAL CONDITIONS AND PROGRAM CONSTANTS -----
      CALL DATAIN(NN,NSTEPS,KRAIN,KDRAIN,KOD1,KOD4,DELT,DELMIN,DELMAX,
     1TMAX,PRDEL,PULSE,EPSI,DRAIN,TMINIT,TOL1,TOL2)
C
C     ----------
      NN2=2*NN
      N1=NN2-1
      NE=NN-1
      NITT=0.
      TMINF=0.
      PRTIME=0.
      SUMT=DELT
      ISTEP=1
      RAIN=BC(2,SUMT)
      DO 8 I=1,NN2
    8 PE(I)=P(I)
C
C     ----- DYNAMIC PART OF MODEL -----
    9 NIT=0
   10 NIT=NIT+1
      NITT=NITT+1
      IF(SUMT.GE.PULSE) KRAIN=1
      IF(KOD4.EQ.2) GO TO 26
      DO 12 I=1,NN2
   12 T(I)=PE(I)
      CALL MATEQ(NN,KRAIN,KDRAIN,DELT,SUMT,EPSI,RAIN,DRAIN)
      IF(KOD1.EQ.1.OR.KOD1.EQ.2) WRITE(6,1002) NIT,DELT,ISTEP,SUMT,(PE(I
     1),I=1,NN2)
C
C     ----- CHECK ITERATIVE PROCESS -----
      DO 14 I=1,NN2,2
      TOL=TOL1+TOL2*ABS(T(I))
      IF(ABS(PE(I)-T(I))-TOL) 14,14,16
   14 CONTINUE
```

```
      GO TO 26
   16 IF(NIT-NITMAX) 10,18,18
   18 DELT=0.5*DELT
      WRITE(6,1006) NIT,SUMT,DELT
      IF(DELT.GE.DELMIN) GO TO 22
      WRITE(6,1008) DELT,DELMIN,SUMT
      DO 20 I=1,NN
      J=2*I-1
   20 WRITE(6,1010) I,X(I),P(J),P(J+1),PE(J),PE(J+1)
      CALL EXIT
   22 SUMT=SUMT-DELT
      DO 24 I=1,NN2
   24 PE(I)=0.5*(P(I)+PE(I))
      GO TO 9
C
C     ----------
   26 PRTIME=PRTIME+DELT
      IF(KRAIN) 28,28,30
   28 MAT=ISPR(1)
      EL=X(2)-X(1)
      P1=.957031*P(1)+.042969*P(3)+EL*(.095703*P(2)-.013672*P(4))
      PE1=.957031*PE(1)+.042969*PE(3)+EL*(.095703*PE(2)-.013672*PE(4))
      P2=.65625*(P(3)-P(1))/EL+.546875*P(2)-.203125*P(4)
      PE2=.65625*(PE(3)-PE(1))/EL+.546875*PE(2)-.203125*PE(4)
      RAIN=0.5*(SPR(MAT,2,P1)*(1.-P2)+SPR(MAT,2,PE1)*(1.-PE2))
   30 IF(KDRAIN.EQ.2) GO TO 32
      MAT=ISPR(NN)
      DRAIN=0.5*(SPR(MAT,2,P(N1))*(1.-P(NN2))+SPR(MAT,2,PE(N1))*(1.-PE(N
     1N2)))
   32 TMINF=TMINF+(RAIN-DRAIN)*DELT
      IF(KOD4.EQ.1) GO TO 60
C
C     ----- END OF FLOW PART, START TRANSPORT PART OF MODEL -----
C
C     ----- ASSEMBLE AND SOLVE GLOBAL MATRIX EQUATION FOR TRANSPORT -----
      CALL MATSO(NN,KRAIN,DELT,SUMT)
      IF(KOD1.GT.1) WRITE(6,1012) DELT,ISTEP,SUMT,(C(J),J=1,NN2)
   60 CONTINUE
C
C     ----- CHECK FOR POSSIBLE PRINT-OUT -----
      IF(PRTIME.LT.(PRDEL-0.5*DELT)) GO TO 34
      PRTIME=PRTIME-PRDEL
      IF(KRAIN.EQ.0) RAIN=SPR(ISPR(1),2,PE1)*(1.-PE2)
      IF(KRAIN.EQ.1) RAIN=BC(2,SUMT)
      IF(KDRAIN.NE.2) DRAIN=SPR(ISPR(NN),2,PE(N1))*(1.-PE(NN2))
C
C     ----- PREPARE FOR THE NEXT TIME STEP -----
      CALL PRINT(NN,RAIN,DRAIN,SUMT,ISTEP,NIT,NITT,DELT,TMINIT,TMINF)
   34 IF(SUMT.GE.TMAX.OR.ISTEP.GE.NSTEPS) GO TO 42
      DELS=DELT
      IF(NIT.LE.2) DELT=1.25*DELT
```

```
      IF(NIT.GE.6) DELT=0.80*DELT
      IF(KOD4.EQ.1) GO TO 38
      DO 36 L=2,NE
      MAT=ISPR(L)
      J=2*L-1
      P1=ABS(SPR(MAT,2,PE(J))*(1.-PE(J+1)))/(SPR(MAT,1,PE(J))+SPS(MAT,1)
     1*SPS(MAT,4))+1.E-06
   36 DELT=AMIN1(DELT,0.25*(X(L+1)-X(L-1))/P1)
   38 DELT=AMIN1(DELT,DELMAX)
      IF(KOD4.EQ.2) DELT=DELS
      DELCH=DELT/DELS
      SUMT=SUMT+DELT
      ISTEP=ISTEP+1
      DO 40 J=1,NN2
      PE1=DELCH*(PE(J)-P(J))
      P(J)=PE(J)
   40 PE(J)=P(J)+PE1
      GO TO 9
   42 WRITE (6,1014) SUMT,ISTEP
      STOP
C
C     ---------------
 1002 FORMAT(/11X,'PE(I) DURING ITERATION (NIT=',I3,' DELT =',F10.6,' IS
     1TEP =',I4,' SUMT =',F10.5,')'/(10X,10F11.3))
 1006 FORMAT(//11X,10(1H*),'NO CONVERGENCE AFTER',I3,' ITERATIONS AT TIM
     1E =',F8.5,' NEW DELT =',E11.4,1X,10(1H*))
 1008 FORMAT(//11X,8(1H*),'DELT =',E11.4,', IS LESS THAN DELMIN (=',E11.
     14,'), EXECUTION TERMINATED AT TIME =',E11.4,1X,10(1H*)//11X,'LAST
     2CALCULATED VALUES'/11X,22(1H*)/11X,'NODE',5X,'DEPTH',9X,'P(I)',6X,
     3'GRADIENT',9X,'PE(I)',6X,'GRADIENT')
 1010 FORMAT(11X,I4,F10.2,2(3X,2F12.4))
 1012 FORMAT(//11X,'C(I) FOR DELT =',F10.6,', ISTEP =',I4,', SUMT =',
     1F10.5/(10X,10F11.4))
 1014 FORMAT(//11X,10(1H*),'NORMAL TERMINATION AT TIME =',F13.5,' DAYS A
     1ND STEP NUMBER =',I5)
      END
```

BANSOL

```
      SUBROUTINE BANSOL(NEQ)
C
C     PURPOSE: TO SOLVE THE GLOBAL MATRIX EQUATION
C
      COMMON /TWO/ S(60,4), F(60)
      N1=NEQ-1
      DO 4 I=1,N1
      J=I-1
      M=MINO(4,NEQ-J)
      P=S(I,1)
      DO 4 L=2,M
      C=S(I,L)/P
      II=J+L
      JJ=0
      DO 2 K=L,M
      JJ=JJ+1
    2 S(II,JJ)=S(II,JJ)-C*S(I,K)
    4 S(I,L)=C
      DO 6 I=1,N1
      J=I-1
      M=MINO(4,NEQ-J)
      C=F(I)
      F(I)=C/S(I,1)
      DO 6 L=2,M
      II=J+L
    6 F(II)=F(II)-S(I,L)*C
      F(NEQ)=F(NEQ)/S(NEQ,1)
      DO 8 I=1,N1
      II=NEQ-I
      J=II-1
      M=MINO(4,NEQ-J)
      DO 8 K=2,M
      L=J+K
    8 F(II)=F(II)-S(II,K)*F(L)
      RETURN
      END
```

BC

```
      FUNCTION BC(K,SUMT)
C
C     PURPOSE: TO CALCULATE TRANSIENT BOUNDARY CONDITIONS
C
      DIMENSION T0(2),R0(2),T1(4),R1(4),S0(4),Q0(4),S1(4),Q1(4)
      DATA N1/2/, T0/0.,100./, R0/-14.4954,-14.4954/
      DATA N2/4/, T1/0.,0.99,1.01,100./,R1/25.,25.,-0.5,-0.5/
      DATA N3/4/, S0/0.,0.116666 ,0.116667,10./,Q0/209.,209.,0.,0./
      DATA N4/4/, S1/0.,0.49,0.51,100./, Q1/20.,20.,0.,0./
C
C     ----------
      GO TO (1,6,12,18),K
    1 DO 2 I=2,N1
      IF(T0(I)-SUMT) 2,4,4
    2 CONTINUE
    4 BC=((T0(I)-SUMT)*R0(I-1)+(SUMT-T0(I-1))*R0(I))/(T0(I)-T0(I-1))
      RETURN
    6 DO 8 I=2,N2
      IF(T1(I)-SUMT) 8,10,10
    8 CONTINUE
   10 BC=((T1(I)-SUMT)*R1(I-1)+(SUMT-T1(I-1))*R1(I))/(T1(I)-T1(I-1))
      RETURN
   12 DO 14 I=1,N3
      IF(S0(I)-SUMT) 14,14,16
   14 CONTINUE
   16 BC=((S0(I)-SUMT)*Q0(I-1)+(SUMT-S0(I-1))*Q0(I))/(S0(I)-S0(I-1))
      RETURN
   18 DO 20 I=1,N4
      IF(S1(I)-SUMT) 20,20,22
   20 CONTINUE
   22 BC=((S1(I)-SUMT)*Q1(I-1)+(SUMT-S1(I-1))*Q1(I))/(S1(I)-S1(I-1))
      RETURN
      END
```

```
      SUBROUTINE DATAIN(NN,NSTEPS,KRAIN,KDRAIN,KOD1,KOD4,DELT,DELMIN,DEL
     1MAX,TMAX,PRDEL,PULSE,EPSI,DRAIN,TMINIT,TOL1,TOL2)
C
C     PURPOSE: DEFINE GEOMETRY AND INITIAL CONDITIONS
C
      COMMON /ONE/ X(30), ISPR(30), INT(30), P(60), PE(60), C(60)
      DIMENSION TITLE(20), A(7), B(5)
C
C     ---------------
      WRITE(6,1000)
      DO 1 I=1,3
      READ(5,1002) TITLE
    1 WRITE(6,1004) TITLE
      WRITE(6,1006)
      READ(5,1008) NN,NSTEPS,DELT,DELMIN,DELMAX,TMAX,PRDEL,PULSE,DRAIN
      READ(5,1010) KRAIN,KDRAIN,KOD1,KOD2,KOD3,KOD4,EPSI,TOL1,TOL2
      WRITE(6,1012) NN.NSTEPS,DELT,DELMIN,DELMAX,TMAX,PRDEL,PULSE,EPSI
      WRITE(6,1014) TOL1,TOL2,KRAIN,KDRAIN,KOD1,KOD2,KOD3,KOD4
C
C     ----- READ INITIAL CONDITIONS -----
      DO 6 I=1,NN
      READ(5,1016) K,MAT,X(K),Z1,Z2,Z3,Z4,INT(K)
      IF(K.EQ.I) GO TO 2
      WRITE(6,1018) I
      CALL EXIT
    2 J=2*I-1
      J1=J+1
      C(J)=Z3
      C(J1)=Z4
      ISPR(I)=MAT
      IF(KOD2.EQ.0) GO TO 4
      P(J)=Z1
      P(J1)=Z2
      GO TO 6
    4 P(J)=SPR(MAT,4,Z1)
      P(J1)=Z2/SPR(MAT,3,P(J))
    6 CONTINUE
C
C     ----- REDEFINE SURFACE BOUNDARY VALUES -----
      NN2=2*NN
      EL=X(2)-X(1)
      IF(KRAIN.EQ.0) GO TO 12
      RAIN=BC(2,0.)
      MAT=ISPR(1)
      WRITE(6,1020)
      Z1=SPR(MAT,1,P(1))
      Z2=Z1
      Z3=SPR(MAT,3,P(1))*P(2)
      Z4=AMIN1(0.5,100./EL)
      P(2)=1.-RAIN/SPR(MAT,2,P(1))
      I=0
```

```
    8 I=I+1
      IF(I.LT.40) GO TO 10
      WRITE(6,1022)
      CALL EXIT
   10 WC=Z1-EL*(SPR(MAT,3,P(1))*P(2)-Z3)/6.
      Z2=(1.-Z4)*Z2+Z4*WC
      P(1)=SPR(MAT,4,Z2)
      P(2)=1.-RAIN/SPR(MAT,2,P(1))
      WRITE(6,1024) I,Z2,P(1),P(2)
      IF(ABS(Z2-WC)-0.0001) 11,11,8
   11 Z1=RAIN/SPR(MAT,1,P(1))
      DSP=SPS(MAT,2)+SPS(MAT,3)*ABS(Z1)
      Z2=AMAX1(0.,Z1)*BC(4,0.)
      C1=(6.*DSP*C(1)+Z2*EL+DSP*EL*C(2))/(6.*DSP+Z1*EL)
      IF(C1.LT.C(1))  C2=AMAX1(C1,BC(4,0.))
      IF(C1.GT.C(1)) C2=AMIN1(C1,BC(4,0.))
      C(1)=C2
      C(2)=(Z1*C(1)-Z2)/DSP
      GO TO 14
   12 P(1)=BC(1,0.)
      P(2)=3.*(P(3)-P(1))/EL
      C(1)=BC(3,0.)
      C(2)=3.*(C(3)-C(1))/EL
      RAIN=SPR(ISPR(1),2,P(1))*(1.-P(2))
   14 IF(KDRAIN-2) 15,16,15
   15 DRAIN=SPR(ISPR(NN),2,P(NN2-1))*(1.-P(NN2))
      GO TO 17
   16 P(NN2)=1.-DRAIN/SPR(ISPR(NN),2,P(NN2-1))
C
C     ----- WRITE INITIAL CONDITIONS -----
   17 TMASS=0.
      TMINIT=0.
      WRITE(6,1026)
      NE=NN-1
      DO 18 L=1,NE
      G1=.7407407-.0740741*FLOAT(INT(L))
      G2=1.-G1
      MAT=ISPR(L)
      MAT1=ISPR(L+1)
      I=2*L-1
      I1=I+1
      I2=I+2
      I3=I+3
      EL=.3333333*(X(L+1)-X(L))
      P1=.7407407*P(I)+.2592593*P(I2)+.2222222*EL*(2.*P(I1)-P(I3))
      C1=.7407407*C(I)+.2592593*C(I2)+.2222222*EL*(2.*C(I1)-C(I3))
      P2=.2592593*P(I)+.7407407*P(I2)+.2222222*EL*(P(I1)-2.*P(I3))
      C2=.2592593*C(I)+.7407407*C(I2)+.2222222*EL*(C(I1)-2.*C(I3))
      Z1=SPR(MAT,1,P(I))
      Z2=G1*SPR(MAT,1,P1)+G2*SPR(MAT1,1,P1)
      Z3=G2*SPR(MAT,1,P2)+G1*SPR(MAT1,1,P2)
```

```
      Z4=SPR(MAT1,1,P(I2))
      WRITE(6,1028) L,X(L),P(I),P(I1),P1,P2,Z1,Z2,Z3,C(I),C(I1),C1,C2,MA
     1T,INT(L)
      TMASS=TMASS+EL*(0.5*(Z1*C(I)+Z4*C(I2))+Z2*C1+Z3*C2)
   18 TMINIT=TMINIT+EL*(0.5*(Z1+Z4)+Z2+Z3)
      WRITE(6,1030) NN,X(NN),P(I2),P(NN2),Z4,C(I2),C(NN2),MAT1
      WRITE(6,1032) RAIN,ORAIN,TMINIT,TMASS
C
C     ----- CALCULATE MATERIAL PROPERTIES -----
      IF(KOD3.EQ.0) GO TO 28
      I1=(KOD3-1)/5+1
      DO 22 I=1,I1
      I2=5*I
      I3=I2-4
      WRITE(6,1034) (K,K=I3,I2)
      I2=MIN0(I2,KOD3-I2+5)
      Z1=-0.5
      DO 20 J=1,54
      Z1=Z1*1.2
      DO 19 J1=1,I2
      K=J1+5*(I-1)
      A(J1)=SPR(K,1,Z1)
   19 B(J1)=SPR(K,2,Z1)
   20 WRITE(6,1036) Z1,(A(K),B(K),K=1,I2)
   22 CONTINUE
      WRITE(6,1038)
      DO 26 MAT=1,KOD3
      DO 24 J=1,7
   24 A(J)=SPS(MAT,J)
   26 WRITE(6,1040) MAT,(A(J),J=1,7)
      WRITE(6,1042)
   28 CONTINUE
C
C     ---------------
 1000 FORMAT(1H1,10X, 82(1H*)/11X,1H*,80X,1H*/11X,1H*,9X,'ONE-DIMENSIONA
     1L UNSATURATED TRANSPORT',34X,1H*/11X,1H*,80X,1H*)
 1002 FORMAT(20A4)
 1004 FORMAT(11X,1H*,20A4,1H*)
 1006 FORMAT(11X,1H*,80X,1H*/11X,82(1H*))
 1008 FORMAT(2I5,7F10.0)
 1010 FORMAT(6I5,5F10.0)
 1012 FORMAT(//11X,'INPUT PARAMETERS'/11X,16(1H=)/11X,
     1'NUMBER OF NODES......................(NN)...............',I5/11X,
     2'MAXIMUM NUMBER OF TIME STEPS.......(NSTEPS).............',I5/11X,
     3'INITIAL TIME STEP..................(DELT)............',F10.5/11X,
     4'MINIMUM ALLOWABLE TIME STEP........(DELMIN)..........',F10.5/11X,
     5'MAXIMUM ALLOWABLE TIME STEP........(DELMAX)..........',F10.5/11X,
     6'MAXIMUM SIMULATION TIME............(TMAX)............',F10.5/11X,
     7'PRINT DELT FOR OUTPUT..............(PRDEL)...........',F10.5/11X,
     8'PULSE LENGTH FOR 1ST-TYPE BC.......(PULSE)...........',F10.5/11X,
     9'WEIGHTING COEFFICIENT..............(EPSI)...........',F10.5)
```

```
1014 FORMAT(11X,
    1'ITERATION TOLERANCE.................(TOL1)...........',F10.5/11X,
    2'ITERATION TOLERANCE.................(TOL2)...........',F10.5/11X,
    3'KRAIN........................(RAINFALL CODE)..........',I5/11X,
    4'KDRAIN.......................(DRAINAGE CODE)..........',I5/11X,
    5'KOD1..............(OUTPUT FOR EVERY ITERATION)..........',I5/11X,
    6'KOD2............(INPUT VARIABLE IS PRESSURE HEAD).......',I5/11X,
    7'KOD3...............(WRITE MATERIAL PROPERTIES)..........',I5/11X,
    8'KOD4...........(SOLVE ONLY FOR FLOW OR TRANSPORT).......',I5)
1016 FORMAT(2I5,5F10.0,I5)
1018 FORMAT(//5X,8(1H*),'ERROR ENCOUNTERED WHILE READING INITIAL CONDIT
    1IONS, CHECK NODE',I4,1X,'EXECUTION TERMINATED',9(1H*))
1020 FORMAT(//11X,'REDEFINED SURFACE VALUES'/11X,24(1H=)/11X,'ITERATION
    1',7X,'MOIST. CONT.',4X,'PRESSURE',4X,'GRADIENT')
1022 FORMAT(//8X,9(1H*),'PROBLEMS ENCOUNTERED WHILE REDEFINING SURFACE
    1PRESSURE VALUES, EXECUTION TERMINATED',1X,9(1H*))
1024 FORMAT(13X,I4,3X,F11.4,4X,F10.3,3X,F10.4)
1026 FORMAT(//11X,'INITIAL CONDITIONS'/11X,18(1H=)/23X,9(1H-),'PRESSURE
    1 HEAD',9(1H-),5X,3(1H-),'MOISTURE CONTENT',3(1H-),5X, 9(1H-),'CONC
    2ENTRATION',8(1H-),2X,'INDICES'/11X,'NODE  DEPTH  FUNCTN   GRAD  F
    3(1/3)  F(2/3)    FUNCTN  F(1/3)  F(2/3)    FUNCTN   GRAD   F(1/3
    4)  F(2/3) MAT  INT')
1028 FORMAT(10X,I4,F7.1,1X,4F8.2,3X,3F8.4,3X,4F8.3,1X,2I3)
1030 FORMAT(10X,I4,F7.1,1X,2F8.2,19X,F8.4,19X,2F8.3,17X,I3)
1032 FORMAT(//11X,'INITIAL INFILTRATION RATE',11X,F11.4,/11X,'INITIAL D
    1RAINAGE RATE',15X,F11.4/11X,'INITIAL AMOUNT OF MOISTURE IN PROFILE
    2',F10.4/11X,'INITIAL MASS IN SOLUTION',12X,F11.4)
1034 FORMAT(1H1,10X,'SOIL-HYDRAULIC PROPERTIES (MOISTURE CONTENT AND HY
    1DRAULIC CONDUCTIVITY)'/11X,71(1H=)//13X,5(16X,'SOIL')/11X,'PRESSUR
    2E',10X,I3,4(17X,I3)/13X,'HEAD',5(8X,'WC',6X,'COND'))
1036 FORMAT(10X,E10.3,1X,5(2X,F7.4,E11.3))
1038 FORMAT(//11X,'PHYSICAL AND CHEMICAL SOIL PROPERTIES'/11X,37(1H=)//
    110X,'MATERIAL',5X,'BULK',6X,'DIFFUSION',20X,'ADSORPTION',3X,11(1H-
    2),'DECAY COEFFICIENTS',12(1H-)/12X,'INDEX',4X,'DENSITY',4X,'COEFFI
    3CIENT',4X,'DISPERSIVITY',4X,'CONSTANT',4X,'0-TH(LIQUID)',2X,'1-ST(
    4LIQUID)',4X,'1-ST(SOLID)')
1040 FORMAT(11X,I4,3X,7(F10.4,4X))
1042 FORMAT(1H1)
     RETURN
     END
```

MATEQ

```
      SUBROUTINE MATEQ(NN,KRAIN,KDRAIN,DELT,SUMT,EPSI,RAIN,DRAIN)
C
C     PURPOSE: TO CALCULATE THE GLOBAL MATRIX EQUATION
C
      COMMON /ONE/ X(30), ISPR(30), INT(30), P(60), PE(60), C(60)
      COMMON /TWO/ S(60,4), F(60)
      DIMENSION FE(4,3), DX(4,3), SR(10), T(60)
      DATA FE(1,1),FE(3,3),FE(1,2),FE(3,2),FE(1,3),FE(3,1),DX(1,1),DX(1,
     13),DX(1,2),DX(3,2),DX(3,1),DX(3,3)/2*.9208488,2*0.5,2*.0791512,2*-
     2.4285714,-.75,.75,2*.4285714/
C
C     ----------
      NE=NN-1
      NN2=2*NN
      N1=NN2-1
      N2=NN2-2
      N3=NN2-3
      NEQ=N1-KDRAIN/3
      EPSM=EPSI-1.
      DO 2 I=1,NN2
      T(I)=0.5*(P(I)+PE(I))
      DO 2 J=2,4
    2 S(I,J)=0.
C
C     ----- CONTRIBUTIONS OF NODAL INTEGRATION POINTS -----
      DO 4 I=1,NN
      LL=2*I-1
      L1=LL+1
      II=MAX0(I-1,1)
      JJ=MIN0(I+1,NN)
      EL1=0.05*(X(JJ)-X(II))
      MAT=ISPR(I)
      CAP1=SPR(MAT,3,T(LL))*EL1/DELT
      COND1=SPR(MAT,2,T(LL))*EL1
      S(LL,1)=CAP1
      S(L1,1)=EPSI*COND1
      F(LL)=CAP1*P(LL)
    4 F(L1)=(EPSM*P(L1)+1.)*COND1
C
C     ----- ELEMENT LOOP; CONSTRUCT GLOBAL MATRIX -----
      DO 12 L=1,NE
      LL=2*L-2
      L1=LL+1
      L2=LL+2
      L3=LL+3
      L4=LL+4
      EL=X(L+1)-X(L)
C
C     ----- CALCULATE HERMITIAN BASIS FUNCTIONS -----
C
      FE(2,1)=.1181895*EL
```

```
      FE(2,2)=.125*EL
      FE(2,3)=.0246676*EL
      DX(2,1)=.1993777*EL
      DX(2,2)=-.125*EL
      DX(2,3)=-.1279491*EL
      DO 6 K=1,3
      FE(4,K)=-FE(2,4-K)
    6 DX(4,K)=DX(2,4-K)
C     ----- CALCULATE MATERIAL PROPERTIES AT LOBATTO POINTS -----
      W1=FE(1,1)*T(L1)+FE(2,1)*T(L2)+FE(3,1)*T(L3)+FE(4,1)*T(L4)
      W2=FE(1,2)*T(L1)+FE(2,2)*T(L2)+FE(3,2)*T(L3)+FE(4,2)*T(L4)
      W3=FE(1,3)*T(L1)+FE(2,3)*T(L2)+FE(3,3)*T(L3)+FE(4,3)*T(L4)
      MAT1=ISPR(L)
      MAT2=ISPR(L+1)
      G2=.0861869+.101835*FLOAT(INT(L))
      G1=1.083889-G2
      COND1=(G1*SPR(MAT1,2,W1)+G2*SPR(MAT2,2,W1))/EL
      COND2=.7111111*(SPR(MAT1,2,W2)+SPR(MAT2,2,W2))/EL
      COND3=(G2*SPR(MAT1,2,W3)+G1*SPR(MAT2,2,W3))/EL
      EL1=.25*EL/DELT
      CAP1=(G1*SPR(MAT1,3,W1)+G2*SPR(MAT2,3,W1))*EL1
      CAP2=.7111111*(SPR(MAT1,3,W2)+SPR(MAT2,3,W2))*EL1
      CAP3=(G2*SPR(MAT1,3,W3)+G1*SPR(MAT2,3,W3))*EL1
C
C     ----- ADD ELEMENT CONTRIBUTIONS TO GLOBAL MATRIX -----
      K=0
      DO 10 I=1,4
      II=LL+I
      DO 10 J=I,4
      W1=DX(J,1)*DX(I,1)*COND1+DX(J,2)*DX(I,2)*COND2+DX(J,3)*DX(I,3)*
     1COND3
      W2=FE(J,1)*FE(I,1)*CAP1+FE(J,2)*FE(I,2)*CAP2+FE(J,3)*FE(I,3)*CAP3
      JJ=J+1-I
      K=K+1
      S(II,JJ)=S(II,JJ)+W1*EPSI+W2
   10 SR(K)=W1*EPSM+W2
C
C     ----- CONSTRUCT RHS VECTOR ----
      EL1=.2142857*EL*(COND1+1.75*COND2+COND3)
      F(L1)=F(L1)+SR(1)*P(L1)+SR(2)*P(L2)+SR(3)*P(L3)+SR(4)*P(L4)-EL1
      F(L2)=F(L2)+SR(2)*P(L1)+SR(5)*P(L2)+SR(6)*P(L3)+SR(7)*P(L4)+
     1(.0996889*COND1-.0625*COND2-.0639746*COND3)*EL*EL
      F(L3)=F(L3)+SR(3)*P(L1)+SR(6)*P(L2)+SR(8)*P(L3)+SR(9)*P(L4)+EL1
   12 F(L4)=F(L4)+SR(4)*P(L1)+SR(7)*P(L2)+SR(9)*P(L3)+SR(10)*P(L4)+
     1(.0996889*COND3-.0625*COND2-.0639746*COND1)*EL*EL
C
C     ----- INCLUDE BOUNDARY CONDITIONS -----
      IF(KRAIN.EQ.1) GO TO 22
      S(1,1)=1.
      F(1)=BC(1,SUMT)
      DO 20 I=2,4
```

```
      F(I)=F(I)-S(1,I)*F(1)
   20 S(1,I)=0.
      GO TO 24
   22 S(2,1)=1.
      W1=AMIN1(0.5,100./(X(2)-X(1)))
      W2=1.-BC(2,SUMT)/SPR(ISPR(1),2,PE(1))
      F(2)=(1.-W1)*PE(2)+W1*W2
      RAIN=(BC(2,SUMT)+BC(2,SUMT-0.5*DELT)+BC(2,SUMT-DELT))/3.
      F(1)=F(1)-S(1,2)*F(2)+RAIN
      F(3)=F(3)-S(2,2)*F(2)
      F(4)=F(4)-S(2,3)*F(2)
      S(1,2)=0.
      S(2,2)=0.
      S(2,3)=0.
   24 CONTINUE
      IF(KDRAIN-2) 26,28,30
   26 IF(KDRAIN.EQ.0) GO TO 27
      MAT=ISPR(NN)
      DRAIN=0.5*(SPR(MAT,2,P(N1))+SPR(MAT,2,PE(N1)))*(1.-P(NN2))
      GO TO 29
   27 F(N1)=F(NN2)-S(N1,2)*P(N1)
      F(N2)=F(N2)-S(N2,2)*P(N1)
      F(N3)=F(N3)-S(N3,3)*P(N1)
      S(N1,1)=S(NN2,1)
      S(N2,2)=S(N2,3)
      S(N3,3)=S(N3,4)
      GO TO 32
   28 PE(NN2)=0.5*(PE(NN2)+1.-DRAIN/SPR(ISPR(NN),2,PE(N1)))
   29 F(N1)=F(N1)-S(N1,2)*PE(NN2)-DRAIN
      F(N2)=F(N2)-S(N2,3)*PE(NN2)
      F(N3)=F(N3)-S(N3,4)*PE(NN2)
      GO TO 32
   30 F(N2)=F(N2)-S(N2,2)*P(N1)-S(N2,3)*P(NN2)
      F(N3)=F(N3)-S(N3,3)*P(N1)-S(N3,4)*P(NN2)
   32 CONTINUE
C
C     ----- SOLVE FOR UNKNOWNS -----
      CALL BANSOL(NEQ)
C
C     ----------
      DO 34 I=1,N2
   34 PE(I)=F(I)
      IF(KDRAIN.EQ.0) PE(NN2)=F(N1)
      IF(KDRAIN.EQ.1) PE(N1)=F(N1)
      RETURN
      END
```

MATSO

```
      SUBROUTINE MATSO(NN,KRAIN,DELT,SUMT)
C
C     PURPOSE: TO ASSEMBLE AND SOLVE GLOBAL MATRIX EQUATION FOR TRANSPORT
C
      COMMON /ONE/ X(30), ISPR(30), INT(30), P(60), PE(60), C(60)
      COMMON /THREE/ S(60,7), F(60)
      DIMENSION FE(4,3), DX(4,3)
      DATA FE(1,1),FE(3,3),FE(1,2),FE(3,2),FE(1,3),FE(3,1),DX(1,1),DX(1,
     13),DX(1,2),DX(3,2),DX(3,1),DX(3,3)/2*.9208488,2*0.5,2*.0791512,2*-
     2.4285714,-.75,.75,2*.4285714/
C
C     ----------
      NE=NN-1
      NN2=2*NN
      NEQ=NN2-1
      DEL=2./DELT
      DO 2 I=1,NN2
      DO 2 J=1,7
    2 S(I,J)=0.
C
C     ----- CONTRIBUTIONS OF NODAL INTEGRATION POINTS -----
      DO 4 I=1,NN
      LL=2*I-1
      L1=LL+1
      MAT=ISPR(I)
      II=MAX0(I-1,1)
      JJ=MIN0(I+1,NN)
      EL1=.025*(X(JJ)-X(II))
      RTK=SPS(MAT,1)*SPS(MAT,4)
      DIF=SPS(MAT,2)
      DSP=SPS(MAT,3)
      DL1=SPS(MAT,6)
      DS1=SPS(MAT,7)
      QN=SPR(MAT,2,PE(LL))*(1.-PE(L1))
      QO=SPR(MAT,2,P(LL))*(1.-P(L1))
      WCN=SPR(MAT,1,PE(LL))
      WCO=SPR(MAT,1,P(LL))
      S(LL,4)=EL1*(WCN*(DEL-DL1)+RTK*(DEL-DS1))
      S(L1,3)=-EL1*QN
      S(L1,4)=EL1*(WCN*DIF+DSP*ABS(QN)-.1666667*QN*QN*DELT/(WCN+RTK))
      F(LL)=EL1*C(LL)*(WCO*(DEL+DL1)+RTK*(DEL+DS1))+EL1*(WCO+WCN)*SPS(MA
     1T,5)
    4 F(L1)=EL1*(QO*C(LL)-C(L1)*(WCO*DIF+DSP*ABS(QO)+.1666667*QO*QO*DELT
     1/(WCO+RTK)))
C
C     ----- LOOP OVER ELEMENTS -----
      DO 10 L=1,NE
      LL=2*L-2
      L1=LL+1
      L2=LL+2
      L3=LL+3
```

```
      L4=LL+4
C
C     ----- CALCULATE REMAINING HERMITIAN BASIS FUNCTIONS -----
      EL=X(L+1)-X(L)
      FE(2,1)=.1181895*EL
      FE(2,2)=.125*EL
      FE(2,3)=.0246676*EL
      DX(2,1)=.1993777*EL
      DX(2,2)=-.125*EL
      DX(2,3)=-.1279491*EL
      DO 6 K=1,3
      FE(4,K)=-FE(2,4-K)
    6 DX(4,K)=DX(2,4-K)
C
C     ----- CALCULATE MATERIAL PROPERTIES -----
      MAT1=ISPR(L)
      MAT2=ISPR(L+1)
      G1=.9208488-.093522*FLOAT(INT(L))
      G2=1.-G1
      RTK=G1*SPS(MAT1,1)*SPS(MAT1,4)+G2*SPS(MAT2,1)*SPS(MAT2,4)
      DIF=G1*SPS(MAT1,2)+G2*SPS(MAT2,2)
      DSP=G1*SPS(MAT1,3)+G2*SPS(MAT2,3)
      DL1=G1*SPS(MAT1,6)+G2*SPS(MAT2,6)
      DS1=G1*SPS(MAT1,7)+G2*SPS(MAT2,7)
      P1=.9208488*P(L1)+.0791512*P(L3)+FE(2,1)*P(L2)+FE(4,1)*P(L4)
      PE1=.9208488*PE(L1)+.0791512*PE(L3)+FE(2,1)*PE(L2)+FE(4,1)*PE(L4)
      P2=.8571429*(P(L3)-P(L1))/EL+.3987554*P(L2)-.2558982*P(L4)
      PE2=.8571429*(PE(L3)-PE(L1))/EL+.3987554*PE(L2)-.2558982*PE(L4)
      BN1=0.5*(G1*SPR(MAT1,2,PE1)+G2*SPR(MAT2,2,PE1))*(1.-PE2)
      BO1=0.5*(G1*SPR(MAT1,2,P1)+G2*SPR(MAT2,2,P1))*(1.-P2)
      WCN=G1*SPR(MAT1,1,PE1)+G2*SPR(MAT2,1,PE1)
      WCO=G1*SPR(MAT1,1,P1)+G2*SPR(MAT2,1,P1)
      CN1=0.25*EL*(WCN*(DEL-DL1)+RTK*(DEL-DS1))
      CO1=0.25*EL*(WCO*(DEL+DL1)+RTK*(DEL+DS1))
      AN1=(DIF*WCN+2.*DSP*ABS(BN1)-.6666667*BN1*BN1*DELT/(WCN+RTK))/EL
      AO1=(DIF*WCO+2.*DSP*ABS(BO1)+.6666667*BO1*BO1*DELT/(WCO+RTK))/EL
      T1=.1361111*EL*(WCO+WCN)*(G1*SPS(MAT1,5)+G2*SPS(MAT2,5))
      RTK=0.5*(SPS(MAT1,1)*SPS(MAT1,4)+SPS(MAT2,1)*SPS(MAT2,4))
      DIF=0.5*(SPS(MAT1,2)+SPS(MAT2,2))
      DSP=0.5*(SPS(MAT1,3)+SPS(MAT2,3))
      DL1=0.5*(SPS(MAT1,6)+SPS(MAT2,6))
      DS1=0.5*(SPS(MAT1,7)+SPS(MAT2,7))
      P1=0.5*(P(L1)+P(L3))+FE(2,2)*(P(L2)-P(L4))
      PE1=0.5*(PE(L1)+PE(L3))+FE(2,2)*(PE(L2)-PE(L4))
      P2=1.5*(P(L3)-P(L1))/EL-.25*(P(L2)+P(L4))
      PE2=1.5*(PE(L3)-PE(L1))/EL-.25*(PE(L2)+PE(L4))
      BN2=0.25*(SPR(MAT1,2,PE1)+SPR(MAT2,2,PE1))*(1.-PE2)
      BO2=0.25*(SPR(MAT1,2,P1)+SPR(MAT2,2,P1))*(1.-P2)
      WCO=0.5*(SPR(MAT1,1,P1)+SPR(MAT2,1,P1))
      WCN=0.5*(SPR(MAT1,1,PE1)+SPR(MAT2,1,PE1))
      CN2=0.25*EL*(WCN*(DEL-DL1)+RTK*(DEL-DS1))
```

```
      CO2=0.25*EL*(WCO*(DEL+DL1)+RTK*(DEL+DS1))
      AN2=(DIF*WCN+2.*DSP*ABS(BN2)-.6666667*BN2*BN2*DELT/(WCN+RTK))/EL
      AO2=(DIF*WCO+2.*DSP*ABS(BO2)+.6666667*BO2*BO2*DELT/(WCO+RTK))/EL
      T2=.088889*EL*(WCO+WCN)*(SPS(MAT1,5)+SPS(MAT2,5))
      RTK=G2*SPS(MAT1,1)*SPS(MAT1,4)+G1*SPS(MAT2,1)*SPS(MAT2,4)
      DIF=G2*SPS(MAT1,2)+G1*SPS(MAT2,2)
      DSP=G2*SPS(MAT1,3)+G1*SPS(MAT2,3)
      DL1=G2*SPS(MAT1,6)+G1*SPS(MAT2,6)
      DS1=G2*SPS(MAT1,7)+G1*SPS(MAT2,7)
      P1=.0791512*P(L1)+.9208488*P(L3)+FE(2,3)*P(L2)+FE(4,3)*P(L4)
      PE1=.0791512*PE(L1)+.9208488*PE(L3)+FE(2,3)*PE(L2)+FE(4,3)*PE(L4)
      P2=.8571429*(P(L3)-P(L1))/EL-.2558982*P(L2)+.3987554*P(L4)
      PE2=.8571429*(PE(L3)-PE(L1))/EL-.2558982*PE(L2)+.3987554*PE(L4)
      BN3=0.5*(G2*SPR(MAT1,2,PE1)+G1*SPR(MAT2,2,PE1))*(1.-PE2)
      BO3=0.5*(G2*SPR(MAT1,2,P1)+G1*SPR(MAT2,2,P1))*(1.-P2)
      WCN=G2*SPR(MAT1,1,PE1)+G1*SPR(MAT2,1,PE1)
      WCO=G2*SPR(MAT1,1,P1)+G1*SPR(MAT2,1,P1)
      CN3=0.25*EL*(WCN*(DEL-DL1)+RTK*(DEL-DS1))
      CO3=0.25*EL*(WCO*(DEL+DL1)+RTK*(DEL+DS1))
      AN3=(DIF*WCN+2.*DSP*ABS(BN3)-.6666667*BN3*BN3*DELT/(WCN+RTK))/EL
      AO3=(DIF*WCO+2.*DSP*ABS(BO3)+.6666667*BO3*BO3*DELT/(WCO+RTK))/EL
      T3=.1361111*EL*(WCO+WCN)*(G2*SPS(MAT1,5)+G1*SPS(MAT2,5))
C
C     ----- ASSEMBLE GLOBAL MATRIX EQUATION -----
      DO 8 I=1,4
      II=LL+I
      F(II)=F(II)+T1*FE(I,1)+T2*FE(I,2)+T3*FE(I,3)
      DO 8 J=1,4
      JJ=J+4-I
      S(II,JJ)=S(II,JJ)+.5444444*(AN1*DX(J,1)*DX(I,1)+AN3*DX(J,3)*DX(I,3
     1)-BN1*FE(J,1)*DX(I,1)-BN3*FE(J,3)*DX(I,3)+CN1*FE(J,1)*FE(I,1)+CN3*
     2FE(J,3)*FE(I,3))+.7111111*(AN2*DX(J,2)*DX(I,2)-BN2*FE(J,2)*DX(I,2)
     3+CN2*FE(J,2)*FE(I,2))
    8 F(II)=F(II)-C(LL+J)*(.5444444*(AO1*DX(J,1)*DX(I,1)+AO3*DX(J,3)*DX(
     1I,3)-BO1*FE(J,1)*DX(I,1)-BO3*FE(J,3)*DX(I,3)-CO1*FE(J,1)*FE(I,1)-
     2CO3*FE(J,3)*FE(I,3))+.7111111*(AO2*DX(J,2)*DX(I,2)-BO2*FE(J,2)*DX(
     3I,2)-CO2*FE(J,2)*FE(I,2)))
   10 CONTINUE
C
C     ----- END OF ELEMENT LOOP -----
      IF(KRAIN) 12,12,16
   12 DO 14 J=1,7
   14 S(1,J)=0.
      S(1,4)=1.
      F(1)=BC(3,SUMT)
      GO TO 20
   16 F(1)=F(1)+(AMAX1(0.,BC(2,SUMT-DELT)*BC(4,SUMT-DELT))+AMAX1(0.,BC(2
     1,SUMT-0.5*DELT)*BC(4,SUMT-0.5*DELT))+AMAX1(0.,BC(2,SUMT)*BC(4,SUMT
     2)))/3.
      DO 18 J=1,7
   18 S(2,J)=0.
```

MATSO

```
      MAT=ISPR(1)
      S(2,3)=BC(2,SUMT)/SPR(MAT,1,PE(1))
      S(2,4)=-SPS(MAT,2)-SPS(MAT,3)*ABS(S(2,3))
      F(2)=AMAX1(0.,S(2,3))*BC(4,SUMT)
C
C     ----- LOWER BOUNDARY CONDITION (DC/DX=0)
   20 S(NEQ,5)=0.
      S(NEQ,4)=S(NEQ,4)+0.5*SPR(ISPR(NN),2,PE(NEQ))*(1.-PE(NN2))
      F(NEQ)=F(NEQ)-0.5*SPR(ISPR(NN),2,P(NEQ))*(1.-P(NN2))*C(NEQ)
      S(NEQ-1,6)=0.
      S(NEQ-2,7)=0.
C
C     ----- SOLVE FOR NEW SOLUTION VECTOR -----
      CALL SOLVE(NEQ)
      DO 22 I=1,NEQ
   22 C(I)=F(I)
      RETURN
      END
```

PRINT

```
      SUBROUTINE PRINT(NN,RAIN,DRAIN,SUMT,ISTEP,NIT,NITT,DELT,TMINIT,TMI
     1NF)
C
C     PURPOSE: PRINT PRESSURE HEAD, MOISTURE CONTENTS AND CONCENTRATIONS
C
      COMMON /ONE/ X(30), ISPR(30), INT(30), P(60), PE(60), C(60)
C
C     ---------------
      NE=NN-1
      SUMT1=SUMT*24.
      SUMT2=SUMT1*60.
      WRITE(6,1002) SUMT,SUMT1,SUMT2,DELT,ISTEP,NIT,NITT
      TMASS=0.
      TMIN=0.
      DO 4 L=1,NE
      G1=.7407407-.0740741*FLOAT(INT(L))
      G2=1.-G1
      I=2*L-1
      I1=I+1
      I2=I+2
      I3=I+3
      MAT1=ISPR(L)
      MAT2=ISPR(L+1)
      EL=.3333333*(X(L+1)-X(L))
      P1=.7407407*PE(I)+.2592593*PE(I2)+.2222222*EL*(2.*PE(I1)-PE(I3))
      P2=.2592593*PE(I)+.7407407*PE(I2)+.2222222*EL*(PE(I1)-2.*PE(I3))
      C1=.7407407*C(I)+.2592593*C(I2)+.2222222*EL*(2.*C(I1)-C(I3))
      C2=.2592593*C(I)+.7407407*C(I2)+.2222222*EL*(C(I1)-2.*C(I3))
      Z1=SPR(MAT1,1,PE(I))
      Z2=G1*SPR(MAT1,1,P1)+G2*SPR(MAT2,1,P1)
      Z3=G2*SPR(MAT1,1,P2)+G1*SPR(MAT2,1,P2)
      Z4=SPR(MAT2,1,PE(I2))
      WRITE(6,1004) L,X(L),PE(I),PE(I1),P1,P2,Z1,Z2,Z3,C(I),C(I1),C1,C2
      TMASS=TMASS+EL*(0.5*(Z1*C(I)+Z4*C(I2))+Z2*C1+Z3*C2)
    4 TMIN=TMIN+EL*(0.5*(Z1+Z4)+Z2+Z3)
      WRITE(6,1006) NN,X(NN),PE(I2),PE(I3),Z4,C(I2),C(I3)
      TMINCR=TMIN-TMINIT
      WRITE(6,1008) RAIN,TMINF,DRAIN,TMINCR,TMIN,TMASS
C
C     ----------
 1002 FORMAT(//11X,109(1H*)/11X,'ELAPSED TIME',5X,'DAYS',6X,'HOURS',7X,'
     1MINUTES',8X,'DELT',31X,'ISTEP',5X,'NIT   NITT'/24X,2F10.4,2E14.4,2
     24X,3(4X,I4)//23X,9(1H-),'PRESSURE HEAD',8(1H-),6X,'---MOISTURE CON
     3TENT---',4X,8(1H-),'CONCENTRATION',9(1H-)/11X,'NODE    DEPTH    FU
     4NCTN    GRAD  F(1/3)  F(2/3)',6X,'FUNCTN  F(1/3)  F(2/3)    FUNCTN
     5   GRAD   F(1/3)  F(2/3)')
 1004 FORMAT(10X,I4,F9.1,3X,4F8.2,4X,3F8.4,2X,4F8.3)
 1006 FORMAT(10X,I4,F9.1,3X,2F8.2,20X,F8.4,18X,2F8.3)
 1008 FORMAT(/11X,'INFILTRATION RATE',16(1H.),F8.3,15X,'MOISTURE ADDED T
     1O PROFILE',12(1H.),F8.3/11X,'DRAINAGE RATE',20(1H.),F8.3,15X,'MOIS
     2TURE INCREASE IN PROFILE',9(1H.),F8.3/11X,'TOTAL MOISTURE IN PROFI
     3LE',8(1H.),F8.3,15X,'TOTAL MASS IN SOLUTION',15(1H.),F8.3)
      RETURN
      END
```

SOLVE

```
      SUBROUTINE SOLVE(NEQ)
C
C     PURPOSE: TO SOLVE THE GLOBAL MATRIX EQUATION FOR TRANSPORT
C
      COMMON /THREE/ S(60,7), F(60)
C
C     ---------------
      N1=NEQ-1
      DO 6 K=1,N1
      P=1./S(K,4)
      KK=K+1
      KC=4
      DO 4 I=KK,NEQ
      KC=KC-1
      IF(KC) 6,6,2
    2 C=-P*S(I,KC)
      S(I,KC)=C
      II=KC+1
      L=KC+3
      DO 4 J=II,L
      JJ=J+4-KC
    4 S(I,J)=S(I,J)+C*S(K,JJ)
    6 CONTINUE
      DO 14 I=2,NEQ
      JJ=5-I
      II=1
      IF(JJ) 8,8,10
    8 JJ=1
      II=I-3
   10 SUM=0.0
      DO 12 J=JJ,3
      SUM=SUM+S(I,J)*F(II)
   12 II=II+1
   14 F(I)=F(I)+SUM
      F(NEQ)=F(NEQ)/S(NEQ,4)
      DO 18 K=1,N1
      I=NEQ-K
      JJ=I
      M=MIN0(7,4+K)
      SUM=0.0
      DO 16 J=5,M
      JJ=JJ+1
   16 SUM=SUM+S(I,J)*F(JJ)
   18 F(I)=(F(I)-SUM)/S(I,4)
      RETURN
      END
```

```
      FUNCTION SPR(MAT,N,PR)
C
C
C     PURPOSE: TO CALCULATE THE SOIL-HYDRAULIC PROPERTIES
C
      DIMENSION WCR(9), WCS(9), ALPHA(9), RN(9), CONDS(9), SS(9)
      DATA WCR/.200,.250,.170,.1611,.150,.140,.1311,.1244,.120/,WCS/.540
     1,.400,.470,.4611,.450,.440,.4311,.4244,.420/,ALPHA/.005,.009,.010,
     2.01036,.01080,.01120,.01156,.01192,.0120/,RN/1.60,3.0,2.0,2.178,2.
     340,2.60,2.778,2.911,3.00/,CONDS/25.,10.,75.,132.8,205.,270.,327.8,
     4371.1,400./,SS/4.E-07,5.E-08,1.E-07,1.E-07,1.E-07,1.E-07,1.E-07,
     51.E-07,1.E-07/
      DATA CONDM/1.E-08/
      IF(PR)1,10,10
    1 P=ABS(PR)
      A=ALPHA(MAT)
      R=RN(MAT)
      S=1.-1./R
      THETA=(1.+(A*P)**R)**(-S)
      IF(N-2) 2,4,6
    2 SPR=WCR(MAT)+(WCS(MAT)-WCR(MAT))*THETA
      RETURN
    4 T=1.-THETA*(A*P)**(R-1.)
      IF(THETA.LT.0.04) T=S*THETA**(1./S)
      COND=CONDS(MAT)*SQRT(THETA)*T*T
      SPR=AMAX1(COND,CONDM)
      RETURN
    6 T=1.+(A*P)**R
      WC=WCR(MAT)+(WCS(MAT)-WCR(MAT))*THETA
      SPR=(WC-WCR(MAT))*(R-1.)*A*(A*P)**(R-1.)/T + WC*SS(MAT)/WCS(MAT)
      RETURN
   10 GO TO (12,14,16,18),N
   12 SPR=WCS(MAT)
      RETURN
   14 SPR=CONDS(MAT)
      RETURN
   16 SPR=SS(MAT)
      RETURN
   18 THETA=(PR-WCR(MAT))/(WCS(MAT)-WCR(MAT))
      R=RN(MAT)
      S=R/(1.-R)
      IF(THETA.GT.0.999999) GO TO 20
      SPR=-(THETA**S-1.)**(1./R)/ALPHA(MAT)
      RETURN
   20 SPR=0.
      RETURN
      END
```

SPS

```
      FUNCTION SPS(MAT,N)
C
C     PURPOSE: TO SUPPLY PHYSICAL AND CHEMICAL SOIL DATA
C
C     N=1 :  BULK DENSITY
C     N=2 :  DIFFUSION COEFFICIENT
C     N=3 :  DISPERSIVITY
C     N=4 :  ADSORPTION CONSTANT
C     N=5 :  ZERO ORDER LIQUID PHASE DECAY CONSTANT
C     N=6 :  FIRST ORDER LIQUID PHASE DECAY CONSTANT
C     N=7 :  FIRST ORDER SOLID PHASE DECAY CONSTANT
C
      DIMENSION RHO(9), DIF(9), DSP(9), ADC(9), DLO(9), DL1(9), DS1(9)
      DATA RHO/1.22,1.60,1.41,1.43,1.46,1.49,1.51,1.53,1.54/, DIF/ 9*0.6
     17/, DSP/3.50,2.00,3.00,2.73,2.40,2.10,1.83,1.63,1.50/, ADC/.500,.2
     200,.300,.247,.180,.120,.067,.027,0./, DLO/1.,0.,.8,.45,.15,.03,.00
     32,0.,0./, DL1/-.10,0.,-.092,-.072,-.048,-.024,-.009,-.002,0./, DS1
     4/-.05,0.,-.046,-.036,-.024,-.012,-.0045,-.001,0./
C
C     ----------
      GO TO (2,4,6,8,10,12,14),N
    2 SPS=RHO(MAT)
      RETURN
    4 SPS=DIF(MAT)
      RETURN
    6 SPS=DSP(MAT)
      RETURN
    8 SPS=ADC(MAT)
      RETURN
   10 SPS=DLO(MAT)
      RETURN
   12 SPS=DL1(MAT)
      RETURN
   14 SPS=DS1(MAT)
      RETURN
      END
```